

**TEACHERS' ORIENTATIONS TOWARD AND INTERACTIONS WITH
STUDENTS' WRITING IN HIGH SCHOOL MATHEMATICS**

by

Ethan P. Smith

A dissertation submitted to the Faculty of the University of Delaware in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Education

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ABSTRACT

The purpose of this study is to investigate different aspects of teachers' orientations toward students' writing in high school mathematics. I used an embedded case study design (Yin, 2018) across the cases of three teachers whose instruction had previously been found to be dialogically focused – i.e., made use of high-level tasks, presented opportunities to share multiple representations or strategies, and included a focus on student discourse (Munter et al., 2015). I first interviewed each teacher to capture their described orientations toward writing in regard to math teaching and learning. I then conducted four observations of each teacher's instruction to detail the types of interactions that they had with students' writing. Finally, I conducted post-observations with each teacher to gain insight into their own interpretations of their observed lessons. My findings indicate that these teachers recognized writing as a tool to support student communication and reasoning, but that they situated writing as an embedded practice informed by their views on discourse and student self-efficacy. I also found that these participants enacted a range of interactions across different types of writing tasks, indicating that – while certain types of tasks may support more student-directed interactions – the teacher still plays a significant role as a facilitator of classroom discourse. This dissertation contributes to the field by suggesting approaches for better capturing factors that influence math teachers' orientations toward writing, and by indicating how noticing activities that include multiple mediums of evidence of student thinking might help such teachers to adopt a stance of inquiry (van Es & Sherin, 2021) toward students' writing.

Chapter 1

INTRODUCTION

Students' construction of written work, and the ways that a teacher can notice and interact with students' writing, is an important aspect of school mathematics. Writing arises as one of multiple language demands in the mathematics classroom, as students must grapple with how to read, write, speak, and listen to mathematical ideas as they engage with their learning (Aguirre & Bunch, 2012). As Reeves (1990) succinctly describes, "language is the essential vehicle for transmitting and understanding mathematics in school, for turning experience into thinking and learning" (p. 213). It is perhaps no surprise then that reform movements and professional organizations for mathematics education have long included an emphasis on effective student communication and reasoning – through both speaking and writing – as a key goal of mathematics education. For instance, the National Council of Teachers of Mathematics (NCTM; 2000) describes the importance of challenging students "to think and reason about mathematics and to communicate the results of their thinking to others orally or in writing" (p. 60). They go on to describe how providing opportunities for students to write and speak in mathematics classes affords students with "dual benefits: they communicate to learn mathematics, and they learn to communicate mathematically" (National Council of Teachers of Mathematics, 2000, p. 60). These interwoven goals of communication and reasoning are expanded upon in the Common Core State Standards for Mathematics (CCSSM), which include an explicit call that mathematically proficient students should have opportunities to

“Construct viable arguments and critique the reasoning of others” (Common Core State Standards Initiative, 2010, p. 6).

However, while supporting students’ spoken communication and reasoning through discourse has been a frequent focus of research in mathematics education research (e.g., Moschkovich, 2007; Munter et al., 2015; Webb et al., 2014), written communication and reasoning has not received the same level of attention (Casa et al., 2020; Morgan, 1998; Pugalee, 2004). This relatively limited attention on writing is concerning because mathematical ideas are communicated not only through spoken language, but within and across the multiple sign systems of natural language (written words, phrases, and sentences), mathematical symbolism, and visual imagery – and these systems are often represented through written texts (O’Halloran, 2000, 2008). In other words, writing in mathematics can be viewed as *multisemiotic*, or expressed within and across these multiple sign systems (O’Halloran, 2008).

What’s more, the language demands of the mathematics classroom extend and connect across multiple modalities (e.g., speaking, listening, reading, writing), and these different modalities inform how mathematical ideas or representations are created, communicated, and understood (Aguirre & Bunch, 2012). Therefore, it is important to understand students’ writing in mathematics as it arises and is interacted with (by students and/or the teacher) over the course of mathematics instruction. Such a focus on writing in relation to other modalities is also paramount given that the language of mathematics itself evolved in part “as a response to the functions which were fulfilled symbolically and visually” (O’Halloran, 2008, p. 14). Therefore, mathematical ideas are not only communicated multisemiotically through writing, but these written sign systems also help to inform the structure of spoken mathematical

discourse. For these reasons, I define *writing in mathematics* broadly to entail the construction of any written text encompassing natural language, mathematical symbolism, or visual imagery that is used to understand or communicate mathematical ideas.

Past research has delved into writing in relation to school mathematics. For instance, the writing-to-learn movement that arose in the 1980s considered how writing – and explicit instruction tied to improving students’ writing – could improve students’ learning of mathematics (Connolly, 1989; Morgan, 1998). Borasi and Rose (1989), for example, described how journal writing about their mathematical problem solving encouraged students “to become introspective of how they do and learn mathematics, and consequently be brought to identify more general heuristics to solve mathematics problems as well as realize the possibility of alternative approaches to the same learning task” (p. 356). More recently, research has similarly focused on what Powell et al. (2021) call *mathematics writing*, or “a writing activity in which students write about mathematics concepts or procedures” (p. 418). Casa et al. (2016) similarly refer to this sort of activity as *mathematical writing*. This research has included the use of teacher surveys to broadly investigate how, when, and under what conditions teachers in the United States use such writing in their classroom (Powell et al., 2021), syntheses of how writing in mathematics has been approached in relation to mathematics assessments and interventions in empirical research (Powell et al., 2017), and overall recommendations for mathematics writing for the elementary grades (Casa et al., 2016).

One emerging area of focus in research on mathematics or mathematical writing is also the relationship between such writing and desired outcomes for students

beyond the realm of academic achievement. For instance, the synthesis of literature conducted by Powell et al. (2017) shows how mathematics writing may support student self-efficacy toward and enjoyment of mathematics. These outcomes may arise in part because opportunities to write can provide additional means for student participation and opportunities for students to “draw on the full range of their cultural, linguistic, and social resources” (Casa et al., 2020, p. 7). For these reasons, writing may serve as an important instructional tool for supporting student engagement and equity in the mathematics classroom.

Although the aforementioned research indicates many benefits of writing in the mathematics classroom, this prior research is also limited in two ways. First, the definition of mathematics writing in these previous studies primarily focused on the construction using the written word (i.e., words, phrases, and sentences) to describe, explain, or justify mathematical ideas. Although such research recognizes that students can reason and communicate in ways other than (or in addition to) the written word, the definition of mathematics writing from Powell and colleagues (2021) tends to emphasize writing via words, phrases and sentences rather than a multisemiotic approach towards writing. Second, elementary grades have been a major focus of much of previous research studies on writing in mathematics classrooms. For instance, the recommendations from Casa et al. (2016) apply to mathematics writing in the elementary grades. However, content-specific disciplinary literacy skills become increasingly important for students in the context of the specialized nature of secondary school mathematics (Shanahan & Shanahan, 2008). Therefore, it is important to examine writing in mathematics at the secondary level, and to consider

how writing at those grade levels is similar or different to writing that occurs in elementary mathematics classrooms.

The construct of teacher noticing is a helpful frame through which to investigate writing in mathematics. Sherin et al. (2011) define teacher noticing as “Where do teachers look, what do they see, and what sense do they make of what they see?” (p. 3). Therefore, the conditions under which and the ways in which mathematics teachers *notice* students’ writing in their classrooms could also indicate how they understand the purposes of writing in mathematics teaching and learning. Additionally, teacher noticing is a helpful construct for considering not only what and how mathematics teachers notice (and respond to) students’ writing in-the-moment, but also how teachers are able to notice – and potentially develop their practices around – noticing aspects of students’ writing important to mathematics learning. Indeed, previous research on teacher noticing has focused not only on how it arises during teachers’ instruction, but also how teachers can improve aspects of their noticing through professional learning (e.g., Goldsmith & Seago, 2011; Jacobs et al., 2010; Sherin & van Es, 2009; van Es & Sherin, 2002). Therefore, investigating teachers’ noticing of students’ writing in mathematics could shed new light on teacher practices around writing and opportunities for supporting teacher learning around students’ writing in mathematics.

Additionally, teachers may co-generate their noticing of a student’s writing by engaging in inquiry-focused discourse with that student. van Es and Sherin (2021) describe this sort of cogeneration as a dimension of noticing called *shaping*, which “involves teachers constructing interactions, in the midst of noticing, to gain access to additional information that further supports their noticing” (p. 23). Given that the

language demands in the mathematics classroom with which students must grapple arise across both written and spoken modes of language (Aguirre & Bunch, 2012), shaping can be an especially important element of teachers' noticing of students' writing. By both *attending to* and *interpreting* the features of a student's written work – the first two dimensions of noticing according to van Es and Sherin (2002, 2021) – and shaping their noticing through spoken discourse that values the student's contributions – it is possible that mathematics teachers can better support students' navigation across these multimodal language demands.

This approach to the cogeneration of teacher noticing of students' writing is also relevant from an equity standpoint. Louie et al. (2021), for instance, describe how deficit noticing in mathematics tends to frame students as simply receivers of mathematics content, while anti-deficit noticing entails “giving students space to develop their own ideas and approaches *and* leveraging them to advance mathematics learning” (p. 102). van Es et al. (2022) similarly describe how “positioning students as capable and competent through discourse” (p. 125) is an important aspect of noticing for equity. As such, a teachers' shaping amidst their noticing of a students' writing through dialog that recognizes, respects, and builds on that student's understandings might also act as a way to disrupt culturally dominant, deficit forms of noticing.

The purpose of this study was to investigate teachers' orientations toward and interactions with students' writing in high school mathematics. Using an embedded case study approach (Yin, 2018), I examined three high school mathematics teachers' orientations toward writing in mathematics (Chapter 4), the ways in which they interacted with students' writing in class across different types of writing tasks

(Chapter 5), and the ways in which they interpreted classroom artifacts relevant to those interactions (Chapter 6). This analysis revealed that:

1. These teachers generally agreed with Casa et al. (2016) that writing serves as a way to support students' reasoning and communication in mathematics. However, other goals around supporting student ownership over their learning and student self-efficacy were also cited by these teachers as primary drivers behind how they approach (or would like to approach) writing in their own classrooms (See Chapter 4).
2. Informative, explanatory, and potentially exploratory writing tasks may have afforded these teachers more opportunities to shape their noticing of students' writing. However, these teachers appeared to also be able to, at times, shape their noticing of students' writing through discourse with students even with closed-ended, procedural/symbolic writing tasks (See Chapter 5).
3. These teachers could, at times, engage students in spoken discourse regarding their writing in ways that did not appear to be shaping the teachers' noticing of students' thinking, but rather seemed to focus on funneling (Herbel-Eisenmann & Breyfogle, 2005) students towards a preconceived solution or piece of feedback for which the teacher wanted to impart (See Chapter 5).
4. These teachers expressed an overarching desire for greater student involvement and ownership during their interactions with those students' writing, indicating the potential of teacher attunement to the role of shaping in teacher noticing activities and the opportunity of such activities to

strengthen teachers' shaping practices amidst their noticing. The use of both written samples of students' work and video recordings of classroom interactions appeared especially helpful in these activities for fostering the teachers' attention toward the shaping of their understanding of students' writing (See Chapter 6).

In these ways, this dissertation study expands upon previous findings regarding teacher noticing (e.g., Jacobs et al., 2010, Stockero et al., 2017, and van Es & Sherin, 2002, 2021) and teacher questioning (e.g., Boston et al., 2017, Dyer & Sherin, 2016, and Herbel-Eisenmann & Breyfogle, 2005) and helps connect these ideas to research focused on the types of and challenges surrounding student writing and writing instruction in mathematics (e.g., Casa et al., 2016, Powell et al., 2021, and Teuscher et al., 2015). This research and these relationships will be further explored in the literature review (Chapter 2) and discussion (Chapter 7) of this study. My hope is that the contributions offered by this study exemplify the importance of student voice in both the written and spoken discourse of the mathematics classroom, indicate the nuanced and rich ways that high school mathematics teachers think about and incorporate writing into their instruction, and suggest ways that secondary mathematics teachers can be further supported in enacting a range of writing opportunities – and student-directed teacher interactions with students' writing – in their classrooms.

Chapter 2

LITERATURE REVIEW

The ways that students use writing to grapple with and express their understanding of mathematical ideas, and the ways that teachers notice students' understandings as expressed through their writing, is an important and broad area of research in mathematics education. Casa et al. (2016) describe two primary goals of writing in mathematics: “for students *to reason* mathematically and *to communicate ideas*” (p. 3). Powell et al. (2017) similarly describe writing as serving goals of communication and reasoning, while also noting how students' low self-efficacy towards writing can present a compounding challenge impacting students' performance in school mathematics. Given the critical role of writing as a way for students to reason with and communicate mathematical ideas, it is perhaps unsurprising that fostering students' effective communication of mathematical ideas through both writing and spoken discourse is a key goal of educational reform movements and professional organizations in the United States (Common Core State Standards Initiative, 2010; National Council for Teachers of Mathematics, 2000).

In this review of the literature, I describe different ways that writing manifests in school mathematics, and how writing relates to spoken discourse in mathematics. I then illustrate some of the ways that writing in mathematics has been approached historically in the literature, and ways that we might reimagine writing as extending across different sign systems that arise in mathematics. I also address how writing has the potential to support student ownership over their learning and more equitable

student engagement with mathematics. Finally, I explain how teacher noticing is a compelling construct through which teaching that incorporates students' writing in mathematics can be investigated. Together, this review of literature indicates methodological considerations and potential contributions of my present study.

2.1 Types of Writing in School Mathematics

As broader goals of communication and reasoning might be narrowed into a focus on writing in mathematics, so too can writing be narrowed down into different types or forms of writing that arise in mathematics classrooms. Casa et al. (2016) describe different types of writing specifically in the context of elementary mathematics writing, namely (1) exploratory (making sense of a situation or problem), (2) informative/explanatory (describing or explaining something), (3) argumentative (constructing or critiquing an argument), and (4) mathematically creative (expressing or elaborating on original ideas or showing flexible thinking). Together, these different types of writing serve different sub-purposes (shown in parentheses) that in turn align to the overarching goals of reasoning and communicating mathematically. Given the increasing specificity of content-specific literacy as students progress from elementary towards secondary grade levels (Shanahan & Shanahan, 2008), these types of writing may arise somewhat differently in later grades. Indeed, Powell et al. (2021) found that the focus of writing – as understood through the responses to a large survey of mathematics teachers in the United States – was somewhat different in elementary versus secondary mathematics classrooms, as teachers reported to focus less on explanatory writing but more frequently used argumentative writing in the form of writing about mathematics proofs in secondary settings. Besides these limited distinctions, however, Powell et al. did not find further significant differences between

grade levels. Thus, although the four types of writing recommended by Casa et al. (2016) were focused on elementary grades mathematics instructions, it appears as though they could serve as a helpful initial framework through which to categorize different types of writing in mathematics, even at the secondary level.

2.2 Writing and Dialogically Focused Instruction

As noted in the introduction (Chapter 1), students must navigate multiple demands of language in the mathematics classroom across different modes of language (e.g., reading, writing, speaking, and listening; Aguirre & Bunch, 2012). As such, instruction that is *dialogically focused* affords a potentially useful setting through which to investigate students' writing in mathematics and the ways that teachers interact with students' writing. Munter et al. (2015) describe dialogic instruction as that which gives students “opportunities to (a) wrestle with big ideas, without teachers interfering prematurely, (b) put forth claims and justify them as well as listening to and critiquing claims of others, and (c) engage in carefully designed, deliberate practice” (p. 8). In line with such goals, dialogic instruction tends to include the use of (1) high-level, open-ended tasks, (2) opportunities for sharing multiple representations or strategies (e.g., graphs, tables, etc.), and (3) student discourse (Henningsen & Stein, 1997; Munter et al., 2015).

The emphasis on different strategies and representations, and the focus on building connections between different strategies and representations through spoken discourse, make dialogically focused classrooms primed for research on writing in mathematics – especially with regard to the multisemiotic nature of writing. Ultimately, mathematical discourse operates both intra- and inter-semiotically – meaning that there are functions that occur *within* each semiotic system (the written

word, mathematical symbolism, and visual imagery). Because spoken discourse involves the use of language – e.g., mathematical talk (Munter et al., 2015) – dialogically focused classrooms could offer ample opportunities to connect such spoken language inter-semiotically with aspects of the written word, mathematical symbolism, and visual imagery. Likewise, because dialogical instruction involves engaging students across multiple mathematical representations, students’ construction of such representations could offer opportunities for the teacher to interpret and respond to these as examples of mathematical writing. For such reasons, writing appears to play an important underlying role in dialogically focused classrooms, making them prime candidates for research on writing in school mathematics.

2.3 Teacher Orientations Toward Writing in Mathematics

Although teachers whose instruction is dialogically focused can ultimately serve as valuable participants in research on writing in school mathematics for the aforementioned reasons, it is also worth directly considering how teachers explicitly orient themselves towards writing itself. Similar to Remillard and Bryans’ (2004) construct of teacher orientations toward mathematics curriculum materials, I consider teachers’ orientations toward writing in mathematics to arise at the intersection of teacher beliefs, perceptions, and practices. Beyond focusing on how teachers see writing as supporting student reasoning and communication in mathematics (e.g., Powell et al., 2021), I believe that past research focused on “writing-to-learn” (e.g., Connolly, 1989) in mathematics provides some helpful insights into the ways that teacher do (and might) orient themselves towards writing in mathematics. However, there are also limitations in both this approach to thinking about writing in

mathematics as well as research on mathematics writing worth considering in the present study, which I describe below.

2.3.1 The Writing-to-Learn Movement

The writing-to-learn movement – one of the early efforts to strengthen teachers’ orientations toward writing – rested on the idea that students’ conceptual understanding of mathematics can be developed by supporting their use of written mathematical language, rather than the traditional stance that conceptual knowledge was a prerequisite for such writing (Connolly, 1989; Morgan, 1998). For instance, Layzer (1989) described a “peculiar synergy” (p. 131) between writing and learning mathematics, where writing helps students to better grasp abstract mathematical ideas, while a growing conceptual understanding of such ideas in turn allows students to articulate the nature of such concepts more precisely.

Various studies in the vein of the writing-to-learn perspective have shown promising student outcomes. Lesnak (1989), for instance, incorporated writing-to-learn strategies (e.g., students describing their procedures and strategies for solving problems, writing about practical applications of the content, etc.) into a remedial college algebra course, finding that students in the course had significantly higher class averages than a control course. However, Lesnak (1989) reported that the course also helped “raise their [the students’] levels of confidence and created more positive attitudes towards the learning of algebra” (p. 155). This positive affective student response to writing-to-learn interventions was similar to findings pertaining to journal writing interventions – a particular writing-to-learn strategy where students reflect on their mathematical learning in personal journals throughout a course (Borasi & Rose, 1989; Waywood, 1992). Borasi and Rose (1989), for example, demonstrated how

journal writing in a college algebra course allowed students to reflect on their emotional relationship and history with mathematics. Kostos and Shin (2010), on the other hand, indicated that math journaling in an elementary classroom helped students to better communicate their mathematical thinking. Thus, such writing-to-learn strategies have shown promise in supporting both communicative aspects of mathematics learning as well as affective issues related to student engagement in mathematics.

One criticism of earlier writing-to-learn studies was that, although they did indicate communicative and affective benefits, they failed to directly link writing interventions to improvements in mathematics learning (Morgan, 1998). However, subsequent investigations of various writing interventions did suggest such a link. For instance, Pugalee (2004) found that having ninth grade algebra students use writing to describe their problem-solving process resulted in more correct solutions than having students use think-aloud descriptions. Bicer et al. (2013) also found that middle school students developed greater problem-solving skills when placed in an intervention focused on writing mechanics and generating story problems. Such findings indicate that writing itself can support not only students' affective relationship towards mathematics, but also students' evolving abilities to communicate and reason mathematically.

Beyond such specific interventions, some investigations also indicate more broadly how mathematics teachers think about and report to engage with writing instructionally. Powell et al. (2021), for example, conducted a national (U.S.) survey which found that many teachers do engage in some form of writing with students in their mathematics classrooms, and that respondents were more likely to report using

such writing practices when they also reported greater self-efficacy for teaching mathematics writing. This is in line with findings from Cantrell et al. (2008) that showed how teachers' efficacy towards supporting students' literacy in their content was related to their willingness to implement literacy strategies in their classroom. However, Powell et al. (2021) also showed that mathematics teachers primarily used writing as a way for their students to explain their work, while few reported using writing in creative or argumentative ways. This indicated some nuance to how math teachers chose to implement writing in their classrooms. Teuscher et al. (2015), on the other hand, found that just over half (55%) of secondary mathematics teachers in their sample were not even familiar with the idea of writing-to-learn, and those who were familiar with the term indicated that it was often too time-consuming to implement in their own classroom. These self-reported surveys, although limited, indicate that explicit teacher attention towards focusing on and improving student writing may be limited in scope and frequency in mathematics classrooms, and that teachers' self-efficacy towards writing instruction may impact their willingness to use writing in their classrooms.

2.3.2 Goals of Writing in Mathematics

As noted above, previous research on writing in mathematics has emphasized different educational goals that may be supported through writing. For instance, Borasi and Rose (1989) described how journaling activities served as a "therapeutic outlet" (p. 354) for students, giving them space to process their emotions and improve their dispositions towards mathematics. In Waywood's (1994) analysis of journaling activities, he also discussed how creating portfolios of students writing over time can serve as a tangible artifact of students' learning, and thus help situate students as

capable learners of mathematics. Jurdak and Abu Zein (1998) also found that journal writing was both “an enjoyable activity for students and a vehicle for learning and self-expression.” (p. 418). Through such studies, journal writing activities were shown to support a range of positive outcomes, from more positive student experiences with school mathematics, to greater engagement with instruction, to better opportunities for teachers to see students as thinkers and doers of mathematics. Although these outcomes were not explicitly tied to student achievement, they still represented desirable educational goals from the standpoint of student engagement and equity.

More recently, there have been calls for research on writing in mathematics to also more explicitly focus on goals around student achievement and skills around mathematics and writing. For instance, Powell et al. (2017) focused their recommendations on mathematics writing research on the importance of better measuring students’ writing and mathematics skills over time, creating more explicit writing interventions, and measuring students’ self-efficacy and motivation specifically for mathematics writing activities. Although such calls for more research connecting writing practices to measures of student learning are understandable, this does not mean that other goals around student engagement and equity should be cast aside. Casa et al. (2020) explained how future research on writing in mathematics could benefit from applying a similar approach as research on mathematical talk, which “encompasses a wide range of perspectives, methodologies, and units of analysis” (p. 4) and has thus been able to achieve greater impact on teacher practice. By centering writing research in the context of multiple educational goals relevant to both teachers and researchers – such as student achievement, student engagement, and equitable student participation – research on writing in mathematics could potentially

better situate itself relative to research on mathematical talk while also gaining greater prominence among practitioners.

2.3.3 Writing in Mathematics as Multisemiotic

One issue with much research focused on writing-to-learn or mathematical writing is that it privileges the role of the written word as opposed to other sign systems such as mathematical symbolism and visual imagery. Indeed, in much of the literature on writing-to-learn, writing refers specifically to the construction of the written word (typically referred to as “natural” language as it is used to represent spoken language). This is in part because writing-to-learn proponents sought to differentiate themselves from the traditional notion that mathematics instruction involves *only* the manipulation of mathematical symbols. However, in doing so, such proponents at times implied a clear distinction between mathematical activity and writing. Connolly (1989), for instance, described the traditional stance on writing in mathematics as follows:

Given that mathematics has its own symbol system, through which practitioners can think and express themselves, it is not immediately apparent why a second, “natural” written language is helpful to learning how, exactly, it might serve a class in learning mathematics (p. 9).

This perspective is supported by Layzer (1989), who referred to mathematics as “an unnatural language” (p. 125), further contrasting the idea of using the written word and doing mathematics.

These distinctions are at odds with the perspective that mathematics is a multisemiotic activity. As noted by O’Halloran (2000), “The analysis of the language of mathematics classrooms must necessarily be incomplete unless the contributions and interaction of the symbolism and visual display are taken into account” (p. 360).

Ultimately, mathematics can be understood as a combination of language, mathematical symbolism, and visual imagery which interact with one another to “form a single unified system of meaning-making” (Lemke, 2003, p. 215). Indeed, mathematical symbolism is thought to have evolved historically as mathematicians moved from rhetorical algebra (expressed in natural language) towards the written structure of symbolic algebra (O’Halloran, 2000, 2008). Likewise, when considering the nature of mathematical visual imagery (e.g., graphs, tables, etc.), Pimm (1987) notes how “graphs are symbolic artefacts, and pupils have to learn to read and interpret them as much as other mathematical symbols” (p. 96).

Thus, confining the idea of “writing” in mathematics to only that of the written word can appear to ignore the sociohistorical and sociolinguistic nature of mathematical activity. Writing in mathematics warrants investigation from a multisemiotic perspective precisely because mathematics can be viewed as a multisemiotic activity. I therefore support a multisemiotic approach to understanding writing in mathematics similar to O’Halloran (2008), where “writing” can consist of any combination of the written word, construction of visual imagery (e.g., a representation of a function drawn on a graph), and written mathematical symbolism. If writing is intended to serve the dual goals of student reasoning and communication (Casa et al., 2016; Powell et al., 2017), then this multisemiotic approach towards writing in mathematics can also capture and place value on a wider range of reasoning and communication expressed by students. As such, my perspective is that a multisemiotic approach towards understanding writing in mathematics to both better capture the nature of writing in the doing of mathematics and better respect the diverse

ways of navigating the language demands of a mathematics classroom (Aguirre & Bunch, 2012) for the learner.

2.4 Teachers' Noticing of Student Writing in Mathematics

As described in my introduction, teachers' noticing provides a helpful frame through which to investigate writing in mathematics. There are particular aspects of research in teacher noticing that are especially relevant to my current investigation, which focuses on teachers' orientations with writing, the ways that they interact with students' writing, and how they interpret these interactions. Namely, my present investigation is informed by findings regarding the malleability of teachers' noticing, the co-generation of teachers' noticing with student input, and the ways that such co-generation of teachers' noticing aligns with student-oriented approaches to questioning patterns (Herbel-Eisenmann & Breyfogle, 2005) and telling (J. P. Smith, 1996).

2.4.1 The Malleability of Teacher Noticing

A consistent finding with studies on teacher noticing is that noticing is malleable. In other words, the ways in which teachers notice children's mathematical thinking and what exactly they notice can be deepened through professional learning experiences (Jacobs et al., 2010; Sherin & van Es, 2009). In studying teacher practices in the mathematics classroom, teacher noticing itself is an important tool of research given that what teachers notice about a students' verbal or written work informs how they will respond to it (Jacobs et al., 2010). Such noticing also aligns with NCTM's (2014) Mathematics Teaching Practices, namely the idea that "Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend

learning” (p. 53). However, research importantly indicates the malleability of professional noticing: it is a skill which is informed by teachers’ beliefs, experiences, and backgrounds, but can also be developed through teaching experience and professional learning experiences (Jacobs et al., 2010; Stockero & Van Zoest, 2013; van Es & Sherin, 2002). Therefore, understanding the ways in which teachers are noticing students’ writing in mathematics could not only help better describe their orientation toward such writing, but could also serve as a way to strengthen or shift teachers’ orientations toward writing through professional learning.

2.4.2 The Dimensions of Teacher Noticing

Noticing is a complex practice that entails a number of dimensions through which the focus and depth of teachers’ noticing is determined. van Es and Sherin (2002) originally identified three such aspects of teacher noticing:

- (a) identifying what is important or noteworthy about a classroom situation;
- (b) making connections between the specifics of classroom interactions and the broader principles of teaching and learning they represent; and
- (c) using what one knows about the context to reason about classroom interactions. (p. 573).

van Es and Sherin (2021) subsequently summarized the first aspect or component as *attending* and the second two components as *interpreting*. The idea of *attending* is especially important in my present study, as teachers ultimately attend to what they consider to be “significant” (van Es & Sherin, 2002, p. 573). Therefore, in the context of writing, the broader orientation that the teacher has towards writing could ultimately inform (and be indicated by) what the teacher does and does not notice about a students’ written work.

Jacobs et al. (2010) also investigated teachers' noticing of students' mathematical understanding, adding an additional component to "Deciding how to respond on the basis of children's understandings" (p. 172). This final component was an important addition because it recognized the ways in which an internal process of teacher noticing might interact with subsequent teacher actions. Thus, the way that a teacher would respond to a student about their writing could indicate what that teacher noticed about that students' writing. This is valuable because such responses are directly observable through classroom observations and recordings – more so than other aspects of teacher noticing.

Teachers can also directly include the student in how they make sense of what they notice about their written work. van Es and Sherin (2021) describe how teachers can adopt a stance where "interpreting is not only about trying to make sense of a phenomena but also involves seeing observed phenomena as something worth trying to figure out" (p. 22). They describe this as a *stance of inquiry* and note how this in turn can set teachers up to engage in the *shaping* dimension of noticing (i.e., interacting with their environment to further their noticing). The extent to which teachers interpret students' writing as an object of mathematical reasoning or communication and the ways in which they interact with such writing while teaching could therefore speak to the depth of their professional noticing skills regarding students' written work. It could also indicate their stance about whether students' writing is something about which it is worthwhile to interact with students to further their noticing as teachers. This focus on shaping is again valuable because the ways in which teachers verbalize their interpretation of students' writing would be able to be captured in classroom observations or recordings.

2.4.3 Co-Generating Noticing and Student-Directed Interactions with Student Writing

Previous research suggests some ways that teachers' observable interactions with students' writing might be categorized to better indicate the teachers' orientation toward writing. First, if a teacher's observed interactions with students' writing is largely *evaluative* or *descriptive* (i.e., the teacher describing the mathematical meaning they are drawing out from the written product without any inquiry with the student), then, this could indicate that the written product is not seen as something that requires further investigation on the teachers' end and thus also does not require shaping. Louie et al. (2021), for instance, describe how attending to the accuracy or correctness of students' work can reinforce a form of deficit noticing that reinforces the idea of "mathematics learning as absorption of a universal, objective, and fixed body of knowledge" (p. 99). However, such an interaction could also be indicative of a type of judicious telling (J. P. Smith, 1996), as teachers would still be attending to what they found worthwhile about a students' written work and using that basis to interpret how to provide feedback in a way that might advance students towards the goal of the learning. In other words, a descriptive response that still positions the student as competent could be a way for the teacher "in part to reaffirm them [the student] as mathematical thinkers and in part to communicate [the teacher's] expectations" (van Es et al., 2022, p. 121). However, because the teacher is the sole speaker in these sorts of interactions – or at least is not dialoging with the student as a way to further shape their noticing – I consider these interactions to be *teacher-directed*.

Alternatively, a teacher may respond to a student's writing in a way that is more *student-directed*, or that actively integrates the students' ideas and voice into the interactions. Dyer and Sherin (2016) describe three teaching practices for responding

to student thinking that could indicate these sorts of interactions. These are (1) a substantive probe of a students' idea (i.e., asking the student to elaborate or explain their thinking), (2) an invitation for student comment (i.e., asking for students to restate or respond to other students' thinking), and (3) the teachers' uptake of the students' idea (i.e., asking whether their idea holds in other circumstances or asking them to apply their idea in other ways). Such practices are in line with both a stance of inquiry and the shaping dimension of teacher noticing because these teaching practices give space to or inquire about students' thinking.

The teaching practices described by Dyer and Sherin (2016) are also aligned with what Boston et al. (2017) describe as assessing and advancing questions. *Assessing questions*, which are open-ended questions that a teacher uses to assess a student's understanding of the mathematics behind their current work or approach (Boston et al., 2017), can thus be understood as an expression of the teacher shaping their noticing around student writing. This sort of question is in line with the substantive probe of a students' idea or the invitation for student comment as described by Dyer and Sherin (2016). *Advancing questions*, which involve a comment or wonderment meant to extend students' thinking towards the learning goal after which the teacher leaves the interaction (Boston et al., 2017), can similarly be understood as entailing the teachers' uptake of the students' idea as described by Dyer and Sherin (2016). Although this sort of question extends beyond the shaping dimension of noticing (because teachers leave the interactions afterwards), it is still student-directed in that it is "situating student thinking at the center of the intellectual work of the classroom" (Dyer & Sherin, 2016, p. 73). Therefore, teacher interactions with students' writing that includes both assessing and advancing questions might be

ideal in representing “the integrated nature of noticing and teaching” (van Es & Sherin, 2021, p. 24), where teachers are shaping their understanding of students’ written work (through assessing questions) and then using what they have noticed to situate students as the drivers of their subsequent thinking and writing (through advancing questions).

For these reasons, it is worth considering not only whether a teachers’ interaction with students’ writing makes use of *any* student-directed questioning, but also what the patterns of questioning during such an interaction may entail. Herbel-Eisenmann and Breyfogle (2005) refer to the aforementioned series of assessing and advancing question as a *focusing* pattern of questions. This can be contrasted with a *funneling* pattern of questions, which is when “the teacher asks a series of questions that guide the students through a procedure or to a desired end. In this situation, the teacher is engaged in cognitive activity and the student is merely answering the questions to arrive at an answer, often without seeing the connection among the questions.” (Herbel-Eisenmann & Breyfogle, 2005, p. 485). This latter pattern of questioning, as it does not represent a genuine stance of inquiry on the teachers’ part, would not rise to the definition of a student-directed interaction. While still involving some student participation in the dialog, the teacher questions in a funneling pattern of questions do not rise to the level of assessing or advancing questions (i.e., serve to help shape the teachers’ noticing or build from the students’ spoken responses to advance their thinking). Therefore, these sorts of interactions are still more evaluative or descriptive in nature but are within a liminal space that is approaching more student-directed types of interactions.

Such a focus on investigating teachers' interactions with students' writing may also draw out the ways in which teachers' orientations toward writing in mathematics values student autonomy and agency regarding the students' written work. Louie et al. (2021), for instance, described how the ways in which teachers notice students thinking could also indicate whether students are being positioned as primarily receivers of mathematics or as sense makers who deserve opportunities to develop their own mathematical understanding. Furthermore, Wager (2014) showed how teachers' responses about different students' participation (during noticing events in professional learning workshops) also indicated how those teachers positioned themselves relative to equitable mathematics pedagogy. Therefore, equity issues such as student status in the math classroom (Horn, 2012) and situating students as competent (van Es et al., 2022) could be brought to the fore by highlighting some of the ways in which student participation is sought by the teacher during noticing events. These studies demonstrate how more observable aspects of teachers' noticings about students' written work – especially as it relates to when and how students are given space to genuinely participate in the shaping of and response to that noticing – are valuable to consider further in math education research. The ways in which students are (or are not) positioned as active participants during interactions with students' writing can help inform us about the extent to which student autonomy and agency is sought by the teacher during such interactions.

2.4.4 The Medium of Artifacts in Teachers' Noticing Activities

As previously noted, research on teacher noticing has focused not only on in-the-moment noticing as it arises in the classroom, but also on teachers' noticing as a professional learning activity. For instance, van Es and Sherin (2002) described not

only different dimensions that affect teacher noticing in the classroom, but also described how video recordings can “help teachers to develop this ability to notice and interpret classroom interactions” (p. 575). Jacobs et al. (2010), although describing teacher noticing as “decision making that occurs on a daily basis in the classroom when a child offers a verbal- or written-strategy explanation” (p. 173), ultimately analyzed elementary mathematics teachers’ skills at noticing through the teachers’ written responses to video clips and samples of students’ written work. Dyer and Sherin (2016), on the other hand, used videotaped classroom observations and follow-up interviews with teachers about the videos to describe those teachers’ in-the-moment noticing and decision-making. Although van Es and Sherin (2021) focused their research about shaping on observed classroom interactions, they also saw evidence of teachers shaping their noticing during video viewing sessions, such as by asking to watch video segments multiple times to better understand students’ ideas. In these ways, noticing has been investigated both as an in-the-moment occurrence in classrooms and as professional learning activity.

In studies about teacher noticing within professional learning experiences, different mediums of artifacts – especially written samples of students’ work and video recordings of instruction – have been used as a means to capture and evaluate aspects of teacher noticing. Research has shown the benefit of video clubs, where teachers watch recorded video clips of instruction to strengthen their noticing of students’ thinking and their professional vision (Sherin & van Es, 2009; van Es & Sherin, 2002). The investigation by Jacobs et al. (2010) on teachers’ professional noticing of children’s mathematical thinking incorporated both video clips and samples of students’ written work as artifacts of practice for teachers to interpret.

Interestingly, this study pulled from the writing samples to describe their findings about how teachers attended to and decided how to respond to students' strategies (the first and third dimension of their noticing framework), and from the video clips to describe their findings about how teachers interpreted students' understandings (the second dimension of their noticing framework), but did not note the medium of the teachers' noticing (writing samples or video clips) as relevant to their findings. Goldsmith and Seago (2011), on the other hand, addressed differences that arose in teachers' noticing conversations based on the medium used, but ultimately concluded that "working with video artifacts reduced the burden of following students' reasoning more than when working with students' written work" (p. 182). These variety of approaches to investigating teacher noticing leave questions about how a combination of different instructional artifacts – both video recordings and samples of students' written work – might inform what teachers notice and how teachers notice with regards to students' thinking within and across these different mediums.

2.5 Summary of Literature Review

Writing in school mathematics serves dual goals of supporting student reasoning and student communication of mathematical ideas (Casa et al., 2016; Powell et al., 2017). Writing occurs alongside and intertwined with other modes of language such as spoken discourse during mathematics instruction, and it is important for teachers to support students in navigating these different language demands of the math classroom (Aguirre & Bunch, 2012). Because what teachers notice and respond to in the classroom is impacted by what they consider noteworthy (Jacobs et al., 2010; Louie et al., 2021; van Es & Sherin, 2002), understanding mathematics teachers' orientations toward writing and how they interact with students' writing can help

describe how these teachers are attending to the language demands of writing in the classroom. Indeed, research indicates that teachers' beliefs about writing – especially their self-efficacy in supporting writing instruction – impacts their willingness to attend to writing in the classroom (Cantrell et al., 2008; Powell et al., 2021). However, because teachers can deepen their professional noticing over time (Jacobs et al., 2010), developing ways to support teachers with focusing on their noticing of student writing could ultimately support teacher professional learning regarding these aspects of their noticing.

The extent to which teachers actively include the student amidst their (the teacher's) noticing is an especially compelling approach to categorizing and analyzing teacher noticing of students' writing. Teachers' use of shaping can suggest that they hold a stance of inquiry toward students' thinking as expressed in their writing (van Es & Sherin, 2021), indicating the value they place on students as sense-makers in their own learning. Such a stance would be in line with the sort of questioning patterns that promote student agency and autonomy in mathematics (Boston et al., 2017; Dyer & Sherin, 2016; Herbel-Eisenmann & Breyfogle, 2005). Issues of equity are also at play in terms of what teachers notice and how they notice (Louie et al., 2021; Wager, 2014), and so this student-directed approach to investigating teacher noticing of students' writing could attend to such equity issues.

2.6 Research Questions

My study adds to the current literature by adopting the aforementioned student-directed approach to teacher noticing of and responses to students' writing in mathematics, while also drawing connections between such noticings and research

about different ways in which teachers orient themselves towards writing in mathematics. My research questions are as follows:

1. Among a group of high school mathematics teachers whose instruction is dialogically focused, what are their described orientations toward writing in school mathematics?
2. How do these teachers interact with their students' writing instructionally during mathematics lessons in relation to the types of writing prompted by the task?
3. How do these teachers' interpretations of instructional artifacts (samples of students' writing and recorded teacher interactions with students' writing) align or diverge from their described orientation toward writing in mathematics?

These research questions allow me to illustrate and draw connections between how these teachers describe their orientation toward writing in their classrooms, how they are observed interacting with and responding to students' writing, and how they interpret instructional artifacts of those observations. This approach helps me to address the intersection of beliefs, perceptions, and practices that impact teacher orientations (Remillard & Bryans, 2004). By conducting teacher interviews both before and after my observations of the teachers' instruction, this approach also allows me to more directly draw on the teachers' perspectives and interpretations of their classroom practices, better honoring their voice in my interpretations of the data.

