USING DATA ENVELOPMENT ANALYSIS TO IDENTIFY INTERNAL BENCHMARKS IN A LARGE PARCEL DELIVERY SERVICE

by

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This thesis is dedicated to my wife, Tomoko.

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ABSTRACT

It is known that an organization can dramatically improve the efficiency of its operations by applying the practices of its best-performing internal units throughout the organization—a process known as "internal benchmarking." However, identifying the best performers (or "benchmarks") can be difficult to accomplish for a large organization, especially if the operations, conditions, or priorities vary amongst all of its internal subunits. The swift and accurate identification of benchmarks in a large, complex organization will enable the managers to improve the overall efficiency of the system. This study examined the use of data envelopment analysis (DEA) to identify benchmarks in the context of a large internal benchmarking problem. Additional methodologies were also developed and examined in conjunction with DEA. These methodologies were applied to a real-world data set of a large parcel delivery service and the results indicated that data envelopment analysis is a valuable benchmarking tool.

Chapter 1

INTRODUCTION

1.1 Benchmarking Overview

Benchmarking is concerned with acquiring knowledge through comparative study and applying it to improve internal operations or processes (Smith 1997). For example, two automobile manufacturers may agree to a benchmarking arrangement that examines how each company does business. By sharing limited information with each other about (say) production methods or management techniques, both car-makers will benefit even though they are competitors. (Of course, this spirit of co-operation probably would not extend to the sharing of upcoming car designs or engine specifications.) This comparative study of other companies in the same industry is known as "competitive benchmarking." (McGonagle and Fleming 1998) When the various sub-divisions or units being compared are within the organization itself, this is known as "internal benchmarking." (McGonagle and Fleming 1998) For example, an automobile manufacturer may compare its various factories against each other to determine which ones are performing better and why. By comparing the operations of many different units, the company's management can discover how the best-performing units work their magic, and apply those "best practices" to the other, underperforming units. Thus, benchmarking enables organizations to avoid "re-inventing the wheel" for each of their internal units. Rather, the best practices of the organization are advertised and then applied by all of the units to improve the organization as a whole. This thesis will deal solely with the internal benchmarking process, and any further use of the term "benchmarking" should be construed to mean solely internal benchmarking.

1.2 The Benchmarking Process

Benchmarking has been used by many organizations to improve the overall efficiency of their operations by analyzing the operations of successful units and then applying the best practices used by those successful units throughout the organization. Although organizations are different (and each may require a different approach from their benchmarking team), the benchmarking process generally has five phases: planning, data collection, data analysis, implementation, and monitoring.

1.2.1 Planning

A number of questions must be answered prior to beginning a benchmarking project (Fowler 1997). What are the goals of the project? Which results are management attempting to improve? On which processes does the management place the highest priority? What type of units is being benchmarked? What data do the benchmark team need to describe the process? By answering these questions, the benchmarking team can focus on the particular process within the organization that needs improvement.

1.2.2 Data Collection

The next step in the benchmarking process is to collect data from the organization's units. This data can be collected by the benchmarking team or it can be provided to them by the organization's management. This collected data must reflect the process being benchmarked and may include qualitative performance data.

("Which units does everyone think are performing the best?") For these reasons, the data collection phase requires strong participation from the organization's management so that the collected data are relevant to the process being benchmarked (Keehley et al 1997). Additionally, the data collection process must capture any conditions that are specific to each site, such as location, cost of goods, or local demand for a product. This is to avoid penalizing or rewarding units for circumstances outside of their control (Tarricone 1998).

1.2.3 Data Analysis

In the data analysis phase, the collected data is tabulated and processed to provide a picture of the organization and its component sub-units. The performances of the units are compared and this allows the benchmarking team to identify the best-performing units. The qualitative data provided by the management is also used to corroborate the quantitative data analysis (Landry 1993).

1.2.4 Implementation

Once the best-performing units have been identified, the best practices of these units are implemented throughout the organization (Fowler 1997). There are two general methods to accomplish this. The first is to send a benchmarking team to the best-performing units to study their operations (Camp 1995). This team then distributes information about the best practices to the underperforming units to implement. This method is inexpensive and results can be obtained quickly because the benchmarking team is trained and experienced. The main disadvantage to this method is that the best practices are implemented by fiat and without regard for whether or not they will be the best practices for all units, in all cases. The second method of implementation is to send the management of the low-performing units to the benchmarks, enabling the managers to see first-hand how the benchmark units do business. The managers can then implement the best practices, making adjustments based on the actual conditions at their particular unit. The underachieving units apply any lessons learned directly to their own operations, without having to wait for upper management's approval or directive. However, in a large organization it can be expensive and time-consuming for each benchmark unit to host the management of all the other units and for the management of the lower-performing units to travel to the benchmark units.

1.2.5 Monitoring

The monitoring phase entails collecting additional data after the implementation to assess if the organization has improved and by how much. If no improvement is measured, then regressing to one of the earlier benchmarking phases will be appropriate (Camp 1995).

1.3 Difficulties in Applying Benchmarking

To demonstrate some of the problems that can occur within the benchmarking process, it is illustrative to discuss a typical (hypothetical) benchmarking situation. The typical benchmarking problem involves a company or organization with a handful of units (generally on the order of ten or so) that it wishes to benchmark such as factories, company divisions, or sales management teams. These units have many of the same characteristics—location, goals, production—which makes it simple to distinguish which units are performing well and which are not. Having similar characteristics also means that the organization will be able to collect similar data from each of the units. As an example, if the company is benchmarking its factories it is a simple matter to compare the factories' production output, or the number of man-hours used, or the price per unit produced, to determine which factory is performing the best. After the benchmark unit has been discerned, it is fairly easy for the other units to implement the best-practices of the benchmark unit to their own operations because of the small number of units and their homogeneity.

This hypothetical example breaks down, however, when the benchmarking process is applied to the real-world situation of a large service-oriented organization, such as a parcel delivery service.

Unlike the hypothetical example of ten or so units, a parcel delivery system consists of a large number (hundreds or thousands) of offices. Such a large number makes the task of comparing the operations of the units difficult during the data analysis phase of the benchmarking process. While these parcel delivery offices have the similar task of delivering parcels to postal customers, the operations of each office can be quite dissimilar. An office in an urban area might have many more packages to deliver than a rural parcel office. On the other hand, the rural office will be responsible for a much broader geographic area so they may need more delivery personnel despite their lower volume. An office near a factory may have more outgoing parcels then incoming, while the situation would be reversed near a major shopping area as shopkeepers receive their wares. Dissimilar units increase the complexity of the benchmarking because differences between the units must be taken into account. It is not enough to just find the best-performer and use it as a benchmark. The best-practices used at the post office with, say, the largest volume of packages delivered may not improve the operations at a rural post office. It is even conceivable that attempting to shoehorn the “urban” best-practices into every parcel delivery office may lead to a deterioration rather than an improvement in the overall parcel delivery system operations.

Another difference between the idealized hypothetical example and the real-world problem of a postal system is in the nature of the operations. A postal system is a service organization. A service organization is one that is concerned with providing a particular service or services (in this case, the service to deliver parcels from Point A to Point B in a timely manner) to customers, as opposed to a production organization which makes a product (automobiles, cell phones, widgets). A production organization can use simple metrics (such as price per unit produced) for determining which factory has the best production, but determining which post office is providing the best service is more difficult. Multiple inputs and outputs are needed to determine factors such as service quality and customer expectations, as well as more straightforward measurements. As more and more inputs and outputs are used to examine the organization, the complexity of the data collection process increases.

Thus, the parcel delivery system’s large number of units, dissimilar delivery operations, and expanded data collection requires more demanding data analysis techniques than the simple hypothetical benchmarking example.

Additionally, the implementation of the best practices can be difficult in a real-world organization. This is most true for large organizations and organizations that have many disparate types of units.

In large organizations, with perhaps dozens or hundreds of units to compare, it becomes unwieldy to implement a benchmarking program. Data must be collected from every unit, collated, and analyzed. Then the handful of best-performing units must be examined and their best-practices applied to the hundreds of other, lower-performing units. It would not be feasible for a benchmark unit to host such a large number of managers from other units while still operating efficiently. In addition, it may not be practical for a unit to send its management team to the benchmark location because of geographic or time constraints (if the benchmark unit is a long distance away, for example).

Lastly, when dealing with a large organization such as a parcel delivery service there may be such a large difference between the performances of the best- and worst-performing units that it would not be practical to expect the worst units to "jump up" to the level of the benchmark units. It may be more realistic to have the lesser-performing units take smaller steps to improve their performance before attempting to match up with the best units.

1.4 Applications of Data Envelopment Analysis to Benchmarking

Originally developed to examine the efficiency of public school systems (Charnes et al, 1978), data envelopment analysis (DEA) has since been applied to many areas of study. Although the banking (e.g., Schaffnit et al 1997 and Athanassopoulos 1997) and health care professions (for example, Garcia et al 1999 and Fare et al 1995 examined Spanish hospitals and Swedish pharmacies, respectively) have used DEA extensively, numerous other problems ranging from the decline of the German steel foundry industry (Schefcyzk and Gerplott 1998) to the efficiency of Canadian highway maintenance patrols (Cook et al 1990) to a study examining the quality of life in Japan (Hashimoto and Kodama 1997) have utilized data envelopment analysis to great benefit.

This thesis will explore the use of DEA in a benchmarking setting. To date, several research articles have described the use of data envelopment analysis in a formal benchmarking program. Nillesen and Pollitt (2001) describes a benchmarking case study of an electric company in Florida that used DEA for external benchmarking. Donthu et al (2004) applied DEA methods to the benchmarking of the marketing operations of a large company. Homburg (2001) described the use DEA to identify benchmarks by using the efficient units, but did not explore the use of inefficient units for benchmarks. Several articles examined small and simple systems of banks (Schaffnit et al 1997 and Bergendahl 1998) or small businesses (Madu and Kuei 1998) and are of little applicability to a large-scale benchmarking operation. Ozcan (1998) describes a study using DEA to benchmark physicians and analyze practice behavior of 160 American physicians. Although this study only used DEA-efficient physicians as benchmarks (thus losing the advantages of using inefficient DMUs as benchmarks), it did group the physicians by geographic areas. Because of the varying prices and demands amongst areas, this had the effect of grouping similar DMUs together—the same effect as the similarity index method that will be introduced in the next chapter. However, Ozcan's grouping only works if the similar units are known *a priori*; this thesis will introduce a method that does not need this information. Lastly, Sherman and Rupert (2006) used DEA to analyze the <u>results</u> of a benchmarking program consisting of 200 bank branches before and after two major banking systems merged. This study provided valuable insight to how DEA could be used in the monitoring phase of an internal benchmarking program.

1.5 Intent of Thesis and Specific Problem

It is the intent of this thesis to identify and prioritize a set of quantitative methods and heuristics using the operations research technique of data envelopment analysis that will allow benchmarking teams to improve the operations of a large organization.

This study will be conducted in two stages: an exploratory stage and an application stage. The exploratory stage will introduce the data envelopment analysis process and will examine the effects of several heuristics and algorithms on hypothetical internal benchmarking data sets. The application stage will apply these methods to a large, real-life benchmarking data set representing the operations of a parcel delivery service.

The contributions of this thesis to the existing literature are twofold. First, it would appear with its multiple branches or post offices, emphasis on data collection, and ongoing need for efficiency improvement that the postal or package delivery industry would be an ideal candidate for the use of data envelopment analysis. However, the author is not aware of any published use of DEA in a postal or parcel delivery setting. This thesis would extend the use of data envelopment analysis into a previously unstudied sector.

Secondly, this study will extend the literature concerning the use of data envelopment analysis in a benchmarking problem by introducing a new quantitative method for using DEA results to identify suitable benchmarks for the system. Chapter 2 will provide a short primer on data envelopment analysis. Chapter 3 will explore the use of DEA in an internal benchmarking setting. Multiple methods of determining benchmarks will be introduced and discussed. In Chapter 4, the methods will be applied to a real-world benchmarking problem involving the operations of a parcel delivery service.

Chapter 2

LITERATURE REVIEW

This chapter will review the literature of the concept of efficiency and the introduction of the technique of data envelopment analysis.

2.1 Efficiency Ratios

The efficiency of a unit is usually defined as the ratio between a unit's input (such as money, personnel, etc.) and an output, such as revenue or production (Equation 2.1).

$$Efficiency = \frac{Output}{Input} \tag{2.1}$$

For example, an automobile's fuel efficiency can be measured in miles (output) per gallon of gasoline (input). When comparing multiple automobiles, the one with the greatest output to input ratio is the most efficient.

In the case where each unit has one input and one output, this ratio method works well. But problems arise when multiple inputs and outputs are used. For example, suppose that there exists a system of units, each of which has two inputs (personnel and raw materials) and one output (revenue). Now suppose that two of the units have identical revenue, but unit A uses more raw materials than unit B while unit B uses more personnel than unit A. Which is more efficient, A or B? The more inputs, outputs, and units introduced into the system, the more difficult it becomes to determine efficiency. Each unit will have a number of efficiency ratios (equal to the product of the number of inputs and number of outputs) and comparing these ratios amongst all the units becomes an unmanageable problem.

2.2 Farrell Efficiency

One solution to this problem of multiple inputs and outputs was proposed by Farrell (1957). Each of the inputs and output are assigned arbitrary weights (constant throughout the entire system) and the efficiency of each unit is computed as Equation 2.2.

$$Efficiency = \frac{\sum_i v_i y_i}{\sum_j u_j x_j} \tag{2.2}$$

where u_j = weight assigned to input j
v_i = weight assigned to output i
x_j = amount of input j
y_i = amount of output i.

To demonstrate Farrell Efficiencies, Table 2.1 lists the data set for a hypothetical system consisting of ten units, each with two inputs and one output.

With just one output for each unit (all of which are equal) the data for the inputs can be plotted in a two-dimensional graph (Figure 2.1).

The disadvantage to this method is the difficulty in finding a set of weights that treat the units fairly. As shown in Table 2.1, the efficiencies of the units change based on the weights assigned to the inputs (u_1 and u_2). When u_1 is much greater than u_2 (u_1 = 0.9, u_2 = 0.1), the units that use less of Input 1 (such as units C and J) receive the better efficiency scores. But when u_2 is greater than u_1 (u_1 = 0.1, u_2 = 0.9), units that use less of Input 2 (such as units A and E) receive the higher efficiencies. (It may be helpful to examine Figure 2.1 to see how the efficiency scores depend on the units' inputs.) The weights assigned arbitrarily to the inputs and outputs determine which units are the most efficient. This method can be valuable if the entity assigning the weights can determine their values explicitly. In the hypothetical example of Table 2.1, if the values of u_1 and u_2 equal the prices per unit of Inputs 1 and 2 then this method will give reasonable results.

Table 2.1 Hypothetical Data Set (Two inputs/one output)

DMU	Input 1	Input 2	Output	Efficiency $u_1 = 0.5$ $u_2 = 0.5$ $v = 10$	Efficiency $u_1 = 0.9$ $u_2 = 0.1$ $v = 10$	Efficiency $u_1 = 0.1$ $u_2 = 0.9$ $v = 10$
A	4	3	1	2.9	2.6	3.2
B	5	16	1	1.0	1.6	0.7
C	1	18	1	1.1	3.7	0.6
D	13	13	1	0.8	0.8	0.8
E	10	2	1	1.7	1.1	3.6
F	15	8	1	0.9	0.7	1.1
G	17	5	1	0.9	0.6	1.6
H	10	8	1	1.1	1.0	1.2
I	3	13	1	1.3	2.5	0.8
J	2	5	1	2.9	4.3	2.1

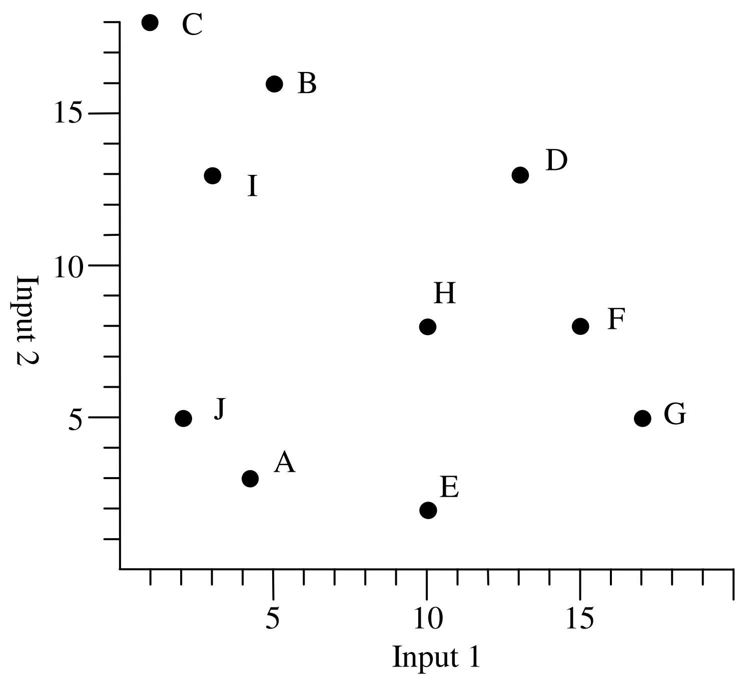

Figure 2.1 Plot of Hypothetical Data Set

However, often the units being compared for efficiency are autonomous. Although the units may have the same general goal (using inputs to produce outputs), they are given the freedom to make decisions based on the local conditions at the unit. Suppose the units being compared in Table 2.1 are hospitals with Inputs 1 and 2 representing the number of doctors and nurses employed, respectively. The administrator of each hospital makes the decision of how many of each profession to employ. He or she staffs the hospital in the most efficient way possible based on the local conditions. If salaries for doctors are very high in the area, then perhaps fewer doctors and more nurses are hired. This may be the situation at Unit C. If the patients at another location have more complex ailments than the norm, then perhaps more doctors and fewer nurses are needed. This may be the situation at Unit G. The relative Farrell efficiencies for these units will change depending on the weights assigned to the values of Inputs 1 and 2. When a high value of u_1 and a low value of u_2 are used, C will be more efficient than G. The opposite will be true when the values of the rates are reversed (Table 2.1). Thus, either unit can appear to be the less efficient one even though the administrator at each hospital is employing doctors and nurses as appropriate to cope with the local, unique conditions. When units are allowed to make its own decisions on how to best use the inputs and outputs available to it, Farrell efficiencies will produce unfair comparisons between the units.