Chapter 3

METHODS

In this chapter, I explain my methods for this study. First, I describe the selection criteria for the three participating teachers in this study and provide contextual information about these teachers and the schools where they teach. Then, I describe my data collection, explaining the process for how I conducted teacher interviews and classroom observations for this study. After this, I explain my approach to analyzing the interview and classroom observation data to answer the research questions. I show how I (1) constructed teacher profiles from interview responses, focusing on the participants' described purposes of writing in math and perspectives regarding writing-to-learn and writing as multisemiotic, (2) coded teacher interactions with students' writing from observation data along dimensions of the type of interaction and the type of writing embedded in the task prompt, and (3) compared teacher interpretations of instructional artifacts (i.e., samples of students' written work and recordings of interactions with students' writing) with their described orientations and observed interactions. Together, my methods recognize teacher orientation as arising at the intersection of beliefs, perceptions, and practices (Remillard & Bryans, 2004). By investigating teachers' orientations and interactions with students' writing through multiple data sources, my methods allow me to better represent these different manifestations of orientation. In essence, I define these teachers' orientations toward writing in mathematics as encompassing their stated beliefs about writing in mathematics, their observed interactions with students' writing in the classroom, and

their described perceptions of these interactions. A summary of my analytic framework is shown in Figure 3.1.

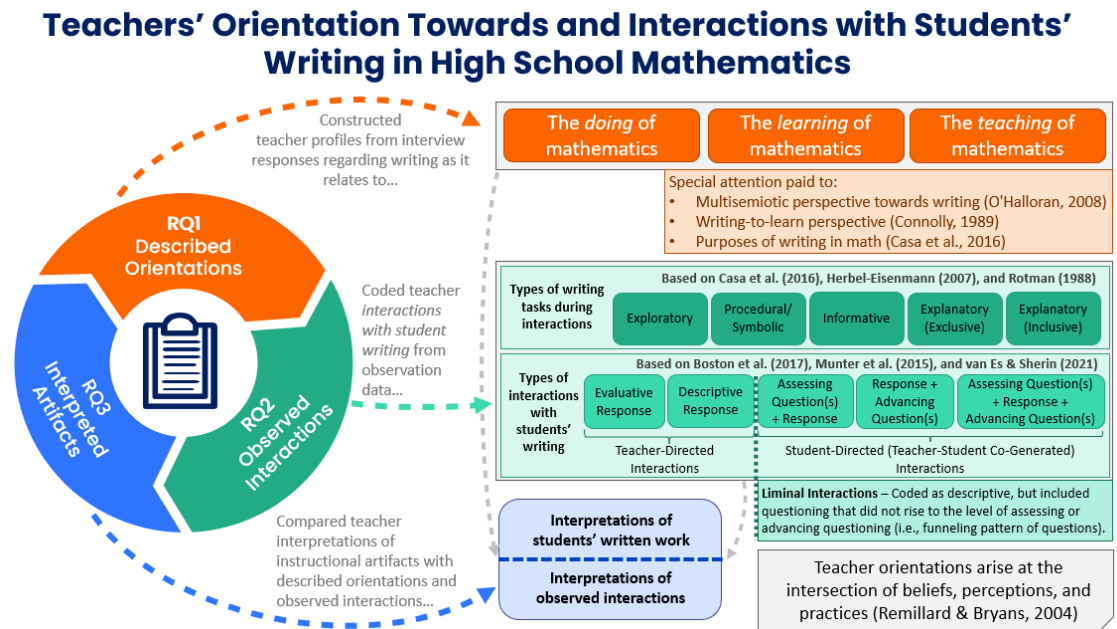


Figure 3.1: A visual summary of the study's analytic framework

3.1 Participants and Context

Because a main focus of my study centered on the ways in which teachers interacted with student writing through spoken discourse in the classroom, I recruited three high school mathematics teachers for this study whose instruction was previously found to be dialogically focused. This entailed instruction that included the use of (1) high-level, open-ended tasks, (2) opportunities for sharing multiple representations or strategies (e.g., graphs, tables, etc.), and (3) student discourse (Henningsen & Stein, 1997; Munter et al., 2015). I also considered the attributes as relevant to the present study given that successful mathematics writing interventions

often incorporated teacher and peer discourse and employed the use of higher-level tasks which would require explanations of strategies and solutions (Powell et al., 2017). Therefore, teachers whose instruction was anticipated to be dialogically focused in these ways were considered to be appropriate participants for the present study.

Additionally, all three of the components of dialogic instruction were previously included in the observation rubrics used in SMiLES (Secondary Mathematics in-the-moment Longitudinal Engagement Study; see Jansen et al., 2021), a study in which I had supported as a graduate research assistant. Using data from SMiLES, I had previously identified a subset of teachers in that study whose instruction was dialogically focused and had found that such teachers also tended to attend to aspects of students' written language instructionally (Smith, 2021). As such, I determined that such teachers – especially those who had participated in the SMiLES research – would be strong candidates for my dissertation study and focused my recruitment efforts on these individuals.

From these criteria, I was able to successfully recruit three high school mathematics teachers to participate in my study with the following pseudonyms: Mrs. Taylor, Mrs. Hudson, and Mrs. Barnett. Mrs. Taylor, whose Geometry classroom I observed, identified as white and reported having 30 years of teaching experience. Mrs. Hudson, whose Algebra II classroom I observed, identified as Black and reported having 16 years of teaching experience. Mrs. Taylor, whose Honors Pre-Calculus classroom I observed, identified as mixed race (Asian and Caucasian) and reported having nine years of teaching experience. All three participants also identified as female. I was keen to observe different high school mathematics courses in order to

observe a range of mathematics topics and how writing arose across a variety of contexts. However, these different course choices also arose out of close consultation with the participants, with each of them indicating that these particular courses would provide an accurate representation of the ways in which they typically “engage students with their written work.”

The variety of courses also allowed me to recruit from teachers at a single school site. All three of the participants were teaching at a high school in a Mid-Atlantic state which served a student population that is approximately 54% white, 27% black, 7.5% Asian, 7.5% Hispanic, and 4% two or more races. For each participant, I asked that I be able to observe a class that they felt was generally representative of this student population at their school and which they also felt was indicative of their typical instructional approach.

3.2 Data Collection

There are three primary sources of data that informed this study. First, I conducted two interviews with each teacher participant – one pre-observation interview and one post-observation interview. Second, over the Spring 2022 timeframe when observations were conducted, I asked participants to complete weekly surveys that described their upcoming lessons and provided their choices of preferred lessons for observation. Finally, I conducted four classroom observations with teachers, once per week, during that Spring 2022 semester. This section describes my data collection process and the sources from which I designed my measurements for the analysis of these data. A summary of these sources, and their alignment to each research question, is shown in Table 3.1.

Table 3.1: Data Collection Sources for Research Questions

Research Question	Data Sources
Among a group of high school mathematics teachers whose instruction is dialogically focused, what are their described orientations toward writing in school mathematics?	Pre-Observation Interview (Questions 3-13, see Appendix A) adapted from Cantrell et al. (2008) and Powell et al. (2021). Post-Observation Interview participant member-check (Candela, 2019; Creswell & Clark, 2017) of draft profiles.
How do these teachers interact with their students' writing instructionally in relation to the types of writing prompted by the task?	Recordings of classroom observations Four observed ~90-minute lessons from Spring 2022 for each teacher. Approximately one observation per week for each teacher. Weekly observation dates determined collaboratively with teacher participants.
How do these teachers' interpretations of instructional artifacts (samples of students' writing and recorded teacher interactions with students' writing) align or diverge from their described orientation toward writing in mathematics?	Post-Observation Interview (Questions 4-10, see Appendix B): Teachers engaged in noticing activity (Goldsmith & Seago, 2011; Jacobs et al., 2010; Sherin et al., 2009) involving students' written work and recorded noticing events from observations. Clip selection and questions informed by Sherin et al. (2009).

3.2.1 Teacher Interviews

I conducted two interviews with each participant. The first interview was conducted prior to the observations and was designed to understand teachers' self-reported orientations toward writing in mathematics and writing instruction, and to

construct profiles which describe such views (in line with my first research question). The second interview served two purposes. First, I used it to allow for member-checking (Candela, 2019; Creswell & Clark, 2017) by the teacher participants to ensure that my analysis of their reported orientations toward writing in mathematics were accurate. Second, I asked teachers to engage in noticing activities of both artifacts of students' writing and video recordings of those teachers' interactions with students about their writing, which was in line with my third research question.

3.2.1.1 Pre-Observation Interview

The first interview drew primarily from two previous investigations concerning teachers' orientations toward literacy and writing. The first of these was a survey conducted by Powell et al. (2021) which captured teachers' reported frequency of writing in mathematics, the ways that they use writing, and their self-efficacy in supporting writing in mathematics. However, this survey had primarily closed questions and centered on a definition which restricts mathematics writing to students' use of written words but *not* writing numerals, equations, or other semiotic systems of mathematics (Powell et al., 2017). Because of this, I adapted my items drawn from this source (see Appendix A) to be more open-ended for the interview format and to exclude references to any particular type of writing. For instance, while the Powell et al. (2021) survey asked participants to select the frequency (e.g., monthly, weekly, etc.) that they use specific writing activities such as "Write explanations of math procedures" (p. 430), my aligned interview item was adapted to ask "How frequently do your lessons require students to write? What do those lessons typically look like? Why is that?" This allowed for a semi-structured interview where the teachers' conceptions of writing could drive their responses, rather than setting pre-defined

parameters about what specific strategies or mathematical texts count as writing. Additionally, these “why” questions allow teachers “to consider information that has been taken for granted” (Zazkis & Hazzan, 1998, p. 432), which – in the context my study – implied how the teacher’s views on writing related to their perceived instructional roles as a mathematics teacher.

The second source of adapted interview questions for this pre-observation interview came from Cantrell et al.’s (2008) investigation of content area teachers’ perceptions of literacy teaching and learning. Because my study focused specifically on writing in mathematics, references to “literacy” (or “literate”) were replaced with “writing” or “writers” and references to teachers’ “subject area” were replaced with “mathematics” or “your classroom.” For instance, “How do you help your students become more literate in (state relevant subject area)?” (Cantrell et al., 2008, p. 94) was adapted to “Specifically, how do you help your students become better writers in your classroom?” Such changes were intended to keep the interview focused on the specific content of the present study.

Finally, there were several items which I created which were intended to gauge participants’ receptiveness to a multisemiotic perspective of writing in mathematics and a writing-to-learn approach to mathematics instruction. For the multisemiotic perspective, participants were asked:

One way of thinking about “writing in mathematics” is to consider not only the written word, but also how students are writing out to solve equations, drawing on graphs, and so on. How is this broader definition similar or different from how you think of students’ writing in mathematics? How, if at all, does that broader definition of writing in mathematics make you think differently about students’ writing in your own classroom?

This item was designed to gather participants' thoughts about the different sign systems of written mathematical discourse, namely language, visual imagery, and mathematical symbolism (O'Halloran, 2008). For the writing-to-learn approach, participants were asked:

Some teachers think that students should mostly be writing towards the end of a learning sequence, while others think that these opportunities should happen mostly at the beginning of a learning sequence, or even throughout the entire sequence. Where in the learning sequence do you think students should be writing, and why?

As described by Connolly (1989), a traditional perspective about writing vis-à-vis learning mathematics would be that the fluent usage of writing in the field of mathematics first requires a conceptual understanding of mathematical content. Writing-to-learn, on the other hand, posits that explicitly building skills around writing can in turn help develop a learner's conceptual understanding of mathematics. By asking about the place of writing in a learning sequence – and the participants' reasoning behind their response – I hoped to draw out whether the teacher was receptive to writing being integrated into the learning process and whether they felt that it could serve as a tool to build students' conceptual understanding (rather than just assess such understanding in a summative manner). Both this question and the question focused on multisemiotic writing were also posed after teachers were asked to provide their thoughts on what “writing in mathematics” meant to them and what their instructional approach vis-à-vis writing entailed. In this way, the questions were meant to provide direct insight into participants thoughts on these aspects of the present study without muddying their initial responses with regard to such aspects.

Teacher responses to these interview questions were used to construct profiles for each teacher which broadly described their orientation toward writing in

mathematics. In particular, these profiles captured – based on teacher responses – the way that each participant understood writing as it related to the doing of mathematics, the learning of mathematics, and the teaching of mathematics. These different aspects of writing in mathematics were drawn out in order to better capture elements of both the teachers’ beliefs about writing in mathematics and their self-reported instructional practices that supported students writing in mathematics. Given that teacher orientations arise at an intersection of beliefs, perceptions, and practices (Remillard & Bryans, 2004), I felt that these different aspects could thus better capture the complexity of teachers’ orientation toward writing in mathematics compared to questions that only asked teachers about their beliefs or their teaching practices.

For how writing relates to the *doing* of mathematics, teacher responses were intended to inform the extent to which the teacher described writing as addressing semiotic systems of natural language, mathematical symbolism, and/or visual imagery, similar to the multisemiotic perspective described by O’Halloran (2000, 2008). Given that traditional notions of writing in mathematics may not account for this array of different semiotic systems (Draper & Siebert, 2010), I anticipated that teachers might focus on particular types of writing, such as explanatory writing (i.e., just focusing on the written word). To this end, the previously described item that I designed to directly ask teachers about their thoughts on a multisemiotic perspective on writing was included as a way to gauge the participants’ receptiveness to this perspective, regardless of their initial interview responses.

Teacher responses regarding the *learning* and *teaching* of mathematics as it relates to writing were intended to capture the extent to which the teacher described a writing-to-learn instructional approach and the ways in which they described the

overall purpose of writing in school mathematics. By asking participants to provide examples of how their lessons require students to write and how they help students become better writers, I hoped to shed light on how they saw writing as supporting their instructional goals with students. Specifically, I was curious about the extent to which their responses reflected the recommendations of Casa et al. (2016) that mathematics writing should serve the joint goals of supporting student communication and mathematical reasoning. Given that the recommendations of Casa et al. (2016) pertained to the elementary grade levels, these items were also intended to show how my participants aligned with these goals, which (if either) goal they emphasized in their descriptions, and whether they indicated any additional goals that were supported by writing in their classrooms.

3.2.1.2 Post-Observation Interview

The second interview with teacher participants (see Appendix B) took place approximately two weeks after the conclusion of the conclusion of teacher observations in Spring 2022. The day before the interviews, teachers were emailed draft copies of the teacher profiles created from the first interview, as well as soft copies of student writing that was to be used during the interview, and were encouraged to review these documents prior to the interview. While teachers were also given an opportunity to review these documents during the interview, this early sharing was meant to provide them with additional time to process these documents.

The first component of this post-observation interview entailed participants member-checking (Candela, 2019; Creswell & Clark, 2017) their profiles that I had drafted. In particular, teachers had an opportunity to identify any perceived inaccuracies with these initial drafts, as well as any aspects of their views on writing in

mathematics that they felt were missing from the profile. These final teacher profiles were used to inform the results of this study. However, it is also worth noting that only Mrs. Barnett requested any adjustment to her initial profile, asking that I also include her belief that learners have “the right to say what makes sense” to them, and that writing is a way to communicate “what I’m thinking and what’s making sense to me” in a way that a learner can “make sense of that to other people.” Mrs. Barnett’s profile, with these additions bolded, is shown in Appendix C, Figure C.1.

Following the profile member-checking, teacher participants were asked to reflect on written student work collected from one of their observed lessons, followed by examination of a video excerpt from that same lesson. The use of both written and video artifact reflection here was intentional. Goldsmith and Seago (2011), who investigated teacher noticing of student thinking, describe how secondary mathematics teachers in their project who were first presented with video artifacts “focused immediately on analyzing students’ reasoning” (p. 181) as observed in the recordings. However, teachers tasked with analyzing written work had to first *interpret* the students’ presumed reasoning process themselves since this was not always immediately clear from the final written product. By beginning this section of the interview with the examination students’ written work, I therefore intended it as an opportunity for teachers to engage in professional noticing (Jacobs et al., 2010) before being potentially influenced by the specific recorded interactions with certain students’ writing. In this way, I hoped that teachers’ noticings could provide a broader and more accurate idea about what elements of student writing the participants considered to be most important – and why. This approach also allowed me to ask teachers to indicate the ways in which the students’ writing reflected the learning

goal(s) of the lesson (which had been shared by the teacher prior to the observation and were restated as the participant examined the written student work) in a way where particular students' writing (the student or students from the subsequent recorded interactions) did not overshadow such reflections.

Similar to Jacobs et al.'s (2010) investigation of mathematics teachers' professional noticing, I selected samples of student work for this activity which appeared to reflect a range of student understanding in relation to the lesson's learning goal(s). As such, I preferred tasks from which I was able to collect and present (via the interview documents) a range of student work which was concise enough for teachers to reasonably interpret in the timeframe of the interview itself (e.g., I avoided using class-wide samples of completed, fully marked-up worksheets). Additionally, I preferred student writing samples from the most recent (fourth) classroom observation to minimize the length of time between when the lesson was enacted and when the interview took place. Indeed, the only participant for whom I drew on an earlier observation was Mrs. Hudson, for reasons related to the types of interactions with student writing observed in her fourth observation (described below). Finally, along these lines, I selected these student samples to align with the video clips of teacher interactions with students writing that were used in the subsequent video viewing segment of the interview. In essence, my goal with these clips was to not only relate teachers' interpretations to their intended goals of the lesson, but also to relate such responses to the video viewing interpretations as well. Because the student written work and the recorded interactions overlapped in this way, the student work also served as a supplementary artifact to aid the teacher's interpretation of the recorded interaction.

The teachers then each participated in a video viewing session (Erickson, 2007) where they observe the recorded interaction with student writing, following a process employed previously by Mirzaei et al. (2021) for the SMiLES project. In order to identify clips that were conducive to such teacher reflection, I adapted the selection criteria proposed by Sherin et al. (2009), who chose clips (1) that show evidence of student thinking, (2) where that evidence is easily understandable, and (3) that focus on substantive mathematical ideas. Given that my study emphasized teachers' orientations toward writing and their interpretation of their own instructional moves, I adapted these criteria in several ways. First, I focused on interactions that showed evidence of teacher questioning – whether this was coded as assessing/advancing questions or as some other type of questioning (e.g., funneling patterns of questioning; Herbel-Eisenmann & Breyfogle, 2005) that did not meet all of the characteristics of assessing/advancing questions. Second, I focused on interactions where I was able to link the spoken language in the recording to some specific aspect of the student written work that the teachers were also interpreting with ease. In other words, I would prioritize examples of the teacher asking "...you said AB is parallel to DC and AD is parallel to BC. How do you know they're parallel?" (see Figure 5.2) over a teacher asking, for example, "what does this mean?" if the student's writing was not legible in the recording. This was done to minimize the amount of focus that the participants would need to devote to simply recollecting what exactly they were discussing with the student (assuming even that they would be able to do so). Finally, also I chose clips that focused on substantive mathematical ideas as reflected in lesson goals (as shared by teachers' pre-observation), and which occurred during whatever activity teachers indicated made the observed lesson an "ideal day to observe your interaction

with students' written mathematical work.” As noted in the selection process for the student writing samples, I also prioritized interactions from the most recent classroom observation, and which – when possible – arose during different tasks during the lesson. Together, these considerations when selecting video clips were intended to provide samples that were aligned with the preceding noticing activity with student writing samples, were likely to be readily interpreted by teachers without additional provided context (beyond the clip and student writing samples), and were varied in content yet aligned with the teachers’ overall considerations of writing-focused mathematics instruction.

Similar to the approach of Sherin et al. (2009), the interview questions that followed the video clip began with a more generalized prompt, “What stands out most to you from this interaction?” After this, the teachers were asked about their recollected intentions with this sort of interaction, and the extent to which this sort of affordance was typical in their instruction. As such, this section of the interview allowed for the teacher’s stimulated recall (Lyle, 2003) of the event, serving as “an indirect method of obtaining evidence of cognitive activity” (p. 872) – i.e., an imperfect but insightful look into the teacher’s recollection of their thinking during the interaction. Additionally, because these questions were designed to help better describe these sorts of instructional moments from the perspective of the teacher, this section also served as an additional member-check (Creswell & Clark, 2017) in validating the analysis of such affordances.

3.2.2 Classroom Observations

Four video observations of each teacher’s classroom instruction were conducted for this study during the Spring 2022 semester, with teachers being

observed on a weekly basis. In order to improve the likelihood of observing instruction that involved student writing (and teachers' interactions with that writing), I asked participants to complete a survey prior to each observation week where they recommended a lesson from that upcoming week (as well as a backup lesson date) when they preferred for me to observe. This was similar to protocols of the SMiLES project, when teachers were asked to choose themselves a potentially engaging instructional activity that they preferred the researchers would analyze out of the full, recorded lesson (Jansen et al., 2021; Smith, 2021). These surveys were also framed in a way to recognize the partnership of the participating teachers in this work and to emphasize my focus on their interactions with students' writing. The survey noted that:

I am looking forward to observing your class and video recording! A goal of my project is to observe ways that you attend to students' writing during your instruction. As a joint problem solver with this research, I am seeking your perspective in deciding on the best days to observe your instruction. For the upcoming week [dates provided], I ask that you please indicate the two best days that you believe would offer an opportunity to observe you working with student's written mathematical work. Additionally, please identify the learning goal and why this is a good time to observe your participating class period.

For the two preferred days, teachers were asked to share the overall lesson topic, the learning goal, why they felt that "this will be an ideal day to observe your interaction with students' written mathematical work," and what samples of students' written work they anticipated that I could collect during and after the lesson. As such, these surveys operated as a way to further understand participants' orientations toward writing in relation to their day-to-day teaching.

On observation days, I used one video camera, one boundary microphone preferably positioned in proximity to a group of assenting students, a clip-on

microphone attached to the teacher, and a backup sound recording device. Since I was focused on teachers' instructional moves for these observations, I worked to ensure that the teacher and any associated presentation or writing surfaces (e.g., a whiteboard or a projected computer screen) were clearly visible during the recording. Additionally, when the teacher appeared to be interacting with students' writing (e.g., providing feedback about something that a student had written), I attempted to best capture these exchanges, including zooming in on students' written work and/or taking pictures of this work once the interaction had concluded. In these ways, I hoped to better preserve multiple modes of data from which I could identify and analyze these interactions with students' writing.

3.3 Data Analyses

I employed an embedded multiple-case study approach (Yin, 2018) for this study. Each of the teachers represented a case, while a subunit of teachers' observed interactions with students' writing was embedded within each of these three cases. Similar to Thompson (1984), employing a case study approach of different teachers in the context of their classroom instruction and interview responses was intended to shed light on the ways that different teacher conceptions (in this study, their described orientations toward writing in mathematics) played out instructionally with students. By employing both teacher interviews and classroom observations, I hoped to construct a more nuanced description of each teacher and their classroom through which I could answer each of my research questions. In essence, the use of classroom observation data, teacher reflections on samples of student written work, and teacher reflections on recordings of their interactions with students' writing allowed me to (1) triangulate (Saldaña, 2013) across different sources of data, (2) pattern match (Yin,

2018) the predicted teacher interactions with and interpretations of students' writing (from pre-observation interviews) with what was observed and described by the participants (from observation data and post-observation interviews), and ultimately (3) better capture the complexity of teachers' orientations toward writing in mathematics and how teachers' stated views on writing in mathematics appear to influence their actions in the classroom.

Table 3.2: Analytical Process for Answering Research Questions

Research Question	Analytical Processes
Among a group of high school mathematics teachers whose instruction is dialogically focused, what are their described orientations toward writing in school mathematics?	Multistage coding scheme (Kauffman et al., 2002). First stage - created narrative profiles for how teachers broadly described the purposes of writing in mathematics. Second stage - coded based on teacher references to writing-to-learn and multisemiotic writing. Performed a member-check (Candela, 2019; Creswell & Clark, 2017) during post-observation interview.
How do these teachers interact with their students' writing instructionally in relation to the types of writing prompted by the task?	Coded classroom observation recordings by types of teacher interactions with student writing (adapted from Boston et al., 2017 and van Es & Sherin, 2021). Achieved 82% interrater agreement. Coded each interactions for the type of mathematical writing task (based on Casa et al., 2016; Herbel-Eisenmann, 2007; and Rotman, 1988).
How do these teachers' interpretations of instructional artifacts (samples of students' writing and recorded teacher interactions with students' writing) align	Created analytic memos (Saldaña, 2013) summarizing teachers' noticings of artifacts. Engaged in pattern matching (Yin, 2018)

or diverge from their described orientation toward writing in mathematics?

to draw out similarities or differences from described orientation profiles and teachers' noticings.

3.3.1 Teacher Interviews

Teacher responses to the pre- and post-observation interviews formed the basis for how I determined teachers' described orientations toward writing in mathematics. Similar to Kauffman et al. (2002), I analyzed these interviews through a multistage coding scheme. This began by creating a narrative profile of each participants' responses by "summarizing prominent topics, uncovering emergent themes, noting memorable responses, and describing overall tone" (Kauffman et al., 2002, p. 277). To draw out distinctions between the participants, I split these profiles into subsections addressing how the teacher viewed the purpose of writing in relation to the (1) *doing of mathematics*, (2) *learning of mathematics*, and (3) *teaching of mathematics*. For the *doing of mathematics* part of the profiles, I identified language that specifically focused on different sign systems that the teacher associated with writing in mathematics (e.g., "I could draw a picture or I could, again, write out an equation, things like that" from Mrs. Barnett when describing how she thinks about writing in mathematics). For the *learning of mathematics* part of the profiles, I identified language that focused on student actions related to the process of writing, or how writing related to student thinking and learning in mathematics (e.g., "I think the biggest obstacle for the writing is the kids knowing what they're thinking. I get, 'I know what I wanna say, but I don't know how to say it'...It's like, 'I don't know how to write it down.'" From Mrs. Taylor's description of students' writing skills in

mathematics). For the *teaching of mathematics* part of the profiles, I identified language that focused on the teachers' actions (or aspirations) for how to support student writing (e.g., “we can help students be able to communicate those ideas to each other, like through the structure of our activities or even just the way we conduct a discussion in our classroom” from Mrs. Barnett’s description of how mathematics teachers can help students acquire and practice their writing skills).

By starting with this first round of coding, I was able to broadly capture teachers' descriptions about how they defined “writing in mathematics” and what they saw as the purpose of writing in the teaching and learning of mathematics. A sample of one of these profiles is provided in Appendix C, Figure C.1. Although some of these described purposes were in line with established goals of mathematical writing as laid out by Casa et al. (2016) – i.e., reasoning with and communicating mathematical ideas – these profiles were also intended to capture other perceived goals (or challenges) that informed how the teachers approached writing in mathematics.

Again following Kauffman et al. (2002), I then engaged in another round of coding based upon “a thorough review of the literature, and preliminary data analysis” (p. 277). In particular, I highlighted teacher comments that referenced writing-to-learn and multisemiotic connections in writing. For the former, I looked for statements that each teacher made about when in the learning process writing is most appropriate for students, and any challenges that the teachers described for integrating writing into their math teaching practices. For the latter, I looked for statements that each teacher made about multiple sign systems being complimentary when writing in mathematics

(e.g., drawing a function on a graph and then describing that function using words, phrases, or sentences).

As previously described in the data collection procedures, I shared the drafts of the initial narrative profiles with each participant prior to their post-observation interview. During that interview I performed a member-check (Candela, 2019; Creswell & Clark, 2017), asking participants about any aspect of these profiles that they would amend, remove, or add in order to more accurately capture their orientations toward writing in mathematics. As noted by Saldaña (2013), such a process can serve as a validation tool during the coding of interview data. Two participants (Mrs. Taylor and Mrs. Barnett) requested no changes to their profile, while Mrs. Barnett requested that I add one additional sentence about the rights of the learner to communicate their thinking. Additionally, given that teachers' experiences can impact what they attend to in their noticing of students' work (Jacobs et al., 2010), I also asked teachers in this post-observation interview whether or not they "always thought about writing in mathematics this way" in order to elicit responses about any events or experiences that may have been perceived to have impacted these teachers' overall described orientations toward writing in mathematics. Together, the written summaries that resulted from this multistage coding scheme (Kauffman et al., 2002) were used as the basis for my findings regarding the first research question (Chapter 4).

The post-observation interview also informed my findings regarding my third research question (teachers' interpretations of instructional artifacts). In these activities, questions were designed to promote teacher reflections related to both students' actions and written products (e.g., "what do you notice about these students'

written work?” and “What stands out most to you from this interaction?”) as well as the teachers’ role in relation to these actions and artifacts (e.g., “What sort of feedback, if any, would you want to give each of these students for how to revise or improve upon the written work shown here if you saw this work during the lesson?” and “What was going through your mind during this clip, to the best of your recollection?”). Using these different questions as a baseline, I created analytic memos (Saldaña, 2013) to describe how each teachers’ responses indicated how the written products and recorded interactions (1) reflected students’ understanding and learning and (2) reflected the teachers’ intended and enacted instructional moves. By employing this process, I was able to essentially engage in a process of pattern matching (Yin, 2018), with the teachers’ described orientation profiles informing the predicted outcomes of the teachers’ noticings about students’ writing in mathematics, and their described interpretations of these artifacts and recordings informing (in part) the ways that these teachers interpret such student writing in practice. The observation data analysis then served to further bolster my interpretation of such patterns.

3.3.2 Classroom Observations

In addition to informing the post-observation interviews, classroom observation data were used to answer the second research question concerning how these teachers interact with their students’ writing instructionally in relation to the writing expectations of the task. I analyzed the recorded observation data both in terms of interactions that arose with the teacher and students’ writing and in terms of the types of writing embedded in the task during such interactions. As such, my unit of analysis in these classroom observation data would be considered the instructional

moments – as represented by the recorded video, transcript, and collected student written work – that encompass such teacher interactions with students’ writing.

These *interactions with student writing* were thus defined as the spoken feedback – via statements or questioning – stated by a teacher to a student or students concerning a written text (written language, mathematical symbolism, and/or visual imagery) created by students. Munter et al. (2015) describe how teacher feedback during more direct instruction often focuses on evaluative assessments of the student’s work and descriptions of suggested changes (e.g., based on mistakes) for the student, while feedback during dialogic instruction focuses not only on such evaluative and descriptive focus, but also on “advancing students’ growing intellectual authority about how to judge the correctness of one’s own and others’ reasoning” (p. 10). Because the participants in my study were selected because of evidence concerning their dialogically focused instruction, I anticipated that this broader and co-generated (between the teacher and student) type of feedback might arise in the data, but also prepared for the possibility that more evaluative or descriptive feedback might at times also arise.

Based on these stipulations, I identified these interactions with student writing based on the teachers’ spoken language. An interaction with student writing was defined as originating when a teacher utters an evaluative or descriptive statement about a student’s written text or poses a question to a student concerning a text that the student has created. These interactions were defined as concluding when the teacher’s spoken exchange with the student concluded, as evidenced by (for example) the teacher moving onto providing feedback for another students’ writing or leaving the area where the interaction took place.

The types of teacher interactions with students' writing, shown in Table 3.3, were based on research concerning teachers' responses to student thinking in mathematics and recommendations for instructional approaches to questioning. These codes also evolved via an emergent process (Saldaña, 2011, 2013) whereby I revised and refined the types of interactions based upon ongoing analysis of the observation data. I opted for this analytical focus with the data given that the teacher participants had been associated with dialogic instruction. The categories created to analyze these data also align with the distinctions between teacher feedback described by Munter et al. (2015) as previously noted.

Table 3.3: Types of Teacher Interactions with Students' Writing in Mathematics

Type of interaction	Description
Teacher-directed interactions	
Evaluative Response	The teacher addresses the perceived correctness of a student's written text without then explaining how they see the writing as communicating mathematical meaning.
Descriptive Response	The teacher addresses the perceived correctness of a student's written text and/or states how they see the writing as communicating mathematical meaning.
Student-directed (or student-teacher cogenerated) interactions	
Assessing Question(s) + Response	The teacher inquires the student about the mathematical meaning of their written text and then states how they see the writing as communicating mathematical meaning.
Response	The teacher states how they see the student's

+ Advancing Question(s)	written text as communicating mathematical meaning and then ends the interaction with a question or suggested next step to advance or revise the student's writing.
Assessing Question(s) + Response + Advancing Question(s)	The teacher inquires the student about the mathematical meaning of their written text, states how they see the writing as communicating mathematical meaning, and then ends the interaction with a question or suggested next step to advance or revise the student's writing.

Firstly, an interaction could entail a teacher-directed response to a students' writing. This could involve a purely *evaluative* response (e.g., "Good job, that is correct" or "No, the solution is 12"), or a response that evaluates the student writing but also *describes* how the teacher sees (or do not see) their writing as communicating mathematical meaning (e.g., "I appreciate that you wrote out each step as you applied different operations to solve this equation, as that helps me understand how you arrived at your solution."). Importantly, while such interactions are coded as "teacher-directed" given that the teacher dominates the spoken language of the interaction itself, the student's voice – via the teacher interpreting the mathematical meaning in what the student has written – is not absent from descriptive interactions. This category could rather be seen as a type of judicious telling (J. P. Smith, 1996) that uses students' own written or spoken ideas in order to focus those students onto specific mathematical ideas relevant to the learning goals. Given that I identified a number of such interactions in a prior investigation of SMiLES project data (E. P. Smith, 2021; E. P. Smith et al., 2023), I anticipated that such interactions would arise in the present investigation.

As noted by Munter et al. (2015), teachers and students could also be "co-participants" (p. 10) in these interactions around students' writing. As previously

noted, teachers could more actively involve students in these interactions through a process that van Es and Sherin (2021) call *shaping*, which “involves teachers constructing interactions, in the midst of noticing, to gain access to additional information that further supports their noticing” (p. 23). This is in turn aligned with what Boston et al. (2017) describe as *assessing* and *advancing* questions, with the former entailing the teacher assessing students’ understanding of the mathematics behind their written work (e.g., “Where did this ‘ $\times 4$ ’ come from in your equation here?”) and the latter entailing a noticing or wonderment meant to extend students’ thinking (and writing) towards the learning goal (e.g., “Why did you draw the slope of this function as steeper than this other function? What about the function equation informs you how steep the slope might be when you graph it?”). With advancing questions – which I expanded in this study to include suggestive statements that serve a similar point as questions (e.g., “See what happens to the function rule when you change the slope.”) – the teacher is meant to then leave the student(s) to continue their thinking independently.

In line with these definitions, I only coded certain questions from the teacher as assessing or advancing questions. Because, like van Es and Sherin (2021), I see assessing questions as in line with the noticing dimension of shaping, there needed to also be evidence that the teacher was gaining additional information about their noticing from the student (e.g., the student answering the question or providing a gestured response or statement indicating their uncertainty about providing an answer) for questions to be coded as assessing questions. Alternatively, for questions to be coded as advancing questions, this required that the teacher ended the interaction after asking the question. I also did not code instances where the teachers posed a series of

closed questions to students (e.g., “What is it called when two lines have the same slope?”) as assessing questions, as this is in line with the idea of funneling pattern of questions (Herbel-Eisenmann & Breyfogle, 2005). Because funneling patterns of questions are intended to guide students to a predetermined end – rather than shedding further insight into the students’ understandings – I did not consider such patterns of questions to rise to the level of assessing questions. Together, these coding considerations demonstrate how only questions that met a particular set of requirements were described as assessing or advancing questions.

In addition to categorizing the types of interactions with student writing, I also focused the second research question on understanding the type of writing embedded in the task itself. These categories of writing task, shown in Table 3.4 were based on the work of Casa et al. (2016), and categorized the writing as either exploratory (to make personal sense of a situation or problem), procedural/symbolic (to calculate a numeric or algebraic solution by using procedures with mathematical symbolism), informative (to describe something), exclusive explanatory (to explain something, students executing “already-known” ideas), or inclusive explanatory (to explain something, students as active participants in forming ideas).

Table 3.4: Categories of Writing Tasks in Mathematics

Type of writing task	Focus of writing and example
Exploratory	The written task prompts students to “personally make sense of a problem, situation, or one’s own ideas” (Casa et al., 2016, p. 6). Students may be asked to broadly describe any patterns, relationships, or solutions that they encounter, or to consider what such rough draft thinking might mean. There is not an expectation for precision or

	<p>accuracy in the prompt (e.g., asking “What do you notice about...” rather than “What is true about...”).</p> <p>Example: Look at the focus and directrix of each parabola. How does the distance between the focus and the directrix affect the shape of the parabola?</p>
Procedural/ Symbolic	<p>The written task prompts students to find a particular numeric or algebraic solution, which can be accomplished solely by performing procedures, arithmetic, etc.</p> <p>Example: Solve for x: $\log_6 x = 2$</p>
Informative	<p>The written task prompts students to describe features or aspects of some given or found mathematical idea, such as “attributes, observations, definitions, and representations” (Casa et al., 2016, p. 9). There is some expectation for precision or accuracy in the prompt. This includes asking students to describe procedures or “steps” to solving a problem <i>without</i> asking them to explain or justify their thinking.</p> <p>Example: How do we find the horizontal asymptotes of a rational function?”</p>
Explanatory (Exclusive)	<p>The written task prompts students to describe “mathematical concepts, strategies used to solve problems, reasoning about processes and procedures, and mathematical connections and comparisons” (Casa et al., 2016, p. 9). This entails more than being asked to simply list procedures or steps. The writer is framed as executing a defined set of statements to arrive at an already-known conclusion.</p> <p>Example: Prove that $x = 7$ is a valid solution to the given equation.</p>
Explanatory (Inclusive)	<p>The written task prompts students to describe “mathematical concepts, strategies used to solve problems, reasoning about processes and procedures, and mathematical connections and comparisons” (Casa et al., 2016, p. 9). This entails more than being asked to simply list procedures or steps. The writer is framed as an active participant in the living activity of “doing” mathematics.</p> <p>Example: How might we find a solution to the following equation?</p>

Note. These categories are based on Casa et al. (2016), Herbel-Eisenmann (2007), and Rotman (1988).

The differentiation among types of explanatory writing tasks as either “exclusive” or “inclusive” was derived from the curriculum-focused work of Rotman

(1988) and Herbel-Eisenmann (2007), where differences were noted in whether mathematical task prompts framed the reader (e.g., the student completing the task) as merely executing a defined set of statements to arrive at an already-known conclusion (exclusive, e.g., “Prove that $x = 7$ is a valid solution to the given equation.”) or as an active participant in the living activity of “doing” mathematics (inclusive, e.g., “How might we find a solution to the following equation?”). This involved the consideration of imperatives versus hedging verbs and the inclusion of personal pronouns in the writing prompts (“Prove...” versus “How might you prove...”), with the latter components indicating a more inclusive stance towards the explanatory prompt. The distinction between these explanatory types of writing were pursued under the conjecture that the framing of the prompt could foster different types of teacher interactions with the writing that students might produce during their engagement with such tasks.

To analyze the observation data instances of interactions with student writing, I reviewed all observations, transcribing each example of such interactions as they arose and collecting associated samples of student writing for each interaction (e.g., screen captures of the student writing if displayed on the board, pictures of student writing taken after the conclusion of the interaction, collected samples of students’ submitted writing, etc.). A sample of my codebook is shown in Appendix C, Figure C.2 and Figure C.3. For each interaction, I listed exactly when in the lesson it occurred, the length of the interaction, the setting in which it took place (with an individual student, in a small group setting, or in a whole group setting), and whether it was initiated by the teacher or the student (e.g., the student called the teacher over to look at or answer a question about their written work). I also bolded parts of the interaction transcript

that I identified as being descriptive responses and bolded and italicized assessing and advancing questions (see the last column in Appendix C, Figure C.2). In addition to the transcription and associated visual evidence, I also wrote a brief justification for each coding decision (see the first column in Appendix C, Figure C.3).