2.3 Data Envelopment Analysis

The method of data envelopment analysis was developed by Charnes, Cooper, and Rhodes (1978) to overcome this problem. DEA allows each decision-making unit (or “DMU”) to choose its own weights to be applied to the inputs and outputs so as to maximize its efficiency when compared to the other DMUs:

For a set of n DMUs, let the DMU being evaluated be designated as DMU_0 and

$$\text{Maximize} \quad \theta = \frac{\sum_i v_i y_{i0}}{\sum_j u_j x_{j0}} \tag{2.3}$$

$$\text{subject to} \quad \frac{\sum_i v_i y_{ik}}{\sum_j u_j x_{jk}} \leq 1 \qquad (k = 1,2,\ldots, n), \tag{2.4}$$

$$v_i \geq 0, \tag{2.5}$$

$$u_j \geq 0. \tag{2.6}$$

There will be n constraints (one for each DMU). The constraints and the objective function ensure that θ will be between zero and one for each DMU. Essentially, Equation 2.3 finds the highest Farrell efficiency for each DMU by adjusting the weights applied to the inputs and outputs. This fractional program is equivalent to the following linear program:

$$\text{Maximize}\, \theta = \sum_i v_i y_{i0} \tag{2.7}$$

$$\text{subject to} \sum_j u_j x_{j0} = 1 \tag{2.8}$$

$$\sum_i v_i y_{ik} \leq \sum_j u_j x_{jk} \qquad (k = 1,2,\ldots, n) \tag{2.9}$$

$$v_i \geq 0, \tag{2.10}$$

$$u_j \geq 0. \tag{2.11}$$

The linear program is solved separately for each DMU in the system (i.e., it will be solved n times).

For each DMU, θ is the measurement of efficiency. If $\theta = 1$, then the DMU is efficient. If $\theta < 1$, then the DMU is inefficient, and $(1-\theta)$ is the amount of improvement available in the DMU's performance (e.g., if $\theta = 0.75$, then the DMU must either increase its outputs or decrease its inputs by 25% in order to become efficient).

2.3.1 Data Envelopment Analysis Example

To illustrate how DEA works, it is useful again examine the sample data set from Table 2.1 with two inputs and one output. (This example is based on a sample problem from Charnes et al, 1994)

The linear program for DMU A is:

$$\text{Maximize } \theta = v \tag{2.12}$$
$$\text{subject to} \quad 1 = 4u_1 + 3u_2 \tag{2.13}$$
$$v \le 4u_1 + 3u_2 \tag{2.14}$$
$$v \le 5u_1 + 16u_2 \tag{2.15}$$
$$v \le u_1 + 18u_2 \tag{2.16}$$
$$v \le 13u_1 + 13u_2 \tag{2.17}$$
$$v \le 10u_1 + 2u_2 \tag{2.18}$$
$$v \le 15u_1 + 8u_2 \tag{2.19}$$
$$v \le 17u_1 + 5u_2 \tag{2.20}$$
$$v \le 10u_1 + 8u_2 \tag{2.21}$$
$$v \le 3u_1 + 13u_2 \tag{2.22}$$
$$v \le 2u_1 + 5u_2 \tag{2.23}$$
$$u_1 \ge 0 \tag{2.24}$$
$$u_2 \ge 0 \tag{2.25}$$
$$v \ge 0 \tag{2.26}$$

The optimal solution to this linear program is (u_1^* = 0.045, u_2^* = 0.273, v^* = 1.0, θ^* = 1). Because the efficiency of DMU A (θ) equals one, A is said to be efficient. This means that when DMU A is allowed to choose the values of the weights for the inputs and outputs in the system, it is able to select values that portray its operations in a better light than any other unit.

For Unit B, the linear program is:

$$\text{Maximize } \theta = v \tag{2.27}$$
$$\text{subject to} \quad 1 = 5u_1 + 16u_2 \tag{2.28}$$
$$v \le 4u_1 + 3u_2 \tag{2.29}$$
$$v \le 5u_1 + 16u_2 \tag{2.30}$$
$$v \le u_1 + 18u_2 \tag{2.31}$$
$$v \le 13u_1 + 13u_2 \tag{2.32}$$
$$v \le 10u_1 + 2u_2 \tag{2.33}$$
$$v \le 15u_1 + 8u_2 \tag{2.34}$$
$$v \le 17u_1 + 5u_2 \tag{2.35}$$
$$v \le 10u_1 + 8u_2 \tag{2.36}$$
$$v \le 3u_1 + 13u_2 \tag{2.37}$$
$$v \le 2u_1 + 5u_2 \tag{2.38}$$
$$u_1 \ge 0 \tag{2.39}$$
$$u_2 \ge 0 \tag{2.40}$$
$$v \ge 0 \tag{2.41}$$

The optimal solution to this linear program for DMU B is (u_1^* = 0.160, u_2^* = 0.012, v^* = 0.383, θ^* = .383). The DEA-efficiency of DMU B is less than one, which means that Unit B is not efficient. This means that when DMU B chooses the weights of the system, there is no combination of values that it can select that will make itself appear better than the other DMUs in the system. In fact, the best it can do is to choose weights that make it appear to be only 38.3% as efficient as the DEA-efficient DMUs (such as DMU A).

Table 2.2 summarizes the results for the ten DMUs in the system.

Table 2.2 Data Envelopment Analysis of Hypothetical Data Set

DMU	x_1	x_2	y	θ^*	u_1^*	u_2^*	v^*	Reference Units
A	4	3	1	1.000	0.045	0.273	1.000	A
B	5	16	1	0.383	0.160	0.012	0.383	C, J
C	1	18	1	1.000	0.419	0.032	1.000	C
D	13	13	1	0.269	0.038	0.038	0.269	A, J
E	10	2	1	1.000	0.045	0.273	1.000	E
F	15	8	1	0.349	0.016	0.095	0.349	A, E
G	17	5	1	0.468	0.021	0.128	0.468	A, E
H	10	8	1	0.389	0.056	0.056	0.389	A, J
I	3	13	1	0.596	0.250	0.019	0.596	C, J
J	2	5	1	1.000	0.419	0.032	1.000	J

DMUs A, C, E, and J all have θ^* equal to 1, which means that they are considered DEA-efficient. All the other DMUs are inefficient to varying degrees, ranging from D at 0.269 to DMU I's θ^* at 0.596. To illustrate what each DMU's DEA-efficiency (the θ^*'s in Table 2.2) represents, it is necessary to refer to Figure 2.2.

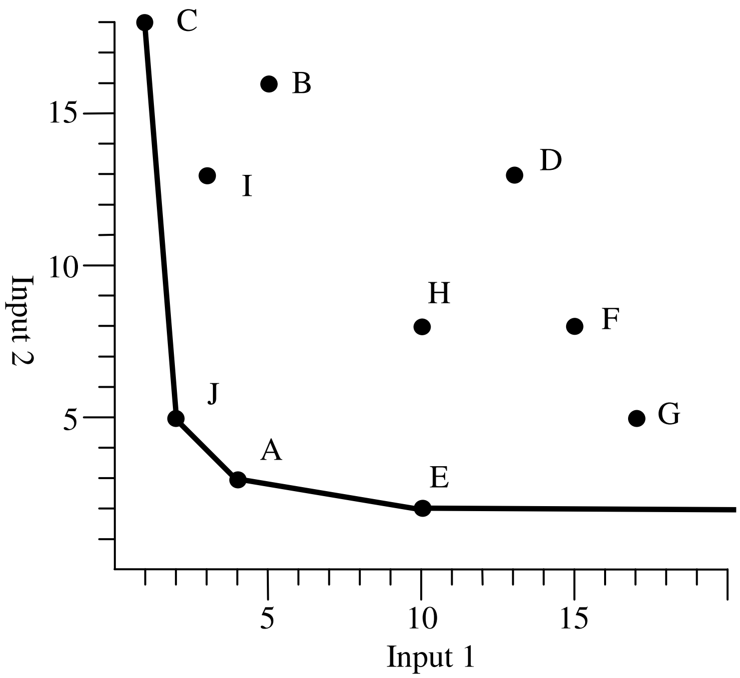

Figure 2.2 Plot of Hypothetical Data Set and Efficient Frontier

Line segments connecting the efficient DMUs, A, C, E, and J, and additional segments parallel to the Input 1 and 2 axes define the “efficient frontier.” The efficient frontier envelops (from whence the name Data Envelopment Analysis is derived) all the DMUs and measures their efficiency. The closer the DMU is to the efficient frontier, the more efficient the DMU.

For each DMU, there is a point on the efficient frontier that represents that DMU’s theoretical performance if it were to eliminate all of its inefficiencies. This “ratio efficiency point” is found at the intersection of a line segment drawn from the origin to the DMU and the efficient frontier. In Figure 2.3, DMU H* is the ratio efficiency point for DMU H. The ratio of the distance from the origin to DMU H* and the distance from the origin to DMU H equals the efficiency of DMU H:

$$\frac{OH^*}{OH} = 0.389 = \theta_H^*$$

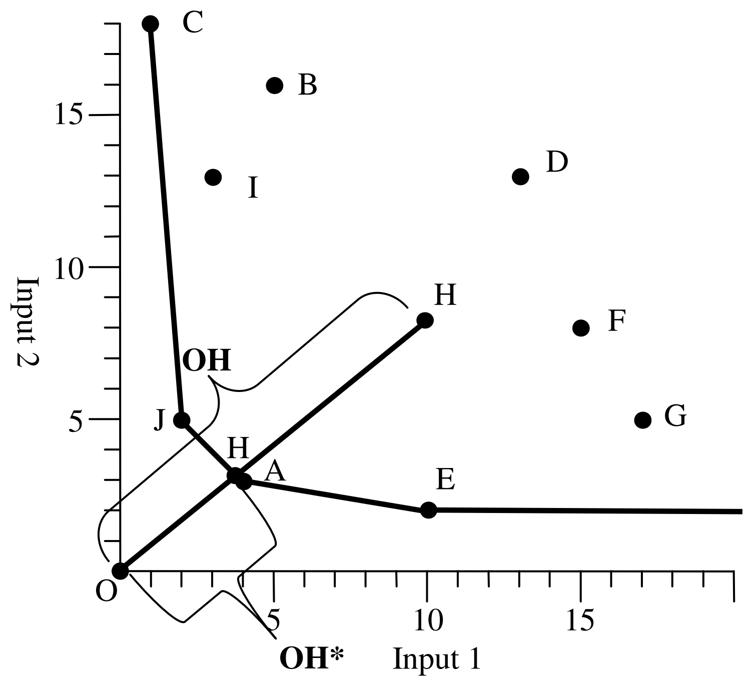

Figure 2.3 Construction of Efficiency Ratios

If DMU H reduces its consumption of Inputs 1 and 2 by 61.1% each (1-0.389), it will "slide" down the OH line until it reaches H*, at which point it will be efficient. The same will be true if it increases its production of the output by 61.1%. (It should be noted that this hypothetical example used in this paper uses two inputs and one output in order to represent with two-dimensional diagrams the concepts behind data envelopment analysis. As shown by the basic equations and linear programs, all of these concepts are as applicable when any number of inputs and outputs are used.)

2.3.2 Reference Sets and the Dual DEA Problem

In the DEA linear program, Equation 2.9 will produce one inequality constraint for each DMU in the system. If the optimal solution for the input/output weights (u_i* and v_j*) is plugged into these constraints, one or more of them will result in strict equalities. The DMUs corresponding to these equalities in the constraints make up the "reference set" for the DMU in question. For example, if DMU B's optimal solution ($u_1^* = 0.160$, $u_2^* = 0.012$, $v^* = 0.383$) is used in Equations 2.28 through 2.38 then Equations 2.31 and 2.38 (corresponding to DMUs C and J, respectively) are strict equalities. Therefore the reference set for DMU B consists of DMUs C and J. Reference sets can only be composed of efficient units (called "reference units"), and the ratio efficiency point of the DMU can be constructed using a linear combination of the reference set. In a two-dimensional system, the reference set for a DMU consists of the efficient DMUs on either end of the line segment containing the DMU's ratio efficiency point. (See Figure 2.4.)

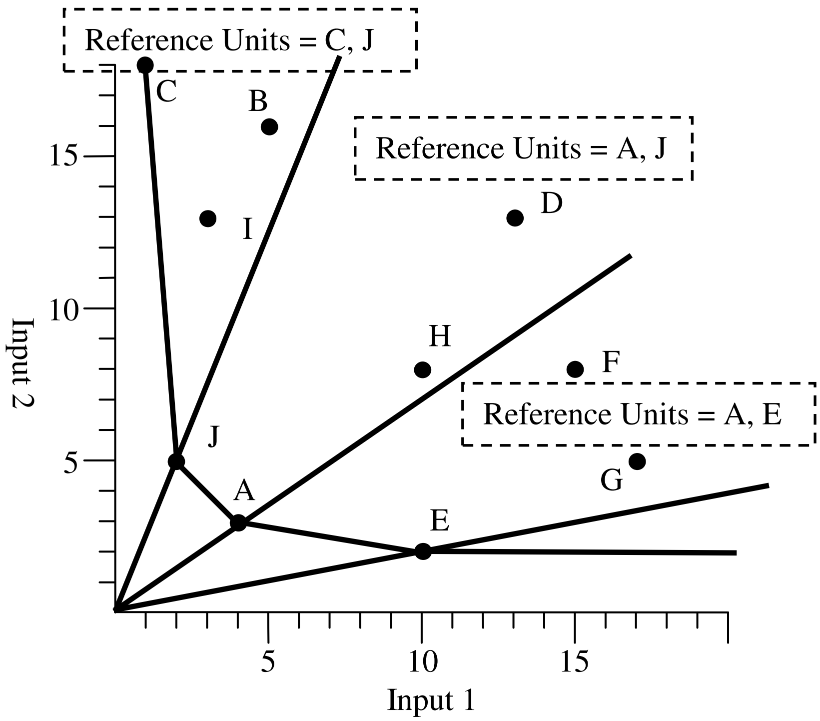

Figure 2.4 Reference Sets

Equation 2.9 will produce *n* constraints, with *n* equal to the total number of DMUs in the system. Therefore, this linear program will solve for $(i + j)$ unknowns with $n + 1$ constraints. If the system has many DMUs with relatively few inputs and outputs (as is often the case) then solving the linear program will become computationally labor-intensive. The dual of the linear program comprising Equations 2.7-2.11 will often be quicker and more useful, solving for *n* unknowns with $(i + j + 1)$ constraints. For a set of *n* DMUs, let the DMU being evaluated be designated as DMU_0 and let the reference set of DMU_0 be represented by a vector, $\boldsymbol{\lambda_0}$, where $\boldsymbol{\lambda_0} = (\lambda_{01}, \lambda_{02}, \lambda_{03}, ..., \lambda_{0n})$. If a DMU is in the reference set of DMU_0, then its respective “reference weight” (λ value) will be a positive number; otherwise it equals zero. The DEA dual linear program for DMU_0 is:

$$\begin{array}{llr} \text{Minimize} & \theta & (2.42) \\ \text{subject to} & \sum_n \lambda_{0n} x_{np} - \theta x_{0p} \leq 0 \quad (\text{for p} = 1, 2, ..., \text{i}) & (2.43) \\ & \sum_n \lambda_{0n} y_{nq} - y_{0q} \geq 0 \quad (\text{for q} = 1, 2, ..., \text{j}) & (2.44) \\ & \sum_n \lambda_{0n} = 1 & (2.45) \end{array}$$

In addition to its computational advantages, the dual program is usually easier to analyze. By solving for the reference set, the user can see for each DMU which reference units are most similar. Table 2.3 lists the DEA-efficiencies and reference weights for each of the DMUs in the example set. As expected, the DEA-efficiencies are the same as in the primal linear program. By comparing the reference weights listed in Table 2.3 and plot of the DMUs in Figure 2.4, the relationship between the inefficient DMUs and their reference set is ascertained. In DMU H’s reference set, for instance, DMU A has a reference weight over sixteen times as great as the reference weight for DMU J. This corresponds to the location of DMU H* in Figure 2.3. Therefore, if DMU H can improve its performance enough to place itself in a new position on the efficient frontier, it will be much closer in appearance to A than J. The opposite holds true for DMU B. Reference weights will play a major role in the benchmarking methodology described in Chapter 3.