Because the categories of interactions were created for the purpose of this study – as compared to the categories of writing tasks being largely adapted from Casa et al. (2016) – I called upon a fellow graduate student with experience coding observation data from the SMiLES project to check the reliability of this coding scheme. After training this colleague using my preliminary coding results for one observation, she independently completed this coding process for three additional observation results (one for each teacher participant), identifying the type of interaction from the collected transcripts and visual evidence. Comparing her codes to my own for those same observations showed an 82% interrater agreement, indicating sufficient reliability of this coding scheme.

With this interrater reliability confirmed, I continued with the aforementioned coding process for the remaining observations. This resulted in 189 observed interactions with students' writing, each coded with an associated type of writing task, transcription of the interaction, justification of the coding, and visual evidence. These collected results were then primarily used to inform my findings associated with the classroom observations data, the focus of Chapter 5.

3.4 Summary of Methods

I made use of teacher interviews and classroom observation data in order to answer my three research questions. The first research question regarding teachers' described orientations toward writing in mathematics was primarily answered based

on teacher interview responses from an initial, pre-observation interview. I used a multistage coding scheme (Kauffman et al., 2002) to create profiles and summaries of these responses, and then engaged in member checking (Candela, 2019; Creswell & Clark, 2017) and follow-up questions in a second, post-observation interview to answer this question. The second question regarding teachers' interactions with students' writing was analyzed by coding types of teacher interactions with students' writing (based on Boston et al., 2017; Munter et al., 2015; van Es & Sherin, 2021) and then coding these interactions in terms of the type of writing embedded in the tasks during these interactions (based on Casa et al., 2016; Herbel-Eisenmann, 2007; Rotman, 1988). The third question regarding teachers' interpretations of instructional artifacts was analyzed using pattern matching (Yin, 2018) to compare teachers' responses regarding noticing activities (Jacobs et al., 2010) to their orientation profiles.

Chapter 4

RESULTS: TEACHERS' DESCRIBED ORIENTATIONS TOWARD WRITING IN MATHEMATICS

In this chapter, I address my first research question: Among a group of high school mathematics teachers whose instruction is dialogically focused, what are their described orientations toward writing in school mathematics? I present cases of three teachers and report on how each teacher described the purpose(s) of writing in mathematics, as well as instances where each teacher referenced their approach towards (or challenges with) writing-to-learn and writing as a multisemiotic activity. I also share the experiences that teachers describe as informing their orientation toward writing in mathematics. After this, I describe my findings of a cross-case analysis (Yin, 2018) that synthesizes results across all three of the participants descriptions.

Through these results, I show how each teacher broadly spoke to the different purposes of writing in mathematics (i.e., communicating and reasoning; Casa et al., 2016), but I also illustrate that they reported distinct nuances in how they understood writing in relation to the doing, learning, and teaching of mathematics. These distinctions were that Mrs. Taylor writing as a tool to support students' explanations and justifications, Mrs. Hudson focused on writing as a way to build students' mathematical vocabulary and thus mathematical understanding, and Mrs. Barnett focused on writing as a multisemiotic tool to support students' communication of mathematical ideas. Such distinctions indicate the complexity of writing in the context of high school mathematics, the reflective experience of the teacher participants in

unpacking that complexity, and the way that past experiences for these teachers (both in and out of the classroom) are described as informing these orientations.

4.1 Case 1: Mrs. Taylor’s Described Orientation Toward Writing in Mathematics

Mrs. Taylor’s described orientation toward writing in mathematics is shown in Table 4.1. While Mrs. Taylor focused many of her responses around how writing supports students’ reasoning in mathematics (specifically, their skills at explaining and justifying their thinking), she did also indicate that writing helps to promote communication – both with peer learners and with the teacher. Because she repeatedly discussed the role of writing as a way to explain or justify mathematical processes and solutions, Mrs. Taylor’s description can be seen as multisemiotic in that it attended to similarities and differences in how students use mathematical symbolism and the written word to reason in school mathematics. While Mrs. Taylor indicated general support for the idea that writing can be woven throughout to support the learning process (e.g., writing-to-learn), she also indicated that the instructional time costs of asking students to write and for the teacher to interpret and respond to each students’ writing was a major barrier in the feasibility of using writing consistently in math instruction.

4.1.1 Mrs. Taylor’s Described Purposes of Writing in Mathematics

Mrs. Taylor’s descriptions about the purpose of writing in mathematics referenced writing as benefiting both student reasoning and communication. Many of her responses focused on writing as a way for students to reason by explaining or justifying their work, or “how did you get from A to B.” She emphasized the distinction of these two types of writing, where explaining is “just showing your work,

[making] sure I understand that you understand by showing your work,” whereas justifying means that “you need to explain how you know that this answer’s correct, so it’s a deeper dive into ‘this is how I got my answer.’” She stated that such justification writing is important because being able to write about mathematics topics (via the written word) indicates how well you understand a topic (“what is it about what you did that tells you that this is gonna be correct?”). In this way, she also noted that “kids that know the math better write more often” and vice versa. Thus, students’ ability to use the written word to explain or justify the process through which they arrived at a solution is typically an indicator of their mathematical ability.

Table 4.1: Summary of Mrs. Taylor’s Described Orientation Toward Writing in Mathematics

Aspect of Writing in Mathematics	Summary of Teacher’s Described Orientation
The purpose of writing in mathematics	Writing is a way for students to explain and justify their mathematical thinking through the written word.
References to writing-to-learn	It is challenging to use writing as a tool for learning in mathematics because of the time it takes for students to write and to provide students with feedback.
References to writing as multisemiotic	Producing explanations or justifications through the words, phrases, and sentences can help students in more deeply understanding the reasoning behind how their strategies for solving problems using other sign systems (e.g., solving for an unknown in an equation using mathematical symbolism), and it is beneficial for students to engage in these different types of writing.

Experiences that were perceived to have impacted described orientation

Standardized assessments that required written student responses spurred professional learning workshops around pushing students to explain their thinking in writing.

Additionally, Mrs. Taylor described how a focus on procedures and algorithms can actually “get in their [students’] way,” as they may not then be focusing on the conceptual structures that underly such procedures. However, by using the written word to explain such work, she noted that learners can then be “processing that [the mathematics behind the procedure] in their head.” She stated that this in turn can help learners “have a little bit more idea of what’s going on” at a later point “when they have to write out a generalization” or apply the mathematical concepts in a different way. In this way, she is describing how the written word connects to other more abstract sign systems in mathematics but can also improve students’ reasoning with such systems.

In terms of writing as a tool to reason with and communicate mathematical ideas, Mrs. Taylor described how it is important that the written word not only make sense to its author but to others who may be reading or interpreting that writing. Because she saw writing as being a tool for helping to justify, she described how it is important to ask “if somebody else came up and looked at this [writing], would they understand what it was that you meant?” In this way, writing is a way to “make sure [that] I understand that you understand by showing your work justifying.” Thus, writing (via natural language) could additionally demonstrate competency that is otherwise hard to grasp from mathematical symbolism alone.

There were perceived similarities and differences that Mrs. Taylor noted between spoken and written communication in the context of math instruction. She

stated that “if you can explain it [a mathematical idea] to the person sitting next to you, you can write it down.” In recounting her instructional approach towards writing, she similarly noted how “I don’t want them [students] to get off the hook” with spoken explanations, and so if students did state that they could explain an idea she oftentimes would respond with “well, what would you say? Write it down.” However, she also saw challenges in pushing students to communicate through writing. She described such her push for students to write out their explanations as a “shift [that] needs to take place” in math classrooms, as otherwise “they [students] think of it [writing] as an English thing.” Thus, Mrs. Taylor saw writing as an important tool of communication that is underutilized and underappreciated in math classrooms.

Mrs. Taylor’s description about the role of writing in the teaching of mathematics indicated further connections that she saw between writing as a tool for students’ reasoning and as a way for students to communicate such reasoning with their teacher and peers. She supported the idea of teachers using “tasks that require a little bit more writing rather than just strict computation...where they [students] have to explain what it was that they did rather than just rote” computation, and that such tasks should then be followed by opportunities for students “to share what they wrote” and to also see their peers’ written justifications. Mrs. Taylor felt that such activities help the students to reflect on the quality of their writing and give the teacher opportunities to provide students with feedback on how to improve their written explanations.

4.1.2 Mrs. Taylor’s References to Writing-to-Learn

Mrs. Taylor reported that, although writing-to-learn activities could support student learning, they were time intensive and so she did not rely on them frequently

in her classroom. She described how the appropriateness of using writing-intensive tasks “depends on the lesson” and on “where it lends itself.” For examples, she discussed how writing prompts could be helpful as a “warm-up” because students could then “look and see what other people wrote” and thus better know where they need to improve their own written explanations. She also noted how writing could be used “at the end of a learning sequence” or even “in the middle,” and provided another example of asking students to “write out what they were thinking and explain or justify why this particular situation was true.” Such responses show how Mrs. Taylor saw opportunities for students to be actively learning from writing, especially with the sharing of and discourse around such writing that could then follow.

Despite providing these examples, Mrs. Taylor also indicated that writing activities do take up valuable instructional time, and she saw this lack of sufficient instructional time for such activities as a major challenge for teachers otherwise hoping to provide such writing opportunities for their students – including herself. As she noted, “that’s the part that goes when you’re on the time crunch.” She contrasted the logistical challenge of enacting writing activities with the relative ease of supporting mathematical talk in the classroom, noting that “I can ask what they [students] are saying and things like that, but I can really only – unless I’m collecting everything and reading everything – it’s almost impossible to do that.” Therefore, she saw writing as something that could be cut if instructional time was an issue.

4.1.3 Mrs. Taylor’s References to Multisemiotic Connections in Writing

While Mrs. Taylor frequently referred to writing (via the written word) occurring after students’ problem solving with other sign systems (e.g., solving an equation), this coupling of the written word with other sign systems shows a

multisemiotic orientation toward writing in math. Throughout Mrs. Taylor’s responses, she repeatedly drew a distinction between mathematical symbolism – and related procedures and algorithms that make use of such language – and “writing,” which she saw as using the written word to explain or justify mathematical processes and solutions. As previously noted, she saw writing as “a deeper dive into ‘this is how I got my answer.’” Writing (through the written word) would therefore be complimentary to and builds upon students’ reasoning using mathematical symbolism. However, Mrs. Taylor’s descriptions of writing through this lens would also imply that use of the written word generally follows the use of mathematical procedures and algorithms, rather than preceding or co-occurring with that sign system.

When pressed about a multisemiotic perspective on writing in mathematics, Mrs. Taylor did note that “part of the writing [in mathematics] is being able to show their [students’] work in coming up with an answer.” She ultimately concluded that “students need to reflect more in writing on what it was that they did [in coming up with an answer],” and so she thought that “they’re both valuable,” referring to reasoning through other sign systems and explaining such thinking through the written word. As such, Mrs. Taylor described these different sign systems of writing as distinct but complimentary to supporting students’ learning in math.

4.1.4 Mrs. Taylor’s Description of Past Experiences that Impacted Her Responses

Mrs. Taylor recounted how changing formats of standardized tests gave rise to her first shift in how she thought about and attended to writing as a mathematics teacher. Building on a story she had shared in her first interview, she talked about how the Partnership for Assessment of Readiness for College and Careers (PARCC)

assessment – designed via a multi-state collaboration to reflect new college and career readiness standards – included an open-ended performance task component as part of a state assessment for the first time that she could recall. Previous assessments had only ever been multiple choice or been graded on a specific numeric solution, but this new assessment would require students to write out justifications for how they arrived at their solutions. In response, she and other teachers in her school district began teaching students about “sentence starters” that they could use when writing out such justifications. The issue here, she found, was that after students would write the sentence starter, the rest of their justifications “didn’t make sense.” In essence, such writing “got very rote” and students “didn’t understand what it was that they were [doing].” She similarly names how her current curriculum (Illustrative Mathematics) also is “asking kids to think about writing,” albeit with guiding questions such as “What is it that you did?” rather than specific question starters or stems.

Ultimately, Mrs. Taylor did not seem to have found these instructional shifts spurred by policy and curriculum to have been successful in shifting how students engage with writing in mathematics. She noted that “trying to get the kids away from those one- or two-word answers is tricky” because students were “still conditioned to coming up with an answer, not how they came up with the answer or why that answer works, or even why they decided on that particular method for the solution.” This concern seemed in line with Mrs. Taylor’s earlier descriptions about the amount of time that writing in mathematics (specifically, creating written justifications for mathematical strategies and solutions) required.

4.1.5 Mrs. Taylor’s Described Orientation Summary

In sum, Mrs. Taylor described writing as relating primarily to aspects of mathematical reasoning, especially explanation and justification. However, students’ ability to then communicate that writing with their teacher and peers was also seen as a valuable learning experience that could also improve students’ understanding of the mathematics. While she implied that other sign systems actually represent “the math,” she did note the importance of the written word in helping students to understand and explain their understanding of such mathematical ideas. Along these lines, she indicated that writing should occur throughout the learning process, although the time-consuming nature of writing limited how frequently she actually used writing in her own instruction. She ascribed her thinking about writing in mathematics to be influenced by the way that written explanations or justifications have been embedded within standardized tests, and how that has in turn funneled down to district policy and professional learning. Through her responses, Mrs. Taylor appeared oriented towards writing as a tool for supporting mathematics learning, but not necessarily an essential tool in that regard.

4.2 Case 2: Mrs. Hudson’s Described Orientation Toward Writing in Mathematics

Mrs. Hudson’s described orientation toward writing in mathematics is shown in Table 4.2. Her interview responses focused on how writing can support both reasoning and communication in mathematics. She repeatedly described how writing in mathematics is relevant in the context of vocabulary, and she noted that mathematical concepts are inherently linked with the language that we use to describe such ideas. As such, she felt, by having opportunities to engage in writing, students could better understand the nature of (and therefore better reason with) the

mathematical concepts described by such vocabulary. Additionally, students could translate their writing skills into mathematical talk. Mrs. Hudson also emphasized another perceived purpose of writing (especially in whole class situations) in that it helps foster a supportive classroom environment where students feel comfortable sharing their ideas with one another. With her primary focus on vocabulary, Mrs. Hudson's responses emphasized the written word, although she also alluded to the ways that other sign systems are connected to the terms that are used to describe mathematical concepts or representations. Finally, she indicated that such writing opportunities – as they can support students' conceptual understanding and spoken discourse around the mathematics content – should be present throughout the learning cycle, indicating a belief in writing-to-learn.

4.2.1 Mrs. Hudson's Described Purposes of Writing in Mathematics

Mrs. Hudson frequently focused her responses about writing in mathematics towards the realm of academic vocabulary, which she described as a way to help students with “seeing the connections [that] are being made across the board” with mathematics content. In other words, such vocabulary was seen as a tool to support student reasoning in mathematics. A key aspect of such vocabulary that Mrs. Hudson found important in both the doing and learning of mathematics was the word parts (e.g., prefixes and suffixes) that compose such language. For instance, she described how the prefix in “monomial” could be used to help students better understand the nature of monomials and how they relate to other mathematical concepts (e.g., binomials or polynomials). Because students may be familiar with other words that share similar prefixes – for example, she recalled how some students remembered the idea of “mononucleosis” from their science course – they can use their broader

language skills to understand mathematical concepts through a reading of the vocabulary. Thus, by “breaking down English words and translating” them, students can recognize “how they [the words] affect the math world.”

Table 4.2: Summary of Mrs. Hudson’s Described Orientation Toward Writing in Mathematics

Aspect of Writing in Mathematics	Summary of Teacher’s Described Orientation
The purpose of writing in mathematics	By focusing on academic vocabulary, writing is a way for students to communicate mathematical ideas more precisely and to understand connections between mathematical concepts and the language we use to describe such concepts.
References to writing-to-learn	Writing activities should be integrated throughout the learning process as this can help students better connect with the mathematics.
References to writing as multisemiotic	Writing is a way for students to draw connections between words or phrases used to describe mathematical ideas and other sign systems that we use to represent those ideas.
Experiences that were perceived to have impacted described orientation	Her father taught her (and she excelled at) a more procedural approach to learning math, but when she became a teacher, she saw that helping students explain their thinking using precise academic vocabulary helped students to conceptually understand the mathematics as well.

Mrs. Hudson also reported that writing was another way for students to communicate their thinking with one another. For instance, she emphasized the

connections between writing and mathematical talk in the context of teaching mathematics, describing how writing allows students to take their knowledge about mathematics vocabulary and apply it “into some kind of math talk.” She then went on to describe a recent example where she used a “polygraph” activity from Desmos (<https://teacher.desmos.com/collection/601980ced441a40d2179ea77>) where students would write yes-or-no questions to a random partner regarding a number of function graphs on their screen with the goal of identifying which graph was also displayed on that partner’s screen. She noted that “I like to start my lessons off again with the rough draft talk, but it [this example] was through Desmos,” echoing language from Jansen (2020) that indicated how she saw parallels between this writing activity and similar activities that also emphasize spoken discourse.

Mrs. Hudson also used debriefing about students’ writing to support students with increasing their comfort in sharing their thinking with their peers, particularly at the whole class level. She expressed concern that students often hold reservations about writing in mathematics if they have not had much exposure to such opportunities, as they are concerned about being “wrong” with their written work. However, if students have consistent opportunities to practice writing, Mrs. Hudson indicated that they will grow more confident in their writing and be able to make mathematical connections. She specifically noted a “Shout it Out” routine that she would use where she would “have students type their responses to explanations” using a SMART board application and then she would “display it [students’ responses] to the entire class at one time.” Through this process, she described how this shifted students’ perspective on their writing from a focus on “if I’m correct or not” to instead an interest in sharing what they know and learning from the language used by their

peers. In this way, writing was described as serving an additional purpose of building a collaborative classroom environment.

4.2.2 Mrs. Hudson's References to Writing-to-Learn

When asked to define when writing should occur in the learning process, Mrs. Hudson responded in a way that indicated a writing-to-learn orientation. She explained that writing "should go throughout" the learning cycle, describing how she would:

start out with just, again, that rough draft, but just open it up to doing it more often. It's in a warmup, might pop up again to shout it out in a check your understanding, and then maybe shout out again at an exit ticket.

By integrating writing throughout the learning process, students can move from "making their own descriptions" (i.e., "descriptive words" about mathematical ideas) and eventually "move into more formal words," again emphasizing the importance of mathematical vocabulary in informing when and how writing should be used in such instruction.

In addition to these direct responses about integrating writing throughout the learning process, Mrs. Hudson's aforementioned focus on the affective purpose of writing may also indicate a writing-to-learn orientation. Her "Shout it Out" routine appeared to underscore the importance of making student thinking (i.e., writing) visible, and orienting students away from an evaluative perspective (e.g., "if I'm correct or not") to a more learning-oriented approach to their learning. As such, her responses indicated a view that students learn through writing, especially if that writing is tied to some public sharing and revision of such written work as a classroom community.

4.2.3 Mrs. Hudson’s References to Multisemiotic Connections in Writing

Mrs. Hudson’s responses indicated that, while she primarily was focused on writing as natural language construction that included mathematical terminology, she also recognized how such writing supported students’ understanding of other sign systems. Mrs. Hudson’s description of the Desmos “polygraph” activity, for example, showed how written words and phrases could be used to describe and interpret the visual imagery of function graphs. She described the activity as follows:

So, it's a polygraph where they pick a picture of a graph, but they have to talk about they have to use yes or no questions as descriptions to be able to figure out which graph they have. So, they start out with "the hills," "the little squiggly line," "what's this one," but at the end I bring it up again and I encourage them to use vocabulary. So, they may say, "does your graph have a positive x-intercept?" Or, you know, "are your end behaviors up, down or down up?" You know, so to see where they started and then go to the end to push the end.

However, it is worth noting that her responses did not emphasize students’ writing across different sign systems other than natural language. Rather, writing was presented as a way for students to describe or explain their thinking or their understanding of mathematical ideas – ideas which would involve comprehending different sign systems.

4.2.4 Mrs. Hudson’s Description of Past Experiences that Impacted Her Responses

When asked about what has informed her current orientation toward writing in mathematics, Mrs. Hudson spoke to her own experiences as a mathematics student and learner, as well as her transition to being a mathematics teacher. She noted how she originally came to like math as a student based on how her father taught her the content, which focused on a very procedural approach. As such, the way that she would try to teach students reflected this, where she “would just say, ‘you just do this’

or ‘you kind of just go over here and do that’.’. However, she then described how she began to notice that, “when it came to something that was more formal, students didn’t know what to do,” and so she “had to start to utilize the vocabulary more for the understanding.” She recounted how “I had to start to utilize the vocabulary more for the understanding, which I felt a lot of gaps were filled once I started doing that and just breaking down the words and what they actually mean.” She therefore saw this focus on vocabulary as a “big help in their [students’] success as far as understanding when it came to [how to] approach a problem.” Thus, she came to see how students benefitted from not only proficiently manipulating mathematical symbols, but also from being pushed to precisely communicate their mathematical problem solving.

4.2.5 Mrs. Hudson’s Described Orientation Summary

In sum, Mrs. Hudson’s focus on writing in mathematics centered around aspects of communication and especially the development of academic vocabulary. While she emphasized the importance of academic vocabulary in learning mathematics, the primary focus in her described work with students was in building their comfort and confidence with academic vocabulary. Her responses also emphasized how she saw academic vocabulary as a way for students to connect the word parts and meanings behind such words and phrases to other sign systems in mathematics. This indicates how, although natural language (via academic vocabulary) was centered in Mrs. Hudson’s descriptions of her instructional practices, she did recognize multisemiotic connections between different descriptions and representations of mathematical ideas. Through all of these responses, Mrs. Hudson’s described orientation promoted a rough draft (Jansen, 2020) nature of writing in mathematics, where the teacher and students engage in iterative routines of practice

meant to ever increase the precision of the mathematical communication happening in the classroom and connect such language to the mathematical meaning-making. These iterations to improve communication suggest her alignment with a writing-to-learn orientation.

4.3 Case 3: Mrs. Barnett’s Described Orientation Toward Writing in Mathematics

Mrs. Barnett’s described orientation toward writing in mathematics is shown in Table 4.3. The role of communication was particularly prevalent in Mrs. Barnett’s described orientation, and her description was also the only one of the three teachers that explicitly emphasized the multisemiotic nature of writing in mathematics. Mrs. Barnett highlighted the importance of peer communication in particular as being a key purpose of writing in mathematics, as well as the opportunity for students to revise their writing over time. Such an emphasis on communication and revision suggested that Mrs. Barnett was in alignment with a writing-to-learn orientation. However, she also described how writing typically arises in school mathematics as a way for students to “summarize” their learning, which could run counter to a more formative writing-to-learn orientation. Together, Mrs. Barnett’s responses suggest an orientation of writing that is flexible in terms of the forms that writing can take, the mathematical precision to which it must adhere, and the ways in which it supports student learning.

Table 4.3: Summary of Mrs. Barnett’s Described Orientation Toward Writing in Mathematics

Aspect of Writing in Mathematics	Summary of Teacher’s Described Orientation
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The purpose of writing in mathematics	Both writing and mathematics are “intertwined” because they both can serve a goal of trying to communicate ideas. By writing out and sharing ideas with one another, we can also strengthen our reasoning over time through revision.
References to writing-to-learn	While writing is often used as a summative tool in school mathematics, it can go anywhere in the learning process. Students can learn through writing through a focus on revision.
References to writing as multisemiotic	Because writing is a tool of communication, communicating mathematical ideas may entail the use of different sign systems.
Experiences that were perceived to have impacted described orientation	Discussions with her family members have shown her the importance of communicating mathematical ideas in clear and comprehensible ways across diverse audiences.

4.3.1 Mrs. Barnett’s Described Purposes of Writing in Mathematics

In Mrs. Barnett’s described views, “math is a form of communication in and of itself,” where “I have this idea that I want to communicate to people, this pattern that I’ve noticed...and I just need to be able to demonstrate it for someone else in however way I see fit with words, visuals, [or] symbols.” Because writing is also “a way of communicating,” it naturally arises as a way to communicate mathematical ideas. For instance, with proof-writing “we must write words in order to prove something,” but also “we can prove things using visuals, using diagrams, using even symbols.” In this way, different types of writing “are very much all kind of intertwined” in the doing of mathematics, as they can all be used to communicate mathematical ideas.

Because Mrs. Barnett felt that writing serves primarily as a method of communication, she also downplayed the importance of mathematical precision with writing in the learning and teaching of mathematics, so long as the writing can be understood by the audience (e.g., peer learners). She described writing as "the ability to communicate an idea to someone else," "to transport what I'm thinking in my brain to you in some way, either through words or a picture or an equation." Because of this, a learner knowing how "to put down their thinking...in order for someone else to understand it" is an important skill in learning mathematics. In Mrs. Barnett's words, "the terminology [of the written word] does not have to be perfect," because "sometimes when there is fancy jargon...it [the idea] gets lost." Additionally, learners also have "the right to say what makes sense" to them, and so writing can be a way for them to communicate "what I'm thinking and what's making sense to me" in a way that can also "make sense of that to other people." Therefore, both the comprehensibility of a writer's written work and their right to express ideas in a way that makes sense to themselves hold value.

To support such learning, Mrs. Barnett discussed providing students "with sentence stems" and creating public spaces for students to "see similarities between what they said and their classmates said." She stated that teachers could use writing in helping students to "revise or restructure the way that [they are] initially" describing mathematical ideas, allowing students to "refine" their thinking as they are learning. She noted that such routines could develop students' "life skill" of "I want to be able to talk with you, but I don't want to offend you...I just want to be able to communicate my thinking to you in a way that you're going to be able to understand it." In this way, Mrs. Barnett was drawing connections between how such activities

could apply in the context of written or spoken discourse and could support both modes of communication.

4.3.2 Mrs. Barnett's References to Writing-to-Learn

Mrs. Barnett generally indicated an agreement that students could learn through writing, but she also described a summative archetype of how writing typically arises in school mathematics. While Mrs. Barnett did describe using writing for students to “revise or restructure” student’s ideas over time, she also indicated that writing should often occur at the end of a learning sequence “to summarize.” However, despite this initial response, she did later note that “you could justify it [writing] anywhere in the learning process” as well. For instance, she described previous approaches she had taken of having students “write about what they know beforehand” (i.e., activating prior knowledge) or even having students “try to write everything that they knew” about a particular word related to the learning goal. Such responses indicated that, although Mrs. Barnett may not have perceived writing-to-learn activities as being common in school mathematics, she did recognize their place and reported having used such activities in her own teaching at times.

4.3.3 Mrs. Barnett's References to Multisemiotic Connections in Writing

As described above, Mrs. Barnett spoke about writing in mathematics as occurring across different sign systems (e.g., “words, visuals, symbols”), which indicated her orientation toward a multisemiotic perspective on writing. She described how the comprehensibility of communicating mathematical ideas was more important than the form (e.g., “I need to be able to transport what I'm thinking in my brain to you in some way, either through words or a picture or an equation”). Compared to Mrs.

Taylor, who saw writing as a way to explain mathematical ideas, Mrs. Barnett saw both mathematics and writing *as* ways of communicating, and thus inherently “intertwined.” Mrs. Barnett’s described orientation thus appeared highly multisemiotic, if sparse on details about when or how different sign systems may be more or less effective at communicating different mathematical ideas in different contexts.

4.3.4 Mrs. Barnett’s Description of Past Experiences that Impacted Her Responses

Mrs. Barnett noted that her thinking about writing in school mathematics evolved largely through conversations that she had outside of the classroom. For instance, she described talking with her brother – a research scientist – about the importance of “communicating his findings.” While she used to think that such work was “just about solving a problem,” she began to understand an issue of “how much does my solution really mean unless it makes sense to others?” This led her to reflect on her own role as a mathematics teacher, where the content “can make sense to me, but it needs to make sense to others in order to be effectively communicated.” She also described how these sorts of communication skills are important for students as well, as they need to “explain what they’re thinking to others.” However, she went on to state how such communication skills are “where a lot of the challenge lies” for students, indicating how she saw the comprehensibility of writing or spoken discourse as a challenge with which both she and her students grapple.

4.3.5 Mrs. Barnett’s Described Orientation Summary

In sum, Mrs. Barnett’s described orientation toward writing emphasized the importance of communication (with oneself and others) and the flexibility in which we

can communicate mathematical ideas through different sign systems. Her responses alluding to the different sign systems through which we can communicate such ideas (e.g., “words, visuals, symbols”) indicated a multisemiotic orientation toward writing. She described how writing can allow for opportunities to revise students’ thinking over time, suggesting her alignment with writing-to-learn as well. However, she also noted that writing typically happens at the end of a learning sequence as a way to for students to “summarize” their learning, which somewhat contrasted with these other more formative descriptions tying writing to opportunities to revise.

4.4 Cross Case Analysis: Teachers’ Describe Orientations Toward Writing in Math

In this section, I synthesize how teachers describe their orientation toward writing in mathematics. I focus first in laying out the differences in how the teachers speak to the purposes of writing across goals of communicating and reasoning with mathematical ideas. I then consider several themes that arose in relation to teachers’ descriptions about writing in the teaching and learning of mathematics, including challenges concerning writing-to-learn and connections across different written sign systems. Such findings illustrate both the complexity of how writing is perceived as relating to these participants’ work as teachers of mathematics, and the nuanced understanding of such complexity that each participant describes as attending to in their work.

4.4.1 Differences in Describing the Purposes of Writing in Mathematics

Casa et al. (2016) describe mathematical writing as supporting both communication and reasoning in mathematics. Additionally, Powell et al. (2021) describe how communicating through writing can not only serve to benefit discourse

and argumentation in mathematics, but also serves as a tool for assessing students' mathematical knowledge and competence. All three of the teachers who participated in this study alluded to both goals of communication and reasoning when describing writing in mathematics. However, there were differences in how the teachers understood these purposes and how they related to their approaches to supporting student writing in their own classrooms.

A summary of the participants' views on writing as a means of communicating mathematical ideas is shown in Table 4.4. While each teacher described writing as broadly serving a role in communicating, their descriptions suggested different purposes for writing to communicate with others in the context of learning and teaching mathematics. Mrs. Taylor, who spoke repeatedly about writing in the context of explaining and justifying, described how students' writing needs to be comprehensible to its audience, and that having students' share such writing with their peers can help them to get feedback about the correctness of what they wrote. Mrs. Barnett also spoke about the importance of comprehension with students' writing but described how "fancy jargon" can actually get "lost in translation." Mrs. Barnett focused more on communication as a means to facilitate the sharing of ideas, while Mrs. Taylor emphasized an additional role of assessment for such communication.

Table 4.4: Teachers' Views on Writing as a Means of Communicating Mathematical Ideas

Teacher	Described Views and Aligned Quotes
Mrs. Taylor	<p>Writing can be used for explaining and justifying to others, which means that it is important to be comprehensible to readers.</p> <p><i>"...if somebody else came and looked at this [written justification], would they understand what it was that you meant?"</i></p> <p>If students are able to share their writing with others, it allows them to understand the correctness of what they wrote.</p> <p><i>"I think one of the other things that has to happen is that the kids have to share what they wrote. If they just write it down, who's gonna see it? They don't know whether they did it right, whether they did it wrong, they don't know whether what they wrote makes sense, because some kids are just writing stuff to get it down on paper so Mrs. Taylor leaves them alone."</i></p>
Mrs. Hudson	<p>Students can sometimes feel self-conscious or uncomfortable with sharing their writing. However, by encouraging writing opportunities early and frequently, students become more comfortable communicating their thinking.</p> <p><i>"I would also say encourage it often, as far as writing or expressing themselves in some way mathematically...In the beginning, I anonymize their names, so the fear goes away of maybe knowing if 'I'm good,' if 'I'm correct or not,' or, 'if this doesn't sound good, but at least I can display it to the entire class at one time.' ... just being comfortable with it in the beginning, they were like, "I don't really like this," but then at the end now they're like, 'okay, I'm gonna see what I have to say.' And they don't feel limited, you know, to what they want to say there too."</i></p>

Mathematics and writing are forms of communication. We write in mathematics as a way to communicate mathematical ideas with one another.

Mrs.
Barnett

“I think again, writing, it's a way of communicating and, I mean, very literally, like equations can be written. Graphs, you know, we can write on a graph. So, I believe that all of those forms of communication fall under that category of writing, especially again, in this context of, in math. Because I wanna communicate something to you. I could draw a picture, or I could, again, write out an equation, things like that. So, I think that they are very much all kind of intertwined.”

The precision of written language is not as important as is its comprehensibility to whomever is reading what was written.

“...so I'm thinking, the terminology does not have to be perfect. It doesn't have to be, like, fancy. In fact, sometimes when there is fancy jargon, it gets lost in translation. It's the ability to communicate an idea to someone else. I need to be able to transport what I'm thinking in my brain to you in some way, either through words or a picture or an equation. So as far as writing skills, people need to be able to put down their thinking in order for someone else to understand it.”

Mrs. Hudson also put forward a different perspective about the challenges and affordances of asking students to communicate through writing. She described how student comfort in sharing their ideas with one another through writing takes time to develop, and that one of the fears that students have is about whether or not they are “correct or not.” Therefore, while she did describe her “Shout it Out” activity as a way to strengthen students’ precision (in particular, their use of academic vocabulary) over time, she spoke about how this was accomplished by pushing students to *want* to communicate different ideas even if they are not sure about the correctness of such ideas (i.e., students feel comfortable to “throw different things out” through “rough draft” writing activities). Thus, the goal of such writing in the classroom would

presumably not always center on the correctness of what was written, as Mrs. Hudson hoped to move students beyond such a focus on their writing.

The teacher participants also had different stated views on how writing was a means of reasoning with mathematical ideas, as shown in Table 4.5. Mrs. Taylor spoke about how writing, in the form of written explanations or justifications, helps students process their thinking and can help them develop conceptual understanding of mathematics topics better than if they were simply working through procedures or algorithms to come up with solutions. Mrs. Hudson also saw writing as a way to support students' conceptual understanding of mathematics topics, but specifically because it helps students draw connections by considering the vocabulary that they use in their writing. Therefore, while Mrs. Taylor spoke broadly about how writing helps students to “process the thinking” of the mathematics – in essence, writing as a metacognitive activity – Mrs. Hudson specifically saw word parts and etymology as a way to draw connections for students to other sign systems.

Compared to the other two participants, Mrs. Barnett spoke sparingly about the use of writing as a tool of reasoning – her responses largely centered on the role of writing as a tool to support student communication of mathematical ideas. However, she did discuss how writing can be helpful as a way for students to have a tangible artifact of their thinking. In this way, as student “revise or restructure” their original, “not quite put together ideas,” written work can help indicate how students' thinking has changed over time. Mrs. Barnett's responses indicate an idea of writing as a form of rough draft thinking (Jansen, 2020) to be refined over time, rather than as a summative artifact of student learning.

Table 4.5: Teachers' Views on Writing as a Means of Reasoning with Mathematical Ideas

Teacher	Described Views and Aligned Quotes
Mrs. Taylor	<p>Writing is a way to explain and justify mathematical reasoning and helps students to process such thinking.</p> <p><i>"...where I'm trying to push them [my students] now is, as they're getting ready now to go into Algebra II, getting them, pushing them into doing that, thinking themselves, because I think that writing part helps them to process the thinking."</i></p>
	<p>Writing helps students to develop generalizations of mathematical ideas beyond being able to produce solutions to specific problems with procedures and algorithms.</p> <p><i>"The algorithms I think sometimes gets in their [students'] way. And I even with solving equations and things like that, I try to get them out of that algorithm...so that as they're going along, if they're processing that in their head now, then when they have to write out a generalization later, they have a little bit more idea of what's going on, hopefully."</i></p>
Mrs. Hudson	<p>Writing, with a focus on using academic vocabulary, helps develop students' conceptual understanding of mathematical ideas.</p> <p><i>"I remember when I first started teaching, I thought vocabulary wasn't important...So I would just kind of dummy down the vocab sometimes and [I eventually realized] that it makes more sense to include it, because when it's seeing the connections [that] are being made across the board, [that is] when that vocab comes in."</i></p>
Mrs. Barnett	<p>Writing is a way for students to tangibly reflect on and revise their reasoning with mathematical ideas over time.</p> <p><i>"...if they [students] have those kind of not quite put together ideas...we could kind of just revise or restructure the way that you were initially saying that. So again, I want them to write down what they're thinking and then we can refine it as we go."</i></p>

4.4.2 Challenges with Implementing Writing-to-Learn

Although all three teachers expressed support of the notion that students can learn through writing, and that writing should occur throughout the learning process, each of them also noted some concerns with implementing writing as a tool for student learning. These challenges indicate tangible concerns that may be acting as barriers to these teachers' use of writing (especially the written word) as a way for students to communicate and reason with mathematics. Such challenges also seem to align with the overall descriptions that teachers shared about what they view as the purpose of writing in school mathematics.

Table 4.6: Teachers' Challenges with Implementing Writing-to-Learn

Teacher	Described Views and Aligned Quotes
Mrs. Taylor	<p>Although writing can be used for learning, it is difficult to make time for it. Because it is important for students to get feedback on their writing to know if it “makes sense” or is correct, it is difficult to use writing in the classroom as it is time intensive.</p> <p><i>“So, I’ve been more worried about getting through the curriculum than spending more time writing, because that does take time and that’s where, unfortunately, I think teachers can pull away from writing more because it does take time. And how do I get to see 30 kids’ writing? You know? I can look over their shoulders. I can ask what they’re saying and things like that. But, you know, I can really only, unless I’m collecting everything and reading everything, I can’t – it’s almost impossible to do that. So that’s one of the hard parts about having the kids write in mathematics is that I’m not taking papers home and looking at them like an English teacher is, I’m ‘are you getting it? Are you not?’ I don’t know.”</i></p>

Mrs.
Hudson

Students need to feel comfortable about their writing in order to engage with activities that include writing. Because of this, building from “rough draft” writing activities to more frequent, “formal” writing activities can help attend to students’ growing comfort with writing in math.

“I think they [writing activities] should go throughout [the learning sequence]. I’m a day to day, always sharing in any kind of way, whether it’s, like I said, from rough draft to more formal. I think once you see a building process and they [students] are getting more comfortable, I think it [writing] should happen all the time, just so that the comfort is there and that they know the expectation is to go from one type of talk from math to another.”

Mrs.
Barnett

Writing is something that Mrs. Barnett does not feel that she does frequently, and she sees it as something that can happen to summarize students’ learning.

“So, honestly, I have to be completely honest with you. I haven’t been doing [writing] as much and that’s why I was feeling a little insecure about being a part of this study, but I’m hoping that it’ll encourage me to have this more as a structure in my teaching. A lot of times I would have students write like a summary of a lesson or even just like a, ‘this is what I took away from the lesson at the end.’ I mean, trying to put the understanding of the lesson into a sentence or into words, that [is something that] I want to be able to do more. I feel like mostly it would be in like summarizing though as a part of the lesson.”

For Mrs. Taylor, she discussed concerns about the amount of time required to provide students with feedback on their writing. Given her aforementioned emphasis on an evaluative purpose of writing, it is understandable that she would see feedback as an important component of any sort of writing-based activity. She described how writing is “the piece that gets pushed to the side when I’m running out of time” in part because of the amount of instructional time that it takes to provide students with feedback on their writing, and that she perceives this as being a reason overall why “teachers can pull away from writing” in math.

While Mrs. Hudson did not explicitly state a challenge that she felt that she had with using writing in her mathematics classes, her responses again referenced the importance of building student comfort with writing over time. She stated that writing “should happen all the time” once students “are getting more comfortable,” indicating some degree of scaffolding required in successfully implementing writing-based activities. Her responses spoke to the importance of building classroom expectations and norms about how writing fits in with other sorts of activities, such as mathematical talk.

Mrs. Barnett was candid in describing her hesitation to participate in this study because of how infrequently she felt that she used writing in her classroom. However, she did then describe how she tried to build in summative activities where students write “a summary of the lessons” or a response about what they “took away from the lesson at the end.” Because she had also discussed different types of and places for writing in the learning sequence (as previously described), these responses may indicate that her perceived difficulties with using writing in the classroom are particularly in relation to this more summative type of writing activity.