Table 2.3 Reference Weights for Hypothetical Data Set

DMU	θ^*	λ_A^*	λ_C^*	λ_E^*	λ_J^*
A	1	1	0	0	0
B	0.383	0	0.086	0	0.914
C	1	0	1	0	0
D	0.269	0.750	0	0	0.250
E	1	0	0	1	0
F	0.349	0.794	0	0.206	0
G	0.468	0.340	0	0.660	0
H	0.389	0.944	0	0	0.056
I	0.596	0	0.212	0.788	0
J	1	0	0	0	1

2.4 Advantages and Disadvantages of DEA

The main advantage of data envelopment analysis is that the input/output weights used for each unit are optimal for that unit. Thus, they are fair for each unit and no DMU can blame the chosen weights for the inputs and outputs for its efficiency shortcomings. Each DMU has only itself to blame for its decisions that have made it either efficient or inefficient. This concept of a "decision-making unit" is a much more useful idea than that of the "black box" that takes inputs and spits them out somehow as outputs. Placing the onus of inefficiency onto each unit's decision-maker is a more realistic representation of a real-life organization with a management structure.

Unfortunately, there are a number of disadvantages or quirks of the system which the operations researcher must be aware of when using data envelopment analysis.

Due to the nature of the DEA linear programs, a DMU that minimizes its use of any single output to input ratio will be efficient. In the hypothetical data set of Table 2.2, DMUs C and E are both efficient as they each have the highest output to input ratio of one of the inputs in the system (Input 1 for DMU C, Input 2 for DMU E). The linear programs for DMUs C and E find the optimal solutions by placing a very large weight on the output and much smaller weights on the two inputs (Compare u_1*, u_2*, and v* for A and J in Figure 2.2). This feature of the DEA linear programs is a trivial problem when dealing with a two-input/one-output system, but it can cause some unusual outlying units when multiple inputs and/or outputs are used. With *i* inputs and *j* outputs, a unit may perform well compared to the rest of the DMUs on only one of the (*i* times *j*) ratios and still appear to be DEA-efficient. An operations research must examine all of the efficient units to identify potential outliers, and then decide if this unit's efficiency is valid or not. (However, this feature of the linear program becomes less of a problem when DEA is used in a benchmarking setting, as will be demonstrated in the next chapter.)

The linear program of each DMU assigns weights each input and output without making any *a priori* assumptions. This is usually a selling point of the procedure, as the weights will not be tainted with any assumptions from management on which input or output is the most important. Often, however, the weights of the inputs/outputs used by the DMUs are known at least in relative terms. If the DMUs are alchemy factories which use a (presumably proprietary) process to transform the inputs of straw and silver ingots into the output of gold bricks, it would be reasonable to assume that the silver-ingot input would receive a greater weight than the straw input. Otherwise, the DMU that minimizes its use of straw per gold brick will appear to be efficient (regardless of the amount of silver it uses). In order to take advantage of this bonus information about the data set, additional constraints need to be added to ensure that the input/output weights correspond with what is known (i.e., $u_{silver} > u_{straw}$).

Another, related, difficulty that arises when using DEA is the use of irrelevant inputs and/or outputs. Although the DEA process does not use any *a priori* assumptions about the weights assigned to the inputs or outputs, the mere fact that specific ones are used will give implicit weights to the data. As an extreme example, the number of paperclips used could be used as an input when comparing the operations of barber shops. Even though one can assume that there is no relationship between the number of paperclips consumed and the efficiency of barber shops (and thus $u_{paperclips}$ should equal zero), by using this as an input in the DEA linear program it is guaranteed to be given a weight greater than zero. Additionally, the barber shop that uses the fewest number of paperclips will be rated as efficient regardless of its actual performance. Conversely, if important inputs or outputs are omitted from the analysis, they will be implicitly assigned a weight of zero. To guard against this problem, the operations researcher must understand the operations of the organization being studied and coordinate his or her efforts with the organization's management to ensure that the right data is collected and used in the analysis.

2.5 Chapter Summary

The efficiency of decision-making units in an organization is measured by creating ratios of the units' outputs to their inputs. When multiple inputs and/or outputs are measured in the system, the efficiency of the units is difficult to measure. Although Farrell's method of using input/output weights provides a way to overcome this difficulty, it introduces questions of fairness in applying arbitrary weights to the DMUs. The data envelopment analysis method takes the Farrell method one step further by allowing each DMU to select the weights that position itself in the best possible light, relative to the other DMUs. The dual program also measures the efficiency of the DMUs, but by optimizing for the weights of the units in the DMU's reference set. Although there are some features of DEA that can skew the results of an efficiency analysis, a prudent operations researcher can cope with them. The next chapter will examine the application of DEA to a benchmarking problem.

Chapter 3

BENCHMARKING METHODOLOGY

This chapter will examine various ways and methods data envelopment analysis can be used during an internal benchmarking project. Each method will be explored by applying it to a hypothetical DMU data set, with an eye on how to apply the method to a larger, more realistic data set.

3.1 Basic DEA Model

Using DEA in a benchmarking environment with no modifications can yield valuable insight into the operations of the organization. The senior management can ascertain which DMUs are performing at their theoretical best efficiency and which are not. If the DEA-efficient units do not correspond with management's preconceived notion of the benchmark units, then this discrepancy can be useful to adjust management's mindset or to adjust the DEA linear program. For the example listed in Table 2.2, if prior to the data envelopment analysis management believes that DMUs D and F are the best candidates for benchmarking, they will be surprised to discover that they are inefficient when compared to DMUs A, C, E, and J. By comparing D and F with the four efficient DMUs, management can decide why they felt the way they did, and either improve the analysis by adding other inputs and outputs (to better describe the superiority of DMUs D and F) or adjust their opinions to agree with the analysis (i.e., that A, C, E, and J are the most efficient DMUs).

Conversely, the DEA method will inform inefficient units what they need to do in order to attain full efficiency: each inefficient DMU must either increase all outputs or decrease all inputs by a factor equal to $(1 - \theta)$. This will move the DMU down to the efficient frontier. For example, DMU H uses 10 units of Input 1, 8 units of Input 2, produces one unit of Output, and has a DEA-efficiency of 0.389. To become efficient, DMU H can decrease both inputs by 61.1%; this reduces its consumption of Inputs 1 and 2 to 3.89 and 3.11 units, respectively. Alternatively, it can increase its production of Output by 61.1%; this brings DMU H's output up to 1.61. Either of these two options will bring DMU H to its ratio efficiency point of H*. (See Figure 2.3.)

The main limitation to using DEA in this fashion is obvious. The inefficient DMUs will know that they are inefficient and that they need to reduce their consumption/increase their production (indeed, they may have known this prior to conducting the DEA study), but this basic method gives them no information on <u>how</u> to become efficient. This is the whole goal of the internal benchmarking process, and the basic DEA methodology cannot help with that. New methods utilizing DEA need to be examined.

3.2 Using the Efficient Units as Benchmarks

The first-order solution is to use the efficient DMUs as benchmarks for the system. This is a two step method:

1. Use data envelopment analysis to identify the efficient DMUs in the system.

2. Benchmark the system with the efficient units as the benchmarks.

(As discussed in Chapter 1, "benchmarking" in this context means sending personnel from inefficient units to the benchmark units to experience how they do business.)

For the example system, there are four benchmarks: the efficient DMUs A, C, E, and J. The six inefficient DMUs send personnel to these four units to see what they should do to improve their efficiency.

Although this method may work well with small systems, it has a number of shortcomings. First, if the managers of the inefficient DMUs travel to each of the benchmark units, there is a possibility that they will try to benchmark themselves against a unit with which they have little in common. Using this method, DMU G (for example) will travel to all four of A, C, E, and J, even though an examination of Figure 2.2 shows that DMU E seems to be most similar to DMUs A and E. The operations at DMUs G, A, and E appear to revolve around a low consumption of Input 2, while DMUs C and J consume less of Input 1. The operating decisions made by the four efficient DMUs are probably different and it is likely that the operations of DMUs C and J will not yield much useful information to the management of the inefficient unit G. It is even conceivable that by adopting the methods of DMUs C and J, E will become <u>less</u> efficient than if it does nothing at all.

As discussed in Chapter 2, the DEA process will often designate units as efficient solely because they limit their use of one input or maximize their production of one output. These outliers will have little in common with the "normal" DMUs, and sending personnel to study the outliers will often serve little purpose.

Lastly, this method requires each of the benchmark units to host the management of the inefficient DMUs in order to demonstrate how they operate. While this may not be much of a burden in a small system, with a large system it will be impractical for a benchmark unit to host managers from a hundred or more inefficient units.

3.3 Using the most popular efficient units as benchmarks

Another method that can be used for designating benchmarks is:

1. Use data envelopment analysis to identify the efficient DMUs in the system.
2. Determine which efficient units are used most often in the construction of the inefficient DMUs' reference sets.
3. Benchmark the system with the most commonly-used efficient units as the benchmarks.

Using this method, the outlier efficient units can be removed from consideration as benchmarks. This can reduce the time spent benchmarking by the inefficient units. The following system of equations counts the number of referents for each efficient DMU:

$$\text{For each DMU with } \theta = 1,\quad \left.\begin{array}{l} r_s = 0, \quad \text{if } \lambda_s = 0, \\ r_s = 1, \quad \text{if } \lambda_s > 0, \text{ then} \end{array}\right\} \quad (3.1)$$

$$referentnumber = \sum_{s=1}^{n} r_s \qquad (3.2)$$

Although there are no hard and fast rules for deciding what constitutes a "popular" DMU, by tabulating or plotting the *referentnumber* for each efficient DMU a good estimate can be made. A referent percentage can also be constructed, consisting of the percentage of inefficient DMUs that have a particular efficient DMU as a reference unit (*referentnumber* divided by the number of inefficient units).

Even without benchmarking *per se*, finding the popular DMUs can be useful. These DMUs are popular as members of reference sets because they are similar to so many other units and therefore they can give shed light on why the other units in the system are not performing at their theoretical efficiency.

Figure 3.1 is the plot of another hypothetical two-input/one output system. There are four efficient DMUs (A, B, C, D) and seventeen inefficient DMUs, with the dashed lines representing the boundaries between the reference zones that are defined by each of the possible reference sets. Table 3.1 shows that the efficient DMUs B and C appear much more often in the inefficient DMUs' reference sets than either A or D. Thus, it can be assumed that B and C are more representative of the system at large, and that therefore they should be used as the benchmark units.

For this example, using this method instead of the basic method described in the previous section will reduce the total number of benchmarking visits from 68 to 34—a significant improvement in the use of managers' time and money. Additionally, by eliminating the outliers A and D as benchmarks, they will no longer be "corrupting" the inefficient DMUs.

However, this method has its drawbacks. The problem still remains that each benchmark unit will have to host every inefficient DMU's management, which is generally not feasible. (However, one possibility is for the senior management of the entire system to have a benchmarking team study and document the operations of the benchmark units and then have the results of the study implemented by the inefficient units.)

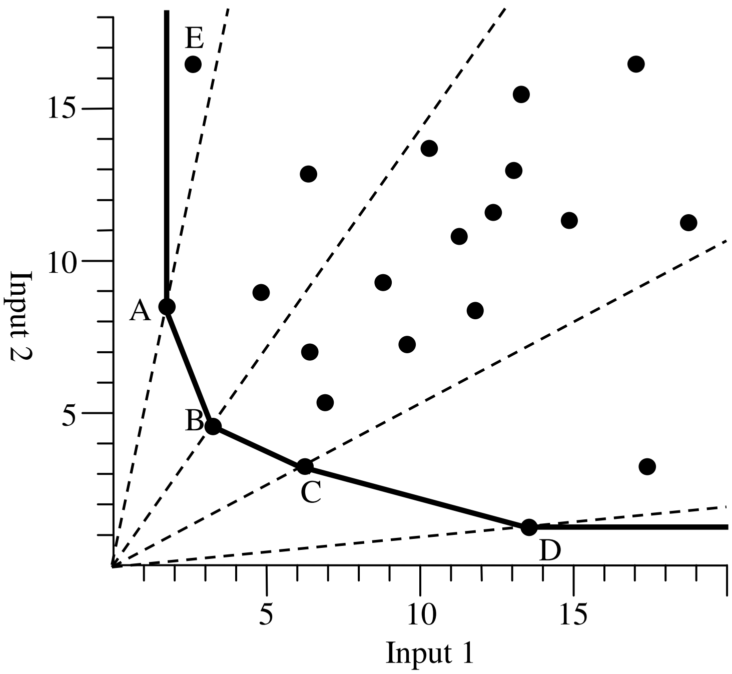

Figure 3.1 Counting the Referent Units

Table 3.1 Referents for Popular DMU example

Efficient DMU	Number of Referents
A	3
B	15
C	14
D	1

Another problem with this method is that it is still possible for a DMU to be benchmarked against a unit with which it that has nothing in common. In Figure 3.1, DMU E only has one unit in its reference set, DMU A. If this method of determining benchmarks is used, then outlying *inefficient* DMUs are discarded as well as efficient outliers. This will often be a problem for the practical benchmarker because the goal of benchmarking is to improve the operations of the offbeat, inefficient units.

Using the popular efficient DMUs as benchmarks can provide valuable insight into the nature of the system, but the method still leaves much to be desired.

3.4 Using Inefficient Units as Benchmarks

One solution to the problem of having too many inefficient units for the benchmarks to handle is to use more benchmarks. Instead of using just the efficient units as benchmarks, allow all units (efficient or inefficient) to be potential benchmarks for less efficient DMUs.

For the two previous methods, every inefficient DMU used the same benchmarks. But in order to benchmark the system using inefficient DMUs, the concept of a "benchmark set" needs to be introduced. A benchmark set for any particular DMU is defined as the subset of all DMUs in the system from which a benchmark can be chosen. Essentially, any individual subject DMU starts off with all DMUs in the system as a potential benchmark and then as DMUs are eliminated (for whatever reason) the list of potential benchmarks becomes smaller until what remains is the set from which the benchmarks are chosen for this unit.

To begin with, all DMUs with DEA-efficiencies less than the subject DMU are eliminated from consideration. By definition, a DMU cannot use a unit with a lower efficiency score as a benchmark.

Next, all DMUs with different reference sets than the subject DMU are eliminated from consideration as a benchmark. As discussed in section 3.2, it is important that the benchmark units have commonalities with the units they are attempting to improve. It is reasonable to assume that units that have the same reference units are similar to each other. This paring down of the benchmark set will eliminate units that are have better efficiency but are dissimilar to the subject DMU.

In a set of n DMUs, for each DMU_m let there be a vector $\boldsymbol{\mu}_\text{m}$ representing the reference units, where $\boldsymbol{\mu}_\text{m} = (\mu_{\text{m}1}, \mu_{\text{m}2}, \mu_{\text{m}3}, ..., \mu_{\text{mn}})$. Each element μ_{mk} of $\boldsymbol{\mu}_\text{m}$ is defined as:

$$\left.\begin{array}{l} \mu_{mk} = 0, \text{ if } \lambda_{\text{mk}} = 0 \\ \mu_{mk} = 1, \text{ if } \lambda_{\text{mk}} > 0 \end{array}\right\} \quad (3.3)$$

Every $\boldsymbol{\mu}_\text{m}$ vector will thus consist of at least one element with a value of one and the remainder with values of zero.