4.4.3 Teachers’ Descriptions of Writing as Multisemiotic

As laid out in the analysis of their individual described orientations toward writing and summarized in Table 4.7 below, each of the teacher participants referenced different ways that they saw writing as related across different sign systems in mathematics. Such responses are interesting in that they suggest different perceptions between the teachers about the relationship between the written word and other sign systems in mathematics. These responses also suggest how the teachers see

writing (via the written word) as supporting students' math learning either concurrently with students' use of other sign systems or following such work.

Table 4.7: Teachers' References to Multisemiotic Writing

Teacher	Described Views and Aligned Quotes
Mrs. Taylor	<p>Explanatory and argumentative writing serves as an extension of operations and procedures performed primarily with mathematical symbols, where students are reflecting on their thinking around how and why they symbolically worked through their tasks.</p> <p><i>"So I think part of the writing is being able to use the, I don't wanna say algorithm really, but [students] being able to show their work in coming up with an answer, not necessarily the prose writing, but being able to show how they came up with [an answer]...maybe that word [reflect] needs to be in there more as they, the students, need to reflect more in writing on what it was that they did."</i></p>
Mrs. Hudson	<p>Because academic vocabulary in mathematics corresponds with representations that use other sign systems besides the written word, focusing on students' precision with such vocabulary can improve their understanding of how the written word corresponds with those other sign systems as well.</p> <p><i>"As far as their writing, I know my students in particular, I would say as they're writing or solving problems, I may use corrections and use definitions. Like for instance, I was grading a polynomials test and the students graphed the polynomial, but they missed a multiplicity. So, I didn't say "you missed the U" or "you forgot a hump here." I might say "you missed a multiplicity of two at $x = 3$," and the students understand what that means. So, I try to do that often as I'm walking around throughout the class, I'll start using the vocab a lot more so they're understanding how that word or term is applying in their graphing or in their table, but not just their writing."</i></p>

Communication is a central goal of mathematics, and writing within or across multiple sign systems can be used to communicate mathematical ideas.

Mrs.
Barnett

“I think again, writing, it's a way of communicating and, I mean, very literally, like equations can be written. Graphs, you know, we can write on a graph. So, I believe that all of those forms of communication fall under that category of writing, especially again, in this context of, in math. Because I wanna communicate something to you. I could draw a picture or I could, again, write out an equation, things like that. So, I think that they are very much all kind of intertwined.”

For Mrs. Taylor, for instance, writing (explanatory and argumentative) was described as an extension of students' operative work (i.e., performing mathematical operations; O'Halloran, 2008) with other sign systems such as mathematical symbolism. She described how the written word can be used to “show how they [students] came up with” an answer, indicating that such writing can be a way for students to reflect on more procedural or symbolic aspects of their work (i.e., using algorithms to arrive at solutions). In this way, Mrs. Taylor was describing writing as something that would follow students' work in math class, as it would be a way for them to explain or justify a solution that they had found.

Mrs. Hudson, on the other hand, described how her attention to precise academic vocabulary could help students as they are writing across different sign systems during class. She recounted how, as students would be working with graphing a polynomial, for instance, she would try to give precise feedback about their work using such vocabulary. She saw this approach as helpful to supporting students' understanding across contexts of different sign systems, as students would be able to know how such language is (for instance) “applying in their graphing or in their table, but not just their writing.” While her responses did not emphasize the actual activity of

students writing through words or phrases like Mrs. Taylor, it did indicate that she saw natural language overall as being a tool for strengthening students' work across different sign systems.

As previously addressed, Mrs. Barnett also alluded to writing (via the written word) as generally occurring at the end of a lesson to summarize (see Table 4.6), which could potentially indicate some alignment between her orientation and Mrs. Taylor's description of explanatory or argumentative writing following more procedural or symbolic work. Mrs. Barnett also described how mathematical ideas can be communicated across different sign systems or representations. As such, she recognized that the written use of mathematical symbolism or visual imagery can serve a goal of such mathematical communication. Unlike Mrs. Hudson, however, Mrs. Barnett did not explicitly speak to the idea of connecting ideas across those different sign systems (besides perhaps the aforementioned references to summative writing), but rather that different sign systems can be used as a means of communicating mathematical ideas.

4.4.4 Summary of Analysis on Teachers' Described Orientations Toward Writing in Math

These findings illustrate how, although the teacher participants held some broad similarities in how they described the purpose and form of writing in mathematics, their responses indicated meaningful differences in how they oriented themselves towards writing as teachers of mathematics. While each teacher mentioned how writing can serve as a means of communicating mathematical ideas, Mrs. Taylor focused on the element of feedback and assessment of student writing, Mrs. Hudson focused on using as a tool to build student comfort with communicating in math class,

and Mrs. Barnett focused on how different sign systems can allow more flexibility in how students communicate mathematical ideas. The teachers all described how writing can support student reasoning with mathematics, but Mrs. Taylor focused on writing as a way to support students' explanations and argumentation, Mrs. Hudson focused on writing as a way to improve students' understanding of academic vocabulary, and Mrs. Barnett focused on writing as a tool for revision.

The teachers also each described different perceived challenges to using writing as a tool for learning in mathematics, including the time intensive nature of providing feedback to students' writing (Mrs. Taylor), the time it takes to build students' comfort with writing in math (Mrs. Hudson), and the difficulty of integrating writing throughout a lesson (Mrs. Barnett). Mrs. Taylor and Mrs. Hudson also explicitly spoke to connections between written and spoken modes of language, while all participants described relationships between different semiotic systems in the context of mathematics. Such responses show the complexity in how writing relates to the broader ideas of teaching mathematics, and how lines can at times be blurred between different modes of language and different sign systems in the teaching and learning of mathematics.

In sum, these findings show that these teachers had some distinct aspects to how they described their orientation toward writing in mathematics. Each teacher was also able to offer up personal anecdotes about how past experiences (both in and out of the classroom) influenced their orientation toward writing in math. The nuanced manner in which each teacher was able to describe their orientations toward writing in mathematics also showed the highly reflective nature of the participants in considering how writing relates to their role as teachers of mathematics. This suggested that

writing is a complex construct that these teachers are considering in their broader work, and with which they are ultimately approaching in distinct ways.

Chapter 5

RESULTS: TYPES OF WRITING PROMPTS AND TEACHER INTERACTIONS WITH STUDENTS' WRITING

In this chapter, I address my second research question: How do these teachers interact with their students' writing instructionally in relation to the types of writing prompted by the task? First, I used observation data to show that teacher-directed interactions (evaluative and descriptive) were the most frequently observed interactions across all teachers, although Mrs. Taylor's instruction did include about twice as many student-directed interactions (assessing and/or advancing questions interactions) compared to the other two participants. Mrs. Taylor's results are also portrayed as having been unique in that she was the only teacher for whom explanatory writing tasks – rather than procedural/symbolic tasks – were the most common type of writing tasks during that lessons that I observed. I describe how such findings indicated a connection between the types of writing tasks used and the types of interactions with student writing that occurred during these observations. Specifically, I focus on how the use of explanatory writing tasks at any point in a lesson appeared to support the use of more student-directed interactions (i.e., those with assessing and/or advancing questions).

I also report on multiple examples of interactions from each teacher's lessons, which were used to exemplify different types of writing tasks and interactions. These examples also helped illuminate some salient themes from the data both within the individual cases and across cases. Although the specific selection criteria for these

examples are further described in my methods (Chapter 3), overall these clips were intended to both represent portions of activities that the teachers felt best showed how they interact with students' writing (based on pre-observation forms) while also addressing my research focus on how questioning and discourse play out in these interactions. As such, these clips were not meant to describe the most *common* type of interaction observed, but rather interactions that addressed the research question and could have served as a noteworthy event of teaching (van Es & Sherin, 2002) that could spur relevant teacher noticings during the subsequent video viewing session (Erickson, 2007) with the teachers.

In particular, these examples illustrate ways that teachers at times displayed funneling patterns of questions (Herbel-Eisenmann & Breyfogle, 2005) rather than assessing or advancing questions, and how some of their assessing questions were "interrupted" to instead result in purely descriptive interactions. Because these interactions held some aspects of student-directed interactions but ultimately fell into student-directed categories, I described these as "liminal interactions." These examples also show how, even when the interactions are more teacher-directed, these teachers were still at times finding ways to place value on student voice as they provided feedback to students on their writing. For the cross-case analysis, I further drew on these examples to reinforce the claim that the type of writing task related to the use of assessing and advancing questions, but also that the teachers as facilitators still held some sway in deciding when and how to interact with students' writing. I also dig further into the occurrence of liminal interactions, suggesting their relevance in the potential pursuit of more student-directed interactions.

Because the selected examples of interactions were those also used in follow-up interviews with teacher participants (see Chapter 6), these findings can also be seen as “setting the scene” for those subsequent teacher reflections. The selected examples throughout this chapter therefore provide a baseline description of students’ work and the teachers’ interactions with that work as it occurred during the lesson, which can then be understood in relation to the interpretations and reflections that teacher describe in interviews (in Chapter 6).

5.1 Case 1: Mrs. Taylor’s Instruction and Interactions with Students’ Writing

Mrs. Taylor’s observed Geometry lessons focused on properties of functions and using related properties to create proofs about the classification of different quadrilaterals. I first describe the overarching types of embedded tasks and interactions with students’ writing observed in Mrs. Taylor’s lessons, noting the prevalence of interactions that occurred during explanatory tasks and how she interacted with students’ writing by using assessing and/or advancing questions (i.e., student-directed). I then provide two examples of interactions she had with students’ writing, focusing on interactions that involved questioning but did not meet other qualifications required to be categorized as student-directed interactions.

5.1.1 Types of Embedded Tasks and Observed Interactions with Students’ Writing

As shown in Table 5.1, Mrs. Taylor enacted a variety of procedural/symbolic, exploratory, informative, explanatory (exclusive), and explanatory (inclusive) writing tasks in her observed lessons. Interestingly, although most of her interactions were evaluative and descriptive in nature, she was observed using about twice as many assessing and/or advancing question interactions compared to the other two teachers in

this study (20 interactions or 33% of total interactions compared to 12 or 17% for Mrs. Hudson and 11 or 19% for Mrs. Barnett). She was also the only participant whose interactions with students' writing mostly occurred in the context of explanatory (exclusive or inclusive) writing tasks rather than procedural/symbolic tasks. One possible reading of this outcome, which I expand upon further in the cross-case analysis of this chapter, is that the more consistent enactment of explanatory writing tasks – and interactions with students' writing during these tasks – created a more conducive environment for engaging with students in discussion around their writing.

There are several conjectures that could explain the difference in these results for Mrs. Taylor's classroom data. For instance, Mrs. Taylor had the greatest years of experience as a math teacher among the participants in this study (30 years compared to 16 years for Mrs. Hudson and 9 years for Mrs. Barnett), so these results could have indicated Mrs. Taylor's experience as a veteran educator in being able to draw out more student-directed discourse during her interactions with students' written work. Given Mrs. Taylor's references to the importance of explanatory writing in her interviews (see Chapter 4), these results could also have been indicative of the value that Mrs. Taylor placed on such writing as a math educator. Although addressing such conjectures falls beyond the scope of this study, they do present opportunities for future research that is aimed at exploring differences in how teachers interact with students' writing in the classroom.

Table 5.1: Types of Writing Tasks and Teacher Interactions with Student Writing, Mrs. Taylor

Interaction	Task Types			Total
Procedural/	Exploratory	Informative	Explanatory	Explanatory

	Symbolic			(exclusive)	(inclusive)	
Teacher-Directed	15% (9)	--	7% (4)	21% (13)	24% (15)	67% (41)
Evaluative	10% (6)	--	5% (3)	8% (5)	11% (7)	34% (21)
Descriptive	5% (3)	--	2% (1)	13% (8)	13% (8)	33% (20)
Student-Directed	3% (2)	2% (1)	4% (2)	9% (6)	15% (9)	33% (20)
Assessing Question(s)	3% (2)	--	2% (1)	3% (2)	7% (4)	15% (9)
Advancing Question(s)	--	2% (1)	--	3% (2)	5% (3)	10% (6)
Assessing + Advancing Questions	--	--	2% (1)	3% (2)	3% (2)	8% (5)
Total	18% (11)	2% (1)	10% (6)	31% (19)	39% (24)	100% (61)

Note. Results are rounded to the nearest percent.

Additionally, Mrs. Taylor's observed interactions also showed the variety of interactions that occurred across different types of tasks. She was observed stating evaluative and descriptive responses across procedural/symbolic, informative, and explanatory writing tasks, and also using some form of assessing and/or advancing questions across all types of tasks. This showed how her interactions were not strictly limited by the sorts of writing tasks with which students were engaged.

5.1.2 Examples of Interactions with Students' Writing, Mrs. Taylor

Below are two examples of Mrs. Taylor's interactions with students' writing from her observed lessons. These examples were chosen in part because they seemed to illustrate examples of liminal interactions. They occurred during students' work on an explanatory (inclusive) writing task in Mrs. Taylor's fourth observed lesson and involved teacher questioning, but not in a way that met the requirements to code these as assessing questions. These interactions potentially offered clues about the

conditions that could limit such liminal interactions to ultimately being more teacher-directed. The first example demonstrated the use of closed-ended funneling questions (Herbel-Eisenmann & Breyfogle, 2005) as opposed to open-ended assessing questions, while the second example demonstrated a “diverted” use of assessing questions and advancing statements. Mrs. Taylor also interpreted the recordings of these two interactions and students’ final written products (a “proof” poster) during her final interview, further discussed in Chapter 6.

5.1.2.1 Mrs. Taylor Interaction 1 – Use of Funneling Questioning

This liminal interaction involved Mrs. Taylor using questioning during an explanatory (inclusive) writing prompt, but not in an open-ended way that would be classified as assessing questioning. This was important to examine because the nature of a teachers’ questions could indicate whether they exhibit a stance of inquiry (van Es & Sherin, 2021) towards their noticing of students’ written work – i.e., a frame “that involves trying to figure out what students mean” (p. 22). In this example, Mrs. Taylor was pushing a student in a small group (Casey) towards a particular written explanation, but her use of funneling questions (Herbel-Eisenmann & Breyfogle, 2005) gave limited opportunity for Casey to negotiate her own thinking or to reflect on what she has written prior to this interaction. Indeed, the similarity of the revised written product that Casey group eventually constructed with the language spoken by Mrs. Taylor, this interaction could have indicated that the “residue” of this interaction was students’ direct replication of Mrs. Taylor’s words (rather than their own written explanation). Mrs. Taylor recalled that she intended for this interaction to support Casey’s mathematical reasoning by taking the procedural/symbolic writing her group had produced and using it as the basis for the explanatory writing (described further in

Chapter 6), which demonstrated an intention to connect to students' thinking. However, this interaction also illustrated an instance where Mrs. Taylor's own spoken language preceded and potentially negated intended opportunities for Casey and her groupmates to grapple with the explanatory writing in their own words. This interaction, then, was liminal because the teacher attempted to connect with students' thinking, but the teacher ended up doing more of the thinking.

This interaction occurred during Mrs. Taylor's fourth observed lesson, when students were asked to create a poster that included a proof for the classification of a quadrilateral (e.g., parallelogram, rectangle, kite) with given vertices on a coordinate plane. This was a small group interaction – initiated by Casey – with a group that had been given coordinates of a rectangle: $(0,0)$, $(3,3)$, $(-3,9)$, $(-6,6)$. One of these students' worksheets for this task is shown in Appendix D, Figure D.1, and the final group poster that was created by the end of the lesson by this group is shown in Appendix D, Figure D.2. Although this task implied the use of procedural/symbolic elements (i.e., determining the slopes and lengths of each side of the quadrilateral), the actual task descriptions included an explanatory (exclusive) prompt (“Use slopes and lengths to determine the type of quadrilateral $ABCD$.”) and explanatory (inclusive) questions (“How can you create parallel lines?”, “How can you create sides that are perpendicular?”, and “How can you create sides that are the same length?”). Because Mrs. Taylor's interaction focused on the explanatory (inclusive) questions, this task type for this interaction was coded as such.

Figure 5.1 shows the interaction that Mrs. Taylor had with Casey, while the other students appeared to observe. The students had found the slopes of the line segments for each side of their quadrilateral, and Casey asked for guidance about the

groups' next steps in actually explaining their work. At this point, Mrs. Taylor engaged in the pattern of closed-ended funneling questions (Herbel-Eisenmann & Breyfogle, 2005) so that Casey could see that two, opposite sides of her quadrilateral sides had a slope of 1 and the other two sides has a slope of -1. Mrs. Taylor recollected that her intent was, in part, to push Casey's group towards recognizing that the opposite sides are parallel and the adjacent sides are perpendicular, thus meaning that their quadrilateral would be classified as a rectangle. Despite repeated questioning from Mrs. Taylor about how Casey's group could describe the opposite sides as parallel (lines 19-20, 22-23, 25-26), Casey struggled with these questions, seemingly calling out different mathematical vocabulary terms ("Reciprocals," line 21; "Congruent," line 24). Finally, Mrs. Taylor opted to answer the question for Casey, stating "No. If the slopes are equal...they're parallel" (line 26). She then asked Casey to identify what that makes the adjacent sides, which Casey correctly responded with "Perpendicular?" (line 33). Although Casey was funneled towards at least identifying the adjacent sides as perpendicular, she did not explicitly state or verbally indicate an understanding regarding Mrs. Taylor's stated goal of recognizing how this information can be used to identify the shape as a rectangle.

1	<i>Casey:</i>	Do we have to write the explanation for [inaudible]?
2	<i>Mrs. Taylor:</i>	Uh, sort of. Like, what is...you have to analyze, what does this mean, so
3		this is the writing part, right? So, here's your slopes, so 3 over 3 is equal
4		to what?
5	<i>Casey:</i>	1
6	<i>Mrs. Taylor:</i>	And -6 over 6 is equal to...?
7	<i>Casey:</i>	1
8	<i>Mrs. Taylor:</i>	Nope, -6 over 6. A negative divided by a positive is...?
9	<i>Casey:</i>	Negative?
10	<i>Mrs. Taylor:</i>	Negative. So this is...?
11	<i>Casey:</i>	-1
12	<i>Mrs. Taylor:</i>	-1. This is...?
13	<i>Casey:</i>	1
14	<i>Mrs. Taylor:</i>	This is...?
15	<i>Casey:</i>	-1
16	<i>Mrs. Taylor:</i>	Ok. So now you need to go into it and say, well opposite slopes are the
17		same, this is 1 and this is 1.
18	<i>Casey:</i>	Oh.
19	<i>Mrs. Taylor:</i>	This is 1 and this is 1. So those opposite sides are negative...? If the
20		slopes are the same, the opposite sides are...?
21	<i>Casey:</i>	Reciprocals?
22	<i>Mrs. Taylor:</i>	No. The slopes are the same, right? So that means that the opposite sides
23		are...?
24	<i>Casey:</i>	Congruent?
25	<i>Mrs. Taylor:</i>	No. What does it mean? If the slopes are the same, what does that mean
26		about the segments?
27	<i>Casey:</i>	[inaudible]
28	<i>Mrs. Taylor:</i>	No. If the slopes are equal...they're parallel. Opposite sides of a rectangle
29		are parallel, right? Not perpendicular, they're parallel, right? So, you need
30		to say that. Opposite sides are parallel because the slopes are the same.
31		Adjacent sides – this is 1 this is -1 – those are opposite reciprocals. So that
32		means the adjacent sides are what?
33	<i>Casey:</i>	Perpendicular?
34	<i>Mrs. Taylor:</i>	Perpendicular, and that's what makes it a rectangle. And the fact that all
35		the sides are not the same length means that it's not a square, so it's just a
36		rectangle. So that's the stuff that you have to write down.
37	<i>Casey:</i>	Ok.

Figure 5.1: Descriptive interaction with funneling questioning from Mrs. Taylor's instruction

This particular task was explanatory, meaning that the expectations of the task prompt involved open-ended student explanations. Despite this, Mrs. Taylor

repeatedly used one-word, closed questions with Casey and eventually explained the mathematical meaning related to those responses herself, which made this a descriptive interaction. Mrs. Taylor's description that "Opposite sides of a rectangle are parallel" (lines 28-29) and "the fact that all the sides are not the same length means that it's not a square, so it's just a rectangle" (lines 34-35) were also almost verbatim what students' written explanations stated on their poster that they then completed after this interaction (see Appendix D, Figure D.2). The similarity between Mrs. Taylor's spoken language during this interaction and students' subsequent writing could have suggested that Casey and her group members were not passive observers during this teacher-directed interaction, but rather were listening carefully to Mrs. Taylor's descriptive language. However, because this subsequent writing occurred after the conclusion of this interaction, any explicit connections between the spoken interaction and the subsequent student writing fall beyond the scope of this study.

What *could* be noted explicitly from this interaction was that, although she posed questions to Casey throughout their exchange, Mrs. Taylor was the dominant voice during this interaction. Each of her questions warranted (and received) a one-word response from Casey, and these questions did not provide an opportunity for Mrs. Taylor to further assess Casey's thinking or leave Casey with a question or wonderment about her next steps. Based upon this observation evidence and Mrs. Taylor's subsequent reflection on this interaction (see Chapter 6), it would appear that Mrs. Taylor was providing this feedback with more evaluative aims in mind.

5.1.2.2 Mrs. Taylor Interaction 2 –Questioning Diverted to Descriptive Interaction

This second interaction shows how assessing questions and advancing questions/statements rely not only on the utterance of the question or statement itself, but also on what follows that utterance. Therefore, problems can arise after the questions or statements are made by the teacher that diminish the intention of such questions. Like the previous example, this interaction also occurred during the final lesson that I observed with Mrs. Taylor’s class but involved a student (pseudonym Erika) in a different group that was tasked with identifying a quadrilateral as a parallelogram. For the assessing question, Mrs. Taylor interrupted Erika’s response to interpret the group’s writing herself. For the advancing statement, instead of Mrs. Taylor leaving to let Erika grapple with how best to proceed (Boston et al., 2017), Erika instead asked for immediate clarification from Mrs. Taylor, which she then provided. As such, this interaction highlighted two potential challenges with enacting student-directed interactions with students’ writing: teachers may not give students sufficient time to actually explain their thinking, and students may press the teacher to describe the next steps in revising their written work for them.

The transcript of this interaction is shown in Figure 5.2 and involves students’ written product of a poster “proof” (see Appendix D, Figure D.3 for the image of this poster as captured at the end of the lesson). Because the assessing question was not responded to by the student and the reply to the advancing statement was given by the teacher herself, this interaction was coded as descriptive in nature, with the task (the same as the first example from Mrs. Taylor’s classroom) being explanatory (inclusive).

1	<i>Mrs. Taylor:</i>	Um, so, you...just to clarify, right? So, you have the slopes and then
2		<i>*pointing at poster paper*</i> you said AB is parallel to DC and AD is
3		parallel to BC. How do you know they're parallel?
4	<i>Erika:</i>	Oh, because they're -
5	<i>Mrs. Taylor:</i>	<i>*cutting student off*</i> Ah, right? And then, just another little bit to say, how
6		do you know that AD is congruent to DC? Those should have line
7		segments over the top of them, also.
8	<i>Erika:</i>	Ok.
9	<i>Mrs. Taylor:</i>	Um, and it could just be because they have the same length, right? And
10		that's because the slopes are the same. So just, how do you know that
11		that's what that is? And, oh, you wrote "parallelogram" at the top. Ok.
12	<i>Erika:</i>	So, do all of these need line segments? Like, even these? Or just -
13	<i>Mrs. Taylor:</i>	Yeah <i>*pointing*</i> , so all of those should have line segments because if you
14		don't have the line segment over top it's the length of AB, and lengths are
15		equal. Line <i>segments</i> are congruent.
16	<i>Erika:</i>	Ok. That makes sense. Even on these?
17	<i>Mrs. Taylor:</i>	That you don't because that is the length of that segment.
18	<i>Erika:</i>	Ok.
19	<i>Mrs. Taylor:</i>	So, that's what that means. But when you say that they're congruent -
20	<i>Erika:</i>	Yeah.
21	<i>Mrs. Taylor:</i>	Then you need the line segment over the top. When you're talking about
22		the slope <i>*pointing*</i> you should have <i>m</i> - lower case <i>m</i> - and then AB, you
23		know what I mean? So, you write it like "slope of AB equals -5," "equals
24		1," "equals so on."
25	<i>Erika:</i>	Oh, ok.

Figure 5.2: Descriptive interaction with terminated questioning from Mrs. Taylor's instruction

At first, Mrs. Taylor appeared to ask an assessing question, saying "How do you know they're [the opposite sides] parallel?" (Figure 5.2, lines 2-3). However, just as Erika began to respond (line 3) Mrs. Taylor interrupted them with an additional assessing question about the justification for the congruence of the opposite sides (lines 5-6). Rather than providing time for Erika to respond, Mrs. Taylor moved on to an evaluative piece of feedback about how the group had written their line segments, stating that "Those should have line segments over the top of them" (lines 6-7). This

interaction appeared to be ending with an advancing statement, namely Mrs. Taylor providing the feedback that “if you don’t have the line segment over top it’s the length of AB, and lengths are equal. Line *segments* are congruent.” This advancing statement could have focused students on their written notation for the line segments and segment lengths because Mrs. Taylor had not evaluatively identified which parts of the written notation required the line segment symbol. However, as Mrs. Taylor turned away from the group and appeared ready to end the interaction, the student instead asked for evaluative feedback in the moment, asking “So, do all of these need line segments? Like, even these?” (line 12). At this point, Mrs. Taylor described which parts of the writing required the line segment symbol and which did not, leading to the ultimate classification of this interaction as descriptive.

Although the first interaction example illustrated Mrs. Taylor using a funneling pattern of questions (Herbel-Eisenmann & Breyfogle, 2005), this example shows some questions that otherwise might have been coded as assessing or advancing had Mrs. Taylor given Erika time to respond (for the former) or if she had left without first providing Erika with an immediate response to her open-ended question (for the latter). Because this interaction did not unfold in these ways, it was also a sort of liminal interaction. This example underscores the importance of the interactions that encompass such questions – it was not sufficient that the question was open-ended or *could* have served to assess or advance student thinking. Rather, the extent to which the teacher appeared to be shaping (van Es & Sherin, 2021) their noticing together with the student could have indicated the assessing quality of such questions, while the decision of when to conclude such an interaction (e.g., by walking away after asking an open-ended question/wonderment or after providing descriptive/evaluative

feedback) can have indicated whether the interaction served to advance students' own thinking about and revisions to their writing.

5.1.3 Summary – Mrs. Taylor's Interactions with Students' Writing

Overall, Mrs. Taylor enacted more student-directed interactions in her observed lessons compared to the other two teacher participants, and explanatory writing tasks were more prevalent task types of these interactions as well. This could have suggested (1) a relationship between the use of explanatory writing tasks and the opportunities for student-directed interactions, (2) Mrs. Taylor's experience as an educator in facilitating classroom discourse, or (3) Mrs. Taylor's stated focus on exploratory writing as a teaching tool. Additionally, Mrs. Taylor was observed enacting both evaluative and descriptive feedback as well as assessing and/or advancing questions across a variety of writing prompts. This could add credence to the idea that Mrs. Taylor, as a veteran math teacher, was able to flexibly interact with different types of students' writing in different ways.

However, the two interaction examples I have shared from Mrs. Taylor's observed lessons also indicated some ways that Mrs. Taylor's interactions did not always assess or advance students' thinking through questioning. The first example showed a funneling pattern of questioning (Herbel-Eisenmann & Breyfogle, 2005) that ultimately was more descriptive than inquisitive in nature. The second example showed how if (1) the teacher does not provide adequate time for a student to share their own thinking in response to an (otherwise) assessing question, or (2) they answer (otherwise) advancing questions in the moment rather than leaving and allowing students to grapple with such questions, then otherwise student-directed interactions can be diverted into more teacher-directed evaluative or descriptive interactions. Such

examples illustrated the importance of both the pattern of questions posed during these interactions and the amount of time afforded students to explain their thinking as important factors in whether students' responses can serve to help shape (van Es & Sherin, 2021) the teacher's noticings about the student's written work.

5.2 Case 2: Mrs. Hudson's Instruction and Interactions with Students' Writing

Mrs. Hudson's observed Algebra II lessons focused on the features of rational functions (for her first two lessons) and evaluating expressions with exponents (in her final two lessons). In this case, I first describe the overarching types of embedded tasks and interactions with students' writing observed in Mrs. Hudson's lessons, describing how many of Mrs. Hudson's observed interactions with students' writing occurred during her repeated use of a whole group "Shout it Out" activity. I also note the prevalence of interactions that occurred during descriptive (inclusive) writing tasks compared to descriptive (exclusive) tasks, suggesting how this may have been in line with Mrs. Hudson's stated focus on building student comfort through revision-focused rough draft writing opportunities. Finally, I provide two examples of interactions that Mrs. Hudson had with students' writing. The first example illustrates Mrs. Hudson's "Shout it Out" activity, which she had indicated in her interview was a key way in which she built students' comfort with writing and having their writing shared publicly (which she had noted as an important goal of writing in mathematics, see Chapter 4). The second example shows how Mrs. Hudson was asking an advancing question during a procedural/symbolic task that appeared in line with a subsequent explanatory writing task, again indicating the connections between such open-ended explanatory writing tasks and interactions with students' writing that might occur during preceding, closed-ended procedural/symbolic tasks.

5.2.1 Types of Embedded Tasks and Observed Interactions with Students' Writing

As shown in Table 5.2, most of Mrs. Hudson's interactions with student writing occurred with procedural/symbolic and explanatory (inclusive) tasks, and across both types of tasks her most common interactions with students were evaluative or descriptive in nature. Unlike the interactions typically seen in Mrs. Taylor and Mrs. Barnett's lessons (which were primarily conducted in small group or individual settings), many of Mrs. Hudson's interactions occurred in a whole group setting, often during her "Shout it Out" activities. During these activities, Mrs. Hudson would share students' written responses to different tasks and provide brief evaluative or descriptive feedback on those responses. The limited amount of assessing and advancing questioning occurring during these interactions may have been intentional, as Mrs. Hudson may have intended for this to be an opportunity to affirm students regarding their written work and to build student comfort with seeing their work shared and discussed whole class. Interpreting the interactions in this way would seem to align with Mrs. Hudson's interview responses where she discussed such whole group activities in similar terms.

Table 5.2: Types of Writing Tasks and Teacher Interactions with Student Writing, Mrs. Hudson

Interactions	Task Type					Total
	Procedural/ Symbolic	Exploratory	Informative	Explanatory (exclusive)	Explanatory (inclusive)	
Teacher-Directed	49% (35)	--	8% (6)	1% (1)	24% (17)	83% (59)
Evaluative	32% (23)	--	6% (4)	--	17% (12)	55% (39)
Descriptive	17% (12)	--	3% (2)	1% (1)	7% (5)	28% (20)
Student-Directed	10% (7)	--	--	1% (1)	4% (3)	17% (12)

Assessing Question(s)	7% (5)	--	--	--	3% (2)	10% (7)
Advancing Question(s)	3% (2)	--	--	1% (1)	1% (1)	6% (4)
Assessing + Advancing Questions	1% (1)	--	--	--	--	1% (1)
Total	61% (43)	--	8% (6)	3% (2)	28% (20)	100% (71)

Note. Results are rounded to the nearest percent.

It is also worth noticing that Mrs. Hudson’s interactions with students’ writing occurred largely during inclusive explanatory tasks, while interactions were only observed twice during exclusive explanatory tasks. Such tasks, which use hedging verbs and personal pronouns to position students as active participants in the living activity of mathematics (Herbel-Eisenmann, 2007; Rotman, 1988), might also have related to Mrs. Hudson’s stated emphasis on the rough draft nature of student writing and on building student comfort with their writing. The formatting of these inclusive explanatory tasks could have served to downplay the focus on correctness and instead emphasize the importance of sharing one’s own, evolving ideas about the mathematics topic.

5.2.2 Examples of Interactions with Students’ Writing, Mrs. Hudson

Both of these examples of interactions occurred during Mrs. Hudson’s third observed lesson. As previously noted, the first example occurred during a whole group “Shout it Out” activity, while the second example occurred during an interaction with an individual student during a procedural/symbolic task later in the lesson.

5.2.2.1 Mrs. Hudson Interactions 1 – Whole Group Writing Interactions

This example entailed a series of successive interactions that were chosen because they occurred during one of the whole group “Shout it Out” writing activities (which were observed in every one of Mrs. Hudson’s lessons), and because this series of interactions contrasted with the typical small group or individual interactions seen with the other two teacher participants. Additionally, Mrs. Hudson indicated in her pre-observation surveys for each observation that she saw the “Shout it Out” activity as being indicative of when student writing would occur during this lesson. Such interactions also aligned with Mrs. Hudson’s stated orientation toward writing, where she indicated her support of using such writing opportunities to gauge student understanding, build their comfort with writing (and the public display of their written work), and connect to opportunities for mathematical talk. Finally, this example was representative of how most of Mrs. Hudson’s evaluative and descriptive interactions arose in her observed lessons, as just over half (31 out of 60) of these occurred during such “Shout it Out” activities, while also showing how she would, at times, integrate assessing questions into such activities.

This series of interactions centered around students’ writing for the problem, $6^m \cdot 6^m \cdot 6^m = 6^{21}$ and focused on why $m = 7$ was a correct response while a combination of any three numbers would be incorrect. This problem was one of eight different problems involving multiplication and division of terms with exponents (see Appendix D, Figure D.4) that students completed concurrently for this task. Because students were prompted to “Be prepared to explain your reasoning” and specifically asked to “Explain Solution” when inputting their written responses in the SMART Board app, this was coded as an explanatory (inclusive) writing task.

As was typical for this type of activity in Mrs. Hudson’s class, she gave a mixture of evaluative and descriptive feedback to multiple students’ responses, displaying each in succession on the whiteboard (see Appendix D, Figure D.5). As shown in Figure 5.3, she first provided some brief evaluative feedback to a student’s response, noting that “I think they meant to put 3” (line 3). She also described one student’s response as “Good” (line 5). Such whole group evaluative feedback to students’ writing was typical during these activities and is reflected in the relatively large number of observed evaluative interactions from Mrs. Hudson’s lessons (Table 5.2).

1	<i>Mrs. Hudson:</i> So, let’s look at number 5, this one’s a little different. You’re multiplying
2	3 numbers. The good thing is that the bases were all the same, but let’s see
3	what we got for m . “I got $m = 7$ because I divided 21 by...” – I think they
4	meant to put 3 – “...and got 7 so m is equal to 7.” “once again adding, $7 +$
5	$7 + 7$ is 21.” “Divide the exponent of 21 by 3 and to get 7.” Good. Oh, let
6	me go back. “When integers are multiplied the exponents are added, so 21
7	divided by 3 is 7.” Question – why in this problem couldn’t I put, like, 7,
8	5, and 2 *Writes “ $7 + 5 + 2$ ” on SMART Board*. I’m sorry, not that.
9	*Erases “ $7 + 5 + 2$ ”* 7, let’s say, I don’t know, 9, and was there a 10 and
10	16 and...5. *Writes “ $7 + 9 + 5$ ” on SMART Board* Why can’t I do 7 and
11	9 and 5? They equal 21.
12	<i>Emory:</i> Because they’re all the same so they should be one.
13	<i>Mrs. Hudson:</i> Right. That’s the big piece I wanted you all to point out there – if they
14	were different variables, you could have any combination that would
15	create 21, but because they’re the same they were all 7s. Alright, so just be
16	careful when they’re different variables.

Figure 5.3: Whole group descriptive feedback during Mrs. Hudson’s instruction

However, these activities also gave Mrs. Hudson opportunities to provide descriptive feedback and, in this instance, pose an assessing question about one student’s written response. She publicly displayed a response which stated that “When

integers are multiplied the exponents are added” (line 6), and then asked “Why can’t I do 7 and 9 and 5? They equal 21” (i.e., why can’t the exponents be 7, 9, and 5 instead of them all being equal to 7?). After a student clarified that “they’re all the same” (line 12), Mrs. Hudson then offered some descriptive feedback about how, since the unknown exponents were all represented by the variable m , that they should all represent the same value (lines 13-16).

This example shows how Mrs. Hudson enacted teacher-directed interactions, but did so in ways that were still rooted in the public display of students’ writing. Although it was predominantly her voice that is driving these discussions in the moment, her focus was on the affirmation of and feedback regarding students’ written work. As such, Mrs. Hudson’s teacher-directed interactions with students’ writing might be seen as distinct from a more explicit, direct instruction model (Munter et al., 2015) because she was not relying on teacher modeling of how to solve these problems in ways that were not integrating her students’ thinking. Although her interactions were largely evaluative and descriptive here, they were still responsive to the written work that students had first produced.

5.2.2.2 Mrs. Hudson Interaction 2 – Advancing Statement

The next interaction presented below in Figure 5.4, from later in the same lesson, involved an individual exchange with a student (pseudonym Alexandra). This interaction demonstrated an advancing statement that, while occurring during a symbolic/procedural task, aligned with a future explanatory (inclusive) task with which students would soon be tasked. This interaction was also student initiated, showing how Mrs. Hudson used Alexandra’s natural curiosity about the pattern she

was seeing in her responses (“it never goes down below zero”) in a way that could potentially advance her thinking (and eventual written explanation) about that pattern.

Students during this task were working on a handout from the Illustrative Mathematics (2023) curriculum (see Appendix D, Figure D.6) that was designed to help them understand how 2^{-n} is equivalent to $1/2^n$ (e.g., $2^{-2} = 1/4 = 1/2^2$). Alexandra initiated this exchange as she had reached the items that were equivalent to values of two with a negative exponent ($\frac{2^5}{2^6}$ and $\frac{2^5}{2^7}$) and noticed a pattern with how “the answer goes down but it never goes down below that zero” (line 3).

1	<i>Alexandra:</i> Is this equal to negative 2? [inaudible] The answer is negative –
2	<i>Mrs. Hudson:</i> [crosstalk] Mm, think of that –
3	<i>Alexandra:</i> ...because the answer goes down but it never goes down below that zero.
4	<i>Mrs. Hudson:</i> Mm, look at that. Question – can you convert this decimal to a fraction
5	<i>*Pointing to where Alexandra had written “.25”*</i>
6	<i>Alexandra:</i> Uh, as a fraction?
7	<i>Mrs. Hudson:</i> .25 as a fraction.
8	<i>Alexandra:</i> Uh, 25 over 100.
9	<i>Mrs. Hudson:</i> Reduce it. Mm-hmm. So, if you look at one over four ($1/4$), and here you
10	have one over two ($1/2$), see if you can see what happens with those, what
11	happened there.

Figure 5.4: Advancing interaction during Mrs. Hudson’s instruction

In response to this, Mrs. Hudson provided some descriptive feedback to Alexandra’s written work, asking if she could convert where she had written “.25” into a fractional form (lines 4-7). She then provided an advancing statement when she told Alexandra that “if you look at one over four [$\frac{1}{4}$], and here you have one over two [$\frac{1}{2}$], see if you can see what happens with those” (lines 9-10). Based on Mrs. Hudson’s interpretation of this interaction (see Chapter 6), this interaction was meant to help Alexandra notice the relationship of 2^{-n} being equivalent to $1/2^n$, which students would

then be asked to explain in an explanatory writing prompt of “How does a negative exponent effect the value?” (see Appendix D, Figure D.7).

Because this subsequent prompt was explanatory in nature, this example could illustrate the potential for explanatory writing prompts to act as anchors from which students can reflect upon preceding symbolic/procedural writing tasks. Mrs. Hudson would have known that the lesson was advancing towards this open-ended explanatory writing task, and indeed she indicated in her post-observation interview that this progression of the lesson was why she was asking this sort of question in that moment. Although it is not possible to know how this interaction may have unfolded during a lesson without that subsequent explanatory writing task, the available evidence suggests that the inclusion of such tasks can serve as at least one of the available resources (in addition to, for instance, teachers’ mathematical knowledge for teaching; Ball et al., 2008) that may support teachers’ employment of questions that might advance students’ thinking towards the learning goal.