The benchmark set for a DMU_0 consists of every DMU_s ($s = 1, 2, 3, ..., n$) in a system of n DMUs which solves the following system of equations:

$$\theta_s \geq \theta_0 \quad (3.4)$$

$$\boldsymbol{\mu}_\mathbf{s} = \boldsymbol{\mu}_\mathbf{0} \quad (3.5)$$

Equation 3.4 ensures that the benchmark units have greater efficiencies than the subject DMU and Equation 3.5 ensures that the benchmark units have the exactly the same units in their reference set as in the subject's reference set.

To illustrate the construction of the benchmark set, it is of use to examine the hypothetical data set of the last chapter. In Table 2.2, the far right-hand column (labeled "Reference Units") can represent the reference units $\boldsymbol{\mu}$ for each DMU. To find the benchmark set for DMU A, first all DMUs with $\theta < 1$ (as DMU A is efficient) are eliminated from consideration. This eliminates all but DMUs A, C, E, and J. Then, any remaining DMU with a $\boldsymbol{\mu}$ vector not equaling $\boldsymbol{\mu}_{\mathbf{A}}$ is removed leaving just the benchmark set. DMUs C, E, and J are eliminated which leaves a benchmark set of just DMU A.

(It can be shown that the benchmark set for any efficient unit, such as A, is simply the unit itself. Since efficient DMUs cannot be improved anyway, this method can be simplified by only finding benchmark sets for inefficient units.)

For DMU B, all DMUs with efficiencies less than 0.383 are removed from the benchmark set. This eliminates DMUs D and F. Then all units with a different reference set than DMU B's reference set of C and J are removed from consideration. This leaves DMUs C, I, and J as DMU B's benchmark set. Table 3.2 lists the benchmark sets for each DMU.

Table 3.2 Benchmark Sets for Hypothetical Example

DMU	θ^*	Reference Units	Benchmark Set	Number of Inefficient Referents
A	1	A	Efficient	4
B	0.383	C, J	C, I, J	0
C	1	C	Efficient	2
D	0.269	A, J	A, H, J	0
E	1	E	Efficient	2
F	0.349	A, E	A, E, G	0
G	0.468	A, E	A, E	1
H	0.389	A, J	A, J	1
I	0.596	C, J	C, J	1
J	1	J	Efficient	4

If each inefficient unit benchmarked itself against every unit in its benchmark set, there would be seven benchmarks for this system (all units used as referents in Table 3.2; A, C, E, G, H, I, and J). These seven benchmarks would host a total of fifteen (the sum of the right-hand column of Table 3.2) benchmarking visits from the other DMUs' management, for an average of 2.1 visits per benchmark. This compares very well against the method introduced in Section 3.2 where four benchmarks hosted each of the inefficient units, for an average of six visits per benchmark. Therefore this method can reduce the amount of time any particular benchmark unit spends improving other DMUs.

The number of benchmarking visits can be reduced even further by limiting the number of visits that each DMU either makes or hosts. For example, suppose the number of benchmarking visits each inefficient DMU makes is restricted to two. Then DMUs B, D, and F would only be required to make visits to a subset of their benchmark set, while still receiving exposure to multiple superior units. The same logic would apply to a limit on the number of DMUs for which a unit can be a benchmark, although this case can be more difficult to calculate (as in the case of a DMU that serves as the sole unit in a benchmark set for multiple DMUs).

The fact that inefficient units are being used as benchmarks should not be seen as a handicap to this method. Because of Equation 3.4, all units in a DMU's benchmark set are still required to have a superior efficiency than that unit, so that there will still be at least some improvement when benchmarking. For example, suppose that DMU B selects DMU I as to be its sole benchmark (note from Table 3.2 that DMU I is a member of B's benchmark set). Then after visiting and using DMU H as a benchmark, DMU B can expect to see a theoretical improvement in its efficiency from 0.383 to 0.596. In addition, because I is itself being benchmarked against another unit (say, J) its efficiency is increasing as well. This "daisy-chain" effect of the inefficient units improving and dragging the others up to better efficiency is a positive consequence of this method.

Another advantage that using the inefficient decision-making units as benchmarks provides is that these units may provide a more reasonable benchmarking target than an efficient DMU. As described in Chapter 2, an inefficient unit attains full efficiency by reducing its inputs or increasing its outputs in such a way that they move towards the origin, becoming more efficient, until they reach the efficient frontier at its ratio efficiency point. However, in practice this may not be reasonable or possible for a unit. Taking DMU B as an example again, basic data envelopment analysis says that all B has to do is to follow the example of the efficient units in its reference set, C and

J, and reduce its consumption of both Inputs 1 and 2 by a mere 61.7%. It would probably be quite a shock to the management of DMU B if they were informed that this was their task! Obviously, there is something unknown which is causing a fundamental difference between the efficient units and the inefficient DMU B. In effect, B is so far behind in terms of efficiency that its operations will not appear anything at all like the efficient units' operations. It would be futile to have DMU B take on the best practices of the efficient units via benchmarking when it is simply not conducting the same operations and it would also be unreasonable to expect that DMU B could "jump up" its efficiency to a level equal to that of the efficient DMUs. Indeed, it is more reasonable to assume that another inefficient unit is conducting its operations in a manner similar to B and that this inefficient unit would be a better benchmark.

The most serious problem with using this method to select a benchmark set is that it is possible for units to be similar (and thus good candidates for benchmarking), even without having the same reference units. Table 3.3 contains excerpted information from Tables 2.2 and 2.3 concerning three units from the hypothetical data set from Chapter 2: DMUs F, G, and H.

Table 3.3 Excerpt from Hypothetical Data Set

DMU	x_1	x_2	θ^*	λ_A^*	λ_E^*	λ_J^*
F	15	8	0.349	0.794	0.206	0
G	17	5	0.468	0.340	0.660	0
H	10	8	0.389	0.944	0	0.056

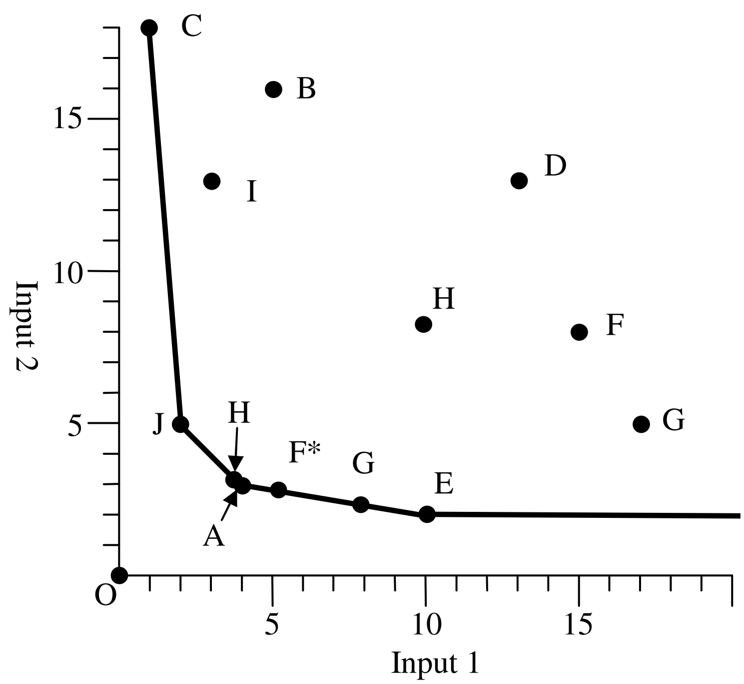

Figure 3.2 Inefficient Units as Benchmarks

Because DMUs F and G share the same reference units (A and E), DMU G appears in the benchmark set of DMU F (see Table 3.2). The fact that they both share the same reference units is shown in Figure 3.2, as the two units' ratio efficiency units, F* and G* appear on the A to E segment of the efficient frontier. However, based on the position of their ratio efficiency units, DMU H appears to be a better choice for a benchmark of DMU F even though that they have different reference units comprising their reference sets (A and E for DMU F; A and J for DMU H). In other words, if both F and H improve their operations enough to move to the efficiency frontier, their operations (i.e., inputs and outputs) will be very similar. Therefore, if an inefficient DMU wants to find a good benchmark unit, then it is certainly possible that limiting the benchmark set to consider only those DMUs with the same reference units will miss some miss quality candidates.

3.5 Common Reference Units Method

If the goal is to increase the number of DMUs that are examined as possible benchmarks, then expanding the benchmark set is the solution. The easiest way to do this to relax Equation 3.5 so that it is no longer necessary for units in the benchmark set to have exactly the same reference units as the subject DMU and instead allow the benchmark set to be comprise any unit that has at least one reference unit in common with the subject DMU.

The benchmark set for a DMU_0 contains every DMU_s (s = 1, 2, 3, ..., n) in a system of n DMUs which solves the following system of inequalities:

$$\theta_s \geq \theta_0 \qquad (3.6)$$

$$\boldsymbol{\mu}_\mathbf{s} \times \boldsymbol{\mu}_\mathbf{0}^\mathbf{T} \geq 1 \qquad (3.7)$$

where $\boldsymbol{\mu}_0$ and $\boldsymbol{\mu}_s$ are defined as in Equation 3.3. (I.e., every element of the $\boldsymbol{\mu}$ vector is zero except that elements corresponding to a reference unit have a value of one.)

As before, the first equation (Equation 3.6) ensures that the benchmark set contains only units with efficiencies greater than θ_0. But now the second equation, Equation 3.7, selects all DMUs that have at least one reference unit in common with

DMU_0. By adjusting the relational symbol (=, >, etc.) and the integer on the right-hand side of Equation 3.7, other conditions can be used.

By applying this system of inequalities to the hypothetical data set, it dramatically increases the benchmark sets for several DMUs as compared to the original benchmark sets (Table 3.4).

Table 3.4 Common Reference Units Method

DMU	θ^*	Reference Units	Original Benchmark Set	Expanded Benchmark Set	Number of Inefficient Referents
A	1	A	Efficient	Efficient	4
B	0.383	C, J	C, I, J	C, H, I, J	1
C	1	C	Efficient	Efficient	2
D	0.269	A, J	A, H, J	A, B, F, G, H, I, J	0
E	1	E	Efficient	Efficient	2
F	0.349	A, E	A, E, G	A, E, G, H	1
G	0.468	A, E	A, E	A, E	3
H	0.389	A, J	A, J	A, G, I, J	3
I	0.596	C, J	C, J	C, J	3
J	1	J	Efficient	Efficient	4

If the management at each inefficient DMU benchmarks their unit against every DMU in their benchmark set there will be a total of 23 benchmarking visits divided amongst the nine benchmark units (every unit except for DMU D will be a benchmark for at least one other DMU), for an average of 2.6 visits per benchmark. For this hypothetical set, this average compares well against the original benchmarking method introduced in Section 3.2 (six visits per benchmark) and is roughly comparable to the 2.1 visits per benchmark average from the previous section. This method still provides a significant reduction in the amount of time any particular unit spends benchmarking.

This expansion of the benchmark sets now allows DMUs to benchmark themselves against similar units that previously were not available. In the previous section, DMU F was not able to use DMU H as a benchmark despite their noticeable similarities. But by using the common reference units method, DMU H is now in DMU F's benchmark set (Table 3.4) and thus eligible as a benchmark.

The effect of this benchmark set construction method is to expand the reach of the benchmark sets, capturing all possible good candidates. At the same time, however, this expansion of the benchmark set also results in the possible capture of some poor candidates—candidates that were not in the original benchmark set.. This is the biggest drawback to this method. The benchmarking team can be assured that it will find suitable benchmarks for each of the inefficient units because all possible candidates are in the benchmark set, yet there is no way of distinguishing between the good and poor candidates in the benchmark set. In order for this common reference method of building benchmark sets to be useful, a technique to make this distinction within each DMU's benchmark set is needed.

3.6 Similarity Index Method

Once the benchmark set for each inefficient DMU is defined, it is useful to rate the units in the set by their suitability as a benchmark, specifically: How similar is each unit to the subject unit? Because their operations resemble those of the subject, the units that are most similar are most likely to pass on their best practices to the subject DMU.

The concept of similarity between two DMUs is defined as simply the distance between the DMUs' ratio efficiency points. As discussed in Chapter 2, the ratio efficiency point is the point on the efficient frontier that an inefficient DMU is projected to be once it is efficient. For example, in Figure 3.3, the point H* is the ratio efficiency point for the inefficient DMU H.

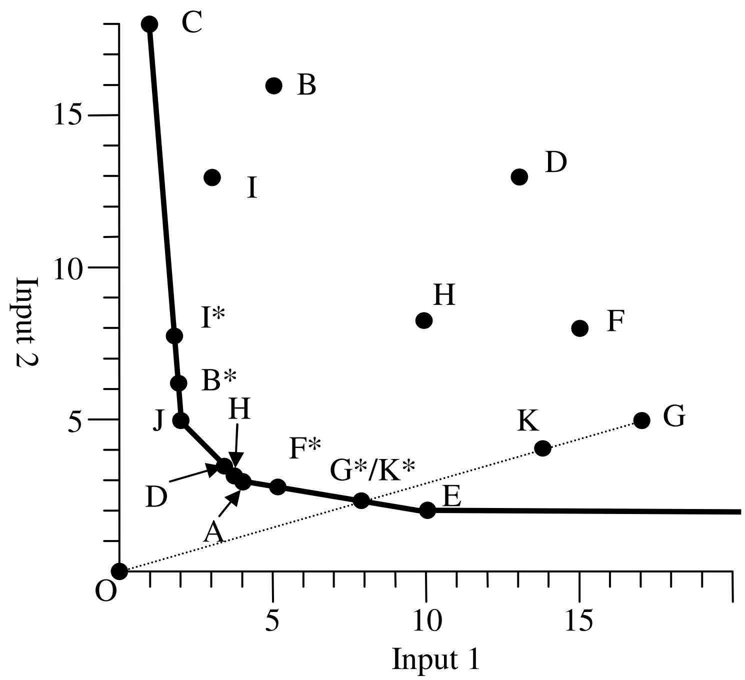

Figure 3.3 Similarities Between Units

As DMU H reduces its inputs or increases its output, it will slide down the OH line segment until it reaches the efficient frontier at H*. The ratio efficiency point for every DMU in our hypothetical data set is plotted in Figure 3.3.

Now suppose there exists another inefficient decision-making unit, DMU K, with inputs and outputs that place it within the current DEA system at some point along the line segment between the origin and DMU G. (See Figure 3.3) By definition, DMU K is more efficient than DMU G and both units' ratio efficiency points (G* and K*) are equal. Therefore, If DMU G were to improve its efficiency, it would "slide down" the OG line segment, eventually reaching DMU K's location. So, in effect, DMU K can be considered to be exactly similar to DMU G, just with the benefit of better efficiency. Another way to visualize this situation is to imagine that the location of DMU K in Figure 3.3 is a "future snapshot" of where DMU G will be once it improves its efficiency (or that G's location is where DMU K used to be, before it got its act together and improved its efficiency). Because of this, DMU K should be a perfect benchmark for DMU G.

This leads to conclusion that the degree of similarity between two DMUs is proportional to the distance between the two units' ratio efficiency points. In Figure 3.3, all ratio efficiency points plotted on the line segments connecting the efficient DMUs A, C, E, and J. By computing the distances from a designated ratio efficiency point (corresponding to a DMU for which a benchmark is sought) to the other ratio efficiency points, the similarity between the units can be measured. For example, if the goal is to find a suitable benchmark to DMU D, a glance at the line efficiency frontier in Figure 3.3 shows that DMU H is the most similar, followed by DMU A. Thus, DMU H would be the best choice as a benchmark for DMU D.

One way to compute these distances (and thus compute the similarity between the respective DMUs) is to calculate the efficient inputs and/or outputs of the ratio efficiency points (using each DMU's θ) and then measure the distances geometrically. The disadvantage to this method is that the scale for the inputs and outputs of each DEA system will be different, so it is difficult to get an idea of what constitutes a “similar” DMU. Sometimes the distance may be 30 units; at other times it may be a single unit. It is better to use the following equivalent equation using the reference weights to calculate the “similarity index” between two DMUs, DMU_i and DMU_j, in a system of *n* DMUs:

$$\text{S.I.}_{ij} = \sqrt{\sum_{s=1}^{n} \left(\lambda_{is} - \lambda_{js}\right)^2} \qquad (3.8)$$

This equation simply calculates the Pythagorean distance between two vectors of *n* elements: the reference sets. Because the reference weights comprising the reference sets of each DMU sum to one, the theoretical possible values of each similarity index range from zero (if all reference weights are identical—equivalent to identical ratio efficiency points) to the square root of two. (The actual maximum similarity index for any specific DMU will depend on how its reference weights are distributed, but it will never exceed the square root of two.)

Table 3.5 lists the similarity indices for DMU D to the other nine DMUs in the system.

Table 3.5 Example Similarity Index for DMU D (λ_A=0.750, λ_J=0.250, λ_C, λ_E=0)

DMU$_i$	θ^*	λ_A	λ_C	λ_E	λ_J	*Similarity Index$_{Di}$*
A	1	1	0	0	0	0.354
B	0.383	0	0.086	0	0.914	1.005
C	1	0	1	0	0	1.275
E	1	0	0	1	0	1.275
F	0.349	0.794	0	0.206	0	0.327
G	0.468	0.340	0	0.660	0	0.816
H	0.389	0.944	0	0	0.056	0.274
I	0.596	0	0.212	0.788	0	1.136
J	1	0	0	0	1	1.061

Although Equation 3.8 sums over the entire set of *n* DMUs, it is easy to see that this equation can be simplified by only using the subset of efficient units, as they are the only units that can receive reference weights (such as λ_A and λ_J).

DMU H is the most similar unit to D, followed by DMU F. The similarity indices for a DMU are proportional, so it can be shown that DMU D is roughly three times more similar to DMU H than the least similar DMUs, C and E (0.274 to 1.275). This confirms what is seen in Figure 3.3.

Because similarity indices can be constructed from a unit to every other DMU in the system (regardless of their efficiency or reference set), it is still necessary to construct benchmark sets using the method described in the previous section (Equations 3.6 and 3.7). The basic conditions of the benchmark still apply: a benchmark must have a greater efficiency and at least one reference unit in common with the subject DMU. Similarity indices are then calculated only for the units in the benchmark set.

The biggest drawback to using a similarity index is that although it allows the units in the benchmark set to be rated or ranked for their suitability as benchmarks, there is still no hard and fast rule to determine what constitutes a “good” index number. The prudent operations researcher must use this index along with the DEA-efficiencies of the units to determine which DMU in the benchmark set will be the best benchmark.

Still, this method of constructing similarity indices allows the operations researcher to determine which DMUs have comparable operations, and thus would make good benchmarks.

3.7 Chapter Summary

Several methodologies of applying data envelopment analysis to a benchmarking problem were discussed. The benefits of using basic DEA techniques were explored. The advantages and disadvantages of the efficient decision-making units as benchmarks for the entire system were discussed, along with the option of using just the most popular efficient units as benchmarks. Then the possibility of using inefficient DMUs as benchmarks was examined and the concept of a benchmark set was defined. Lastly, a method of rating the similarity between a subject unit and the units within its benchmark set was introduced. In the next chapter, these methodologies will be applied to a real-world benchmarking problem involving operations of a large parcel delivery service.

Chapter 4

BENCHMARKING PARCEL DELIVERY OFFICES

The methods of using data envelopment analysis in a benchmarking problem discussed in Chapter 3 are here applied to a real-world data set.

4.1 Problem Description

To test the utility of data envelopment analysis to internal benchmarking, a large data set describing the operations of a large parcel delivery service was used. Various data were collected by the company over the course of a year from the operations of 443 separate parcel delivery offices. Every business day, each office receives packages from a central hub and then sorts and delivers them to the customers in their area of responsibility. Although each office has the same function--the delivery of parcels—they all have different local situations. Some offices are located in cities; some are in rural areas. Certain offices have more parcels to deliver, while other offices tend to have fewer, larger parcels. Factors such as labor costs, road infrastructure, topography, and weather play roles in determining how the managers of each office allocate resources to maximize efficiency.

In light of the complicated operations and special situations of each office it would be practically impossible for upper management to decide which of the 443 DMUs are performing to their full potential without a formal statistical method to determine efficiency. So even without applying any of the specialized benchmarking techniques, using DEA to analyze the operations of the parcel delivery service can pay dividends by identifying the efficient DMUs.

As discussed in Chapter 2, when beginning a data envelopment analysis, the management of the company or other subject matter experts must determine the inputs and outputs that are to be used so that spurious or irrelevant data is not included. Table 4.1 lists the inputs and outputs designated by the parcel delivery service as being of major importance. These data represent monthly averages over a calendar year, and have been randomized for confidentiality. Table 4.2 gives more complete descriptions of the inputs and outputs.

Table 4.1 Inputs and Outputs for Parcel Delivery Service Analysis

I/O	Title	Mean	Median	Minimum	Maximum
Input 1	Number of Delivery Routes	120.19	98.79	21.60	559.04
Input 2	Base Working Time-Delivery	407.07	317.68	73.00	1979.71
Input 3	Base Working Time-Driving Within Route	176.45	140.27	36.80	703.13
Input 4	Base Working Time-Preparation	123.01	100.57	22.85	595.96
Output	Total Volume	17818.32	14067.44	3614.81	87620.64

Table 4.2 Descriptions of Inputs and Outputs

Title	Description
Number of Delivery Routes (Input 1)	Parcels are delivered by personnel along set routes. This is the number of routes each office has.
Base Working Time-Delivery (Input 2)	The amount of time spent actually delivering parcels to customers. This depends on a number of factors, including how many parcels are to be delivered, the size of the parcels, and the number of customers.
Base Working Time-Driving Within Route (Input 3)	The amount of time spent driving from delivery location to delivery location. This input is dependent on both the size of the office's area of responsibility and the number of parcels to be delivered.
Base Working Time-Preparation (Input 4)	The amount of time spent preparing the parcels for delivery. This includes time spent sorting as well as loading the delivery vehicles.
Total Volume (Output)	This is the total weight of parcels successfully delivered by the office.

Table 4.1 shows that the all of the data categories are slightly skewed towards the minimum, and each has a maximum-to-minimum ratio between 19 and 27.

4.2 DEA program

A DEA GAMS model based on Olesen and Peterson (1996) was used to run the analysis. The advantages of GAMS over other commercial linear programming software include quick processing time and the ability to conduct multiple LP matrix operations. Because a linear program must be solved for each DMU, using GAMS avoids the tedious task of manually running the linear program many (for this data set, 443) times. The matrix-based design of GAMS also allows for easy input of data as only a table of inputs and outputs (the coefficients for the constraints) is required; GAMS generates the variables associated with each datum automatically. The major disadvantage to using GAMS is the steep learning curve required, as the language used to program GAMS is difficult and sometimes counter-intuitive when compared with conventional computer programming languages.

The results of the data envelopment analysis were outputted to a data file and then analyzed using Microsoft Excel. Excel was used for its data-manipulation features and the flexibility attained by using Visual Basic for Applications (VBA). However, the size of the data set was just small enough for Excel to handle; a dedicated database application (such as SAS) would be necessary for any larger data set.

4.3 Results of testing

The methods discussed in Chapter 3 are now applied to the data set to test their validity. In lieu of examining the effects of each method on every DMU in the system, decision-making units representative of the data set will be used. Table 4.3 contains the information for six inefficient DMUs that will be used (at various times) to demonstrate these benchmarking methods.

Table 4.3 Sample DMUs

DMU	Output	Input 1	Input 2	Input 3	Input 4	θ
97	60726.52	485.74	1430.42	567.83	409.57	0.749
155	3614.81	29.31	89.05	52.11	33.21	0.901
230	5201.17	63.98	217.50	140.03	74.09	0.416
247	9088.11	65.32	255.63	107.07	71.57	0.550
348	61029.44	362.32	1570.83	381.85	335.36	0.995
418	14113.07	114.90	278.03	163.78	97.85	0.652

The magnitudes of the inputs and outputs give a rough estimate of the nature of operations at each DMU. DMUs 97 and 348 are large operations, (each has an output of approximately 60,000 units) yet their inputs are dissimilar. When compared to DMU 348, DMU 97 uses less of Input 2 but more of the other three inputs. Similarly, DMUs 155 and 230 are small operations, while DMU 247 and DMU 418 are of average size.

4.3.1 Basic data envelopment analysis

The data envelopment analysis of the postal data set identified twenty-one DMUs that are performing at their theoretical best efficiency ($\theta = 1$), while the remaining 422 DMUs are inefficient to varying degrees. The mean efficiency of the entire DMU set (including the 21 efficient DMUs) is $\theta = 0.735$ and the mean efficiency of the inefficient DMUs is $\theta = 0.722$.

As was shown in Chapter 3, in order for an inefficient unit to become efficient it must either increase its output or reduce its inputs by a percentage equal to $(1 - \theta)$. As an example, DMU 97 has $\theta = 0.749$. It must either increase its production of Output (the total weight delivered by the office) by 25.1% or reduce its use of all four inputs by the same percentage.

The basic efficiency measure represented by θ can provide upper-level managers insight into the operations of the system as a whole. For example, the management of this postal system may think that DMU 97 is one of their most efficient units, based solely on its (above-average) output. The use of DEA in this case would illustrate that DMU 97 is, in fact, inefficient and that DMU 97 should be able to increase its output or reduce the consumption of its inputs. The limitation of using DEA in this basic way is that the value of θ by itself does not provide any insights into how DMU 97 can become efficient. The best practices needed to improve the operations of DMU 97 cannot be obtained without comparison to the other DMUs in the system. The goal is to find the benchmarks that can improve the operations of DMU 97.

4.3.2 Efficient DMUs as benchmarks

The basic method discussed in Chapter 3 for benchmarking a system is simply to use the efficient DMUs as the benchmark units. For the postal system, twenty-one efficient DMUs are identified using DEA. Table 4.4 lists these units and their respective inputs and outputs.

Table 4.4 Efficient DMUs (θ = 1) for the Postal System

DMU	Output	Input 1	Input 2	Input 3	Input 4
13	4688.32	25.36	88.34	58.79	23.32
14	4471.92	**21.60**	120.19	39.92	24.93
69	49872.83	299.47	1108.23	293.27	315.63
96	17851.93	97.77	319.96	96.10	84.89
115	4642.69	26.96	**73.00**	55.63	35.24
122	4340.05	26.99	76.26	41.00	32.33
139	46302.53	247.54	961.87	400.53	217.20
171	14929.14	79.65	314.94	59.00	87.03
199	42222.49	211.05	1013.56	476.16	188.20
201	70514.79	371.73	1538.12	451.67	357.60
209	5009.46	30.76	92.05	59.52	**22.85**
214	4690.95	32.42	82.13	37.21	34.81
215	4513.46	34.25	91.22	**36.80**	34.09
241	5708.74	37.61	105.81	39.44	39.18
257	40963.67	178.10	767.88	271.56	208.04
270	**87620.64**	425.59	1281.86	584.77	458.65
362	6333.63	29.53	153.86	39.34	38.27
393	10385.27	38.79	120.16	59.82	44.06
413	4976.43	26.81	89.32	50.07	27.82
424	4398.27	29.80	77.31	46.36	28.19
431	10075.28	44.32	122.85	56.77	51.76

The five boldface inputs and outputs in Table 4.4 are the system-wide minimums (for the four inputs) and maximum (for the output). As discussed in Chapter 2, a DMU that uses the least amount of any input or produces the most of any output will be efficient because the DEA linear program will assign the maximum weight to that input or output while forcing the other weights to zero. During a conventional benchmarking analysis, these five DMUs would be the ones most commonly used as benchmarks because of their superlative nature. However, the other sixteen efficient DMUs have attained their maximum efficiency based on the particular circumstances that exist at each DMU. Each efficient decision-making unit has made the decision to use its available resources to produce at its maximum output.

Use DMU 122 as an example. DMU 122 is producing Output at a very low amount (4340.50 units, close to the system-wide minimum of 3614.81). However, it is also using low amounts of all four inputs (see Table 4.4). Consequently, the DEA program recognizes DMU 122 as efficient. One can easily imagine the situation at DMU 122: the decision-maker at the office recognizes that his office does not have a great amount of Output (total weight of parcels delivered--perhaps this office is located in a rural or industrial area), and he adjusts his office's consumption of the inputs accordingly. The DMU is not penalized for having a low output; it is rewarded for having low inputs.

The advantages of using DEA in this way for an internal benchmarking problem are obvious. It is now a simpler problem. Instead of examining all 443 offices, the management of the parcel delivery business now has to examine the 21 benchmark units—a 95% reduction. DMUs are identified as benchmarks when, in the past, they may have been passed over as a benchmarking candidate. DMU 122 is recognized as efficient based on the circumstances experienced by the decision-maker at that unit, not based on any *a priori* decision by the upper management of the relative importance of a particular input or output. The reverse situation may also be true: the operations of DMUs previously thought by the management to be exemplary may, in fact, turn out to be inefficient when DEA is used.

Now that these twenty-one parcel delivery offices are identified as benchmarks, the next step is for the upper management to study the benchmarks' operations and identify best practices that can be used to improve system efficiency.

The limitations to this benchmarking method were discussed in Chapter 3. Once the best practices of the twenty-one benchmark units are identified, they are distributed to the inefficient DMUs to be implemented. However, the best practices that are in place at the benchmarks may not be suitable for the inefficient units. In fact, the best practices of benchmarks may even be contradictory. Is it reasonable to expect an inefficient parcel delivery office to implement the best practices of both DMU 122 and DMU 270, when DMU 270 has over twenty times the volume of parcels delivered as DMU 122? Obviously, the operations at DMU 122 are very different than the operations at DMU 270.

Additionally, Table 4.4 shows that DMU 270 had the maximum output of any parcel delivery office in the system, thus making it an efficient unit. This "automatic" DEA efficiency caused by maximizing an output or minimizing an input can produce benchmark units that are outliers, with very little in common with the other DMUs in the system.

4.3.3 Using the popular efficient units as benchmarks

In Chapter 3, a method of identifying the most popular benchmarking units was examined. By applying this method to the parcel delivery system, it may be possible to eliminate outlying efficient units as benchmarks and identify those efficient parcel delivery offices which are most similar to the inefficient offices in the system. For each of the efficient DMUs identified in Table 4.4, Equations 3.1 and 3.2 are now applied. This gives the *referentnumber* for each efficient unit (Table 4.5). The *referentnumber* is the number of inefficient DMUs that use the efficient unit as a reference.

Table 4.5 Most Popular Benchmark Units

DMU	*referent number*	*referent percent*
393	390	92.4
270	181	42.9
139	104	24.6
257	98	23.2
171	95	22.5
14	73	17.3
199	64	15.2
201	59	14.0
209	39	9.2
362	38	9.0
96	37	8.8
424	37	8.8
115	32	7.6
214	30	7.1
122	22	5.2
13	18	4.3
431	18	4.3
241	15	3.6
69	7	1.7
215	4	0.9
413	3	0.7

(The average for all *referentnumber* is 65.0 and the median is 37.) From Table 4.5 it is immediately clear that DMU 393 is the most popular efficient unit in the system. Over ninety percent of the inefficient offices use DMU 393 as a reference unit, which suggests that the operations of DMU 393 have much in common with the rest of the system. It makes sense for the upper management to ascertain DMU 393's best practices and apply them throughout their parcel delivery system.

Even though the vast majority of the inefficient DMUs use DMU 393 as a reference unit, there are still 32 that do not. There is no definite rule determining how many DMUs are needed for benchmarks. In other words, how far down Table 4.5 does the management have to go in order to ensure that all of the inefficient are represented by at least one efficient DMU? Analysis of the parcel delivery data set shows that using the ten most popular DMUs is necessary in this case.

Applying to the parcel delivery data set this method of ranking the potential benchmark units based on their popularity has dramatically reduced the number of benchmarks. Now instead of using all 21 DMUs with $\theta = 1$ as benchmarks, the management now only needs to focus on ten. Not only will this make better use of the resources used for the identification of the system's best practices, but it also eliminates outlier benchmarks such as DMU 215 (*referentnumber* = 4) and DMU 413 (*referentnumber* = 3) which potentially have unique situations and operations that are not of much use for benchmarking purposes.

For this data set, the method of choosing benchmarks based on their popularity amongst the DEA-inefficient units provides a list of the parcel delivery offices that should be studied for their best practices. Starting with the original data set of 443 offices, the application of data envelopment analysis quickly identified the 21 efficient decision-making units. The most-popular benchmark method further reduced the number of potential benchmarks to ten—the top 2% of all the offices in the system.

A drawback of the most-popular benchmark method is that a situation can still arise where the benchmark units do not accurately portray the operations at some inefficient units. Table 4.6 lists the reference units and λ's for the six sample DMUs introduced earlier in this chapter. The entries shaded in grey correspond to the ten most popular efficient DMUs identified earlier in this section.

DMU 97 has four reference units—DMUs 139, 201, 270, and 393—and they are all on the most-popular benchmark list. Therefore, the assumption can be made that the best practices identified using the ten benchmark units will be applicable to the operations at DMU 97.

Decision-making unit 155 also has four reference units: DMUs 13, 14, 115, and 122. But only DMU 14 is on the most-popular list. In addition, since λ_{14} only equals 0.022 for DMU 155 it can safely be said that the entire most-popular benchmark list does not accurately reflect the operations of DMU 155. Identifying the best practices using those ten units is unlikely to improve the efficiency of DMU 155. The same can be said of DMU 230.