This interaction also provided an example where the teacher did not ask an assessing question, but their noticing of students’ understanding of their written work was still potentially shaped (van Es & Sherin, 2021) through discourse with the student in the moment. As previously noted, this interaction was student initiated, having been sparked by Alexandra asking Mrs. Hudson a question and describing her own noticings about the task at hand. Although my study classified different interactions with student writing based on the teachers’ actions (e.g., do they engage in assessing or advancing questions with the student about their writing or just evaluate or describe the writing), interactions like this reiterated the importance of both the teacher and the student as potential participants in the construction of teachers’

noticing. Mrs. Hudson had additional information available to her that could have strengthened her understanding of Alexandra's written work (and how she then responded to such noticing via an advancing question) because Alexandra initiated this interaction with a question of her own.

5.2.3 Summary – Mrs. Hudson's Interactions with Students' Writing

Mrs. Hudson was observed mostly enacting procedural/symbolic tasks and facilitating evaluative or descriptive interactions with students' writing, but she also appeared to see this as a step toward supporting students with taking intellectual risks in their future writing. When considering these results in the greater context of the data collected for this study (pre-observation and post-observation interview responses), it appeared that many of these interactions could have been related to Mrs. Hudson's stated goals of building students' comfort and familiarity with writing and having their writing shared publicly. Indeed, many of these interactions occurred during whole group "Shout it Out" activities that Mrs. Hudson had previously described as serving this affective goal.

The first example illustrated one of these "Shout it Out" activities, and showed how, although many of these sorts of interactions were coded as teacher-directed (i.e., evaluative or descriptive), the nature of the activity still was based upon students' own written work. Although Mrs. Hudson was the primary voice throughout these and other such interactions, the interactions were nevertheless built upon public displays of students' written work, rather than the teacher's own worked examples. This showed how the teacher-directed interactions as defined in this study still involve aspects of students' thinking because they are built off of student's written work.

The second example showed how more student-directed interactions that were observed to occur during procedural/symbolic writing tasks at times incorporated questions that were similar to the prompts of subsequent explanatory writing tasks in the same lessons. This suggested the potential value of including such tasks in lessons as a way to encourage more student-directed interactions which might also help connect students' thinking (and written work) on more procedural/symbolic aspects of a task to more open-ended explanatory goals of the lesson. This example also suggested how student-initiated interactions might also inform teachers' shaping (van Es & Sherin, 2021) amidst their noticing of and responses to students' written work.

5.3 Case 3: Mrs. Barnett's Instruction and Interactions with Students' Writing

Mrs. Barnett's observed Pre-Calculus lessons focused on different aspects of rational functions for the first three lessons, while the final lesson involved exponential and logarithmic functions. I first describe the overarching types of embedded tasks and interactions with students' writing observed in Mrs. Barnett's lessons, showing how – although she primarily employed procedural/symbolic tasks and used teacher-directed interactions – her abundance of descriptive interactions nonetheless indicated the value she placed on affirming students' thinking and written work. I also note Mrs. Barnett's use of exploratory writing tasks, which were rarely observed in the other two participants' classrooms.

After this, I provide two examples of interactions that Mrs. Barnett had with students' writing. The first example, which was a whole class interaction, demonstrates some similarities with Mrs. Hudson's "Shout it Out" activity, and it also provides an example of an interaction that occurred during one of the aforementioned exploratory writing tasks. Because this was a descriptive interaction where Mrs.

Barnett chose certain student responses to highlight and connect, it was also helpful in indicating what kind of responses Mrs. Barnett found most relevant to focus on in context of the lesson goals (an area which Mrs. Barnett expanded upon in reflecting on this interaction during her video viewing session, see Chapter 6). The second example illustrates an “assessing + descriptive” interaction, where Mrs. Barnett took the lead in the interaction after presumably assessing that the student was confused. This example is therefore helpful in understanding the sort of conditions that might lead a teacher like Mrs. Barnett to shift away from shaping (van Es & Sherin, 2021) their noticing of student thinking and move onto using judicious telling (J. P. Smith, 1996) with a struggling student.

5.3.1 Types of Embedded Tasks and Observed Interactions with Students’ Writing

Mrs. Barnett’s interactions with student writing during her four observed lessons (Table 5.3) showed a variety both in the type of interaction and the type of writing task in which the interactions took place. In addition to providing evaluative feedback to students’ writing, Mrs. Barnett also appeared to emphasize the mathematical understanding (or potential of such understanding) that she saw and sought in students’ written work. This can first be seen with descriptive responses having been the most common interaction (typically with procedural/symbolic tasks). Such results show how, although most of Mrs. Barnett’s interactions were teacher-directed, she was more frequently providing students with some descriptive explanation about her evaluation of their writing (rather than an evaluation alone). Additionally, there were six examples of Mrs. Barnett using different combinations of assessing and/or advancing questions with procedural/symbolic tasks, showing how

she was able to also engage the students in explaining or reflecting on the mathematical meaning behind what they had written themselves even when the task prompts were more closed ended or procedural/symbolic in nature.

Table 5.3: Types of Writing Tasks and Teacher Interactions with Student Writing, Mrs. Barnett

Interaction	Task Types					Total
	Procedural/ Symbolic	Exploratory	Informative	Explanatory (exclusive)	Explanatory (inclusive)	
Teacher-Directed	65% (37)	4% (2)	9% (5)	4% (2)	--	81% (46)
Evaluative	18% (10)	--	9% (5)	2% (1)	--	28% (16)
Descriptive	47% (27)	4% (2)	--	2% (1)	--	53% (30)
Student-Directed	11% (6)	9% (5)	--	--	--	19% (11)
Assessing Question(s)	7% (4)	5% (3)	--	--	--	12% (7)
Advancing Question(s)	2% (1)	4% (2)	--	--	--	5% (3)
Assessing + Advancing Questions	2% (1)	--	--	--	--	2% (1)
Total	75% (43)	12% (7)	9% (5)	4% (2)	--	100% (57)

Note. Results are rounded to the nearest percent.

Mrs. Barnett also helped students engage in exploratory types of writing and often spent her interactions with such tasks helping students to draw out their own thinking about such activities. Although exploratory writing tasks only made up a small amount of the interactions observed, these were where nearly half of Mrs. Barnett's assessing and advancing questions with students took place, and no evaluative interactions were observed during such tasks. Thus, Mrs. Barnett seemed to

be using these exploratory writing tasks as they might be intended – to push student thinking and reflection about how they are representing mathematical ideas through writing.

In contrast to what is seen with Mrs. Barnett's interactions with students during her exploratory writing tasks, it is also worth noting that the majority of her interactions involved procedural/symbolic tasks, and that most of these were evaluative or descriptive interactions. This shows how that most of the writing work taking place in Mrs. Barnett's classroom during these observations was focused on evaluating numeric or algebraic expressions with closed-ended prompts. Indeed, informative and explanatory tasks were only observed during her first lesson.

Additionally, it is worth noting that, in the limited examples where informative and explanatory tasks were used, Mrs. Barnett's interactions with students' writing were evaluative and (in one instance) descriptive in nature. Both tasks arose at the end of the first lesson as students were describing and explaining their understanding of domain and range based on the lesson in which they had just participated, and Mrs. Barnett's interactions with students' writing in this regard occurred when she was displaying student responses to these tasks as a whole group activity at the conclusion of the lesson. As such, in the few instances where students were prompted to describe or explain their mathematical thinking through the written word, this was enacted as a summative activity with limited, evaluative feedback from Mrs. Barnett to these responses.

5.3.2 Examples of Interactions with Students' Writing, Mrs. Barnett

The following two examples represent some of the interactions that Mrs. Barnett had with student writing during her final observed lesson. The first example is

a whole-group interaction that occurred during an exploratory writing task near the beginning of the lesson, and the second example is an individual interaction that occurred during a procedural/symbolic task later that same lesson.

5.3.2.1 Mrs. Barnett Interaction 1 – Choosing Responses for Descriptive Feedback

In this first interaction, which came from an exploratory writing warm-up activity for her fourth observed lesson, Mrs. Barnett engaged students in a noticing and wondering routine, or at least gave students an opportunity to notice the inverse relationship between an exponential and logarithmic function (see Appendix D, Figure D.8). This interaction was chosen in part because it contrasts with the sort of whole class “Shout it Out” activity from Mrs. Hudson’s classroom, because it is an interaction during an exploratory writing task (which was almost exclusively seen in only Mrs. Barnett’s instruction), and because it demonstrated Mrs. Barnett focusing her descriptive feedback on particular strands of student responses. Thus, this sort of interaction was helpful in indicating approaches to engaging with student writing more specific to Mrs. Barnett’s instruction and in providing a potential window into Mrs. Barnett’s thinking about what sort of student writing to elevate in such situations.

In contrast to Mrs. Hudson’s approach of providing several evaluative or descriptive pieces of feedback to students’ written work, this whole group descriptive interaction (transcribed in Figure 5.5) was different in that Mrs. Barnett revoiced several students’ written responses before providing a single descriptive piece of feedback that summarized the important ideas that she had interpreted across student responses. Although there may be an assumption that she was positively evaluating student responses simply by choosing which ones to call out, she did not initially

provide any explicit evaluative feedback. Rather, she appeared to choose responses that referenced the reflective or inverse nature of the logarithmic and exponential functions, while also restating some students who had instead focused on “the behavior of these curves” (lines 4-5) and even joking about one student who had signed their response as “jazzyboi” (lines 7-8). Although a majority of the responses fell into the first category, she did seem to be restating a variety of students’ written work. This activity therefore appeared to show an instance of how teachers can notice *within* instances of students’ work and can draw from *across* multiple instances of students’ work to facilitate classroom discourse and support student understanding of important mathematics (Stockero et al., 2017).

1	<i>Mrs. Barnett:</i> So, friends, looking at this first graph, some of us say “whoa, they’re
2	reflections of each other across a line in the middle of the two functions.”
3	Ooh, I wonder if a lot of people said that too. “Mirrored over the line of $y = x$.”
4	Ooh, someone even said that they’re inverse functions. A lot of you
5	are talking about the behavior of these curves, like “one of the functions is
6	increasing while the other is decreasing, however they never intersect one
7	another.” How interesting. A lot of you just said that they are inverses or
8	they’re reflections, there is a line of symmetry. Some of us are signing our
9	answers <i>*laughs*</i> . But that’s very good – these are key ideas to notice,
10	friends, that these two functions, they are symmetrical to one another
11	about this line. This line is $y = x$, and it’s true about any function and its
12	inverse that the graph is going to be symmetrical over the line $y = x$.”

Figure 5.5: Noticing activity descriptive summary during Mrs. Barnett’s instruction

Given that these student responses all referenced observable features of the provided graph, and the prompt was simply “What do you notice about these functions,” every student’s written statement was an understandable and appropriate response to the task. Mrs. Barnett’s overarching references to different student

responses might then be understood as an affirmation of student participation in the activity, even though there were particular noticings that she then chose to emphasize in her descriptive response. She noted that the two functions being “symmetrical to one another” across $y = x$, and this being “true about any function and its inverse,” were both “key ideas to notice” in the task (lines 10-12). Thus, although she appeared to be preferencing certain student responses in this descriptive interaction, other students who had offered different noticings were still affirmed in their thinking. Just as the “Shout it Out” activity from Mrs. Hudson’s classroom seemed to address both the perceived correctness of students’ responses as well as affirming their engagement in the task, this interaction in Mrs. Barnett’s classroom also appeared to be attending to both of those aspects as well. Therefore, this sort of interaction, while teacher-directed in who was speaking, was still demonstrating the value in students’ participation in the writing activity.

5.3.2.2 Mrs. Barnett Interaction 2 – Redirecting to Descriptive Feedback after Assessing Question

This second interaction, which also took place during the fourth observed lesson, involved Mrs. Barnett and an individual student, pseudonym Jeri. This interaction showed how Mrs. Barnett used an assessing question and, presumably responding to Jeri’s confusion, shifted towards more descriptive feedback. In this way, this example exhibited at least some characteristics of judicious telling (J. P. Smith, 1996), when Mrs. Barnett based her descriptive feedback on her noticing as shaped (van Es & Sherin, 2021) by Jeri’s gestured response. However, this example did not exhibit the cyclical nature of such telling (see E. P. Smith et al., 2023), which would likely have entailed Mrs. Barnett using subsequent assessing questions to probe what

Jeri had taken away from her feedback, or an assessing question that would have allowed Jeri to build off of the feedback.

Occurring approximately halfway through the lesson, this interaction arose after students had been solving for given variables with logarithmic equations. For instance, students immediately prior to this task had been prompted to “Solve for x : $\log_4 x = 6$ ” on the Desmos activity platform, with an expected solution of “4096” (see Appendix D, Figure D.9). However, Jeri had uniquely responded to this prompt with an interesting written solution of “ $\log = 4^6 = x$,” which indicated some confusion about what “log” meant in the context of the task. Mrs. Barnett had also noticed (as confirmed in her second interview and detailed further in Chapter 6) that Jeri then appeared to be using a “guess and check” strategy for the subsequent prompt of “Solve for x : $5^x = 244140625$.” This could have also indicated that, although Jeri understood how to solve a logarithmic equation for which the argument is unknown (e.g., “ $\log_4 x = 6$ ”) by calculating the solution to 4^6 , she may not have understood how to then use logarithmic form to identify unknown exponents in similar equations.

Mrs. Barnett initiated an individual interaction with Jeri’s writing during this task (Figure 5.6). Mrs. Barnett first affirmed Jeri’s guess-and-check approach (lines 1-5), and then asked an assessing question of how Jeri could alternatively write out the equation “in logarithmic form” (lines 5-7). At this point Jeri gave Mrs. Barnett a confused look and shook her head back and forth, presumably indicating that she was not actually sure how to rewrite the initial problem in logarithmic form. This gestured response from Jeri was naturally limited by its brevity and nonverbal nature and could have warranted further assessing questions to elicit some spoken response from Jeri. However, Mrs. Barnett at this point shifted towards a descriptive response,

demonstrating to Jeri how to use the Desmos graphing calculator to rewrite the problem in logarithmic form (lines 9-19).

1	<i>Mrs. Barnett:</i> Jeri, I saw you had an interesting strategy.
2	<i>Jeri:</i> Who, me?
3	<i>Mrs. Barnett:</i> Mm-hmm, I saw you typing in some values for that one, here, five to the –
4	<i>Jeri:</i> Oh, yeah, I was just using my brain.
5	<i>Mrs. Barnett:</i> <i>*Chuckles*</i> I saw some strategy. I wonder if there is a more direct route,
6	though, so we could just write the logarithmic form of that, right? If we
7	have the five to the x equals that big number, in logarithmic form, how
8	could we write that?
9	<i>Jeri:</i> <i>*Looks at Mrs. Barnett, makes a confused face, and shakes her head ‘no’*</i>
10	<i>Mrs. Barnett:</i> Well, let’s start with log, how about that? So type in “ <i>l-o-g.</i> ” Now, the
11	base of the log is going to be the same as the base of the exponent, and in
12	this case it’s five. I see you have the subscript, that’s great. And now you
13	type in a five. Now, the log base five is gonna – if you take the value that
14	we’re equal to, that 244 million. So, I’ll say the numbers for you. Hit the
15	right arrow. 244,140,625. And that tells you exactly what power I have to
16	raise five to to get that big number. Oh, sorry that’s the previous slide, that
17	was the five that – but, can you go back to your Desmos window? So now,
18	in the input box below, I really just want to prove it to you, can you type
19	five to the twelfth power? And see, you get that same number? So that
20	will help you figure out what the exponent is, just do log of the base of
	whatever the value you’re equal to.
	<i>Jeri:</i> Oh, okay.

Figure 5.6: Descriptive shift after assessing question in Mrs. Barnett’s class

Compared to the liminal interactions from Mrs. Taylor’s instruction, which were liminal in that Mrs. Taylor’s questions did not allow for students to explain their thinking, this interaction might also be considered liminal in how quickly Mrs. Barnett moved from an assessing question to descriptive feedback. While more student-directed types of judicious telling around students’ writing would entail eliciting student feedback about how they might revise their own written work (E. P. Smith et al., 2023), Mrs. Barnett shifted from Jeri’s gestured confusion about her assessing

question to immediately providing descriptive feedback. Although Jeri’s response may have informed what Mrs. Barnett was noticing about her written work, the briefness of the response may have limited the extent to which Mrs. Barnett was able to use it to actually shape (van Es & Sherin, 2021) her noticing. The response may have told Mrs. Barnett that Jeri was struggling to understand how to write the logarithmic form, but not why. By shifting to descriptive feedback, Mrs. Barnett’s subsequent conclusion of this interaction then also did not allow her to unpack why Jeri might have been confused either. This limitation could make this sort of student-directed assessing + descriptive interaction also liminal in nature.

Although the effectiveness of such a “redirected” liminal interaction is beyond the scope of this study, it is also worth noting that Jeri continued to struggle with subsequent tasks in this lesson. As shown in one of these tasks (Appendix D, Figure D.10), Jeri was unique among her peers in writing a solution to an unknown argument of a logarithmic equation (e.g., $\log_6 x = 2$) in the form of $\log_a b = c$ as $\log a^c = b$ (i.e., Jeri’s written response to “Solve for x : $\log_6 x = 2$ ” was “ $\log 6^2 = x$ ”). Thus, she was adding “log” to the beginning of what would otherwise be a correct solution to the problems, potentially in accordance with Mrs. Barnett having previously told her to “start with log” in the descriptive interaction with her strategy for the previous problem. Such evidence from student artifacts – similar to that seen in the first interaction from Mrs. Taylor’s lesson described earlier in this chapter – suggests the need for future research that might consider how the language used by the teacher during these interactions might relate to students’ subsequent writing.

5.3.3 Summary – Mrs. Barnett’s Interactions with Students’ Writing

The overall interactions observed in Mrs. Barnett’s classroom were largely evaluative and descriptive and occurred during tasks that were largely procedural/symbolic. However, Mrs. Barnett also used exploratory writing tasks more frequently than was observed in the other teachers’ classrooms, and almost half of her student-directed interactions occurred during such tasks. Although there were too few of such interactions (5) to draw any concrete conclusions, it could be that such exploratory writing tasks allow for more shaping (van Es & Sherin, 2021) opportunities given that they, like explanatory tasks, are open-ended in nature.

The first example from Mrs. Barnett’s classroom described an interaction that occurred during an exploratory writing task and showed how Mrs. Barnett facilitated a whole-class discussion of students writing – a helpful parallel to the “Shout it Out” example from Mrs. Hudson’s classroom. The second example, on the other hand, showed an interaction that Mrs. Taylor had with her student Jeri, and illustrated a liminal “assessing + descriptive” type of interaction that began with the teacher probing the students’ thinking before shifting to more teacher-directed, descriptive feedback. This example was showed how this type of interaction, while involving an assessing question, could still ultimately have limited opportunities for students’ thinking to shape (van Es & Sherin, 2021) the teachers’ noticing of their written work.

5.4 Cross Case Analysis: Teachers’ Interactions with Students’ Writing

In this section I describe some salient themes that arose when looking across these different cases. I begin by suggesting how the use of informative, explanatory, and potentially exploratory tasks might have supported these teachers in enacting

student-directed interactions by providing explicit prompts which could serve as assessing or advancing questions. Then, I describe how the teachers were still able to enact a variety of interactions with students' writing across a range of writing tasks, indicating the importance of the teachers experience as a facilitator of classroom discourse regardless of the type of task at hand. Finally, I note the preponderance of teacher-directed interactions in these data, and suggest some concerns about the limited observation of interactions that included both assessing and advancing questions.

5.4.1 Informative and Explanatory Writing Tasks and Student-Directed Interactions

From these observed lessons, it appeared that informative and explanatory tasks (both exclusive and inclusive) potentially allowed for more student-directed interactions (i.e., interactions that include assessing and/or advancing questions/statements posed to students) compared to procedural/symbolic tasks. Although assessing and advancing questions were also observed during exploratory tasks, these types of interactions occurred at much lower frequency (if at all) across the participants' observed lessons, limiting cross case analysis of such tasks. As previously indicated, Mrs. Taylor's observed lessons included approximately twice as many explanatory task interactions (or more) compared to the other two participating teachers (43 interactions compared to 22 with Mrs. Hudson and two with Mrs. Barnett), as well as about twice as many student-directed interactions (20 interactions compared to 12 with Mrs. Hudson and 11 with Mrs. Barnett). On top of this, 75% (15 out of 20) of such student-directed interactions occurred during such explanatory tasks during Mrs. Taylor's observed lessons.

But what can we make of the student-directed interactions that occurred during other types of tasks across all of the teachers' lessons? Nearly half of these interactions (7 out of 16) aligned with the prompts of subsequent informative or explanatory tasks within these lessons, although they did not occur during those informative or explanatory writing tasks (see Table 5.4). In other words, many of the assessing and advancing questions that teachers asked during procedural/symbolic tasks seemed to be priming students for some upcoming informative or explanatory writing that followed the procedural/symbolic tasks. Therefore, informative and explanatory writing prompts may have supported opportunities for the teachers to enact assessing and advancing questions by providing an explicit thread connecting the procedural/symbolic work to the sort of student thinking prompted in the subsequent informative or explanatory writing. Alternatively, informative and explanatory writing tasks may have been more formal opportunities to assess students' understanding in ways similar to assessing and advancing questions. Exploratory tasks may also have played a similar role in these regards, but given their scarcity in the lessons I observed in this study, I do not address this type of task here.

Table 5.4: Procedural/Symbolic Interactions and Subsequent Informative/Explanatory Prompts

Teacher and Lesson	Procedural/Symbolic Task Prompt	Assessing or Advancing Questions/Statements Asked During Procedural/Symbolic Task	Subsequent Informative/Explanatory Writing Prompt
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Mrs. Taylor Lesson 4	“Use slopes and lengths to determine the type of quadrilateral $ABCD$ ”	Assessing question: “...if you have to make them [the sides of a square] the same length, what did you <i>do</i> to make them the same length?” Assessing question: “Where’d this come from?” * <i>pointing to a side length of the given shape that a student wrote</i> *	“How can you create sides that are the same length?” “How can you create parallel lines?” “How can you create sides that are perpendicular?”
Mrs. Hudson Lesson 2	“If the speed of the river is r , write an expression for the time it takes to travel 5 kilometers upstream and an expression for the time it takes to travel 10 kilometers downstream.”	Assessing question: “Where’s the 10 coming from, everybody? Where’d they get 10 from? ...That’s the distance. That’s how far they’re going, they’re going 10 kilometers, right? But something happened to their speed. What happened to it?”	“Use your expressions to calculate the speed of the river. Explain or show your reasoning.”
Mrs. Hudson Lesson 3	“Use exponent rules to write each expression as a single power of 2. Find the value of the expression. Record these in the table.”	Advancing statement: “1 over 4 [$\frac{1}{4}$]. See if you can find a pattern from our top value [$\frac{2^5}{2^1} = 2^4 = 16$] to the bottom [$\frac{2^5}{2^7} = 2^{-2} = \frac{1}{4}$]. See if you can find a pattern there.” Advancing statement: “So if you look at 1 over 4 [$\frac{1}{4}$] – and here you have 1 over 2 [$\frac{1}{2}$] – see if you can see what happens with those, what happened there.”	"How does a negative exponent effect the value?"

Mrs. Barnett Lesson 1	"Find the domain of the following rational functions."	Assessing question: "How do you know these answers? I'm curious? (student mentions setting denominator equal to zero but not finding any real number solutions) Oh, so there's no real number that makes this equal to zero?"	"How do we find the vertical asymptotes of a rational function? How does this relate to the domain?"
		Assessing + advancing questions: (Student asks if their solution is correct) "I'm curious about your process?" (Student describes setting the denominator to zero but finding no real number solutions) "You know, $x^2 + 8x + 12$ [denominator of a given rational function]? I'm pretty sure that there's some value of x that might make that equal to zero. How might we figure out - I know you're kind of just subbing in numbers there, and maybe that's more direct because there's just the two, but I think we might be able to find some values of x to make that 0."	

The examples from Table 5.4 show how, even though students were working on procedural/symbolic tasks, the teachers were at times asking assessing and/or advancing questions and statements that were similar to upcoming explanatory writing prompts. For instance, even as students were initially tasked with just using slope and lengths to classify a given quadrilateral (a procedural/symbolic task), Mrs. Taylor was asking them assessing questions to explain their solutions. These questions were asking students to explain what they would do to determine the side length of a shape and to show that two sides of a shape are of equal length, which was the focus of a subsequent explanatory writing prompt.

Similar connections arose in the other two participants' classrooms. For instance, in Mrs. Warren's third observed lesson, students were engaged in a

procedural/symbolic task of converting expressions involving two risen to decreasing powers (e.g., 2^2 , 2^1 , 2^0 , 2^{-1} , 2^{-2} , as shown in Appendix D, Figure D.6). She asked several assessing questions and advancing statements that aligned with a subsequent explanatory (exclusive) writing task of “How does a negative exponent effect the value [of a term]?” Her interaction with one student (Alexandra), described in detail previously (Mrs. Hudson Interaction 2 – Advancing Statement), included the advancing statement “1 over 4 $[\frac{1}{4}]$. See if you can find a pattern from our top value $[\frac{2^5}{2^1} = 2^4 = 16]$ to the bottom $[\frac{2^5}{2^7} = 2^{-2} = \frac{1}{4}]$. See if you can find a pattern there.” Similarly, an earlier interaction that Mrs. Warren has with a different student’s writing includes an advancing statement of “So if you look at 1 over 4 $[\frac{1}{4}]$ – and here you have 1 over 2 $[\frac{1}{2}]$ – see if you can see what happens with those, what happened there.” In both instances, the advancing questions appeared in line with the later explanatory (exclusive) task prompt.

In Mrs. Barnett’s first observed lesson, when students were working on a procedural/symbolic task of “Find the domain of the following rational functions,” Mrs. Barnett also asked specific questions that aligned with a later explanatory (exclusive) prompt of “How do we find the vertical asymptotes of a rational function? How does this relate to the domain?” Twice during the procedural/symbolic task Mrs. Barnett assessed students’ understanding of how to identify exceptions to the domain by setting the denominator equal to zero (i.e., identifying vertical asymptotes). These sorts of assessing questions were then formally taken up with that later explanatory (exclusive) writing task. In this way, the later explanatory (exclusive) task seemingly served as a way to assess *all* students’ understanding of this topic, while the initial

assessing questions during the procedural/symbolic task operated as a more formative assessment of particular students' understandings.

These examples could have indicated a couple of different relationships between the tasks within a lesson and the types of interactions that teachers had with students' writing. First, they could have shown how informative and explanatory writing tasks can support teachers' noticing during earlier procedural/symbolic tasks. Because teachers would have recognized the progression of the lesson towards these later tasks, the prompts of those later tasks could have informed the sorts of questions that they asked students about their writing. Because of the more open-ended nature of such prompts compared to procedural/symbolic tasks (i.e., students may have some variety in the language that they use to describe or explain their thinking), having those prompts in mind could have encouraged teachers' questioning to also be more open-ended (e.g., assessing questions rather than a funneling pattern of questions; Herbel-Eisenmann & Breyfogle 2005).

Second, these examples could have shown how informative and explanatory writing tasks can serve as a way to assess all students in a class about their thinking regarding the lesson topic. Given that teachers' skill at professional noticing can improve with experience (Jacobs et al., 2010), and given that these teacher participants had between nine and 30 years of experience, they may have simply posed these assessing and advancing questions because of their own understanding of the coursework. They may have probed students' thinking in these open-ended ways because they recognized important underlying conceptual understandings behind the procedural/symbolic task that could be drawn out through such discourse. With this reading, the subsequent informative or explanatory tasks could more so have been

considered a means for formally drawing out these understandings from all students in the class through a written prompt.

Of course, these two conjectures are not mutually exclusive. Having explicit informative or explanatory writing tasks within a lesson may have served as a way to orient teachers' interactions with students during earlier procedural/symbolic tasks. Having informative or explanatory tasks within a lesson may also have served to more formally assess students' descriptions or explanations regarding a lesson topic after they have engaged with more procedural/symbolic work. In both instances, both the content of the writing tasks used in the classroom and the teachers' skill at using such tasks (or their own knowledge and experience) to guide their discourse with students would have been important factors.

5.4.2 Teacher Facilitation and Student-Directed Interactions

However, even if some of the observed student-directed interactions that occurred during procedural/symbolic tasks (or both of the two such interactions observed in Mrs. Taylor's case) were influenced by subsequent informative or explanatory writing prompts, there were still other interactions from Mrs. Hudson and Mrs. Barnett's lessons that did not appear directly connected to such prompts – or which arose during lessons that *had* no explanatory writing prompts. This further bolstered the idea that the teacher plays an important role as a facilitator of classroom discourse and relies on more than just an understanding of the lesson's tasks when deciding when and how to interact with students' written work. Informative or explanatory writing tasks may have supported student-directed questioning opportunities, but a range of other variables might also have supported (or hindered) such questioning approaches, including the lesson's learning goals, the teachers'

content expertise, their noticing skills, and their orientations toward writing and discourse in mathematics teaching and learning.

For many of these potential variables that may have impacted the types and frequency of interactions with students' writing, it is beneficial to understand the teachers' interpretation of their own instruction. This is a key area of focus in Chapter 6, in which I investigate teachers' described interpretations of students' written work and recordings of such interactions. However, even my interpretation of the observation data itself indicated that the teachers appeared to be using student-directed questions during procedural/symbolic tasks as a way to make visible potential misconceptions about the lesson's focus areas and to draw out student explanations about their procedural written work. Essentially, although the tasks at hand may have been closed-ended and procedurally focused, these participating teachers still engaged in discourse with students through open-ended questions about their written work. This showed how the teachers could transform or extend a task beyond the explicit procedural/symbolic prompt by asking students to describe, explain, or justify their thinking.

Some of these results also showed how the teachers were potentially facilitating whole class interactions to affirm students' writing and build their self-efficacy towards writing in math. Table 5.5 shows two such examples from the preceding interactions described in this chapter – a series of interactions during a “Shout it Out” activity in Mrs. Hudson's classroom and an interaction that occurred during an exploratory writing task in Mrs. Barnett's classroom. In both of these examples, samples of students' written work was publicly displayed by the teacher and used as the basis for a whole class discussion. It appeared that teachers were using

these whole class interactions to affirm student effort with their writing and to build connections to the big ideas of the lesson across different samples of students' writing. This interpretation was bolstered by teacher recollections of their thinking during these tasks, further described in Chapter 6. Therefore, although many of these whole group interactions may have been teacher-directed, they appeared designed to strengthen student ownership of the learning and self-efficacy around their own writing.

Table 5.5: Teachers' Facilitation with Students' Writing in Whole Class Settings

Teacher and Example Interaction	What this example shows regarding teacher facilitation with students' writing in whole class settings
Mrs. Hudson Interaction 1 – Whole Group Writing Interactions (Section 5.2.2.1)	Mrs. Hudson noted how this “Shout it Out” activity allowed her to give quick feedback to either affirm students or make them consider how to revise their responses, thus building students' comfort with the public display of their writing and pushing them to focus less on correctness. In this example, she enacted a variety of interactions (evaluative, descriptive, assessing question) in line with these described goals.
Mrs. Barnett Interaction 1 – Choosing Responses for Descriptive Feedback (Section 5.3.2.1)	Mrs. Barnett began this interaction by restating (rather than evaluating) a range of student responses. She then affirmed this range of responses (“these are key ideas to notice, friends,” Figure 5.6, lines 8-9), indicating the value of students' written work.

5.4.3 Liminal Interactions and the Preponderance of Teacher-Focused Interactions

Although the above examples show how student-focused interactions involving assessing and advancing questions were observed across different writing task types, ultimately the majority of observed teacher interactions with student

writing were evaluative or descriptive in nature. The summary of all three participants' observed interactions is shown below in Table 5.6. This is not a surprising outcome for several reasons. First, the most common types of writing tasks were procedural/symbolic, so it is understandable that feedback would be more descriptive or evaluative on problems that have a singular solution or anticipated strategy to solve. Second, if the teacher is understood as more than a perpetual questioner, but rather as a facilitator who strategically engaged in judicious telling (E. P. Smith et al., 2023; J. P. Smith, 1996), then evaluative and especially descriptive interactions would be expected at various points across all types of writing tasks.

Table 5.6: Types of Tasks and Teacher Interactions with Student Writing, All Participants

Interactions	Task Type					Total
	Procedural/ Symbolic	Exploratory	Informative	Explanatory (exclusive)	Explanatory (inclusive)	
Teacher-Directed	43% (81)	1% (2)	8% (15)	8% (16)	17% (32)	78% (147)
Evaluative	21% (39)	--	6% (12)	3% (6)	10% (19)	40% (76)
Descriptive	22% (42)	1% (2)	2% (3)	5% (10)	7% (13)	38% (71)
Student-Directed	9% (16)	4% (6)	1% (2)	4% (7)	6% (12)	22% (42)
Assessing Question(s)	6% (11)	2% (3)	0% (1)	1% (2)	3% (6)	12% (22)
Advancing Question(s)	2% (3)	2% (3)	--	2% (3)	2% (4)	7% (13)
Assessing + Advancing Questions	1% (2)	--	0% (1)	1% (2)	1% (2)	4% (7)
Total	51% (97)	4% (8)	9% (17)	12% (23)	23% (44)	100% (189)

Note. Results are rounded to the nearest percent.

Given that descriptive interactions – when the teacher is providing some reasoning behind their evaluation – were almost as commonly observed as evaluative interactions (76 versus 71 interactions, respectively), this may in fact bolster the idea that teachers were focused on supporting students’ mathematical understanding rather than simply the “correctness” of their answers. However, the examples provided throughout the preceding three cases also demonstrate what might be considered “liminal interactions” between teacher- and student-focused interactions, where teachers either engaged in patterns of questioning that were not open-ended in nature, or where their employment of assessing and/or advancing questions was interrupted or diverted to more teacher-focused types of interactions. Therefore, if an instructional goal exists to shift these interactions to be more student-focused – which teacher participants stated was the case in their reflections as described further in Chapter 6 – the circumstances of these liminal interactions may be valuable in indicating surmountable barriers holding back such student-directed interactions.

The use of funneling patterns of questions (Herbel-Eisenmann & Breyfogle, 2005) was observed in both Mrs. Taylor’s interactions and Mrs. Barnett’s interactions. In each of these situations, the teacher used closed-ended questioning to funnel students towards a specific revision in their writing. In the former example, Mrs. Taylor funneled a student towards the idea that two adjacent sides of a shape that have opposite slopes (e.g., 1 and -1) will be perpendicular, while in the second example Mrs. Barnett funneled a student towards the idea that graphical representations of functions can be observed to determine the limits of the function (e.g., if the solution of the function is observed getting “closer” to 0 as x increases, we can assume that $\lim_{x \rightarrow \infty} f(x) = 0$). Had the teachers instead used a focusing pattern of questions, which

“requires the teacher to listen to students’ responses and guide them based on what the students are thinking rather than how the teacher would solve the problem” (Herbel-Eisenmann & Breyfogle, 2005, p. 486), it seems likely that these interactions could have been coded as being more student-directed (i.e., assessing and/or advancing questions would have likely arisen). The explanation of how these examples showed a liminal interaction with students’ writing is further described below in Table 5.7.

Table 5.7: Examples and Explanations of Liminal Interactions

Teacher and Example Interaction	Explanation of how this example shows a liminal interaction
Mrs. Taylor Interaction 1 – Use of Funneling Questioning (Section 5.1.2.1)	Although Mrs. Taylor used questioning during this interaction, these questions were closed ended and appeared focused on funneling (Herbel-Eisenmann & Breyfogle, 2005) students towards a predetermined answer rather than shaping (van Es & Sherin, 2021) Mrs. Taylor’s understanding of the students’ written work.
Mrs. Taylor Interaction 2 – Questioning Diverted to Descriptive Interaction (Section 5.1.2.2)	Although Mrs. Taylor asked an open-ended question that could have been used to further shape (van Es & Sherin, 2021) her understanding of the students’ written work, she did not give the student an opportunity to respond to the question, instead moving on to describe other features that she noticed in the students’ writing.
Mrs. Barnett Interaction 2 – Redirecting to Descriptive Feedback after Assessing Question (Section 5.3.2.2)	Although Mrs. Barnett asked a question that was classified as an assessing question during this interaction, after the student gestured her confusion about how to answer that question, Mrs. Barnett redirected this exchange into a descriptive interaction. The limited feedback that she received from the student after the assessing question (a gesture of confusion) may have provided few actual indications about the students’ understanding of their written work beyond the general confusion.

The initiation of assessing and/or advancing questions that were then diverted towards more teacher-directed interactions were also observed in these observations. Such examples demonstrate how questioning students about their written work requires sufficient wait time and genuine listening from the teacher in order to assess (and then respond to) the student's understanding. For instance, Mrs. Taylor was at one point observed stating an assessing question but then moving into descriptive feedback without giving the student an opportunity to respond (see Mrs. Taylor Interaction 2 – Questioning Diverted to Descriptive Interaction section of this chapter). Mrs. Taylor initially asked the student “How do you know they’re [the two sides of a given quadrilateral] parallel?” (Figure 5.2, lines 2-3), but then – seemingly noticing other aspects of the students’ written work – moved on to providing descriptive feedback before the student could answer the initial question. Had Mrs. Taylor lingered on the initial piece of writing and her initial question with the student, the student may have been able to provide some explanation about their understanding, thus allowing for this to have been classified as an assessing question.

However, even when teacher participants were observed using assessing or advancing questions, they were rarely seen employing both during a single interaction. The actual patterns of questioning in any given interaction thus rarely attended to both assessing student's understanding and leaving students with a question or wonderment to inform their ongoing writing. Across all of the observed lessons, there were 23 *assessing + descriptive* interactions, 13 *descriptive + advancing* interactions, and only seven *assessing + descriptive + advancing* interactions (five of which came from Mrs. Taylor's lessons). Essentially, when assessing questions were asked, these were typically followed with descriptive feedback but not an advancing question. When

advancing questions were asked, these typically were preceded by the teacher's own interpretation of what the student had written (rather than the student explaining their thinking after being asked by the teacher via an assessing question). Therefore, even most of these otherwise student-directed interactions might be seen as liminal themselves, in that the interactions either shifted towards teacher descriptive feedback or arose from teacher's own reading of the student's written work. Rarely were students given the space to both explain their writing and to be left with some open-ended question or wonderment to inform their own subsequent writing (or written revisions).

5.5 Summary: Types of Writing Tasks and Teacher Interactions with Students' Writing

These findings illustrate the ways that different types of tasks and different types of teacher interactions with students' writing arose across these three cases. Ultimately, the findings show how teacher-directed interactions (evaluative and descriptive) were more prevalent in these teachers' observed lessons than student-directed interactions (those involving assessing and or advancing questions). However, the prevalence of both student-directed interactions and explanatory writing task interactions in Mrs. Taylor's lessons indicated that there could be a connection between these two aspects of writing tasks. Additionally, about half of student-directed interactions across all classrooms that arose during procedural/symbolic tasks seemed to be in line with subsequent informative or explanatory writing tasks. These results indicated that informative or explanatory writing tasks may help support teachers in enacting more student-directed interactions with students' writing both during those types of tasks and even during preceding procedural/symbolic tasks.

The results also showed that the teachers played a critical role in facilitating classroom discourse during these interactions. Teachers enacted assessing and advancing questions, or student-directed interactions, across different types of writing tasks, even those that were primarily procedural/symbolic, or tasks that would seem likely to not elicit student-centered interactions. Potentially aided by subsequent informative or explanatory writing tasks or by their own noticing skill, teachers were able to engage students in explaining and advancing their thinking around their written work even with tasks with prompts that were closed-ended and procedural/symbolic in nature. Therefore, the teachers appeared to be crucial players in transforming or extending procedural/symbolic tasks, and in enacting student-directed interactions across all types of writing tasks.