Applying the most-popular method to the parcel delivery system data set gives a good first-order solution to the problem of identifying benchmarks, but it fails to capture some of the nuances inside the data. DMUs such as 155 and 230 will not have representative efficient units against which they can be benchmarked. In all, there are 47 inefficient parcel delivery offices that have a majority of their reference weight (the λ's) fall outside of the ten benchmark units. In essence, not all of the inefficient DMUs follow the lead of the most popular benchmark units. The most-popular set can continue to be expanded but as each new efficient DMU is added it provides a diminishing return on the overall efficiency improvement of the system.

Table 4.6 DEA Reference Weights for Sample DMUs

λ	97	155	230	247	348	418
13	0	0.272	0.814	0.123	0	0
14	0	0.022	0.017	0.042	0	0
69	0	0	0	0	0.380	0
96	0	0	0	0	0	0
115	0	0.079	0.078	0	0	0
122	0	0.626	0	0	0	0
139	0.280	0	0	0	0	0.016
171	0	0	0	0	0.030	0
199	0	0	0	0	0	0
201	0.214	0	0	0	0.591	0
209	0	0	0	0.065	0	0
214	0	0	0	0	0	0
215	0	0	0	0	0	0
241	0	0	0	0	0	0
257	0	0	0	0	0	0
270	0.355	0	0	0	0	0.041
362	0	0	0	0	0	0
393	0.151	0	0.091	0.770	0	0.943
413	0	0	0	0	0	0
424	0	0	0	0	0	0
431	0	0	0	0	0	0

4.3.4 Inefficient Units as Benchmarks

The next method of identifying benchmark units discussed in Chapter 3 was to build a benchmark set for each inefficient DMU consisting of those more-efficient DMUs (but still possibly inefficient) that have the same reference units as the DMU-of-interest. Essentially, it is assumed that the units that make up each benchmark set are good candidates to improve the inefficient DMU because they have better, more efficient practices while having operations that are similar to the inefficient DMU (as shown by the fact that they have the same reference units).

Table 4.7 lists statistics for the benchmark sets for the six representative inefficient DMUs chosen earlier in this chapter.

Table 4.7 Benchmark Sets for Sample Parcel Delivery Service DMUs

DMU	θ^*	**Number of Reference Units**	**Number of DMUs in Benchmark Set**
97	0.749	4	15
155	0.901	4	1
230	0.416	4	1
247	0.550	4	1
348	0.995	3	0
418	0.652	3	57

The strength of this method is that the algorithm represented by Equations 3.3 through 3.5 builds a unique benchmark set for each inefficient DMU, rather than only using the efficient DMUs ($\theta = 1$) as benchmarks, regardless of their applicability to the inefficient DMU. Table 4.7 shows that five of the six representative DMUs have benchmark sets consisting of those DMUs with greater efficiencies and the same reference units. (DMU 348 has an empty benchmark set due to its θ of 0.995. Because few DMUs have a greater efficiency than DMU 348, there are no DMUs that solve both Equations 3.4 and 3.5.) These benchmark sets are unique to each DMU and should provide good candidates for benchmark units. For example, DMU 97's benchmark set consists of 15 DMUs, all of which are more efficient and have at least some similarity to DMU 97; the same applies to the 57 units in DMU 418's benchmark set.

The largest weakness of this method is that each benchmark set may contain DMUs that are only marginally similar to the unit being benchmarked while at the same time the benchmark set may exclude some units that are, in fact, very similar to the DMU being benchmarked. In Table 4.8, the DEA-efficiency and reference weights for DMU 418 (one of the representative DMUs selected earlier) are listed, as are those for DMU 200, DMU 383, and DMU 423. Although both DMU 200 and DMU 383 are in the benchmark set for DMU 418, by comparing the reference weights of the DMUs it is easily shown that DMU 383 is very similar to DMU 418 and that DMU 200 has little in common with DMU 418. On the other hand, DMU 423 is not in the benchmark set, yet it appears to have similar reference weights (and is thus likely to have similar operations) to DMU 418.

Obviously, the complaints raised in Chapter 3 concerning this method remain valid. This method offers no way to rate or rank the benchmarking suitability of DMUs in the benchmark set, while at the same time it ignores units that could be excellent candidates to become benchmarks.

Table 4.8 Example DEA Reference Weights

λ	418	200	383	423
13	0	0	0	0
14	0	0	0	0
69	0	0	0	0
96	0	0	0	0
115	0	0	0	0
122	0	0	0	0
139	0.016	0.833	0.013	0
171	0	0	0	0
199	0	0	0	0
201	0	0	0	0
209	0	0	0	0
214	0	0	0	0
215	0	0	0	0
241	0	0	0	0
257	0	0	0	0
270	0.041	0.080	0.072	0.055
362	0	0	0	0
393	0.943	0.087	0.916	0.945
413	0	0	0	0
424	0	0	0	0
431	0	0	0	0

4.3.5 Common Reference Unit Method

As discussed in Chapter 3, another method to determine the available benchmarks for an inefficient unit is to define the benchmark set for each unit as the set of more-efficient units that have at least one reference unit in common with said inefficient unit. By expanding the benchmark set for each inefficient unit in this manner, the goal is to capture units that would be good benchmarks but would have otherwise been missed.

The expansion of the benchmark sets in this fashion achieves the goal of increasing the pool of possible benchmarks. For example, after using the method from the previous section the benchmark sets for DMUs 230 and 247 each contained just one DMU (Table 4.8). Using the common reference unit method to expand the benchmark sets, they contain 400 and 376 DMUs respectively.

While the common reference unit method solves one problem--the possibility that candidates for benchmarks are excluded from the benchmark set--it does nothing for the problem of ascertaining *which* DMUs in the benchmark set are the best benchmarks. This method gathers up all the benchmark candidates. In the next section, the similarity index method will winnow out the poor candidates.

4.3.6 Similarity Index Method

Once the benchmark set for each of the 422 inefficient DMUs in the data set is built using the common reference unit algorithm, the similarity index method described in Section 3.6 is applied to rate and rank each DMU in the benchmark set based on their similarity to the inefficient DMU.

To demonstrate the entire process, it is helpful to use one DMU as an example. After running the data envelopment analysis on the entire data set, DMU 155 is found to be inefficient with $\theta_{155} = 0.901$. Therefore, in order for the operations at DMU 155 to become efficient, the unit must either increase its output or decrease its inputs by 9.9%. In order to achieve this, the decision-maker at DMU 155 wants to find appropriate units that can serve as benchmarks. There are 55 DMUs in the system that have greater DEA-efficiencies than 0.901, but only 18 of these have at least one reference unit in common with DMU 155. These eighteen DMUs constitute the benchmark set for DMU 155. Equation 3.8 is then applied to each of these units to solve for each similarity index. Table 4.9 lists the results of the similarity index method for DMU 155.

Table 4.9 Similarity Indices for DMU 155 (θ = 0.901)

DMU	θ	SI
381	0.912	0.328
353	0.948	0.374
122	1.000	0.470
429	0.923	0.477
414	0.928	0.492
405	0.911	0.745
417	0.986	0.748
141	0.915	0.754
312	0.910	0.872
218	0.921	0.893
314	0.991	0.898
384	0.994	0.900
330	0.902	0.939
366	0.953	0.948
48	0.948	0.955
13	1.000	0.964
115	1.000	1.147
14	1.000	1.195

The DMUs in Table 4.9 are listed in order of similarity. DMU 381 is the most similar to DMU 155; DMU 14 is the least similar. In fact, the three least-similar DMUs (13, 115, and 14) are efficient which supports the conclusion from Section 3.2 that using only DEA-efficient DMUs as benchmarks is not always appropriate. Table 4.10 lists the reference weights for DMU 155 as well those for the three most- and least-similar units in the benchmark set.

Table 4.10 DEA Reference Weights for DMU 155 and Selected Benchmarks

		Low S.I. (Similar DMUs)			High S.I. (Dissimilar DMUs)		
λ	**155**	**381**	**353**	**122**	**13**	**115**	**14**
13	0.272	0	0	0	1.000	0	0
14	0.022	0.119	0.086	0	0	0	1.000
69	0	0	0	0	0	0	0
96	0	0	0	0	0	0	0
115	0.079	0.116	0.043	0	0	1.000	0
122	0.626	0.615	0.872	1.000	0	0	0
139	0	0	0	0	0	0	0
171	0	0	0	0	0	0	0
199	0	0	0	0	0	0	0
201	0	0	0	0	0	0	0
209	0	0	0	0	0	0	0
214	0	0	0	0	0	0	0
215	0	0	0	0	0	0	0
241	0	0	0	0	0	0	0
257	0	0	0	0	0	0	0
270	0	0	0	0	0	0	0
362	0	0	0	0	0	0	0
393	0	0.150	0	0	0	0	0
413	0	0	0	0	0	0	0
424	0	0	0	0	0	0	0
431	0	0	0	0	0	0	0

From Table 4.10, it is clear that the similarity index method performed reasonably in ranking the benchmark candidates. It is now an easy task for decision-makers at DMU 155 to identify practices that will quickly enable it to improve its efficiency. They just need to benchmark themselves against those DMUs near the top of Table 4.9 (i.e., the units with the smallest similarity indices).

4.4 Chapter Summary

Methods to identify benchmark units were applied to a large, real-world system. The sample data set from a large parcel delivery service was described, and the methods introduced in Chapter 3 were used to analysis the system. The basic data envelopment analysis of the system was useful in identifying offices that were not performing at their theoretical maximum efficiency. The advantages and disadvantages of using only efficient decision-making units as benchmarks were examined, as were the pros and cons of using inefficient DMUs as benchmarks. Lastly, by constructing benchmark sets for every inefficient DMU and solving for each similarity index, the methodology for ranking the suitability of each DMU as a benchmark unit was tested and appeared to be a valid method. The following chapter will discuss possible improvements to the similarity index method and will suggest future areas of study.

Chapter 5

CONCLUSIONS

The overall goal of this study has been to develop a methodology that large organizations can use to identify those internal units that are the most efficient. By examining these benchmark units and identifying the practices they employ, the management of the organization can apply these best practices throughout the entire organization. Alternately, the management of an inefficient unit can independently examine the practices of a similar, yet more efficient, unit and apply them to their own unit with the goal of improving the efficiency of their unit's operations. If every inefficient unit does this, than the overall efficiency of the organization will necessarily improve.

5.1 Solutions

For the upper management of a large organization, conducting a benchmarking survey can be a daunting task. For example, this study examined a system consisting of over 400 separate internal offices, each with multiple, independent inputs and outputs. Without a method to accurately quantify the best-performing units, the upper management will have to choose their benchmark units based on personal opinions and *a priori* decisions of what constitutes an efficient unit. This study has introduced a simple yet powerful methodology to identify the benchmark units and has demonstrated its potential on the real-world data set of a parcel delivery service.

The methodology developed during this study consists of three steps. The first step is to conduct a standard data envelopment analysis of the system. During this stage, the upper management of the organization must work closely with the operations researcher in order to provide insight as to the nature of the organization. Ensuring that the important inputs and outputs are included and weighted correctly is vital to a successful data envelopment analysis. The results of the DEA will give the efficiency and reference weights for each decision-making unit in the system.

The next step is to construct a benchmark set for each inefficient unit. Each DMU's benchmark set consists of those DMUs that have a greater efficiency and at least one reference unit in common. The construction of the benchmark set ensures that each inefficient DMU will be benchmarked against another DMU that is more efficient and has at least minutely similar operations.

The final step in the methodology presented in this study is to calculate the similarity index for each unit in the benchmark set. This index determines how similar each possible benchmark unit is to the examined DMU. By choosing from those in the benchmark set that are most similar, the management ensures that the best practices at the benchmark unit are applicable to the DMU that is being improved.

In this way, the hundreds of potential benchmark units are narrowed down to the most-suitable few.

5.2 Future Study Suggestions

As discussed in Chapter 3, once the benchmark set is constructed for a particular unit there is no definite rule on which member (or members) of the benchmark set is actually used as a benchmark. Because the efficiency of each is essentially independent of its similarity index, simply choosing the most similar DMU as a benchmark may lead to very little improvement in efficiency (i.e., if the most similar DMU has only a marginally better efficiency). Conversely, choosing a benchmark unit from those in the set that have the greatest efficiency may lead to the case where an extremely inefficient DMU is attempting to "jump up" to the level of a much more efficient DMU, instead of having a more realistic goal of incremental efficiency improvement. Or a situation could be imagined where using the apparently perfect benchmark candidate is not cost-effective, perhaps because it is a large distance away. It would be ideal if there was some method to further refine the benchmark set to determine the best benchmark unit for a given situation. One area of possible further study would be to develop a linear or non-linear program for a given system using the similarity indices, DEA efficiencies, and external variables (such as distances between DMUs) as coefficients for the decision variables.

This study dealt only with the use of data envelopment analysis to select the benchmarks for an organization's benchmarking survey. Once the benchmarks have been identified and the best practices of said benchmarks are implemented, DEA has the potential to provide valuable insights into the success of the benchmarking process. By monitoring the efficiencies of the individual DMUs and the efficiency of the system as a whole over time, the upper management can determine how much their organization is improving. One promising method to accomplish this is through the use of Malmquist indices. The Malmquist index was introduced by Caves et al (1982) extending an idea of Malmquist (1953) and was further refined by Fare, et al. (1989). Essentially, the Malmquist index method uses data envelopment analyses of a system from two time periods to calculate the efficiency change for a DMU over the two time periods. The efficiency change is comprised of the technical change component and the productivity change component. The technical change is the increase or decrease in efficiency caused by conditions that apply equally throughout the system, while the productivity change is due to the actions of the DMU. For example, if the cost of one input dropped over the time period, it would appear to increase the efficiency of a given DMU. But since this change in the commodity price applies to all DMUs in the system, this is a technical change so the DMU should not get any credit for improving its efficiency. But any improvement in efficiency over this technical change would be an actual change in productivity due to the decision-making unit's decisions.

Using the method discussed in this study to identify a system's benchmarks coupled with a long-term study of the efficiency change of the system using Malmquist indices is a fertile area of possible study. It would even be possible to "benchmark the benchmarks" by identifying which benchmark units have done the best job in improving the less efficient DMUs.

5.3 Final Summary

Internal benchmarking has been identified as a means of improving the operations of large organizations. Discovering the best, most efficient units in the organization, studying them to determine how they conduct their business, and then distributing those best practices throughout the organization has been proven time and again to dramatically improve the operations of the organization. In practice, however, it can be difficult for large organizations to determine which of their internal decision-making units are operating at top efficiency. In addition, even if these units can be identified it can be difficult or impossible to apply their best practices to other units which may have different situations or priorities. This study was an attempt to solve these problems inhibiting companies and governmental. The use of data envelopment analysis, coupled with the similarity index method discussed in this study, is ideally suited to solve these problems thus enabling organizations to better use the powerful tool of internal benchmarking.

BIBLIOGRAPHY

Athanassopoulos, Antreas D. (1997). "Service Quality and Operating Efficiency Synergies for Management Control in the Provision of Financial Services: Evidence From Greek Bank Branches," *European Journal of Operational Research*. 98: 300-313.

Bergendahl, Goran (1998). "DEA And Benchmarks—An Application To Nordic Banks," *Annals of Operations Research*. 82:233-249.

Camp, Robert C. (1995) *Business Process Benchmarking: Finding and implementing best practices.* USA: ASQC.

Caves, D., Christensen, L., and E. Diewert (1982). "The Economic Theory of Index Numbers and the Measurement of Input, Output, and Productivity," *Econometrica*. 50: 1393-1414.

Charnes, A., Cooper, W.W. and E. Rhodes (1978). "Measuring the Efficiency of Decision-Making Units," *European Journal of Operational Research*. 2: 429-444.

Charnes, A. et al (1994). *Data Envelopment Analysis: Theory, Methodology, and Application*. Boston: Kluwer Academic Publishers

Cook, W. D., Y. Roll, and A. Kazakov (1990). "A DEA Model for Measuring the Relative Efficiency of Highway Maintenance Patrols," *INFOR* 28(2), 113-124.

Donthu, Naveeen, Hershberger, Edmund K. and Talai Osmonbekov (2005). "Benchmarking Marketing Productivity Using Data Envelopment Analysis," *Journal of Business Research*. 58: 1474-1482.

Fare, R., Grosskopf, S., Lindgren, B. and P. Roos (1989). "Productivity Developments in Swedish Hospitals: A Malmquist Output Index Approach," Department of Economics, Southern Illinois University, Carbondale.

Fare, R., Grosskopf S. and P. Roos (1995). "Productivity and Quality Changes in Swedish Pharmacies," *International Journal of Production Economics*. 39: 137-147.