However, the preponderance of teacher-directed interactions was also a key finding from the classroom observations. In some of these interactions, the teachers did appear to be attending to student ownership over the content or self-efficacy by publicly displaying and providing feedback on students' writing in a whole class setting. Many teacher-directed interactions were also liminal in nature, in that they incorporated some form of student questioning, but not in ways that incorporated assessing and advancing questions (Boston et al., 2017), which would lean toward more student-directed interactions. Indeed, interactions that made use of both assessing *and* advancing questions were only observed in seven out of 189 interactions. During these teachers' lessons, teachers were frequently observed interacting to students' own written work, but often in ways that made only partial use of the students' thinking. These mixed findings suggest areas of focus – the types of writing tasks enacted in class and the types of questioning used when interacting with

students' written work – that could afford opportunities for more student-directed interactions with students' writing.

Chapter 6

RESULTS: TEACHERS' INTERPRETATIONS OF INSTRUCTIONAL ARTIFACTS

In this chapter, I address my third research question: How do these teachers' interpretations of instructional artifacts (samples of students' writing and recorded teacher interactions with students' writing) align with or diverge from their described orientation toward writing in mathematics? Results indicate that teachers' interpretations of their students' writing and their interactions with that writing fall largely in line with certain aspects of their previously described orientations toward writing in mathematics. In particular, the purposes of writing in mathematics that each teacher had previously described (see Table 4.4 and Table 4.5) were apparent in each of the participants' reflections during their post-observation interviews. Additionally, each participant provided some level of self-critique that centered around a desire for greater student ownership of such writing, including how the students operate as active participants during teacher interactions with their writing.

To probe the participants about such reflections, each teacher was asked to engage in a professional noticing (Jacobs et al., 2010; Sherin et al., 2011) activity regarding written artifacts from one of their observed lessons. They then each participated in a video viewing session (Erickson, 2007) where they recalled and interpreted at least two of their interactions with students' writing. When possible, the video viewing represented an interaction related to the written artifact that they were asked to interpret in this interview. Each teacher's interpretation of the student

artifacts is discussed in the context of how such interpretations appear to align or diverge from their described orientations.

6.1 Case 1: Mrs. Taylor's Interpretations of Instructional Artifacts

As described previously in Chapter 5, Mrs. Taylor's fourth observed lesson had tasked students with creating a poster to "coordinate proofs to determine the type of quadrilateral they were given." Students were given coordinates of the four vertices of a quadrilateral and asked to use that information to determine the classification of the quadrilateral (e.g., parallelogram, square). Their final written product was a poster that was meant to show their results and their explanations (or "proof") of how they determined their quadrilateral's classification. In this interview, Mrs. Taylor was asked to reflect on the pictures of these student posters, and to interpret two interactions that she had with students during this lesson.

Mrs. Taylor's noticings appeared to align with her described orientation toward writing (see Chapter 4). Below I will show evidence that illustrates how she centered her responses on the perceived correctness and precision of language in students' explanatory writing, while also indicating how she saw the procedural/symbolic tasks of finding side length and slope for the sides of the given quadrilateral as more inherent to the mathematical goals of the lesson and distinct from the explanatory writing. Additionally, her spoken reflections during this interview regarding the recorded interactions also indicated her focus and perceived challenges during these instructional moments, with a stated emphasis on balancing between student ownership of reasoning with or revising their writing versus supporting struggling students. These responses aligned with the challenges that she had previously described regarding writing in mathematics, namely that providing adequate feedback

to students takes time, and that students often mimic the teacher in their writing rather than actually explaining their own thinking. These responses also showed that Mrs. Taylor was acutely aware of the more teacher-directed elements of these interactions and that her intended or idealized approach to such interactions was more student-directed than what was captured in these recordings.

6.1.1 The Correctness and Precision in Students' Explanatory Writing

I coded Mrs. Taylor's poster lesson as first involving procedural/symbolic writing tasks (i.e., calculating slopes and side lengths of shapes with given coordinates) followed by explanatory writing tasks (i.e., explaining or showing reasoning about the classification of the quadrilateral based on those features), and the final poster products that Mrs. Taylor interpreted represented a combination of these different types of tasks. Examples of these posters can be found in Figure D.2, Figure D.3, Figure D.14, and Figure D.15 of Appendix D.

In her interview, Mrs. Taylor focused on the extent to which she felt that the students' written work clearly explained their previous procedural/symbolic work. For instance, she noted that "I kind of like this one [referring to the group poster in Appendix D, Figure D.3] because they went and identified, you know? Every time they came up with something, they told me what it meant." She contrasted this with the written poster from another group (Appendix D, Figure D.11), stating that "I remember this one had some mistakes in what they wrote here, just some vocabulary – 'Our side lengths are opposite reciprocals.' Yeah. No." Such responses were in line with Mrs. Taylor's described orientation toward writing as a way for students to be "able to use the, I don't wanna say algorithm really, but [students] being able to show their work in coming up with an answer," as well as her previous description that

students “need to reflect more in writing on what it was that they did” in class. Her interpretations of the students’ written work appeared to also emphasize such connections between students’ procedural/symbolic solutions and their explanatory writing.

Mrs. Taylor also reflected on her own recollection of teaching during the lesson, pulling out that she had wanted students to “show their work” clearly, stating that students “did a good job with it [creating the poster]. Some didn’t show their work as much as I would’ve liked them to show their work. I really would have preferred that and I kind of did tell them that.” She also recounted her push that students not only justify the classification of quadrilateral their shape would be, but also what types it would *not* be (see Appendix D, Figure D.2), recounting that:

I said, “Sometimes you have to tell me what it’s *not*, because if it’s a rectangle and all you’re doing is telling me that the opposite sides are parallel, that doesn’t make it a rectangle. So, once you’ve got a rectangle, then you need to show me that it’s not a square.” So, a lot of them did that in their work, because they also told me what it wasn’t because of what their information told me.

When pressed about the sort of feedback she would provide to the students about their posters, Mrs. Taylor did seem to address some of the more procedural/symbolic elements of the lesson, if only to frame them as potentially supporting the later written explanations that students were prompted to provide. These responses are also in line with Mrs. Taylor’s described orientation toward writing in math as being a way for students to explain “how did you get from A to B.” In essence, explanatory writing is understood as an extension of students’ more symbolic/procedural work, and can serve as an assessment of that work. Mrs. Taylor noted, for instance, that some of the student groups “got full credit because they did a

fabulous job showing me their work and everything like that,” but that she still provided written feedback asking those students to “make sure you’re using the correct vocabulary” and “make sure you’re showing me how you came up with those answers.” She expanded on this contrast between the symbolic/procedural and explanatory types of writing tasks that composed this lesson, stating that “this is actually a common thing that I’ve seen in these kind of processes – they [students] are using what they know about a parallelogram and shoving it in their proof, even though they didn’t prove it, right?” She continued, stating:

I would tell them [the students who constructed the poster in Appendix D, Figure D.2], “Well, how was it shown in your work?” Right? They just said, “It’s shown in my work,” but they didn’t tell me what does that mean, as far as any of this. They didn’t simplify these to even show, right? So, the sides are not all the same lengths, making the shape a rectangle, not a square. So, the adjacent sides are perpendicular as shown in the slopes and the length, so the length doesn’t show perpendicular anyway. So that would probably be the things that I would tell them. And that’s kind of where I was trying to get them to do is like simplify this.

In this way, Mrs. Taylor was focused on both the correctness of students’ responses as well as the precision of their written justifications for their quadrilateral classification.

6.1.2 Student Mimicry of Descriptive Feedback

After reviewing students’ written artifacts, Mrs. Taylor was shown a video clip of the interaction as transcribed in Figure 5.1, which was enacted with the group creating the poster shown in Appendix D, Figure D.2, and involved what I considered to be a funneling (Herbel-Eisenmann & Breyfogle, 2005) pattern of questions. Mrs. Taylor’s noticings focused on how students’ final written poster product contained language nearly identical to what she stated during this interaction, suggesting that students might have “mimicked” her spoken descriptive talk, rather than writing out a

genuine justification of their own. She also stated some potential questions that she could have asked students during this interaction to have avoided that mimicry. In this way, Mrs. Taylor was making connections between her spoken language (both actual and idealized) and students' subsequent written work.

Mrs. Taylor first described how she was trying to push students during this interaction to explain what their calculations about slopes of the quadrilateral meant in context (i.e., that because opposite sides had equal slopes it meant that those sides were parallel to one another). However, reflecting on the finished written product (Appendix D, Figure D.2), she noted that the students “did the work correctly, but they did not analyze it really in any way, other than in their heads. I mean, they said that it was parallel, but they didn't write as to why it was parallel.” She therefore saw students as not fully achieving her goals for this interaction.

When probed about what she meant by “this is the writing part” (Figure 5.1, lines 2-3), Mrs. Taylor referenced some of the challenges with supporting student writing that she had previously described in her initial interview, namely that students will mimic writing what they are being told to write without actually thinking about writing as their own justification. She described this issue as follows:

So, like I just said, they had done the work, so this is the writing part. This is how you need to analyze what it was that you did here. And so, after we went through all of that, to say that the sides are parallel and those sides are perpendicular, so that makes it a rectangle, that kind of almost seems to me that they just mimicked what I said without finishing with justifying their analysis. “Yeah, these are parallel because that's what my slope said,” but what does that mean? They didn't say “because the slopes are equal,” right? “These are perpendicular because the slopes and the length,” like I said, length doesn't show perpendicularity, slopes do. So, what was it about the slopes that showed that they are [equal]? and [I] went over that, but that - and this is another thing that you see often, which is why I said, I need

to do this more often - is that being able to pick this out and say, this is what this says, what does it mean? And being able to say, “so the lines are parallel.” No, you need to say “the lines are parallel because the slopes are equal.”

Mrs. Taylor’s noticings here suggested that she saw her descriptive feedback as leading to student mimicry, and that she had some idealized questions regarding students’ explanatory writing – questions that appeared in line with the qualifications of assessing or advancing questions. For instance, I see her question of “what is it about the slopes that showed that they are [equal]?” as a potential way to advance student thinking beyond the focus of Mrs. Taylor’s descriptive feedback that “If the slopes are equal...they’re parallel” (Figure 5.1, line 26). She expanded on this line of potential questioning with “this is what this says, what does it mean?” in response to “the lines are parallel,” which I see as an open-ended question could serve to assess students’ understanding of that written response in the moment or could serve to advance students towards more precise written justifications. Thus, these idealized questions could have served as either assessing or advancing questions had they actually been stated during the interaction.

Mrs. Taylor’s spoken reflections on this interaction showed that she saw limitations in her descriptive feedback in that students appeared to mimic such language – but not advance beyond it – in their subsequent writing. Her reflections also indicated that she saw some potential questions that could have served to better assess student understanding and advance their thinking (and writing) beyond what she ultimately saw in their written work. Mrs. Taylor not only noticed the descriptive nature of her interaction with student writing, but voiced potential assessing or advancing questions that she felt might have aligned with her intended learning goals behind this task.

The contrasts that Mrs. Taylor made between her actual and idealized responses showed how the use of both artifacts of students' writing and recorded teacher interactions with that writing could be beneficial for building teachers' skills at noticing. Noticing includes components of how teachers interpret and then decide how to respond to students' understandings (Jacobs et al., 2010; van Es & Sherin, 2002), and Mrs. Taylor's responses here seemed to draw out her focus on at least these components. Her suggestion of potential assessing or advancing questions also suggested her desire for students to play an active role in shaping (van Es & Sherin, 2021) her noticing and subsequent response to what she notices.

6.1.3 Noticing of Liminal Interactions

For her second video viewing, Mrs. Taylor watched the interaction captured in Figure 5.2, which included several instances of what could be considered "terminated" assessing and advancing question opportunities. Mrs. Taylor's termination of those questions had led me to describe the interaction as "liminal," as she had asked open-ended questions (they could have worked as assessing and advancing questions) but did not give the student an opportunity to actually answer or engage with those questions. Mrs. Taylor's responses again suggested that the intentions behind her interactions with explanatory writing tasks like those in this lesson were more student-centered in nature, and that she saw this liminal interaction as an attempt to have the student reason with and revise their written work. Her responses spoke to an underlying perceived challenge of such interactions, namely the balance that she was hoping to strike between guiding the student towards the task goals without explicitly telling the student what to do. Although this liminal interaction was ultimately descriptive in nature (i.e., teacher-directed), Mrs. Taylor's reflections show how her

recollected focus emphasized the sorts of questions that – if stated during the interaction – might have been coded as assessing or advancing questions. Therefore, even though she ultimately provided descriptive feedback to the students about their writing during this interaction, her intentions centered on students using that feedback as a starting point to engage in further explanatory writing on their own.

Compared to the previous interaction, Mrs. Taylor had a more positive reflection on these students' written work, while again emphasizing her focus on using the written word to explain or justify an aspect of mathematical "analysis." She noted the following:

They [the students] have that they [the opposite sides of the quadrilateral] are parallel, [but] how do you know that they're parallel? And that's what I was trying to get all of them to say, "Why is it that they're parallel?", "Why is it that they're perpendicular or not perpendicular?", "How is it that they are congruent or not congruent?" And so they were doing a really good job of doing all of that analysis, but they just needed to incorporate these pieces as in – "How do you know that these are parallel and these are parallel and these are congruent?" – which they did add in it [the final written artifact shown in Appendix D, Figure D.3].

Specifically, Mrs. Taylor reflected on how she was trying to guide students to think about their next steps without telling them exactly what should be written. She mentioned that "I'm going through my head – 'How is it that I can tell them what they need to do without telling them what they need to do?'" This reflection suggested that she hoped to give students opportunities to do the work and thinking, but she was challenged by the need to give them direction so that they would know what she expected.

These responses were notable given the descriptive nature of the interaction and the terminated instances of both assessing and advancing questions from my

analysis of how she interacted with students' writing in the observations of her teaching. Mrs. Taylor's responses in this interview made clear that she was grappling with the balance between teacher and student ownership of the thinking, and when and how she might judiciously tell (J. P. Smith, 1996) students pertinent information without explicitly telling them what writing to produce. This in-the-moment decision making was exemplified through Mrs. Taylor's use of questioning, Erika's posed clarification questions, and ultimately Mrs. Taylor's descriptive response (see Figure 5.2). Although Mrs. Taylor ultimately terminated her assessing and advancing questions and provided descriptive feedback to Erika, her recollected attempt of telling students "what they need to do without telling them what they need to do" was in line with the definition of advancing questions or at least a more student-directed process of reasoning with and revising their writing.

Mrs. Taylor's responses were also again in line with her described orientations around writing from her first interview. Her description of writing as a tool to support student reasoning in mathematics from her first interview seemed especially relevant (see Figure 4.5), as she had described her focus on "pushing them [students] into doing that, thinking themselves, because I think that writing part helps them to process the thinking." Mrs. Taylor's responses regarding this interaction emphasized similar goals, while also demonstrating the perceived challenge in pushing students to share their own thinking through writing.

6.2 Case 2: Mrs. Hudson's Interpretations of Instructional Artifacts

Mrs. Hudson's second interview focused on her interpretations of student writing (and interactions with student writing) from her third observed lesson, which was chosen in part because there were no assessing and/or advancing question

interactions observed in her fourth lesson for her to reflect upon. In this lesson, Mrs. Hudson focused students on solving equations using properties of exponents. This lesson involved a number of activities that involved student writing, including several instances of tasks where students were prompted to provide both procedural/symbolic and explanatory written responses about both solutions to problems involving operations with exponents and their thinking in how they arrived at such solutions. Mrs. Taylor was first asked to interpret on captured student written work (Appendix D, Figure D.4) that she used during a series of whole group explanatory interactions (see Figure 5.3 and Appendix D, Figure D.5). She was then asked to interpret students' solutions to a later task (Appendix D, Figure D.7) that had an initial small group worksheet component (Appendix D, Figure D.6) and a subsequent whole group element focused on rewriting terms with negative exponents. Mrs. Hudson was then asked to interpret two recordings of interactions that she had with student writing – one during the whole group activity aligned with the first artifact, and one during the small group part of the task aligned with the second artifact.

Mrs. Hudson's responses largely reflected her described orientation toward writing from her first interview, especially her emphasis on building student comfort and self-efficacy around writing in math. As described below, this connection was perhaps most apparent in this interview when she stated her intent behind these tasks as building student comfort towards writing in math class. This was also apparent in how she described seeking opportunities to affirm students about their writing, regardless of the perceived precision or correctness of that writing. Indeed, while Mrs. Hudson mostly reflected positively on the recorded interactions from this lesson, her main self-critique was around how she would have liked to have incorporated

students' own writing and thinking even more rather than providing her own descriptive feedback in response to student struggle. Such spoken reflections indicated that Mrs. Hudson deeply valued student ownership and voice in their writing and saw opportunities to expand on this aspect even more in her teaching.

6.2.1 Building Student Comfort with Publicly Sharing Their Thinking

The first artifact (Appendix D, Figure D.4) involved a set of multiplication problems where students were solving for unknown exponents (e.g., $3^b * 3^7 = 3^{11}$) and explaining their reasoning. Mrs. Hudson spoke to the rough draft nature of the task, which was in line with her described orientation of writing in mathematics as being a way to get students comfortable with sharing their thinking through low risk writing activities. For instance, she described how some students used “u” instead of “you” in their responses, but that she was “okay with that” because students were “at least being able to get a rough idea of what’s going on and then transitioning – it seems to be a good start.” She stated that she:

really likes this type of stuff, just [for] the students [to] be able to verbalize their thoughts or write out their thoughts and reasoning in a way that makes sense to them, and then we transition into some vocab that applies to what they're saying.

This also indicated a writing-to-learn orientation, as Mrs. Hudson was framing writing as a sense-making activity that could then be beneficial towards later emphasis on precision of student descriptions about their reasoning.

Mrs. Hudson also described specifically how students shared when they were unsure of their solution, which she appreciated as it indicated to her that students were confident enough to publicly share even when they were not fully confident in their work. For instance, she reacted to a student who wrote that they were “not sure if the

exponents here multiply or add since it's still multiplication technically, but it doesn't have a coefficient so I don't know," describing how she was "okay with things like that because they still are talking about what they are thinking about in parts that are making them confused, and we may be able to clarify it." She concluded by naming how "the confidence to just be able to speak out and try or put out what they're thinking is really big for me." This aligns with her described orientation from her first interview, where she recounted building student comfort with writing and shifting their focus on whether they are "correct or not" to instead being willing to "see what I have to say" during writing activities.

When asked about the sort of feedback she would like to provide students with regards to their written responses shown in the first artifact, Mrs. Hudson described how she "may push some of the students who only gave examples to put it into words and explain that with their vocab." She wanted "those who show the math [the numeric solution] to maybe translate that math into words." When pressed on *why* she would push that sort of feedback, Mrs. Hudson explained that "sometimes just translating it [students' procedural work and solution] into words helps with the math part of it," which suggested that these activities were meant to not only build students' comfort with having their writing displayed and critiqued in a public manner, but to ultimately strengthen students' communication of mathematical ideas.

With the second artifact, where students were asked to describe the pattern then noticed when converting numbers with negative exponents (e.g., finding that $2^5/2^7 = 2^{-2} = 1/4$), Mrs. Hudson reflected on how students were using mathematical vocabulary as they understood it, and how she again appreciated students' use of such language even when it was not a precise or fully accurate description. Promoting

students' comfort with sharing their rough draft thinking again appeared to be a driving factor in Mrs. Hudson's reflections on this written work. Specifically, she talked about how some students described the solution when numbers have negative exponents as being "the inverse of the number," and while Mrs. Hudson stated how she "pushed the word 'reciprocal' later on," she also thought overnight about how students kept referencing "inverse" in their responses, and the following day brought that vocabulary up during a subsequent lesson. She described how "just being able for them to even bring in vocab into their thought process was really good," and that "I wanted to credit them for that, so that's why the next day I said something, so they felt comfortable and confident about it." Student comfort and self-efficacy were thus more important factors in her approach.

Mrs. Hudson continued her reflection on her follow-up to students' use of the word "inverse" versus "reciprocal" in the following lesson and described how she actually came around to students' use of language through that conversation. This was an interesting example of a negotiation of language between the teacher and the students that developed from students' initial rough draft writing, where Mrs. Hudson was curious about students' understandings of written language and willing to recognize the way that students had described the reciprocal as a (multiplicative) inverse. She described how their conversation was focused on building a "consensus" about what a negative exponent "does" to a term, and that "we decided using inverse was more appropriate in that case here, to understand what that negative exponent is doing, but to also understand that it may not always be a value less than one, depending on the locations and different things like that." She said how her "push" was to get at "how the negative exponents affect the values" and also "being able to

understand the complexity of it too, as a numerator or a denominator, and changing in between the two.” Thus, her emphasis in this multiple day exchange with students centered not on the wholesale revision of students’ written (and then spoken) descriptions of how negative exponents operate, but rather ensuring that the language that they used was in line with her larger learning goals for the topic.

6.2.2 Student-Directed Interactions During “Shout it Out” Activity

For her first video viewing, Mrs. Hudson watched the interaction transcribed in Figure 5.3, which was around students’ writing about the problem, $6^m \cdot 6^m \cdot 6^m = 6^{21}$ and focused on why $m = 7$ was the desired response (rather than a combination of any three numbers for the unknown exponents). Her responses again underscored her emphasis on using whole group writing opportunities to build student comfort with sharing their thinking publicly (i.e., the “Shout it Out” activities that she described in her first interview), while also showing how such writing tasks helped her to assess student reasoning in line with her lesson goals.

When she described her interpretation of this interaction (see also Appendix D, Figure D.4 and Figure D.5), Mrs. Hudson noted how this type of activity was common in her instruction, and thus students had grown more comfortable to engage in such public displays of writing. She reflected on how:

most of them [students] are used to shouting out or saying their thoughts in this platform just because they know it's gonna be shared or they can check in, it's a good way to check in to see if they're on the right page. They don't have to feel like they'll be criticized by responding, so I don't think this is any different than a typical day to be able to share out their ideas and thoughts and some misconceptions that come out in this platform.

Such a reflection is again in line with Mrs. Hudson's previous descriptions of her orientation toward writing as a rough draft learning opportunity (i.e., writing-to-learn), as she saw the public display of students writing as a way for students to "check in" on their understanding. She also hinted at a perceived barrier to students' engagement with such writing – namely, their initial discomfort with sharing their developing conceptions in a public space with their peers.

Mrs. Hudson described additional intentions she had behind this interaction, stating how she used her assessing question to gauge student reasoning regarding a potential student misconception that could arise through this problem. She admitted that she "thought that there would be students at this question that were going to give me different combinations of [numbers with a sum of] 21," but when that didn't happen, she decided "to point it out in case they saw something similar again and thought that maybe it could be any combination if we add." In particular, she took a student's written explanation of "When integers are multiplied the exponents are added, so 21 divided by 3 is 7" (Figure 5.3, lines 5-6) in order to still address the conditions under which the unknown exponents would or would not have equal values. Therefore, when the anticipated student misconception did not explicitly arise in students' writing, Mrs. Hudson described how a student-directed interaction was used as a way for her to assess students' understanding. While the results from Chapter 5 described *how* Mrs. Hudson used both teacher- and student-directed interactions during this activity, Mrs. Hudson's responses here provided further context about *why* she recounted enacting a student-directed assessing question during the activity.

6.2.3 Advancing Questions without Assessing Questions

For her second video viewing, Mrs. Hudson described how, when noticing a descriptive + advancing question interaction, she would have ideally interacted with the student's writing in a more student-centered way throughout the interaction (e.g., assessing questions). She reflected on her individual exchange with Alexandra transcribed in Figure 5.4. In this interaction, Mrs. Hudson pushed Alexandra (via descriptive feedback) to "convert this decimal [0.5 and 0.25, obtained from converting 2^{-1} and 2^{-2} , respectively] to a fraction" (Figure 5.4, line 4). When Alexandra did so (resulting in $\frac{1}{2}$ and $\frac{1}{4}$), Mrs. Hudson left her with an advancing statement to "see if you can see what happens with those, what happened here" (Figure 5.4, lines 9-10). As such, this interaction was student-centered because of its inclusion of an advancing statement but also did not include any assessing questions.

Mrs. Hudson's reflection in this interview showed both her desire to value student voice and her recognition that she might not have fully achieved that goal in this particular interaction. She stated how Alexandra "did notice that it was not going below zero, that it was getting smaller and smaller, which was really good there," but also that:

I just don't think she was seeing a pattern with her decimals. So, I wanted her to convert them to a fraction to see if maybe she could see a pattern a little easier there, but I wanted her to have multiple ways to kind of see the pattern.

Mrs. Hudson commented that "I feel like maybe I could have pushed what she had versus pushing her somewhere else...Like pushing, maybe, to find a pattern at the decimal value versus encouraging the fraction," and that "I kind of do that [pushing a student in a different direction] thinking that it might be easier for the kid and it might not always be." This is notable because, although this was an advancing interaction

because of the question with which she leaves Alexandra to consider (Figure 5.4, lines 9-10), it had not been preceded by any assessing questions (only a closed question about converting 0.25 into decimal form).

Through her description, Mrs. Hudson seemed to acknowledge how an interaction that would have included more assessing questions and used those as the basis for the advancing question might have better honored Alexandra's initial written contributions. This idealized interaction appeared more in line with the concept of judicious telling (J. P. Smith, 1996), especially with regards to the recommendations by Lobato et al. (2005) that such moments of judicious telling be initiated as a way to promote student inquiry and elicit students' own interpretations of their work. Mrs. Hudson also appeared to be idealizing the assessing + response + advancing type of interaction, which was only observed seven times out of the 189 interactions captured across all teachers' classrooms, and only one time out of the 71 interactions captured from Mrs. Hudson's lessons (see Table 5.2).

6.3 Case 3: Mrs. Barnett's Interpretations of Instructional Artifacts

In Mrs. Barnett's second interview she was asked to describe her interpretation of students' writing captured from her fourth observed lesson, along with two recorded interactions she had with students' writing during the lesson. Mrs. Barnett's reflections centered largely on the extent to which this lesson focused on procedural fluency versus conceptual understanding. In particular, she discussed her regret over not incorporating more explanatory writing prompts – or at least more assessing questions – that could have extended the more procedural/symbolic aspects of the lesson. This was especially apparent in her reflections of the interactions with student writing, where in both instances she expressed a desire (and even provided some examples) for

how she might have provided more assessing questions to help uncover students' mathematical reasoning behind their written work. This seemed similar to Mrs. Barnett's responses in her first interview, where she stated that learners have "the right to say what makes sense" to them and to communicate "what [they are] thinking and what's making sense to [them]." Mrs. Barnett's responses during her second interview addressed this idea of learners' right to communicate through writing.

Mrs. Barnett described the learning goal in her fourth observed lesson to be "Using the inverse relationship between exponential and logarithmic functions to solve equations involving the two functions. The artifacts used for Mrs. Barnett's interview included student explanatory writing responses (written in the Desmos activity builder) from a warm-up activity (Appendix D, Figure D.8) and student solutions to several tasks from the lesson involving the solving of logarithmic equations (e.g., "Solve for x : $\log_6 x = 2$ "), shown in Appendix D, Figure D.9 and Figure D.10. The interactions transcribed in Figure 5.6 and Figure 5.7 occurred as students engaged with the two tasks from which the writing artifacts were drawn.

6.3.1 Desire for Student-Directed Interactions

Mrs. Barnett's interpretations of samples of student writing and her instruction related to that writing showed a focus on the rough draft (Jansen, 2020) potential of such writing, where she was curious about ways that she might further assess or advance students' thinking beyond what they had written (i.e., the shaping stage of teacher noticing; van Es & Sherin, 2021). She first described a desire to have, in hindsight, planned a more open-ended exploratory writing task that might have allowed for deeper reasoning around the lesson topic. She also reflected on how, with a subsequent procedural/symbolic task, she might have revised the questions to be

more explanatory in nature or could have at least asked questions to get at students' understanding of their work (i.e., assessing questions). Together, such responses show the value that Mrs. Barnett perceived for idealized explanatory writing tasks, or at least assessing and advancing questions that she might have used to extend beyond the written prompt of a procedural/symbolic task.

When asked to interpret a sample of students' written work [from a "What do you notice" task from the beginning of the lesson where students were shown that the logarithmic function $f(x) = \log_b(x)$ is the inverse of the exponential function $g(x) = b^x$, see Appendix D, Figure D.8], Mrs. Barnett indicated that the student responses were in line with her intentions with enacting this task, although the responses still represented emerging conceptions of the lesson topic. She noticed how students had picked up on the inverse relationship between the logarithmic and exponential functions, naming how some students described the $y = x$ line as a "line of symmetry" and how some even said that "they're inverse functions." She also noticed how one student was drawing connections to previous lessons, summarizing their response as "I remember this graph from a function. I believe there is a hole or undefined point at either x or y equals to zero." However, she also stated that "this [the task] is still like an introduction part" that "was early in this lesson." She also shared that:

I'm harsh on myself. Okay. I feel like there was a moderate understanding and I still – I do struggle between the conceptual understanding versus procedural fluency. And I think initially I tried to demonstrate to them that these are inverse functions.

Mrs. Barnett went on to indicate that she could have potentially used specific coordinates to help emphasize the inverse relationship between to functions, which could have then allowed for a discussion about "why those graphs have this symmetry over the line $y = x$." This showed how Mrs. Barnett preferred more student

explanatory writing (i.e., *why* the functions have symmetry rather than just stating that they are inverse functions), or at least revisions to the task that could have afforded her better opportunities to assess students' thinking.

The second artifact Mrs. Barnett reviewed involved students' written work from several subsequent procedural/symbolic tasks in the lesson (Appendix D, Figure D.9 and Figure D.10). In describing her interpretation of these artifacts, Mrs. Barnett continued to delve into the procedural versus conceptual focus of such tasks, and her desire to have tried to assess students' thinking behind their writing. For instance, for the task where students were prompted to solve $\log_4 x = 6$ for x , Mrs. Barnett noticed how some students wrote the solution (4,096) while others wrote " $4^6 = x$ " while others wrote "4,096". Her focus with such noticings was around how such written work might have indicated students' understanding about the purpose of the task ("Why are we doing this? What's the point?"), and Mrs. Barnett again indicated a desire to delve deeper into these students' understandings based on those different responses, stating that "I'm really just trying to delve into their minds, like how they're understanding that different representation." Such responses indicated her desire to interact with students to better understand their thinking, which aligns with the noticing dimension of shaping (van Es & Sherin, 2021).

For a subsequent task of solving $\log_e x = 13$ for x , Mrs. Barnett specifically indicated how she would have liked to have assessed such conceptual understandings. She stated that "I should have maybe delved more into this, but I'm wondering what they understand e to the 13th [power] to be, you know?" This again indicated Mrs. Barnett's interest in wanting to focus more on student thinking behind their written work and less on the procedures to arrive at these solutions. Students' understanding

of Euler's number seemed to be of particular interest to her here, where she questioned how "Because now we have letters on either side, like, is this right? Is this [e] a value here? Or is this another unknown thing like that [x]?" She went on to state that she would have liked to have engaged in more questioning of students' writing, including through the use of an additional explanatory prompt to the task. She recalled that she would have wanted "another follow up question down here [after the "Solve for x " prompt]" about "What is the meaning of e to the 13^{th} equals x " in order to better understand how students were able to connect the different formats of the equation.

Together, such evidence suggests that Mrs. Barnett saw exploratory and explanatory writing tasks as being more conducive to the sort of discourse that can get at students' thinking behind their written work. Based on the sorts of hypothetical questions that Mrs. Barnett provided, such discourse seems in line with the definitions of assessing and advancing questions as well. In the absence of exploratory or explanatory writing tasks, Mrs. Barnett at least indicated her curiosity in how she would have liked to have assessed students' written work to better understand their thinking, which appears in line with the teacher noticing dimension of shaping (van Es & Sherin, 2021). Although Mrs. Barnett's observed interactions with students' writing were mostly teacher-directed (46 out of 57 total), these responses showed how her idealized types of interactions were more student-directed. This would appear to be more in line with her first interview responses which emphasized the right of a learner to "say what makes sense" to them.

6.3.2 Desired Opportunities for Students to Reason with Peers' Writing

In her first video viewing, Mrs. Barnett was asked to describe her interpretation of her interaction transcribed in Figure 5.5, which occurred during a

whole group discussion concerning the warm-up activity. Similar to her reflections in response to students' writing artifacts, Mrs. Barnett was reflective here in her desire that students might have been positioned to lead the conversation more than she ended up doing and noted how she could have provided some additional questioning opportunities to accomplish this outcome. Such a reflection is notable because it shows how – within the context of a descriptive interaction – Mrs. Barnett desired more inquiry and questioning of students' thinking compared to the more teacher-directed nature of the actual interaction. She noted that “I wish I had asked the students to make sense of each other's responses instead of reading through them myself,” which indicated how Mrs. Barnett would have not only wanted students to reason with their own writing, but also the written responses of their peers. She provided some examples of questions she could have asked, namely “What do you think this student meant?” and “How do you think this student noticed this?”, which are both assessing-focused questions, albeit potentially students assessing their understanding of their *peers'* writing rather than their own writing. This would again underscore the value that Mrs. Barnett appeared to be placing on the shaping dimension of noticing (van Es & Sherin, 2021).

6.3.3 Noticing of Teacher-Directed Aspects of Interactions

Mrs. Barnett's responses to her second video viewing showed that she was able to notice the ways in which her descriptive feedback was more teacher-directed, and that she wished that she had used more student-directed questioning during this exchange. She described how the descriptive nature of this interaction, which was the interaction with Jeri captured in Figure 5.6, was in response to her desire to push towards the learning goal of the lesson, but felt that the interaction would have been

more meaningful (or “genuine”) for Jeri if she had first questioned her about the reasoning behind her written work.

Mrs. Barnett recounted her noticing of Jeri using a “guess and check” process for solving logarithmic equations with unknown values, and her desire to affirm Jeri’s thinking while pushing her towards more “systematic” ways of solving these sorts of problems through this interaction. This description yet again centered around Mrs. Barnett’s desire for students to conceptually understand what logarithmic functions mean in relation to exponential functions, even as they engaged in procedural/symbolic writing tasks. She began by stating that “my aim was saying, ‘Well, okay, let’s go towards this learning goal of using a logarithmic function to figure out what that x is in a more systematic way.’” However, she then reflected that “I wish I would’ve asked her [Jeri] more, or kind of just questioned a little more what that representation maybe meant to her.” Mrs. Barnett went on to offer some potential questions that she could have asked, such as “What do you notice about this when you write the log base five of 244 million something, something,” and “What do you notice how that output might connect to the original equation?” Such questions appeared in line with the qualifications of assessing or advancing questions.

In addition to reflecting on some different approaches that she might have taken during this interaction, Mrs. Barnett also recognized that the interaction as it played out was more teacher-directed. When Jeri responded to Mrs. Barnett’s initial assessing question with a gestured indication of her uncertainty (Figure 5.6, line 8), Mrs. Barnett had indeed shifted towards a descriptive response (i.e., a teacher-directed interaction) to Jeri’s writing. Mrs. Barnett reflected that “it would’ve been nice, and I use the phrase, ‘let me prove it to you,’ you know, where maybe it might have been

more genuine if she [Jeri] had made that connection on her own.” In this way, Mrs. Barnett seemed to be indicating her preference – in retrospect – to have engaged in more assessing lines of questioning rather than the shift to a descriptive response that actually occurred.

These responses again indicate how Mrs. Barnett sought more student interaction in the shaping of her noticing (van Es & Sherin, 2021). While most of her observed interactions with students’ writing was teacher-directed, these responses showed how Mrs. Barnett was able to attend (Jacobs et al., 2010; van Es & Sherin, 2002) to the voice of students during these interactions. Her responses also aligned with her descriptions from her first interview about writing serving as an opportunity for students to communicate “what I’m thinking and what’s making sense to me.”

6.4 Cross Case Analysis: Teachers’ Interpretations of Instructional Artifacts

The teachers’ reflections about samples of their students’ writing and recordings of their interactions with student writing are illuminating in several ways. First, these reflections remain largely in line with each teachers’ described orientation toward writing in mathematics from their first interviews. Mrs. Taylor focused many of her responses in both interviews on the way that students were using explanatory writing to accurately describe their problem-solving processes. Mrs. Hudson’s responses across both interviews emphasized the importance of students’ comfort and flexibility as they worked on their writing tasks. Mrs. Barnett’s responses in both interviews spoke to the ways that her tasks and her questioning moves served (or could have better served) for students to use their writing to communicate their understanding of mathematical ideas. Each of these reflections are distinct, and yet

each also aligns to specific aspects of the orientations toward writing that the teachers described in their first interview.

Second, all of the teachers' responses – especially in response to the two video viewing sessions – indicate some level of desire for greater student ownership of student writing and of the interactions with the teacher regarding that student writing. Mrs. Taylor described her desire to push students' writing towards the intended lesson goals through judicious telling (J. P. Smith, 1996) – as well as a regret that some students had appeared to mimic the language of her descriptive interaction in their final written product. Mrs. Hudson appreciated seeing students' effort in engaging in writing and their comfort with having their writing shared, although she indicated at least one instance where she would have liked to have better acknowledged and built off of a students' initial written work before suggesting a different approach to the task at hand. Mrs. Barnett reflected on her efforts to spur greater peer communication of mathematical thinking via writing, but also regretted instances where she felt that she did not give students the space to genuinely communicate the meaning behind their written work. All of these examples demonstrated both the teachers' focus on supporting student ownership during their instruction and the teachers' desire for more such student ownership – and less teacher-directed interactions – in such instruction.

These video viewing responses also added greater context to the examples described in Chapter 5. While those findings explained some of the ways in which teacher interactions with students' writing was more teacher-directed at times, these findings indicated some of the reasons *why* the participants enacted these interactions in these more teacher-directed ways. The repeated recognition from the teachers about how they felt that students could have been more active participants in these

interactions suggests that all of the participants attended (Jacobs et al., 2010; van Es & Sherin, 2002) to student participation in their noticing of these events, and that they valued a more active role for students during these interactions.

Finally, it is worth noting that different sorts of noticings arose through the use of different student artifacts (i.e., samples of students' written work and recordings of teacher interactions with students' writing). A summary of the teachers' reflections and noticings when examining the various instructional artifacts is shown in Table 6.1. The teachers' responses to the recordings of their instruction, as described previously, focused explicitly on ways that the teachers felt that they could have better engaged students as active participants during the interactions. Importantly, the teachers during these video viewing activities also had access to the samples of student work, as they had previously been asked to reflect based only on the written work. In comparing the two different interview activities (reflecting on the student work and reflecting on the video recordings with the student work still available), it appeared that teachers focused more in the initial interview activity (with only the writing samples) on broadly describing how well students were perceived to have met the goals of the relevant instructional activity or lesson. When the teachers had both the writing samples and the video accessible during the second interview activity, they appeared to draw out more of the aforementioned reflections on how they might have better included students in these interactions, and also drew connections that they perceived between the writing samples and the video recordings.

Table 6.1: Focus of Teachers' Responses to Writing Samples and Video Clips

Teacher and Artifact	Focus of response when examining this artifact
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Mrs. Taylor

Samples of student writing – proof posters (see Appendix D, Figures D.2, D.3, and D.11)

Mrs. Taylor focused on the perceived clarity and strength of students' explanatory writing. Particularly, she focused on how well students used the responses to their procedural/symbolic work (finding slopes and lengths of the sides of a quadrilateral) to indicate what type of quadrilateral they had been given.

Video of descriptive interaction – Funneling pattern of questions (see Figure 5.1)

Mrs. Taylor focused on how the students seemed to have only partially justified their proof and suggested that her descriptive feedback might have led the students to mimic her response. She suggested several open-ended questions that she could have potentially asked students to encourage stronger justifications.

Video of descriptive interaction – Terminating assessing and advancing questions (see Figure 5.2)

Mrs. Taylor focused on how she was hoping to have this student reflect on and revise their written explanations, but that she recalled struggling to push the student towards certain revisions without explicitly telling the student what revisions to make.

Mrs. Hudson

Samples of student writing – whole group “Shout it Out” responses (see Appendix D, Figures D.4 and D.7)

Mrs. Hudson focused on the rough draft nature of the task, describing the value she saw in a perceived range of precision in students' language and her appreciation of students' engagement with the writing task.

Video of “Shout it Out” interaction (see Figure 5.3)

Mrs. Hudson focused on how students appeared comfortable engaging with the “Shout it Out” activity, and how she used an assessing question to focus students in on an anticipated common misconception with identifying missing exponents.

Video of advancing question Interaction (see Figure 5.4)

Mrs. Hudson focused on how she would have liked to have been more responsive to the student's developing understandings in her interaction rather than pushing the student towards a different strategy right away.

Mrs. Barnett

Samples of student writing – exploratory task and subsequent procedural/ symbolic task (see Appendix D, Figures D.8 and D.9)	Mrs. Barnett focused on her lesson goals with these tasks, namely balancing students’ procedural and conceptual understanding of the inverse relationship of logarithmic and exponential functions. She also indicated several idealized, open-ended questions she would have liked to have asked students about their writing in hindsight.
Video of exploratory writing task interaction (see Figure 5.5)	Mrs. Barnett focused on how she might have better positioned students to lead during this interaction, including idealized, open-ended questions she might have asked students to help them reflect on their peers’ writing.
Video of assessing question interaction with shift towards descriptive feedback (see Figure 5.6)	Mrs. Barnett focused on how she might have better supported the student to “make the connection” that she stated in her descriptive feedback, and suggested several open-ended questions she might have asked to help achieve that goal.

Mrs. Barnett was the exception to this in that she also described alternative, idealized questions she would have liked to have asked students when she was responding during the first interview activity. She then continued to describe and develop such idealized questions when responding to the video recordings. This could have indicated that she held an especially strong stance of inquiry (van Es & Sherin, 2021) with regards to students’ written work, as she was quick to suggest questions that could help her to better engage with and understand students’ thinking even when examining just the initial samples of their written work.

This is similar to what was seen by Goldsmith and Seago (2011), where the use of written artifacts in teacher noticing activities encouraged broader initial interpretation about students’ reasoning, while the use of video artifacts resulted in

more situationally focused noticing in response to particular students' descriptions and actions. In my study, I was able to identify more direct connections between the teachers' noticing responses and their member-checked described orientations toward writing in math when analyzing the written artifact responses. The responses to the video artifacts (with the writing samples also available), while not misaligned from the broader described orientations of each teacher, provided more nuanced information about the recollected intentions of each teacher when interacting with students' writing, and the idealized sorts of interactions that they held as a goal. This could have indicated that different mediums of artifacts for teacher noticing activities – or a combination of mediums of artifacts – may better draw out different dimensions of teacher noticing. While teachers in my study appeared quite able to attend to and interpret (van Es & Sherin, 2002, 2021) different perceived aspects of students' understandings from the written artifacts alone, the use of both written artifacts and video recordings in the subsequent noticing activities may have helped them to more explicitly reflect on how they were (or were not) shaping (van Es & Sherin, 2021) their noticing of students' understanding during the lesson through interactions with the students.

Given the nature of my other two research questions – explaining the participants' described orientations toward writing in school mathematics and explaining how the participants interact with students' writing instructionally – the use of these different artifacts and the varied participant responses that followed proved beneficial in supplementing my results for those questions. The different types of artifacts appeared to foster different sorts of teacher noticings, and the collective results allowed for the participants' voice to play a greater role in the findings of this

study. These teacher interpretations underscore how the participants were noticing features of students' written work and their own instruction in alignment with their distinct orientations toward writing in mathematics, that they were aware of interactions with students' writing as being more or less student-directed, and that they each idealized interactions that were more student-directed.

Chapter 7

DISCUSSION AND FUTURE RESEARCH

This study centered on understanding teachers' orientations toward writing in high school mathematics, including their self-reported orientations and observations of how the teachers interacted with students' writing during instruction. Given that teachers' orientations arise at the intersection of beliefs, perceptions, and practices (Remillard & Bryans, 2004), I designed this study to focus on a variety of evidence that could illuminate teachers' potentially complex orientations toward writing in mathematics. The results, as informed by teacher interviews and classroom observation data, indicated how the teachers in this study oriented themselves towards writing in mathematics, interacted with students' writing, and perceived the challenges and opportunities that they associated with such writing. Such findings are beneficial in how they lay out distinct ways that these teachers oriented themselves toward writing in mathematics and how they suggest potential opportunities to support teachers' professional learning around their noticing of students' writing in mathematics.

7.1 The Variety of Perceived Purposes and Challenges of Writing in High School Math

My results illustrated the different types of writing that my teacher participants reported promoting through their tasks and their intended rationales for students' writing. The distinctness of each teacher's described orientation toward writing in mathematics demonstrated how, although writing in math may broadly be understood

as supporting student reasoning and communication (Casa et al., 2016), there are different ways to approach such goals in the classroom. Mrs. Taylor focused primarily on writing as a means to support student reasoning and communication through the process of having students explain and justify their mathematical thinking. Mrs. Hudson, on the other hand, saw writing as a tool for strengthening the precision of students' communication of mathematical ideas with academic vocabulary, as well as strengthening their reasoning by connecting that vocabulary to other representations of such ideas. Mrs. Barnett, who described mathematics as ultimately a form of communication, thus saw both writing and engaging in mathematical thinking and learning as being inherently "intertwined." Although these different descriptions ultimately align with broader goals of writing to support reasoning and communication, the distinctness of each teachers' responses suggests substantial differences in how these teachers consider and attend to writing in their classrooms.

The participants' descriptions of their orientations toward writing, as well as their interpretations of classroom artifacts, also suggested that supporting student voice and student comfort in sharing their ideas was an important perceived goal of writing in mathematics to these participants. Mrs. Hudson in particular spoke to the role of consistent, public writing opportunities in fostering a classroom environment where students would feel comfortable in sharing and getting feedback on their mathematical ideas. Mrs. Barnett, on the other hand, referenced how students have "the right to say what makes sense to them," and thus writing can help foster equitable opportunities for students to make use of that right of a learner. Across each of the teachers' interpretations of their interactions with students' writing, they also described their desire for greater student ownership of the discourse surrounding

student's writing in the math classroom. Together, these teachers' responses indicate the perceived value of writing in mathematics as supporting student reasoning and communication not only through the explicit production of written work, but also through the mathematical talk (Munter et al., 2015) or classroom discourse that follows (or intertwines with) such writing.

These findings suggest that the teachers approached writing as an embedded practice within their instruction rather than a standalone activity. Writing was not only described as a means for students to communicate and reason with mathematical ideas (e.g., Casa et al., 2016), but also as a means to support students' self-efficacy as mathematical thinkers and doers and to strengthen classroom discourse. These responses extend beyond the scope of teacher surveys about mathematics writing such as that used by Powell et al. (2021), where they asked teachers explicit Likert-scale agreement questions such as "Students understand math better when they write math explanations" (p. 428). The interview responses from the teachers in my study indicate that teachers' orientation toward writing in mathematics should also be considered in the context of their broader beliefs about instruction and student learning, as the teachers reported their writing practices and beliefs to be strongly influenced by such broader views.

This study also indicated some of the perceived challenges that these teachers felt regarding writing in mathematics, particularly in regard to supporting student learning through writing (i.e., writing-to-learn). Mrs. Taylor, for instance, described how writing was often too time intensive for her to use frequently. This was in part because of the value that she placed on students receiving feedback on their writing: If she did not have time to provide individual feedback to each student's written work

(which she indicated that she did not), then she was not inclined to use writing in her classroom. This is in line with the findings from Teuscher et al. (2015), who also found that teachers were less inclined to use writing-to-learn strategies in mathematics because of how much class time it was perceived to require. My study adds some potential context to these feelings of time constraints. It was not the enactment of actual writing activities that Mrs. Taylor perceived as being too costly with time, but rather the feedback that she would want to provide students in response to their writing.

Mrs. Barnett, however, described a different challenge in that she did not feel efficacious with regards to using writing in her teaching. This seems more in line with the findings from Powell et al. (2021), who found that teachers with higher self-efficacy about using mathematics writing were more likely to report using such writing activities in their classrooms. However, given Mrs. Barnett's broader described orientation of both writing and mathematics being "intertwined" forms of communication that transpire across multiple semiotic systems, it seems that Mrs. Barnett was perceiving her low self-efficacy with writing in part because she was considering writing as solely focused on the written word. This is worth considering in the context of studies like that of Powell et al. (2021), especially given that they narrowly define mathematics writing as "a writing activity in which students write about mathematics concepts or procedures" (p. 418). It stands to reason that, if teachers such as Mrs. Barnett came to associate writing in mathematics as attending to connections across multiple semiotic systems (i.e., a multisemiotic approach), she may have reported more self-efficacy in attending to writing in her teaching.

Mrs. Hudson, on the other hand, did appear confident in her attention to student writing in her classroom, especially through her references to her “Shout it Out” activities. However, she did note the overarching challenge of needing to foster students’ comfort with sharing their thinking through writing over time. Her approach to building that student comfort – by making frequent use of “Shout it Out” activities that involve the public, whole-class sharing of and teacher feedback on students’ written solutions and explanations to instructional tasks – appears generally in line with the recommendation from Powell et al. (2021) that teachers make regular use of writing across a variety of writing activities (e.g., exploratory or explanatory writing tasks). However, while Powell et al. focus more on the connections between the teacher’s self-efficacy towards mathematics writing and the frequency of their use of writing activities, Mrs. Hudson’s responses ask us to consider the importance of attending to *students’* self-efficacy with writing in mathematics. As it was with Mrs. Hudson, students’ comfort (or perceived comfort from the view of the teacher) with writing may play an important role in determining when and under what conditions mathematics teachers are more likely to integrate writing activities into their teaching.

These findings show how the teacher participants, when describing their orientation toward writing in mathematics, also cited broader views regarding discourse and self-efficacy in mathematics as influencing their responses. Mrs. Taylor’s views on the importance of teacher feedback played a role in how she described enacting writing activities as challenging because it would require her to provide feedback to each student. Mrs. Hudson’s views about her students’ self-efficacy towards writing played a role in how she described the frequency and focus of her approach to writing with students (i.e., her rough draft oriented, whole class

“Shout it Out” activities). Mrs. Barnett’s views on what exactly counts as writing in mathematics played a role in how she described her own self-efficacy towards using writing in her classroom. Across each of the teachers’ responses during their video viewing sessions, they repeatedly referenced the perceived importance of student voice and participation in these sorts of interactions with students’ writing. That in turn informed what they were noticing in the videos and how they described alternative, idealized interactions that they could have had with the students’ writing.

Looking across these findings, my study contributed over previous studies in that it situated teachers’ described orientations toward writing and perceived challenges with writing in mathematics within the broader context of teachers’ views on teaching and learning. For instance, Cantrell et al. (2008) described how skeptical secondary mathematics teachers in their study reported content area literacy to be “irrelevant to their subject area, and [they] did not know how to implement the strategies or only did so in highly selective ways” (p. 84). Teuscher et al. (2015) similarly reported how most teachers in their study were not even familiar with the idea of writing-to-learn mathematics, and even though who were familiar with this idea reported that such writing activities took too much class time for them to use in their classrooms. In a similar vein, Powell et al. (2021) noted how, although most teachers in their survey rated mathematics writing as important, many also indicated that they rarely used mathematics writing activities in their own classrooms. In my study, teachers were asked to critically reflect on different views about writing in math (including their own) and consider how such views relate to how writing plays out in their classrooms (see the interview script in Appendix A). This seemed to allow for the

aforementioned interview responses that situated writing within the context of other goals that these teachers held for themselves and their students.

This distinction between my findings and the results from other such studies cited above is notable because it indicated the multifaceted and embedded nature of the participants' orientation toward writing in mathematics. For instance, Mrs. Taylor cited time constraints as a barrier to using writing in her classroom – a result similar to that found by Teuscher et al. (2015) – but her responses also suggested how this concern was rooted in her stated beliefs about the importance of feedback. It was not merely the time required to enact a writing activity that was an issue, but the importance she found in ensuring that all students received feedback on and an opportunity to revise their writing. Likewise, Mrs. Barnett's low perceived self-efficacy with writing instruction was similar to what Powell et al. (2021) found in their survey of teachers, but her other interview responses showed a deep commitment to elevating student voice in the classroom. Her main concerns appeared centered around a pre-conceived notion of that "writing in math" does and does not mean. For Mrs. Hudson, although she reported using writing activities frequently in her classroom, she stated that she did so not only for the perceived learning benefits of such activities, but also because she felt that they supported her desired classroom norms. Such nuances in my findings suggest the importance of understanding not only teachers' stated beliefs about specific types of writing in mathematics, but also how such beliefs relate to teachers' broader orientations toward teaching and student learning. Because of the seemingly multifaceted and embedded nature of these teachers' orientations toward writing in mathematics, future research could benefit from exploring connections between such orientations and teachers' other orientations toward teaching and

learning (e.g., their stance on the role of student inquiry or their attention to supporting student self-efficacy in mathematics).

7.2 The Variety of Observed Writing Tasks and the Teachers' Role as Facilitator

Across different sorts of writing tasks, teachers in this study were observed providing evaluative and/or descriptive feedback about students writing some of the time (i.e., teacher-directed interactions), and engaging students in assessing and/or advancing questions at other times (i.e., student-directed interactions). In some regards, the ways that teachers were able to engage with students' writing in both teacher-directed and student-directed ways across different types of writing tasks (i.e., procedural/symbolic, exploratory, informative, and explanatory) showed the flexibility of the teachers as facilitators of classroom discourse. Although teacher-directed interactions with students' writing were more commonly observed than student-directed interactions, these could be explained at least in part because it takes less time to give evaluative or descriptive feedback compared to the time it takes to engage in discourse with a student to learn more about their writing. Given the purpose of writing in mathematics as supporting both reasoning and communication (Casa et al., 2016), the teachers may have seen it as beneficial that students were receiving both direct feedback on the perceived quality of their written work and, at times, were asked to explain the mathematical thinking that they were trying to communicate through such writing.

Because the teachers were observed using different kinds of interactions across different types of tasks, this indicated that they were not strictly limited in how they interacted with students' writing purely by the type of task. For instance, all three of

the participants were observed asking assessing and/or advancing questions to students in the context of procedural/symbolic tasks, indicating that the closed-question nature of the task prompt did not stop teachers from asking questions to better understand students' thinking behind their written work and/or advance them towards future revisions of that work. In essence, the teacher had an important role to play with the discourse around students' writing in mathematics regardless of the type of writing involved. This would stand somewhat in contrast to previous research that showed how the cognitive demand of mathematics tasks tends to decline from the set-up to the implementation phase of a task enactment (Stein & Lane, 1996). Although my study does not analyze the cognitive demand of teachers' writing tasks or the maintenance of such demands during these interactions, the ability of teachers to take closed-ended procedural/symbolic tasks and engage students in open-ended questions about those tasks suggests that they were able to expand the task beyond the confines of its original prompt (rather than restrict the focus of the task prompt over time). Therefore, further research could better relate the set-up and enactment of these tasks with regards to writing, just as Stein and Lane (1996) explored the cognitive demand of mathematics tasks.

However, this is not to say that the type of writing expectations embedded in a task has no relation to the types of interactions that the teachers have with students' writing. As shown in the cross-case analysis from Chapter 5, explanatory and informative (and perhaps exploratory) writing tasks appeared to promote the use of more student-directed interactions (i.e., teacher-student co-generated interactions with assessing and advancing questions), even during procedural/symbolic tasks that preceded those tasks. For instance, Mrs. Taylor was observed facilitating about twice

as many student-directed interactions with students' writing compared to the other participants, and also had about twice as many interactions occurring during explanatory writing tasks. In Mrs. Barnett's case, about half of her total student-directed interactions (albeit few – five out of 11) occurred during exploratory writing tasks. In all three participants' cases, there were also instances where, even when a student-directed interaction occurred during procedural/symbolic tasks, it nevertheless seemed to be aligned with the written prompts of upcoming informative or explanatory writing tasks within the same lesson (see Table 5.4). As such, informative and explanatory writing tasks may have helped foster more student-directed interactions with students' writing by providing open-ended prompts from which the teachers could directly assess student about their thinking, or by providing explicit connections from procedural/symbolic tasks to later open-ended prompts. This may have also been the case for exploratory tasks but, given that these were rarely observed outside of Mrs. Barnett's observed instruction, such a connection is more tenuous to support from the available data.

Together, this indicated that, for these teachers, the use of informative and explanatory writing tasks seemed to support the teachers' use of shaping practices, which entail “teachers and students engaging in an interaction with each other in the moment” (van Es & Sherin, 2021, p. 24). The use of open-ended, informative, and explanatory focused writing prompts thus may be an effective tool in encouraging more teacher-student co-generation during interactions that focus on tangible written products that the student is producing. These go beyond teacher-directed interactions (evaluative and descriptive interactions) that focus more on the teachers' own

knowledge and experiences to interpret the written work (i.e., attending and interpreting dimensions of the noticing framework; van Es & Sherin, 2002, 2021).

Additionally, it is worth noting that descriptive interactions – which were on par with (for Mrs. Taylor) or more prevalent (for the other two cases) throughout the observed instruction – are not devoid of student voice. Because these interactions entail the teacher stating the ways that they see their students’ writing as communicating mathematical meaning, they still do represent instances of the teacher attending to and interpreting students’ thinking (van Es & Sherin, 2002) and making that noticing visible to the student by talking with them. Therefore, although students may not be an active participant during these interactions (by responding to open-ended assessing questions), the teacher is still basing their responses off of the student’s own writing and communicating their noticing to the student. Such interactions might, in this way, be considered instances of judicious telling (J. P. Smith, 1996) or metacognitive modeling. Indeed, descriptive interactions are in line with the sort of judicious telling that I have previously investigated and recommended as a way to support students’ literacy in mathematics (E. P. Smith et al., 2023). Given the importance of metacognition as a predictor of students’ mathematics performance (Desoete & De Craene, 2019), such interactions may still offer opportunities for students to reflect on their writing approaches and to revise their writing based on the teachers’ feedback.

Activities like the “Shout it Out” activity from Mrs. Hudson’s classroom – although largely coded as teacher-directed – may even be considered a form of shaping (van Es & Sherin, 2021), as it involved an iterative process of students writing out solutions and explanations, Mrs. Hudson curating a discussion around those

responses, and then Mrs. Hudson sharing new writing tasks or groupwork that built upon that discussion. Mrs. Hudson described this sort of activity as a way for her to “check in to see if they’re [students] on the right page.” This description appears in line with the shaping dimension of noticing, which “involves constructing interactions and contexts to gain access to additional information” (van Es & Sherin, 2021). The “Shout it Out” activity, as well as the whole class exploratory writing task observed in Mrs. Barnett’s classroom (see Figure 5.6), were also examples of teachers noticing *within* students’ individual written work and *across* multiple instances of such work (Stockero et al., 2017) in order to orchestrate a whole class discussion and support students’ learning about the main ideas of the lesson. This suggests that, although these interactions were mostly evaluative or descriptive (i.e., teacher-directed), they still involved some instance of shaping teachers’ noticings across the collective written work of all students in the class. As such, these sorts of whole class writing activities represent valuable contexts for future studies focused on understanding how mathematics teachers notice across multiple instances of student thinking.

In sum, both the types of writing tasks used during these observed lessons and the teachers’ experience as facilitators of student discussion appeared to be important drivers of the types of interactions that teachers had with students’ writing. While informative, explanatory, and potentially exploratory tasks may encourage more co-generation during interactions, these teachers did find opportunities to ask open-ended assessing and advancing questions with students even on closed-ended procedural/symbolic tasks. These student-directed interactions may have benefited from later informative or explanatory tasks acting as a referent for potential questions, but the teachers’ decisions to engage students in assessing and advancing questions

around procedural/symbolic tasks warrants further consideration. Because these participants were chosen because previously observed lessons of theirs were found to be dialogically focused (Munter et al., 2015), this sort of orientation toward discourse and mathematical talk could be reflected in these instructional decisions. Even when the task as written was closed-ended and procedural/symbolic – the teachers sometimes used students’ mathematical talk to elevate the focus of discourse around such tasks to be more open-ended and to draw on students’ explanations of their thinking. In essence, they may have tried to make up for the absence of one aspect of dialogic instruction (tasks with multiple entry-points and solution paths) with another such aspect (mathematical talk).

This sort of mathematical talk proved valuable in demonstrating a variety of ways that these teachers appeared to be engaging with the different dimensions of noticing in their classrooms, especially the dimension of shaping (van Es & Sherin, 2021). Indeed, this study showed promise in addressing van Es and Sherin’s (2021) call for research that “entails understanding in what contexts and under what conditions this revised model of noticing [attending, interpreting, and shaping] applies” (p. 25). These cases suggested that the types of writing tasks used in the classroom, as well as the ways that these teachers both chose (in the moment) and desired (in retrospect) to interact with students’ writing, are relevant considerations for research aimed at illustrating this inquiry-focused model of noticing.

7.3 Liminal Interactions and Teachers’ Desire for More Student-Directed Interactions

Although student-directed interactions with assessing and/or advancing questions were not frequently observed in these teachers’ classrooms, there were also

examples of questioning in teacher-directed interactions that did not quite rise to the qualifications of assessing or advancing questions. I describe such interactions as “liminal” in this study because these teacher-directed interactions showed some glimmers of the sorts of teacher-student cogeneration of meaning observed in student-directed interactions. The observation of these liminal interactions (Chapter 5), along with teachers’ stated desire to have enacted these and other teacher-directed interactions in more student-directed ways (Chapter 6), indicated that these teachers were able to notice (in retrospect) the perceived limitations of these interactions in that they did not more actively engage the student as a participant in the noticing event. As such, another contribution of this study is the suggestion that attention towards these sorts of liminal interactions in teacher noticing activities could be a way to strengthen teachers’ noticing skills around attending to students’ inquiry and shaping their noticing interactively with students in the classroom.

These liminal interactions appeared to show how teachers at times used questioning with the intent to engage students in the feedback that they were providing about the student’s writing. However, teachers’ questions in these moments fell into a funneling rather than a focusing pattern (Herbel-Eisenmann & Breyfogle, 2005). A funneling pattern is problematic because it drives students towards an outcome pre-determined by the teacher, rather than genuinely incorporating students’ responses and thinking about their work. Alternatively, if teachers employed more open-ended assessing questions during their interactions with students about their written work, such interactions could have allowed the teacher to better shape their noticing of the students’ understanding, and thus better inform the teacher’s subsequent descriptive feedback and advancing questions regarding the student’s written work. Because the

teachers were able to identify the perceived limitations of these liminal interactions during later video viewing sessions, this suggests that teachers did have an stance of inquiry – i.e., a desire to better “understand the details of student thinking” (van Es & Sherin, 2021, p. 23) – during such sessions. The disconnect, as self-identified by the teachers, lay in the differences between how they were noticing in the moment versus in those later video viewing session.

To this end, the teacher participants did interpret the interactions that I identified as liminal as not being as student-directed as they would have preferred, and they offered suggestions for how they might have better incorporated student ownership and student voice into these interactions. Because of this, the use of video viewing sessions centered around teachers’ liminal interactions with students’ writing could represent a fruitful means to support teachers’ professional learning about how to engage in the shaping stage of teacher noticing *in-the-moment*. Although the teacher interviews in the present study were not designed to support teachers’ efforts to improve their noticing in-the-moment, the fact that the participants were able to notice the lack of teacher-student co-generation during certain recorded interactions and relate this to their desired orientation toward student writing is noteworthy. Just as Sherin and van Es (2009) have shown how video viewing clubs can influence teachers’ learning around noticing, engaging teachers more systematically around videos of interactions with students’ writing warrants further investigation as a potential tool for improving teacher noticing specifically regarding students’ writing.

7.4 Reflections on Student-Directed Interactions with Students’ Writing

To summarize, there are several important considerations regarding teachers’ interactions with students’ writing that were illustrated through this study. The above

results indicated some of the ways that the teachers in this study actively included students in discourse around their written work in the midst of noticing during their instruction. These results also indicated some of the challenges and affordances of using different types of writing activities as perceived by these teachers. However, the results ultimately showed that, regardless of such challenges, each of these teachers indicated the value that they placed on student voice in these interactions, and offered suggestions for how they might have further involved students in discourse around their writing. This suggested that these teachers each held a stance of inquiry (van Es & Sherin, 2021) towards their students' writing, in that they were "seeing observed phenomena as something worth trying to figure out" (p. 22).

Some of these interactions were also liminal, in that teachers were posing questions to students, but not in a way that seemed to be shaping (van Es & Sherin, 2021) their noticing of the students' written work. These examples were beneficial to highlight in this study because they offered illustrations of how different types of teacher questioning could serve different ends. A funneling pattern of questions, for instance, is more focused on steering students towards a predetermined solution rather than providing the teacher with more information about the students' understandings (Herbel-Eisenmann & Breyfogle, 2005). Likewise, even if an open-ended question is posed to a student, if the student is not then given an opportunity to respond to the question, then it may lose its potential in providing additional information to inform the teachers' noticing as well. As such, examples of liminal interactions can help illustrate the limitations of these interactions in supporting teachers' shaping of their understanding amidst their noticing (van Es & Sherin, 2021) with students' writing.

All of this is not to say that the teacher-directed interactions observed in this study were inherently less desired than student-directed interactions. I previously described how many such interactions – especially descriptive interactions – could have indicated the teachers’ use of judicious telling (E. P. Smith et al., 2023; J. P. Smith, 1996) to advance students towards the learning goals of the lesson. Indeed, if a teacher is using a mathematical writing task that requires students to describe or explain their thinking in writing, the information that teacher might otherwise capture through spoken discourse with the student might be made apparent through the student’s writing. This is in part because writing tasks can operate as a means for the teacher to assess students’ mathematics knowledge (Powell et al., 2017). As such, the goal of my classifications of teacher- and student-directed interactions was not intended to solely advocate for one of the other type of interaction. Rather, these distinctions are meant to describe – and advance discussion around – contexts and situations that give rise to different types of interactions with students’ writing.

However, I do believe that there are some desirable characteristics of student-directed interactions worth considering. As described throughout this study, such interactions could demonstrate a teacher’s stance of inquiry and support the shaping of their understanding about students’ thinking in the midst of noticing. The teachers in this study also indicated during video viewing sessions that their idealized interactions with students’ writing included the use of more assessing and advancing questions, which could suggest the viability of student-directed interactions as a relevant area of focus for supporting teachers’ professional learning of their noticing over time. Teachers’ attention to student participation in their noticing can also support their enactment of equitable mathematics pedagogy (Wager, 2014), and positioning

students as capable thinkers and doers of mathematics is an important element of anti-deficit noticing (Louie et al., 2021). Because student-directed interactions are in line with these different aspects of noticing, such interactions could also potentially support equitable and anti-deficit teacher practices. As such, this study not only illustrates relevant distinctions in teachers' interactions with students' writing useful for teacher education, but also can benefit future research focused on teacher noticing and writing in mathematics.

7.5 Implications for Teacher Education Practice

This study has several implications for practice, especially in the context of teacher education. First, it suggests that teachers' interactions with students' writing in-the-moment was more teacher-directed, but that their noticing during video viewing sessions captured greater interest in engaging students as active participants in these interactions. The implication, then, is that teachers can learn from viewing and reflecting on video of their instruction, with the goal of generating more student-directed interactions with students' writing in their classrooms. These participants were selected because their lessons had been found to be dialogically focused (Munter et al., 2015), but most of the mathematical talk around students' written work in the observed lessons was teacher-directed. For mathematics teachers to engage students more actively in discourse around their own written work, they could benefit from adopting and enacting a stance of inquiry (van Es & Sherin, 2021) that situates the student's written work as something that holds mathematics meaning worth discussing. The use of both samples of students' written work and videos of teachers' interactions with students' writing in noticing activities appeared to spur teacher inquiry in this regard. This may have been further supported by participants'

understanding that this study was focused on students' writing. Therefore, employing multiple modes of artifacts to capture students' thinking during teacher's professional learning activities might serve to better help teacher participants in recognizing evidence of students' thinking across these different modalities.

Given the goal of supporting students' communication across both written and spoken modalities in mathematics education (Common Core State Standards Initiative, 2010; National Council for Teachers of Mathematics, 2000), this study's findings also suggests that teachers could benefit from more explicit discussion about both connections and distinctions between students' writing and spoken discourse in the classroom. It appeared that the use of more informative, explanatory, and potentially exploratory writing tasks supported teachers' use of assessing and advancing questions regarding students' writing. Therefore, mathematics teacher educators should attend to the types of writing embedded in teachers' instructional tasks as a tool for building teachers' inquiry stance. For instance, if a task is explanatory, teacher educators could encourage teachers to not only evaluate students' written responses in-the-moment, but also to use that task prompt as a baseline for assessing and advancing questions that ask students to expand upon their explanatory writing in their own words. If a task is more procedural/symbolic, teacher educators could refer teachers to later informative or explanatory task as similar baseline prompts for assessing or advancing questions. These implications extend the work of Casa et al. (2016), who did not address the role of procedural/symbolic tasks in their recommendations around mathematics writing in the elementary grades.

However, this study also indicated that the task alone does not determine the type of interactions that teachers have with students around their writing. The teachers

in this study employed a range of interactions with students' writing across a range of writing types, showing that their skills at facilitating classroom discourse influence their interactions. To this end, teacher educators could also potentially benefit from asking teachers to anticipate different student approaches to a task and identify the sorts of questions that they could ask to assess and advance student thinking. The multimodal noticing activities described previously could again serve as a way for teachers to develop this skill by identifying idealized questions that they would like to ask students, based upon the perceived limits of their interpretations of students' written work and recorded responses during interactions.

In summary, this study demonstrated the iterative nature of teacher noticing with regards to students' written work in high school mathematics. The types of writing tasks employed by teachers, the teachers' experience as facilitators of student learning, and the teachers' stance towards inquiry all appeared to be salient factors in determining when and how teachers chose to interact with students' writing. By providing teachers with opportunities to reflect on their interactions with students' writing through multimodal noticing activities, teacher educators might be able to strengthen teachers' stance of inquiry and shaping dimension of noticing. In turn, teachers' noticing in-the-moment might also then shift to include students as active participants more consistently as teachers interact with and respond to students' written work.

7.6 Limitations of Study

This study has several limitations to consider. First, as a case study, the findings regarding these three teacher participants are not generalizable. While their described orientations, observed interactions with students' writing, and

interpretations of instructional artifacts may indicate future directions for research, the results themselves do not demonstrate generalizable trends in how teachers orient towards and attend to student writing in high school mathematics. Additionally, because these participants were selected because of prior observations of dialogically focused instruction in their classrooms, this sample represents a particular set of cases. This study, especially the analysis of observation data, focused heavily on connections between writing and spoken discourse in the classroom. As such, teachers' observed interactions – and their interpretations of those interactions – may have been influenced by their broader dispositions towards teaching and learning. Because mathematical talk is a key aspect of dialogically focused instruction (Munter et al., 2015), these teachers may have been more prone to engage students in discourse about their written work (i.e., student-directed interactions). Alternatively, if these teachers did greatly value mathematical talk, they may have emphasized spoken discourse throughout their instruction without always tying such discourse to some written work by the student. In either case, the selection of participants could be considered a limitation because these classrooms offered a window into a particular type of mathematics instruction.

The analysis of teacher interactions with students' writing also specifically centered upon the observation of assessing and advancing questions (Boston et al., 2017). Previous research has focused on different aspects of teachers' noticing, such as the extent to which teachers attend to students' strategies "with substantial detail about the mathematically important aspects of that strategy" (Jacobs et al., 2010, p. 183). This study did not assess the *quality* of teachers' evaluative or descriptive feedback, or the quality of their assessing and advancing questions, merely which of

these the teacher used in their interactions with students' writing. As such, there is a possibility that teachers' evaluative or descriptive feedback could have at times more thoroughly addressed important aspects of the lesson goals compared to the use of assessing or advancing questions. Additionally, as seen with the student-initiated interaction in Mrs. Hudson's classroom (section 5.2.2.2), there may have been ways that the teachers' noticing of students' understanding was shaped without the need to pose an assessing question. This study's focus on the observation of particular types of teacher questions as indicating student-directed interactions is therefore limited in how it does not address other means by which teachers may have been interacting with students' writing.

Finally, teachers' interpretations of instructional artifacts (samples of students' written work and recordings of teacher interactions with students' writing) were confined by which artifacts I chose to include for the post-observation interview. I took multiple steps to ensure that such artifacts would provide for a robust interview and analysis of teachers' interpretations, but ultimately each teacher was only asked to interpret two class samples of students' written work and two recordings of interactions with students' writing. Given that there were 189 interactions with students' writing identified across all teacher observations, this means that most interactions were not included in this interpretation and analysis. Alternative approaches to the selection of such instructional artifacts, such as a focus on selecting the most commonly observed type of interaction or type of writing task, might have yielded different teacher interpretations.

7.7 Implications for Future Research

Although the nature of this study limits the generalization of its findings, these results do offer implications for future research. First, the teachers' described orientations and interpretations of instructional artifacts indicated how their views on writing were interwoven into other views about teaching and learning mathematics. These teachers described various goals around providing students with timely feedback (Mrs. Taylor), supporting students' self-efficacy in school mathematics (Mrs. Hudson), and recognizing the rights of the learner (Kalinec-Craig, 2017) to express their developing conceptions of mathematical ideas (Mrs. Barnett). These in turn were cited as informing how these teachers then attended to (or saw challenges with) writing in their classroom. These teachers in their interpretation of instructional artifacts all indicated their desire to engage students in ongoing sensemaking and revision with their writing (i.e., student-directed interactions with students' writing). Because of this, research investigating mathematics teachers' beliefs about writing should also consider broader teacher perspectives about classroom discourse and mathematics identity. Although past research has focused on teacher beliefs specifically about mathematics writing (Powell et al., 2021) and writing-to-learn mathematics activities (Teuscher et al., 2015), these investigations have not identified differences between teachers along these other perspectives. Although mathematics writing has been framed as supporting goals of student communication and reasoning (Casa et al., 2016; Powell et al., 2017), the teachers in this study described how their other instructional goals either supported or restricted their willingness to use writing in their classrooms. Each of the teachers in this study described how they generally agreed that writing supported student communication and reasoning, but their willingness to use writing in their activities – and the ways in which they did describe

using writing – was ultimately informed by these additional instructional goals. Therefore, future research could benefit from better situating mathematics teacher beliefs and practices regarding writing in the context of teachers’ broader beliefs about teaching and learning mathematics. This study provides a potential example of how this could be accomplished.

Second, this study suggests the importance of attending to the medium of teachers’ noticing activities. Previous research has shown the benefit of using recordings of classroom instruction as a way to strengthen mathematics teachers’ noticing (Sherin & van Es, 2009; van Es & Sherin, 2002), while Jacobs et al. (2010) used both video clips and samples of students’ written work as artifacts of practice for teachers to interpret but did not comment on whether any differences in teacher noticing was captured across these different mediums. My study indicates a potential benefit of using *both* student writing samples and video recordings of instruction to support teachers’ noticing. Because teachers were able to reference students’ preceding or subsequent written work when examining their interactions with students’ writing, this allowed them to draw connections between their practices and suggested student outcomes. Therefore, future research should consider how the use of multiple mediums during noticing activities might better support teachers’ development of how they attend to and then interpret students’ thinking.

Additionally, the attention to the medium (or modes) through which teachers are noticing in-the-moment is another area worth consideration in future research. The participants for this study were selected because their instruction had previously been found to be dialogically focused. Because this sort of instruction situates students as “co-participants’ in classroom discourse” (Munter et al., 2015, p 10), I had

conjectured that this would provide the basis for capturing a sizeable variety of student-directed interactions with students' writing. However, out of the 189 interactions with students' writing observed (see Table 5.5), only 22 of these involved assessing questions, 13 involved advancing questions, and only 7 involved both assessing and advancing questions. Over three-fourths of the observed interactions were either evaluative (76) or descriptive (71) teacher-directed interactions. This suggested that, when teachers were specifically interacting with students' *written* work (e.g., not just students' initial understanding of a provided task or of their teacher's or peers' spoken discourse, but rather some tangible product that the student had written themselves), students were rarely included as co-participants during those interactions – especially in a way that involved both assessing and advancing questions.

Future research could better determine whether the focus or extent of inquiry involved with teachers' noticing of students' thinking in-the-moment is related to the medium of that noticing. It could be that student-directed interactions arise more frequently when a teacher is trying to initially probe a students' understanding of a provided task, while teachers may act more frequently as evaluators and interpreters of students' thinking when examining their written work. This sort of future investigation could be a way to address van Es and Sherin's (2021) call for research that "entails understanding in what contexts and under what conditions this revised model of noticing [attending, interpreting, and shaping] applies" (p. 25).

Ultimately, future research should consider further the role of writing and spoken discourse in the classroom and how these relate to teacher noticing of students' mathematical thinking. For instance, Wager (2014) described how teachers' focus on student participation in their noticing supported their enactment of more equitable

mathematics instruction. Similarly, Louie et al. (2021) spoke to the importance of framing students as thinkers and doers rather than merely receivers of mathematics as an aspect of anti-deficit noticing. Such positions towards noticing appear in line with several of my results, such as how the teachers emphasized a desire for greater student involvement in their noticing activities. Future research could consider how attention to different mediums at play during teacher noticing events might relate to teachers' equitable and anti-deficit noticing, or how barriers towards such type of noticing might be related to the medium of the noticing.

7.8 Conclusion

This study used an embedded case study approach (Yin, 2018) to investigate teachers' orientations toward and interactions with students' writing in high school mathematics. I interviewed three teachers whose instruction had previously been found to be dialogically focused (Munter et al., 2015) to capture their described orientations toward writing in mathematics, and then observed and categorized their interactions with students' writing in each of their classrooms over the course of four lessons in Spring 2022. These teachers were then asked to interpret instructional artifacts from these observed lessons in order to gain better insight into their perspectives on students' writing. Together, these data inform my findings.

First, I found that the teachers' described orientations toward writing in mathematics drew on other goals that they shared regarding teaching and learning mathematics. Namely, the teachers emphasized goals around classroom discourse, student ownership over their learning, and student self-efficacy as important drivers in determining the form and frequency through which they used writing in their own classrooms. Second, I found that informative, explanatory, and potentially exploratory

tasks may have supported teachers in enacting student-directed interactions with students' writing – even during preceding procedural/symbolic tasks – by providing an open-ended writing prompt from which teachers could anchor their discourse with students. However, the range of teacher interactions with students' writing (evaluative responses, descriptive responses, and combinations of assessing and/or advancing questions) observed across different types of writing tasks (procedural/symbolic, exploratory, informative, exclusive explanatory, and inclusive explanatory) also underscored the importance of these teachers as facilitators of classroom instruction. These teachers' interactions with students' writing were not strictly beholden to the type of writing expected from a particular task.

Finally, my results showed how these teachers were able to notice ways in which selected interactions with students' writing were more teacher-directed, and also expressed a desire to engage in more student-directed questioning during such interactions. The use of both samples of students' writing and video recordings of these interactions during interview noticing activities appeared to support these sorts of reflections, as teachers were able to compare the students' written work with the spoken discourse of the interactions. In other words, the combined use of these two mediums of instructional artifacts appeared to draw out these teachers' stance of inquiry (van Es & Sherin, 2021) amidst their noticing of their recorded instruction.

These findings offer a number of implications for both practice and research. First, they suggest the benefit of using both written samples of students' writing and video recordings of classroom instruction as artifacts to support noticing activities for teachers' professional learning. Second, they indicate that attending to the type of writing embedded within different mathematics tasks might be a helpful tool for

strengthening teachers' stance of inquiry towards students' written work in the moment of classroom instruction.