Farrell, M.J. (1957). “The Measurement of Productive Efficiency,” *Journal of the Royal Statistical Society*, 120(3): 253-290.

Fowler, Alan (1997). “How To Use Benchmarking,” *People Management,* 3(12): 38-40. June 12. (UK).

Garcia, Florencio, Marcuello, Carmen, Serrano, Diana and Olga Urbina (1999). “Evaluation of Efficiency in Primary Health Care Centres: An Application of Data Envelopment Analysis,” *Financial Accountability and Management*. 15(1): 67-83.

Hashimoto, Akihiro and Migaku Kodama (1997). “Has Livability of Japan Gotten Better for 1956-1990?: A DEA Approach,” *Social Indicators Research*. 40: 359-373.

Homburg, Carsten (2001). “Using Data Envelopment Analysis to Benchmark Activities,” *International Journal of Production Economics*. 73: 51-58.

Keehley, Patricia, Medlin, Steven, MacBride, Sue and Laura Longmire (1997) *Benchmarking for Best Practices in the Public Sector*. San Fransisco, California: Jossey-Bass.

Landry, Pete (1993). “Benchmarking Strategy,” *Executive Excellence*, 10(6):8-9 June.

Madu, Christian N. and Chu-Hua Kuei (1998). “Application of Data Envelop Analysis in Benchmarking,” *International Journal of Quality Science*. 3(4): 320-327.

Malmquist, S. (1953). “Index Numbers and Indifference Surfaces,” *Trabajos de Estatistica*. 4: 209-242.

McGonagle, John J and Denise Fleming (1998). “Options in Benchmarking,” *Journal for Quality and Participatio*n. 21(2):38-42. March/April.

Nillesen, Paul H.L. and Michael G. Pollitt (2001). “Becoming a Best-Practice Company: The FPL Story,” *The Electricity Journal*. 14(9): 96-101.

Olesen, O.B. and N.C. Petersen (1996). “A Presentation of GAMS for DEA,” *Computers in Operations Research*. 23(4): 323-339.

Ozcan, Yasar A. (1998). “Physician Benchmarking: Measuring Variation in Practice Behavior in Treatment of Otitis Media,” *Health Care Management Science*. 1:5-17.

Schaffnit, Claire, Rosen, Dan and Joseph C. Paradi (1997). "Best Practice Analysis of Bank Branches: An Application of DEA in a Large Canadian Bank," *European Journal of Operational Research*. 98: 269-289.

Schefczyk, Michael and Torsten J. Gerpott (1998). "Determinants of Corporate Efficiency in a Declining Industry—An Empirical Analysis of German Foundries," *Management International Review*. 38(4): 321-344.

Sherman, H. David and Timothy J. Rupert (2006). "Do Bank Mergers Have Hidden or Foregone Value? Realized and Unrealized Operating Synergies in One Bank Merger," *European Journal of Operational Research*. 168: 253-268.

Smith, Steve (1997). "Benchmarking: Lessons for Disciplined Improvement," *IIE solution*s. 29(11): 40-45. 1997 Nov (Unilever; pharmaceuticals industry)

Tarricone, Paul (1998). "Best Practices Make Perfect"; *Facilities Design and Management*, 17(3): 50-52. March.

Appendix A

GAMS CODE FOR THE DEA MODEL

The following GAMS code is based on Olesen and Petersen (1996). The program was run using the PC version of GAMS, GAMSIDE. Even with the GAMS output suppressed as much as possible, the final output file for the parcel delivery service data set was 2249 pages long.

```
$ONINLINE
$OFFUPPER
$OFFSYMXREF
 OPTION LIMROW=0; OPTION LIMCOL=0;
 OPTION WORK=10000000;
SETS
 IO1                     /0*4/      five inputs and outputs
 OUTPUT(IO1)             /0/        one output
 INPUT(IO1)              /1*4/      four inputs
 UNITS                   /D1*D443/  443 DMUs
 SUBUNITS(UNITS)         /D1*D443/
 ACTUNIT(UNITS)          ;
ALIAS (IO1,IO2);
ALIAS (SUBUNITS, SUBUNITS1);

TABLE IOACT(UNITS, IO1)

          0          1             2             3             4
D1        31378.80   237.09        655.09        447.69        185.60
D2        33603.78   218.45        910.98        338.71        175.05
D3        30119.43   198.29        755.22        274.58        189.52
.         .               .             .             .             .
.         .               .             .             .             .
              For complete DEA data set, see Appendix B
.         .               .             .             .             .
.         .               .             .             .             .
D441       14475.56   81.52        292.29        190.03        89.17
D442         9245.01   58.46        213.17        117.39        54.94
D443         6896.13   46.24        173.05         76.44        52.56
;

POSITIVE VARIABLES
 WEIGHT(UNITS)  weights;
VARIABLES
 THETA          efficiency score;

SCALAR
```

```
 COL            column number /19/;

EQUATIONS
 PRCONSTR1(INPUT, UNITS)   DEA constraint for each input
 PRCONSTR2(OUTPUT, UNITS)  DEA constraint for each output
 REFTECH                   Reference technology;

 PRCONSTR1(INPUT, ACTUNIT)  .. SUM(SUBUNITS,
WEIGHT(SUBUNITS)*IOACT(SUBUNITS, INPUT))-
                             THETA*IOACT(ACTUNIT, INPUT)=L=0;
 PRCONSTR2(OUTPUT, ACTUNIT)  .. SUM(SUBUNITS,
WEIGHT(SUBUNITS)*IOACT(SUBUNITS, OUTPUT))-
                             IOACT(ACTUNIT,OUTPUT)=G=0;
 REFTECH                   .. SUM(SUBUNITS, WEIGHT(SUBUNITS))=E=1;
PARAMETERS
 EFFSCORE(UNITS)        report of efficiency scores
 OPTWEIGT(UNITS, UNITS) report of weights
 OPTPRICE(UNITS, IO1)   report of optimal virtual multipliers;


MODEL PRIMALDEA / PRCONSTR1, PRCONSTR2, REFTECH/;
OPTION SOLVEOPT=REPLACE;
LOOP(SUBUNITS1,
     ACTUNIT(UNITS)           =NO;
     ACTUNIT(SUBUNITS1)       =YES;
     SOLVE PRIMALDEA MINIMIZING THETA USING LP;
     EFFSCORE(SUBUNITS1)              = THETA.L;
     OPTWEIGT(SUBUNITS1, UNITS)       = WEIGHT.L(UNITS);
     OPTPRICE(SUBUNITS1, INPUT)       = SUM(ACTUNIT,
PRCONSTR1.M(INPUT, ACTUNIT));
     OPTPRICE(SUBUNITS1, OUTPUT)      = SUM(ACTUNIT,
PRCONSTR2.M(OUTPUT, ACTUNIT));

);


$ONTEXT
The following section outputs the DEA results to a file in a format
suitable for importing into a spreadsheet or database
$OFFTEXT


DISPLAY EFFSCORE, OPTWEIGT, OPTPRICE;
FILE PRACTICE /PRACTICE.txt/ ;
PUT PRACTICE ;
PRACTICE.ND=3;
PRACTICE.NW=6;
PRACTICE.LW=6;
PRACTICE.PS=500;
PRACTICE.PW=255;
PUT @1#2
LOOP(SUBUNITS1, PUT SUBUNITS1.TL, PUT @5, PUT EFFSCORE(SUBUNITS1),
PUT /);
PUT #1 ;
LOOP(SUBUNITS1,
  PUT /,
```

```
  PUT @11,
  LOOP(SUBUNITS,
       PUT$(EFFSCORE(SUBUNITS) GT 0.990) OPTWEIGT(SUBUNITS1,
SUBUNITS)));
PUT @12#1 ;
LOOP(SUBUNITS1, PUT$(EFFSCORE(SUBUNITS1) GT 0.990) SUBUNITS1.TL);
```

Appendix B

PARCEL DELIVERY SYSTEM DATA SET

DMU	Output	Input 1	Input 2	Input 3	Input 4
1	31378.80	237.09	655.09	447.69	185.60
2	33603.78	218.45	910.98	338.71	175.05
3	30119.43	198.29	755.22	274.58	189.52
4	8235.93	53.33	180.09	85.38	43.15
5	24175.53	148.38	458.83	324.60	119.83
6	17923.26	122.29	383.34	272.23	118.13
7	18228.51	162.01	501.61	260.26	127.19
8	21425.09	142.01	406.32	180.29	119.03
9	19016.16	136.64	460.71	190.83	128.86
10	13340.35	97.99	396.26	170.43	92.97
11	21879.97	141.99	498.89	246.58	143.65
12	26306.10	175.22	545.88	218.03	136.84
13	4688.32	25.36	88.34	58.79	23.32
14	4471.92	21.60	120.19	39.92	24.93
15	7961.50	42.52	200.46	82.90	54.44
16	14340.16	102.10	371.12	273.90	115.87
17	19644.38	144.06	526.89	159.08	124.15
18	52288.64	303.94	1132.87	406.50	354.97
19	20311.27	173.59	578.60	206.13	150.83
20	13218.66	110.03	340.97	146.67	95.71
21	16673.16	142.94	367.49	173.95	129.67
22	10367.77	56.25	270.57	104.18	76.98
23	11545.15	86.51	314.00	161.21	77.38
24	15220.67	114.92	375.76	159.05	95.08
25	40036.07	232.74	957.68	298.87	262.43
26	26572.82	159.97	567.05	216.05	191.53
27	14885.40	119.15	345.07	152.20	115.20
28	19677.13	120.66	457.84	154.06	113.26
29	14335.37	107.09	364.74	171.19	105.96
30	9579.35	77.73	232.54	82.41	79.95
31	18520.13	151.46	490.61	213.13	131.09
32	14521.50	140.08	440.30	158.90	101.47
33	17965.68	114.63	420.11	169.20	122.86
34	17632.71	134.42	403.37	201.89	131.45
35	9026.97	49.29	227.82	131.79	64.36
36	13700.74	90.27	313.49	149.10	80.04
37	33653.51	215.76	740.00	389.68	197.44
38	9974.11	69.63	295.62	136.84	71.03
39	22619.76	134.55	573.25	272.42	131.83
40	13723.97	87.84	313.01	130.08	99.52
41	11862.48	75.94	275.88	146.68	80.28

DMU	Output	Input 1	Input 2	Input 3	Input 4
42	40805.55	218.25	1080.47	386.03	215.28
43	26797.08	187.39	633.06	338.22	179.00
44	7227.39	37.31	149.28	102.50	51.36
45	19068.67	101.77	363.18	268.95	91.61
46	10381.14	55.00	194.52	107.64	69.52
47	8490.27	48.04	132.20	68.45	50.17
48	5200.15	36.28	108.48	41.04	36.25
49	11665.06	95.58	211.58	114.73	76.45
50	17299.39	116.77	568.13	187.49	122.06
51	9883.91	66.49	226.75	135.22	59.53
52	36393.99	276.94	926.96	470.62	247.85
53	50395.34	359.00	1336.64	477.99	367.21
54	22711.57	159.32	610.36	273.12	146.78
55	22601.20	158.44	592.44	305.35	221.15
56	31126.18	273.83	854.61	319.64	263.94
57	10496.35	73.03	228.47	111.62	91.66
58	19248.81	127.67	484.80	244.39	118.88
59	4505.67	37.61	108.29	49.63	40.13
60	5911.46	60.19	156.71	106.35	53.06
61	12302.43	92.49	314.50	185.70	100.25
62	5917.53	50.27	164.21	72.08	45.63
63	19758.05	127.12	428.42	204.22	117.00
64	7365.60	51.31	162.66	90.26	44.61
65	13987.11	91.11	250.26	129.96	94.53
66	21988.75	152.55	400.00	190.61	113.90
67	18331.83	152.67	718.47	169.38	174.07
68	21006.10	136.27	479.72	139.18	140.63
69	49872.83	299.47	1108.23	293.27	315.63
70	37216.18	439.60	1256.81	492.08	396.63
71	15221.89	98.65	399.84	128.18	111.65
72	31326.04	179.20	738.38	335.41	220.40
73	39173.28	239.41	872.64	287.79	240.73
74	38183.85	378.75	1476.14	391.14	340.74
75	22654.45	120.55	436.45	139.03	156.81
76	7655.62	47.82	181.45	66.95	53.12
77	20437.44	133.48	365.56	177.32	136.92
78	8145.17	52.02	161.91	65.31	58.64
79	10938.88	96.73	236.20	130.99	75.12
80	14114.45	109.54	356.75	145.99	95.79
81	5305.52	61.34	140.18	75.32	44.99
82	5841.04	58.10	169.29	94.96	44.14
83	6836.19	60.65	170.31	84.23	59.04
84	10205.92	99.80	226.82	98.26	87.44
85	26771.38	244.12	665.85	322.49	219.84
86	21283.80	204.48	596.10	216.06	199.77
87	8349.34	83.77	250.43	131.83	62.70
88	13410.37	107.63	447.50	190.41	136.39
89	9546.28	78.35	222.56	92.57	75.31
90	24457.74	238.96	663.08	286.56	209.03
91	16364.71	105.57	354.65	250.14	108.96
92	11231.97	55.71	184.27	61.91	63.93

DMU	Output	Input 1	Input 2	Input 3	Input 4
93	18881.40	115.17	412.32	121.22	110.49
94	14313.36	115.58	317.06	105.37	103.71
95	18154.86	107.87	349.23	109.91	101.66
96	17851.93	97.77	319.96	96.10	84.89
97	60726.52	485.74	1430.42	567.83	409.57
98	14345.53	79.92	265.81	117.52	100.57
99	59527.81	559.04	1979.71	703.13	576.69
100	13224.32	84.59	280.09	149.25	104.28
101	19812.71	123.94	447.74	187.48	136.95
102	28556.43	230.17	595.89	315.67	221.28
103	10079.12	141.17	448.74	242.82	120.68
104	5856.88	43.83	127.17	80.21	36.11
105	9741.22	86.01	277.53	138.20	76.02
106	37462.92	270.42	951.46	393.84	295.45
107	10433.54	77.65	239.90	94.80	86.80
108	10418.17	55.07	173.13	82.41	69.22
109	18267.05	131.29	356.00	140.59	116.84
110	11046.77	54.19	177.07	92.43	75.52
111	17224.01	121.27	352.32	179.74	158.37
112	8286.24	68.60	180.76	125.89	69.16
113	9598.67	85.83	218.33	126.70	88.33
114	6578.66	50.49	129.33	84.65	53.74
115	4642.69	26.96	73.00	55.63	35.24
116	4963.67	43.76	129.05	46.82	46.18
117	7797.00	183.22	521.06	267.31	170.73
118	7861.83	72.09	166.90	116.34	78.73
119	19673.72	146.87	436.10	224.81	191.41
120	9228.02	74.98	266.04	112.22	101.90
121	7278.61	115.57	311.78	221.62	157.26
122	4340.05	26.99	76.26	41.00	32.33
123	6393.10	56.31	157.35	108.84	61.00
124	8691.84	52.96	182.47	74.12	60.74
125	7537.40	42.27	167.99	80.92	61.81
126	14280.70	104.68	327.01	155.17	109.26
127	31542.72	248.12	940.12	310.79	326.47
128	10655.14	108.98	339.16	164.86	110.23
129	32252.31	211.09	627.99	350.97	175.80
130	15501.93	90.84	266.40	113.30	110.23
131	12095.07	85.82	195.72	104.15	76.15
132	10644.18	68.31	247.08	87.76	72.29
133	58323.57	338.05	918.28	441.08	372.28
134	6828.73	62.86	185.70	60.58	55.67
135	14686.72	81.60	367.97	104.81	100.37
136	8658.77	61.25	194.72	87.75	71.10
137	36313.65	222.94	682.96	254.61	229.03
138	23466.15	115.55	486.83	169.30	158.64
139	46302.53	247.54	961.87	400.53	217.20
140	12730.61	75.69	313.99	103.80	68.73
141	6257.93	30.91	117.08	52.92	40.81
142	11131.44	73.47	222.82	127.19	75.06
143	9439.52	49.64	184.03	90.65	59.01

DMU	Output	Input 1	Input 2	Input 3	Input 4
144	7366.83	41.57	134.84	70.11	53.65
145	7581.54	44.37	198.69	69.98	56.46
146	18410.91	165.61	406.49	215.16	139.92
147	19850.83	146.38	474.25	203.50	158.65
148	46025.07	247.45	985.57	438.03	242.35
149	13087.64	99.14	293.06	122.07	87.79
150	7257.05	42.58	137.51	67.70	54.39
151	7988.38	58.25	162.29	80.88	51.75
152	9973.32	80.39	228.44	118.41	76.92
153	25097.28	146.77	498.50	294.76	150.93
154	19935.06	109.54	386.54	161.32	148.44
155	3614.81	29.31	89.05	52.11	33.21
156	5560.56	39.57	104.46	51.00	42.49
157	11957.17	97.01	227.88	142.23	114.98
158	21422.25	117.76	449.62	206.93	162.41
159	9129.38	59.03	195.68	81.07	59.21
160	12226.87	107.14	283.98	152.92	108.84
161	30753.66	243.09	572.10	345.81	231.05
162	13225.39	100.22	334.85	166.12	115.83
163	17384.90	106.69	445.79	171.71	157.10
164	4551.07	30.70	109.53	43.23	40.19
165	4812.85	28.09	122.99	55.94	34.97
166	7473.29	59.02	171.63	81.22	50.62
167	6883.44	50.12	129.34	86.50	58.12
168	9600.92	65.48	229.03	91.03	76.85
169	21897.32	123.23	546.01	212.32	101.20
170	47574.34	314.07	949.88	456.55	270.02
171	14929.14	79.65	314.94	59.00	87.03
172	11144.88	61.81	215.02	173.49	57.98
173	13873.61	96.10	331.35	144.61	87.92
174	11966.67	80.10	301.59	106.33	94.36
175	14729.31	137.70	353.87	122.21	108.60
176	17390.16	113.43	457.84	203.94	125.04
177	27397.61	184.64	913.69	402.06	185.78
178	4679.68	38.03	156.48	56.49	37.44
179	9404.91	70.67	188.33	121.43	54.64
180	5115.17	44.76	154.67	59.76	44.87
181	14146.89	130.98	299.40	245.49	100.68
182	5360.43	55.57	123.97	71.81	42.66
183	14771.74	144.30	374.39	176.07	121.62
184	23812.61	163.92	539.56	373.62	158.32
185	48960.00	322.38	1063.96	477.60	360.12
186	19735.14	129.66	399.37	193.08	131.02
187	17000.65	88.85	315.34	194.28	116.74
188	15350.19	86.41	348.09	147.37	92.22
189	33387.96	253.49	690.74	355.79	292.89
190	62290.13	342.15	1359.84	477.20	384.31
191	18172.78	111.40	419.15	244.08	131.61
192	17223.72	101.19	388.61	182.40	122.82
193	5600.23	32.67	123.39	57.61	39.40
194	19387.31	142.70	500.58	198.28	156.18

DMU	Output	Input 1	Input 2	Input 3	Input 4
195	28091.06	166.76	607.49	313.07	204.03
196	12262.30	100.36	366.83	231.56	103.74
197	14810.58	112.61	325.43	188.24	131.78
198	11508.86	67.79	186.54	131.39	82.60
199	42222.49	211.05	1013.56	476.16	188.20
200	46508.47	350.78	1144.48	608.37	277.27
201	70514.79	371.73	1538.12	451.67	357.60
202	6060.39	35.13	163.66	61.73	31.55
203	23486.81	160.70	654.80	285.02	139.75
204	31505.87	152.91	728.29	292.02	176.07
205	21627.64	161.08	578.09	262.37	119.81
206	41215.74	217.93	1142.48	473.44	206.90
207	10691.50	57.44	212.53	113.96	63.66
208	9318.87	67.85	185.56	113.66	50.56
209	5009.46	30.76	92.05	59.52	22.85
210	8356.32	54.33	151.08	72.18	58.82
211	39993.32	266.07	819.12	310.03	301.79
212	9820.40	69.86	174.68	67.35	61.57
213	5888.26	31.43	133.04	47.88	37.74
214	4690.95	32.42	82.13	37.21	34.81
215	4513.46	34.25	91.22	36.80	34.09
216	15993.84	102.46	346.63	127.42	97.78
217	13763.41	101.06	346.47	127.16	105.62
218	5602.31	38.31	114.64	44.79	36.77
219	5928.85	30.85	130.99	52.37	37.58
220	31973.10	233.60	796.46	338.33	190.75
221	8355.83	53.86	192.38	102.13	51.48
222	11097.56	85.83	274.54	125.92	86.79
223	31499.78	431.88	1596.24	485.54	461.55
224	50527.10	274.62	1214.96	492.86	291.05
225	21106.78	123.78	397.93	181.31	109.75
226	19794.95	125.21	368.01	237.76	132.59
227	15643.56	106.11	376.61	236.17	123.34
228	42263.12	261.87	733.83	435.51	211.68
229	22206.32	128.02	409.10	286.63	136.24
230	5201.17	63.98	217.50	140.03	74.09
231	11854.16	69.02	226.75	131.64	65.99
232	12570.13	81.32	225.25	116.98	63.21
233	37001.96	241.39	699.64	254.91	207.98
234	20037.81	137.90	394.91	162.35	109.10
235	11586.83	98.93	346.82	144.72	108.56
236	6434.11	68.44	162.07	81.96	47.20
237	36586.05	293.02	868.06	347.50	270.88
238	19244.94	109.34	345.79	141.41	111.11
239	12255.06	80.12	341.18	128.28	89.83
240	49970.09	301.55	1133.02	430.41	324.58
241	5708.74	37.61	105.81	39.44	39.18
242	20492.45	119.75	358.28	141.75	155.29
243	20061.81	166.11	442.74	184.14	180.33
244	46166.08	255.68	952.63	366.39	327.07
245	41107.89	190.66	811.21	314.56	213.26

DMU	Output	Input 1	Input 2	Input 3	Input 4
246	12452.98	71.46	258.05	73.92	84.73
247	9088.11	65.32	255.63	107.07	71.57
248	6371.47	67.93	163.95	58.60	63.96
249	36395.91	220.02	767.75	278.36	222.10
250	14865.17	109.13	330.87	140.66	108.68
251	19777.28	202.22	563.59	276.38	197.80
252	8354.13	57.00	181.62	56.93	55.71
253	8756.75	63.35	196.68	115.92	71.14
254	28149.96	137.02	590.17	214.66	169.78
255	8947.32	98.90	364.31	170.09	89.19
256	23848.79	266.18	960.28	427.88	211.73
257	40963.67	178.10	767.88	271.56	208.04
258	64884.49	433.52	1471.53	567.00	418.23
259	24044.08	162.86	579.86	183.56	167.87
260	19344.58	124.81	361.64	200.73	126.04
261	8084.94	43.48	167.46	72.34	50.73
262	8138.13	60.75	242.70	89.07	58.11
263	19521.50	127.01	343.54	211.82	125.24
264	19333.43	124.51	385.20	126.56	126.26
265	17929.91	96.64	372.43	149.42	137.45
266	11183.24	71.92	233.48	74.95	65.52
267	20257.47	123.99	454.91	189.84	145.69
268	32229.07	250.92	799.90	293.39	198.50
269	16409.13	102.11	389.32	167.21	120.78
270	87620.64	425.59	1281.86	584.77	458.65
271	5513.12	31.80	134.56	66.89	45.22
272	22219.24	141.81	562.56	216.32	156.29
273	25924.85	179.42	647.81	269.04	212.83
274	20339.74	108.95	372.75	159.80	162.88
275	21670.89	111.21	472.62	186.48	145.80
276	13174.85	95.00	259.41	140.01	103.82
277	36241.48	210.21	762.38	352.57	222.11
278	11273.60	67.28	226.49	105.63	96.04
279	9274.59	48.38	250.31	94.93	72.14
280	37616.38	191.86	729.25	296.41	225.33
281	14859.71	78.53	231.49	139.03	85.64
282	17757.25	160.53	446.74	295.05	152.24
283	22554.87	131.07	351.49	204.95	128.22
284	9323.75	63.56	179.88	128.29	57.99
285	13504.45	100.83	279.52	169.18	90.08
286	14970.08	134.37	491.09	254.68	147.07
287	13612.55	104.06	357.19	135.68	94.44
288	37870.96	226.08	777.10	302.68	231.95
289	7511.01	56.66	151.71	61.85	55.43
290	8701.16	56.64	220.55	75.92	63.22
291	10404.00	58.12	241.83	112.06	84.64
292	24132.90	146.35	500.27	203.98	144.95
293	46463.16	281.14	1066.69	358.12	278.99
294	15885.99	96.32	282.61	139.56	89.91
295	43334.72	261.35	890.06	345.28	244.86
296	14338.35	111.97	298.44	156.69	91.13

DMU	Output	Input 1	Input 2	Input 3	Input 4
297	23735.26	119.02	359.89	155.75	128.53
298	6002.56	48.11	158.82	64.67	47.81
299	16974.77	118.07	396.26	131.90	127.33
300	8306.61	62.13	151.62	90.75	57.33
301	10613.83	65.75	217.12	70.19	55.62
302	8827.81	57.44	230.54	81.14	62.48
303	8102.79	51.16	156.94	61.68	50.20
304	12332.63	78.92	254.25	116.47	85.33
305	12943.99	93.01	320.83	137.54	73.64
306	6424.20	40.38	122.59	59.49	35.18
307	18657.24	132.91	422.73	166.36	121.94
308	14664.45	84.81	276.47	110.03	96.02
309	23170.92	169.15	475.67	165.63	154.04
310	11104.13	90.44	251.37	93.53	82.79
311	8409.10	67.39	218.03	96.62	68.72
312	5574.54	34.71	117.42	54.52	30.59
313	4338.57	36.77	112.16	58.11	31.51
314	4579.76	30.79	102.30	46.92	25.56
315	6044.94	50.76	138.51	63.43	51.08
316	14741.20	85.41	309.46	189.74	78.00
317	73491.63	481.78	1802.63	586.93	595.96
318	33773.72	210.61	651.53	257.08	224.58
319	21818.20	111.97	352.73	156.70	102.06
320	12551.79	85.34	262.03	127.03	89.75
321	34207.78	231.45	758.87	332.98	229.34
322	7912.32	62.76	189.30	85.17	74.12
323	19917.04	119.97	517.52	160.42	172.60
324	21344.49	122.59	388.98	211.47	147.41
325	12875.95	84.84	297.50	145.19	107.42
326	21948.94	127.44	477.03	218.85	147.60
327	20672.23	111.68	328.09	175.21	109.12
328	14710.80	78.93	279.57	130.38	70.78
329	9852.72	56.73	209.22	69.64	66.02
330	4792.97	31.97	116.93	44.84	32.12
331	24488.13	155.79	528.85	216.05	139.35
332	4677.12	39.09	116.41	38.39	37.03
333	31621.74	219.44	771.99	347.84	218.91
334	55091.47	351.34	1281.52	408.00	374.64
335	9796.27	54.84	238.49	75.97	68.47
336	9190.17	62.10	264.14	82.57	76.04
337	5934.82	48.20	155.02	69.49	46.66
338	27341.55	201.88	887.99	360.00	317.73
339	10097.04	81.41	221.51	72.85	59.59
340	8890.58	50.43	190.13	85.25	48.26
341	14020.20	72.58	321.28	129.29	79.99
342	21411.05	173.27	614.03	184.37	153.52
343	14077.65	109.44	307.58	124.19	110.14
344	49254.05	228.12	1103.96	373.88	275.78
345	21774.01	131.72	431.31	147.01	127.16
346	10827.20	78.76	310.16	126.46	72.61
347	48760.95	232.26	891.11	338.47	281.13

DMU	Output	Input 1	Input 2	Input 3	Input 4
348	61029.44	362.32	1570.83	381.85	335.36
349	54768.29	366.78	1372.04	440.22	430.72
350	39084.65	302.92	894.04	372.13	300.14
351	22068.13	94.95	329.58	181.24	147.53
352	7479.82	41.90	142.54	65.89	47.17
353	3948.09	27.97	84.24	43.79	37.80
354	10930.31	75.25	218.52	122.80	112.46
355	11566.12	58.42	190.23	86.64	82.19
356	21460.08	132.60	317.27	157.19	111.98
357	18677.70	113.73	346.67	142.18	149.89
358	8577.17	64.33	218.41	100.24	54.28
359	61877.74	404.10	1386.15	424.28	375.39
360	26757.17	181.57	714.38	225.36	206.97
361	19466.33	100.65	425.84	140.27	148.79
362	6333.63	29.53	153.86	39.34	38.27
363	10674.38	56.62	211.69	72.66	78.59
364	12928.64	82.29	266.56	102.01	107.26
365	9502.57	68.11	234.04	88.98	78.11
366	4441.56	31.69	118.57	39.62	33.96
367	12790.65	78.21	301.75	88.47	99.90
368	3949.30	47.27	213.46	73.29	56.44
369	15053.51	81.69	320.71	131.55	101.35
370	15436.74	113.81	415.58	120.70	124.07
371	12416.27	64.94	258.36	92.64	83.36
372	5662.14	46.87	144.20	55.28	48.91
373	11565.62	65.87	244.02	110.07	63.48
374	14116.31	72.67	265.96	118.06	90.65
375	14945.91	102.98	348.38	154.42	116.45
376	10859.70	51.99	171.25	77.59	57.56
377	12896.24	85.25	279.84	140.53	79.27
378	5850.28	39.97	108.47	60.68	46.91
379	5005.94	35.91	111.24	66.25	33.93
380	7527.97	67.17	147.66	101.94	64.90
381	5296.15	30.81	96.12	49.75	38.68
382	21518.17	130.24	425.39	187.85	133.42
383	16371.44	98.79	291.03	162.75	103.27
384	7443.98	39.36	142.36	47.59	37.60
385	14416.86	108.28	412.18	138.35	99.44
386	12694.18	82.42	375.96	112.14	87.52
387	11224.49	73.47	248.43	78.46	67.63
388	27513.66	166.63	572.79	246.05	149.30
389	54500.40	375.66	1110.18	441.59	392.41
390	19247.24	116.10	325.89	176.75	121.37
391	13565.20	105.22	262.32	133.90	116.38
392	9625.67	52.18	146.21	70.88	55.75
393	10385.27	38.79	120.16	59.82	44.06
394	6735.57	38.26	106.58	66.00	43.77
395	23046.38	112.82	360.48	166.02	127.04
396	15760.80	123.43	320.00	193.13	98.71
397	10790.46	102.05	264.06	99.79	101.00
398	25434.24	213.92	614.19	332.83	170.01

DMU	Output	Input 1	Input 2	Input 3	Input 4
399	22391.33	188.33	514.46	302.52	153.81
400	5160.18	39.78	135.10	75.64	32.04
401	16563.85	125.25	404.63	240.38	127.86
402	13390.83	88.81	302.43	172.36	85.09
403	9309.66	51.65	219.60	106.92	51.46
404	15726.69	106.45	314.48	197.68	91.78
405	4097.12	28.31	92.18	57.29	34.24
406	7692.00	61.62	186.44	125.10	68.78
407	16398.49	130.05	498.40	212.18	153.51
408	5666.69	61.92	159.95	74.91	54.30
409	10849.67	102.60	263.98	129.24	77.48
410	13827.45	76.97	281.13	130.75	79.03
411	8227.78	52.11	186.69	127.99	64.49
412	19500.91	131.83	358.00	212.76	141.67
413	4976.43	26.81	89.32	50.07	27.82
414	4673.47	31.26	83.39	50.95	34.95
415	4519.85	28.91	110.52	59.28	31.62
416	5649.02	31.61	142.54	67.22	41.76
417	3945.90	25.94	86.82	56.01	29.19
418	14113.07	114.90	278.03	163.78	97.85
419	21828.52	170.02	410.83	213.82	163.43
420	8010.67	44.18	157.48	73.99	46.51
421	10605.71	73.83	254.53	107.63	92.16
422	29394.42	195.95	627.42	332.75	227.67
423	14658.21	90.50	247.39	134.09	96.57
424	4398.27	29.80	77.31	46.36	28.19
425	6686.91	64.19	144.43	76.41	71.30
426	5765.41	44.01	110.03	60.34	36.26
427	6842.09	56.07	157.74	86.40	61.48
428	6975.48	51.20	138.56	64.87	42.81
429	4321.84	27.10	99.47	46.72	35.23
430	11944.68	99.77	296.05	146.77	108.07
431	10075.28	44.32	122.85	56.77	51.76
432	13838.12	92.02	353.35	172.55	70.43
433	15364.06	96.70	324.66	127.42	102.25
434	27800.17	187.85	676.32	310.30	152.96
435	17850.14	158.29	374.88	201.57	133.32
436	14067.44	80.85	317.68	170.28	75.47
437	6091.62	37.22	179.76	70.52	45.50
438	13155.93	87.53	366.41	139.45	105.30
439	13059.54	104.77	344.41	133.08	102.69
440	13587.12	83.20	264.82	138.23	70.60
441	14475.56	81.52	292.29	190.03	89.17
442	9245.01	58.46	213.17	117.39	54.94
443	6896.13	46.24	173.05	76.44	52.56