There were also several contributions from my study that have implications for future research. For mathematical writing or writing-to-learn, my study suggested that researchers could go further to capture the role of broader teacher views on teaching and learning in order to better understand affordances and constraints to teachers' use of writing in mathematics. For teacher noticing, my study suggested that researchers might also better attend to the medium of teachers' noticing – both in-the-moment and in professional learning settings – to better understand the conditions under which different aspects of teacher noticing arise in school mathematics.

Writing is an important tool for supporting students' reasoning and communication in mathematics (Casa et al., 2016). However, writing is not necessarily an isolated practice that arises outside of the greater context of mathematics instruction. This study illustrated how student writing activities are intertwined with the types of spoken discourse, classroom norms, and instructional practices found in high school math classrooms like those of Mrs. Taylor, Mrs. Hudson, and Mrs. Barnett. These teachers' orientations toward and interactions with students' writing demonstrated the perceived challenges and affordances of supporting students' writing in high school math while also underscoring the respect that these teachers held towards honoring their students as thinkers and doers of mathematics. As such, the implications for practice and future research that arise from this study's findings are built upon – and hopefully honor – the multifaceted and student-oriented beliefs and practices held by Mrs. Taylor, Mrs. Hudson, and Mrs. Barnett as mathematics educators.

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Appendix A

TEACHER PRE-OBSERVATION INTERVIEW SCRIPT

I would like to thank you for participating in this study. I am interested in learning more about how you think about and approach different types of student writing in your math classroom. Everything that you choose to share with me today will remain confidential, and if I use any quotes from this interview in my reporting you will be assigned a pseudonym. Please feel free to ask me any questions as we go through the interview if anything is not clear. I will be recording the interview – is that okay with you? [confirm] Ok, I will now start the recording and begin the interview.

[turn on recording devices]

This is Ethan Smith on [date] speaking with [interviewee name]

Introductory questions

1. I would like to begin by learning a bit more about your experience generally as an educator. How long have you been teaching?
2. How do you enjoy teaching math?

Questions about writing in mathematics

3. When I say a phrase like “writing in mathematics,” what comes to mind? Why is that?
4. What particular writing skills, if any, do students require to be successful in mathematics? Why is that?
5. What role(s) can mathematics teachers play in helping students acquire and practice these sorts of writing skills? Why is that?

Questions about writing instruction

6. How frequently do your lessons require students to write? What do those lessons typically look like? Why is that?
7. Specifically, how do you help your students become better writers in your classroom?
8. Some teachers think that students should mostly be writing towards the end of a learning sequence, while others think that these opportunities should happen mostly at the beginning of a learning sequence, or even throughout the entire sequence. Where in the learning sequence do you think students should be

writing, and why? (**Probe:** Does student writing look different at different parts of the learning sequence?)

9. How would you describe your students' skill at writing? Why is that?
10. How well equipped do you feel to deal with students' writing difficulties? Why is that? (**Probe:** Have you previously received any professional development or support around supporting student writing in mathematics?)

Questions about multisemiotic perspective of mathematical writing

11. One way of thinking about "writing in mathematics" is to consider not only the written word, but also how students are writing out to solve equations, drawing on graphs, and so on. How is this broader definition similar or different from how you think of students' writing in mathematics?
12. How, if at all, does that broader definition of writing in mathematics make you think differently about students' writing in your own classroom?

Closing question

13. Was there anything else that we did not cover today that you wanted to share before we end our interview?

Appendix B

POST-OBSERVATION INTERVIEW SCRIPT

Prior to interview: Participants will be emailed a draft description of their beliefs about the nature of writing in mathematics, function of writing in mathematics, and writing instruction. This will be written by the investigator following the first interview. The following email will be attached to the description:

During our interview tomorrow, I would like to get to know more about your reflections about your instruction during the lessons that I observed, and your reaction to students' work that they made during the lesson. This will also be a chance for me to learn more about your thoughts on writing in mathematics. In preparation for this conversation, I have written up and attached a summary of your responses from our previous interview, indicating my draft interpretation of how you think about (1) the nature of writing in mathematics, (2) the function or purpose of writing in mathematics, and (3) how you think about instruction that supports students' writing. Before we meet tomorrow, I ask that you take a look through this description. Tomorrow I will ask you for your reaction, specifically around whether it seems accurate, whether I have missed anything important to your thoughts, or whether anything seems inaccurate in what I wrote. I will also bring a copy of the description for your convenience tomorrow. I look forward to meeting with you then!

Thank you again for your participation in this study. As a reminder, I am interested in learning more about how you think about and approach different types of student writing in your math classroom. Today, I would like for us to look back on a recording from when I observed your classroom and get your feedback about (1) the way that you attended to students' writing during the recorded activity and (2) the writing produced by some of those students. Everything that you choose to share with me today will remain confidential, and if I use any quotes from this interview in my reporting you will be assigned a pseudonym. Please feel free to ask me any questions as we go through the interview if anything is not clear. I will be recording the interview – is that okay with you? [confirm] Ok, I will now start the recording and begin the interview.

[turn on recording devices]

This is Ethan Smith on [date] speaking with [interviewee name]

1. First, I would like to discuss the summary of writing beliefs that I emailed you yesterday. Here is a copy of this summary for your reference. Did you have a chance to read these yet? (If not, give additional time for reading).
2. Looking over this description, is there anything that you feel is inaccurate or could better reflect how you think about writing in mathematics?
3. Is there anything about your beliefs on the nature of writing, the purpose of writing, or how you think about writing instruction that is missing in this description?
4. Have you always thought about writing in mathematics this way, or did you ever think about it differently?

Next, I would like to take a look at some student work that I collected in your class on [observation date]. Prior to this lesson you had stated that your learning goal for the day was [learning goal]. (Share student work samples).

5. In thinking about the lesson from that day to what extent do you feel that students achieved the learning goals? Why is that?
6. What do you see in this student work that helps inform you of that reflection?
7. More broadly, what do you notice about these students' written work?
8. What sort of feedback, if any, would you want to give each of these students for how to revise or improve upon the written work shown here if you saw this work during the lesson?

Finally, I would like us to watch an excerpt from this lesson. As you watch, I want you to think about how you are attending to students' writing during this segment and what you think of the student response to your instruction. I have a few questions for you once we finish watching the clip, but you are also welcome to ask me to stop the clip at any time to share what you are noticing about students, your instruction, or how students are responding to your instruction.

[Play back video recording of writing affordance]

9. What stands out most to you from this interaction?
10. This clip was selected because you appeared to be responding to something that a student (or students) had written. What was going through your mind during this clip, to the best of your recollection?
11. How is this sort of interaction similar or different from a typical lesson with your students? Why is that?

Appendix C

SAMPLES FROM DATA ANALYSIS PROCESS

Mrs. Barnett “Writing in Mathematics” Profile

<p>What does “writing in mathematics” mean?</p>	<p>“Math is a form of communication in and of itself...I have this idea that I want to communicate to people, this pattern that I've noticed...and I just need to be able to demonstrate it for someone else in however way I see fit with words, visuals, symbols.” There are some aspects of mathematics that require writing, such as proofs, because “we must write words in order to prove something.” However, in mathematics “we can prove things using visuals, using diagrams, using even symbols.” In this way, these different types of writing “are very much all kind of intertwined” in mathematics. Writing is also “a way of communicating,” and these different types of writing (writing out an equation, writing on graphs, drawing a picture) can help us communicate about mathematics.</p>
<p>What is the role of writing in the learning of mathematics?</p>	<p>Writing allows a learner to “communicate [their] thinking to you in a way that you're going to be able to understand it.” It is “the ability to communicate an idea to someone else,” “to transport what I'm thinking in my brain to you in some way, either through words or a picture or an equation.” Because of this, a learner knowing how “to put down their thinking...in order for someone else to understand it” is an important skill in learning mathematics. Because of the importance of communication, “the terminology [of the written word] does not have to be perfect,” because “sometimes when there is fancy jargon...it [the idea] gets lost.” Additionally, learners have “the right to say what makes sense” to them, and writing is a way to communicate “what I'm thinking and what's making sense to me” in a way that I can “make sense of that to other people.”</p>
<p>What is the role of writing in the teaching of mathematics?</p>	<p>Writing can be used to “help students be able to communicate those ideas (mathematical ideas) to each other.” As such, it is important for teachers to attend to the “structure of [their] activities” in order to support that sort of student communication. This can include “provid[ing] them with sentence stems” or creating public spaces for students to “see similarities between what they said and their classmates said.” Teachers can also use writing to help students to “revise or restructure the way that [they are] initially” describing mathematical ideas, allowing students to “refine” their thinking as they are learning. While writing should often occur at the end of a learning sequence “to summarize,” “you could justify it anywhere in the learning process” as well. For instance, students could write about what they currently know in relation to the current mathematical topic or even “try to write everything that they” know about some mathematical vocabulary as an introduction to using the term.</p>

Figure C.1: Mrs. Barnett’s profile, with her member checked additions bolded

Teacher			Hudson		pt1 - 23:01, pt2 - 33:26, pt3 - 16:42
Obs #			3		
Date			5/18/2022		
Video Clip Location	Start (seconds)	Start (percent)	Length (seconds)	End (percent)	Transcription
pt2 9:43-9:58	1964	44.7%	15	45.1%	Reading off a students' answer for task. Prompt: "Find the value of each variable that makes the equation true. Be prepared to explain your reasoning." Part 2: $3^a b^3 = 3^{11}$ T: "11-7 = 4. You have to subtract the given exponent from the answer exponent to get the remainder which is the answer for b. Good.
pt2 9:58-10:03	1979	45.1%	5	45.2%	b equals 4 because 7 plus four is equal to 11. Good
pt2 10:03-10:12	1984	45.2%	9	45.4%	11 minus 7 equals 4, so you would get 3 to the 6th power. Close, 3 to the 4th there, you did explain it though.
pt2 12:18-12:24	2119	48.3%	6	48.4%	Part 3: $5^8 / 5^d = 5^2$ 2 minus 8 equals 6. Other way around, just be careful with those terms because that would give you -6.
pt2 12:39-13:30	2140	48.8%	51	49.9%	'look at the factors that are shared by 8 and 2 to find d. 8 and 2 both share the factor 4 which is d. To check your work you can do 8 divided by 4 which is 2.' 'Look at the factors that are shared by 8 and 2 to find d...' So, be careful with the word 'factor.' If we multiply these they should equal...4 times another number should give us that value. So if we do four times another integer or whole number, unless it is 1/2 we cannot get to 2, so we just want to be careful with that.
pt2 13:31-13:53	2192	49.9%	22	50.4%	'Because the bases are the same and we are dividing, we subtract the powers/exponents. $8-d=2$ ' Ooh, this person decided to solve it. ' $8-d=2$ so $8-2=d$ which, d is equal to 6.' Good job, they added d on both sides, subtracted 2.
pt2 14:03-14:16	2220	50.6%	13	50.9%	So yes, if you had something similar this - 'when dividing fractions' make sure, the one piece I didn't notice in this part is that the bases have to be the same.
pt2 18:19-18:32	2480	56.5%	13	56.8%	Part 5: $6^m * 6^m * 6^m = 6^{21}$ 'I got m=7 because I divided 21 by' I think they meant to put 3 'and got 7 so m is equal to 7.'
pt2 18:34-18:43	2495	56.8%	9	57.1%	'Divide the exponent of 21 by 3 and to get 7' Good.
pt2 18:46-19:46	2507	57.1%	60	58.5%	T: 'When integers are multiplied the exponents are added, so $21/3$ is 7.' Question – why in this problem couldn't I put, like, 7, 5, and 2. I'm sorry, not that. 7, let's say, I don't know, 9, and was there a 10 and 16 and...5. Why can't I do 7 and 9 and 5? They equal 21. S: Because they're all the same so they should be one. T: Right. That's the big piece I wanted you all to point out there – if they were different variables, you could have any combination that would create 21, but because they're the same they were all 7s. Alright, so just be careful when they're different variables.

Figure C.2: A sample from the observation codebook, columns 1-6

Type of Interaction	Type of mathematical writing	Justifications	Setting	Initiation
Evaluative	Explanatory (inclusive)	States "good" after reading student response	Whole Group	Teacher
Evaluative	Explanatory (inclusive)	States "good" after reading student response	Whole Group	Teacher
Evaluative	Explanatory (inclusive)	Corrects the student that it is 3 to the 4th power.	Whole Group	Teacher
Descriptive	Explanatory (inclusive)	Notes how 2-8 is incorrect because it would give you -6 not 6	Whole Group	Teacher
Descriptive	Explanatory (inclusive)	Says how we are not necessarily just looking for factors, but rather a missing value that will give us the solution.	Whole Group	Teacher
Evaluative	Explanatory (inclusive)	States that it is a good job.	Whole Group	Teacher
Evaluative	Explanatory (inclusive)	States that the answer is missing something about "the bases have to be the same"	Whole Group	Teacher
Evaluative	Explanatory (inclusive)	Mentions how the student missed writing a 3 in their solution	Whole Group	Teacher
Evaluative	Explanatory (inclusive)	States that it is a good.	Whole Group	Teacher
Assessing Question + Descriptive	Explanatory (inclusive)	Explains how, since it is a single variable, responses should mention how it needs to be the same value, while it could be different numbers if they are different variables.	Whole Group	Teacher

Figure C.3: A sample from the observation codebook, columns 7-11

Appendix D

STUDENT WRITTEN WORK SAMPLES

You will be putting your results on a piece of poster paper, and we will complete a gallery walk tomorrow.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad m = \frac{y_2 - y_1}{x_2 - x_1}$$

You may use GeoGebra to help with your quadrilateral if you like.

Use slopes and lengths to determine the type of quadrilateral $ABCD$.

	slope	length
AB	$\frac{3}{6}$	$\sqrt{18} = 4.24$
BC	$\frac{-6}{6}$	$\sqrt{72} = 8.48$
CD	$\frac{3}{6}$	$\sqrt{18} = 4.24$
AD	$\frac{-6}{6}$	$\sqrt{72} = 8.48$

$A=(0,0) \quad B=(3,3) \quad C=(3,9) \quad D=(6,6)$

How can you create parallel lines?

How can you create sides that are perpendicular?

How can you create sides that are the same length?

You do not need all these, but you need to know how to create these sides and angles. Use only what you need (what is required) to create your figure.

AB

$$\frac{(0-3)^2 + (0-3)^2}{9 + 9 = \sqrt{18}}$$

BC

$$\frac{(3-3)^2 + (3-9)^2}{36 + 36 = \sqrt{72}}$$

CD

$$\frac{(-3-6)^2 + (9-6)^2}{9 + 9 = \sqrt{18}}$$

AD

$$\frac{(0-6)^2 + (0-6)^2}{36 + 36 = \sqrt{72}}$$

Figure D.1: Student written work from the interaction described in Figure 5.1

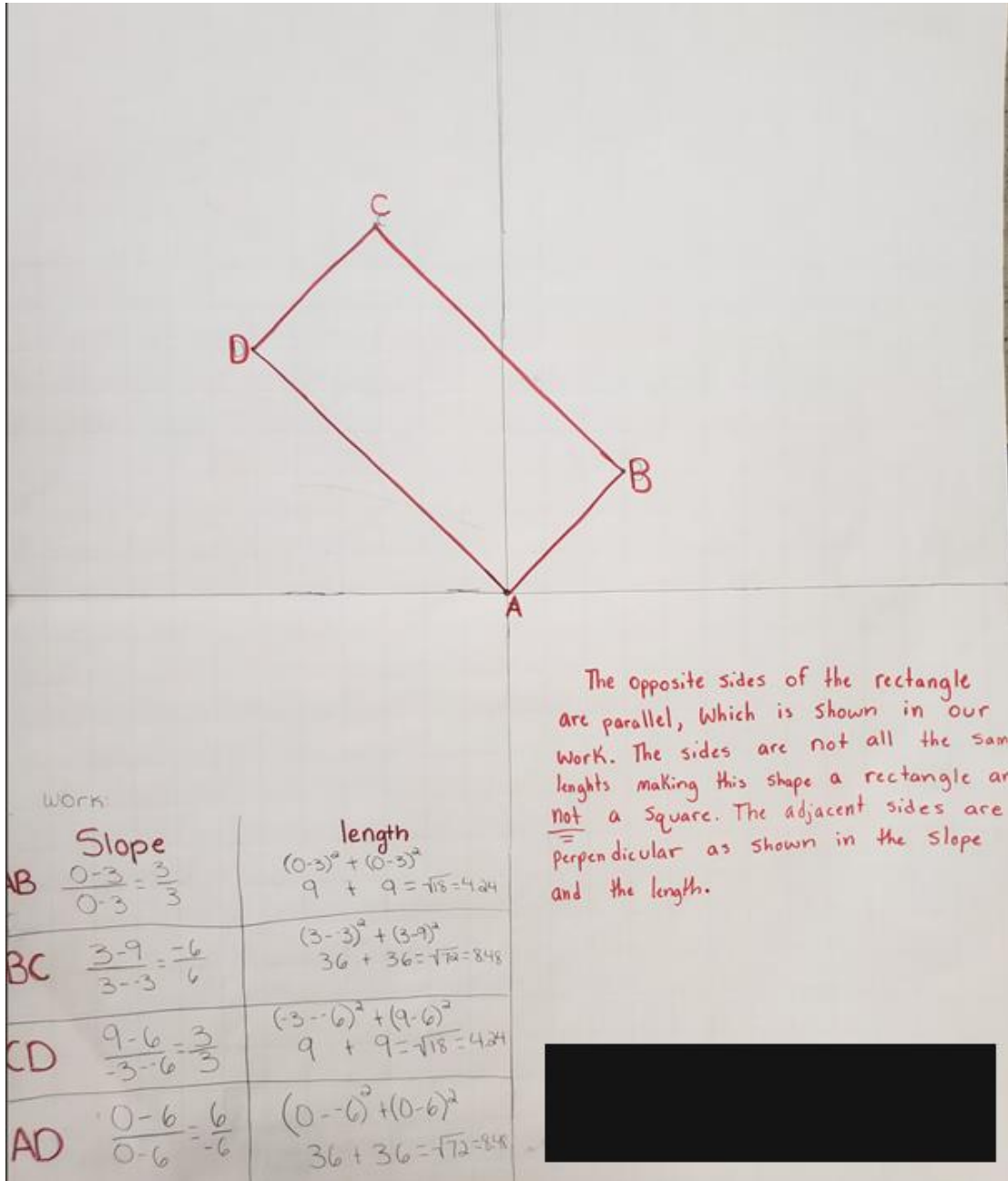


Figure D.2: The end-of-class group poster from students in Figure 5.1 transcript

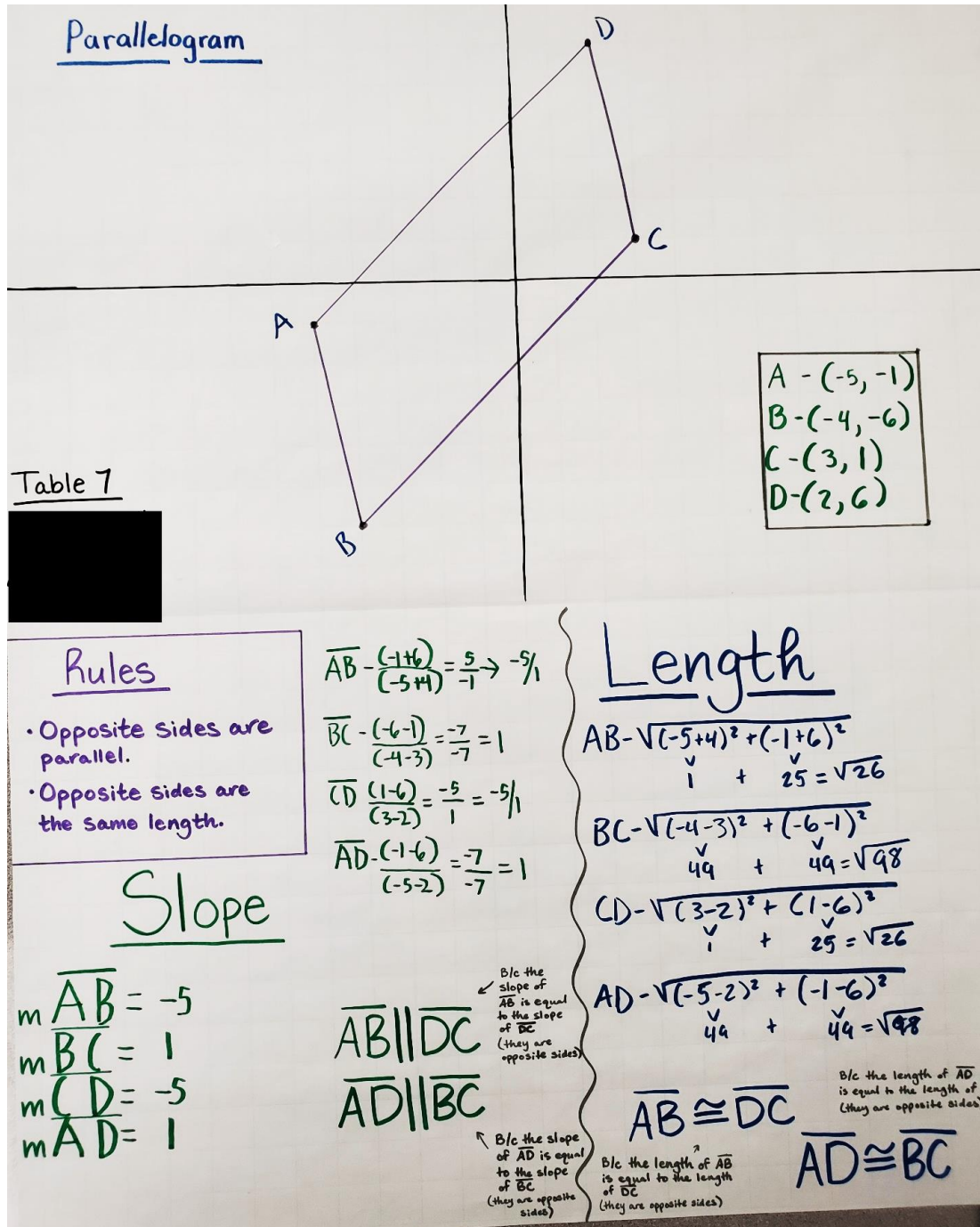


Figure D.3 The end-of class group poster from students in Figure 5.2 transcript

Activity 1.2: Name That Power!

Find the value of each variable that makes the equation true. Be prepared to explain your reasoning.

$1. 2^3 \cdot 2^5 = 2^a$ $a=8$
 $2. 3^b \cdot 3^7 = 3^{11}$ $b=4$
 $3. \frac{4^3}{4^2} = 4^c$ 4 4^1 $c=1$
 $4. \frac{5^8}{5^d} = 5^2$ 6
 $5. 6^m \cdot 6^m \cdot 6^m = 6^{21}$ 7
 $6. (7^n)^4 = 7^{20}$ $n=5$
 $7. 2^4 \cdot 3^4 = 6^s$ 4
 $8. 5^3 \cdot t^3 = 50^3$ 10

$\frac{390,625}{9,165,625} = 25$
 $\frac{5^8}{5^{10}} = 5^2$
 $8-6=2$
 5^8
 5^8
 $d-8=2$
 $d-10$

Explain Solution to...
 3^4 because if the solution is 3^11, you would want to subtract 7 from 11 to get the third exponent of 4.
 Reset activity?
 Resetting this activity will clear the activity results.
 not sure if here multiply

Explain Solution to...
 3^4 because if the solution is 3^11, you would want to subtract 7 from 11 to get the third exponent of 4.
 11-7=4
 Seeing as the bases are the same, we add the bases, b+7=11, 11-7=b, which is equal to 4. b=4
 3^4 11-7=4 11-7=4
 I got b=4 because I added 7+4 which equals 11 so that's how I knew that b=4.

Explain Solution to...
 I got d=6 because I subtracted 8-2 and got 6
 powers/exponents. 8-d=2, 8-2=d, 6=d
 3-2=1 2-8=6 6
 look at the factors that are shared by 8 and 2 to find d. 8 and 2 both share the factor 4 which is d. TO check the work u can do 8/4 which is 2
 5^6 because you would do 8-2.

Explain Solution to...
 I got m=7 because I divided 21 by and got 7 so m=7
 The total exponents is 21 and there are three missing exponents. figure out what times 3 equal 21 and the "what" is 7. to check do 7+7+7=21
 7+7+7=21 7
 When inter...
 6^7 because you need to know what times 3 equals 21.

Explain Solution to...
 Either 16 or 5, not sure if the exponents here multiply or add since it's still multiplication technically but it doesn't have a coefficient but idk
 20-4=16 5x4=20
 I got n=5 because I had to figure out what times 4 equals 20 and 20/4=5 so n=5
 5
 7^16 because 20-4 equals 16.

Figure D.4: Captured SMART Board slides from Mrs. Hudson's classroom

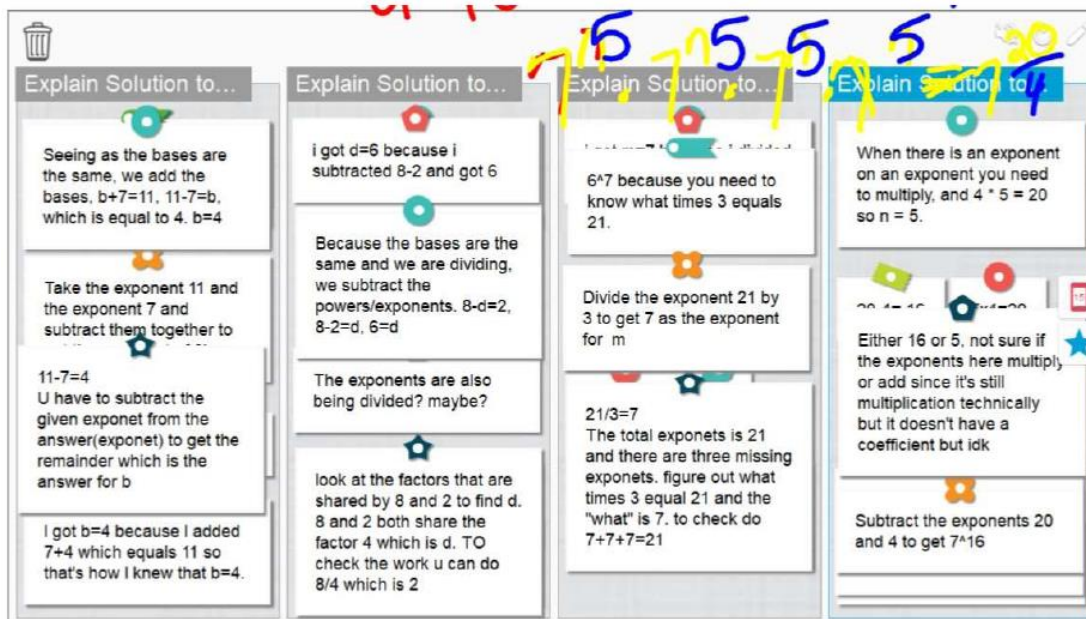


Figure D.4 (continued)

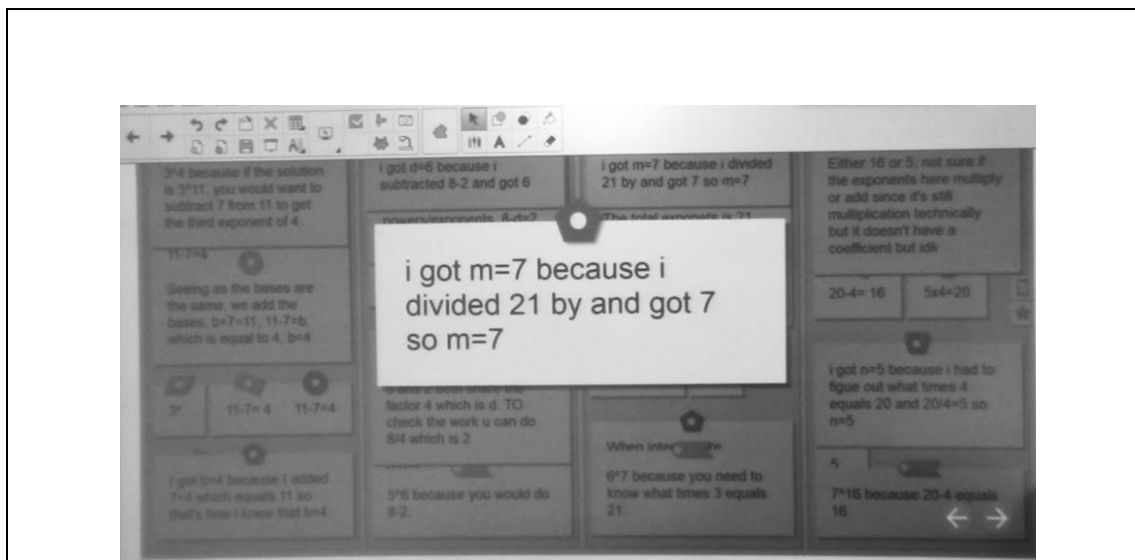
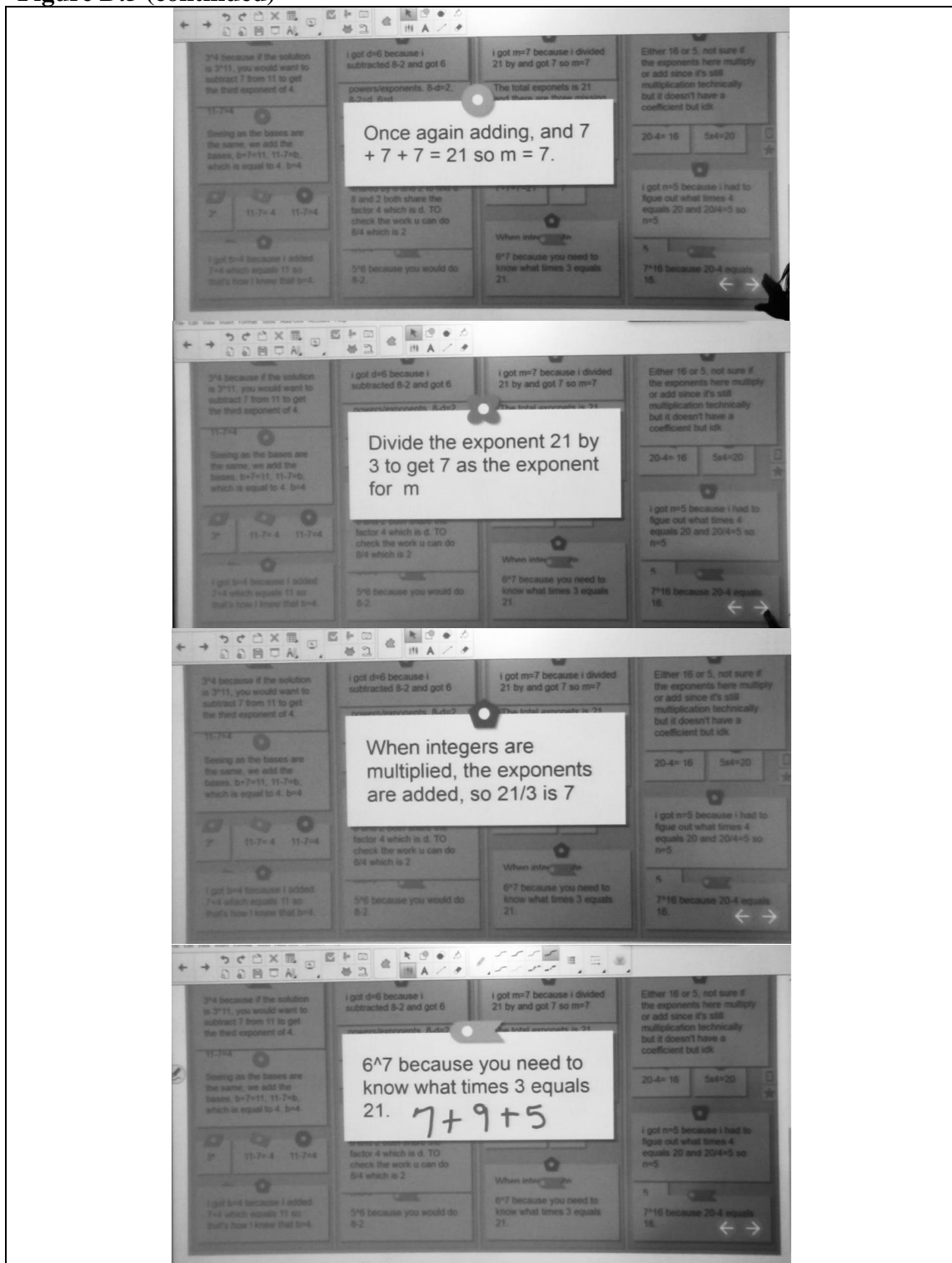


Figure D.5: Captured SMART Board slides from Mrs. Hudson's classroom

Figure D.5 (continued)



Activity 1.3: The Power of Zero

Use exponent rules to write each expression as a single power of 2. Find the value of the expression. Record these in the table. The first row is done for you.

expression	power of 2	value
$\frac{2^5}{2^1}$	2^4	16
$\frac{2^5}{2^2}$		
$\frac{2^5}{2^3}$		
$\frac{2^5}{2^4}$		
$\frac{2^5}{2^5}$		
$\frac{2^5}{2^6}$		
$\frac{2^5}{2^7}$		

Figure D.6: Negative exponents activity from Illustrative Mathematics (2023) used in Mrs. Hudson's class

Use exponent rules to write each expression as a single power of 2. Find the value of the expression. Record these in the table. The first row is done for you.

expression	power of 2	value
$\frac{2^5}{2^1}$	2^4	16
$\frac{2^5}{2^2}$	2^3	8
$\frac{2^5}{2^3}$	2^2	4
$\frac{2^5}{2^4}$	2^1	2
$\frac{2^5}{2^5}$	2^0	1
$\frac{2^5}{2^6}$	2^{-1}	0.5
$\frac{2^5}{2^7}$	2^{-2}	0.25

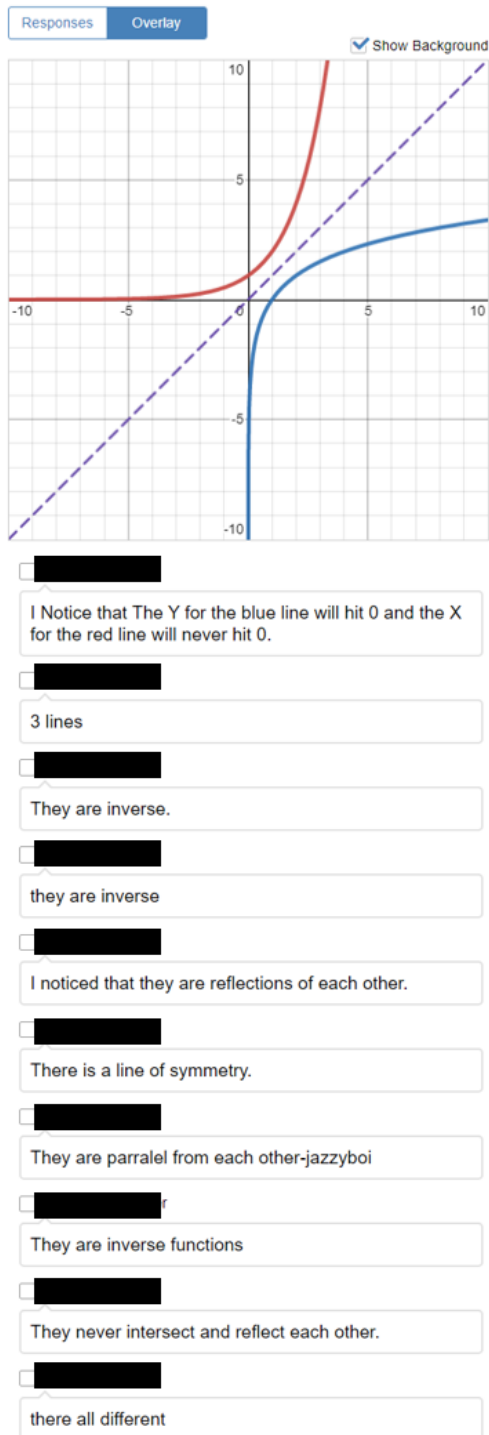
Handwritten notes on the SMART Board include:

- A vertical note on the left: $\frac{1}{2^1}$
- Red arrows connecting the 'power of 2' column to the 'value' column for each row.
- Handwritten values: 16, 8, 4, 2, 1, 0.5, 0.25.
- Handwritten powers of 2: $2^4, 2^3, 2^2, 2^1, 2^0, 2^{-1}, 2^{-2}$.
- Handwritten fraction: $\frac{1}{4}$.

SMART Board annotations:

- A negative exponent divides instead of multiplying, but it starts at 1.
- A negative exponent will take us to the inverse of the number.
- it makes it less than 1.
- It becomes the inverse of a number, $2/1 \rightarrow 1/2$.
- It's division rather than multiplication. I think it divides from 1.
- It divides it in half.
- it becomes the inverse of the number.
- It divides the exponent number in half.
- it will make the number smaller.
- when the exponent is negative, it makes it less than 1 but greater than zero.

Figure D.7: Student responses written on SMART Board for the exponent activity in Mrs. Hudson's class



Blue line is $f(x) = \log_b(x)$

Red line is $g(x) = b^x$

Purple dashed line is $y = x$

What do you notice about these functions?

- [Redacted]
- They are reflections of each other across a line in the middle of the two functions.
- [Redacted]
- I notice that there is a straight line running right through the origin. And the blue line goes through the x-axis, and the red one runs through the y-axis.
- [Redacted]
- I remember this graph from a function, i believe there is a hole or undefined point at either x or $y = 0$ believe that one may be the log and the other may be the exponent.
- [Redacted]
- The functions are mirrored over $y=x$.
- [Redacted]
- I notice that one function is increasing while the other is decreasing, however, they never intercept one another.
- [Redacted]
- there is one dotted line. there is one line (red) increasing and one line (blue) decreasing.
- [Redacted]
- They are inverse functions.

Figure D.8: Students' written responses to the warm-up activity in Mrs. Barnett's class

Follow the example and solve for x.

Consider the following logarithmic equation: Solve for x: $\log_4 x = 6$

$$\log_3 x = 5$$

We can re-write this in exponential form:

$$3^5 = x$$

So,

$$x = 243$$

Responses Summary

- [Redacted] 4096
- [Redacted] 4096
- [Redacted] 4096
- [Redacted] 4096
- [Redacted] 4096
- [Redacted] $4^6 = x$
- [Redacted] 4096
- Jeri $\log = 4^6 = x$
- [Redacted] 4096
- [Redacted] 4096

- [Redacted] $\log_4 x = 6$
 $4^6 = x$
 $x = 4096$
- [Redacted] $4^6 = x$
 $x = 4096$
- [Redacted] $4^6 = x$
 $4^6 = 4^6 + 6$
 $4096 = x$
- [Redacted] $4^6 = 4096$
- [Redacted] $4^6 = x$

Figure D.9: Student responses to the task preceding Mrs. Barnett's interaction with Jeri

Solve for x!

Solve for x : $\log_6 x = 2$

36

Responses Summary

Expression

Students

36



$\log 6^2 = x$

Jeri

Responses Overlay

<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
$6^2 = 36$ $7^3 = 343$ $5^8 = 390625$	$6^2 = 36$ $7^3 = 343$ $5^8 = 390625$	$6^2 = x = 36$ $7^3 = x = 343$ $5^8 = x = 390625$

Solve for x : $\log_7 x = 3$

343

Responses Summary

Expression

Students

343



34



$\log 7^3 = x$

Jeri

Solve for x : $\log_5 x = 8$

390625

Responses Summary

Expression

Students

390625



$\log 5^8 = x$

Jeri

Figure D.10: Student responses to tasks following Mrs. Barnett's interaction with Jeri

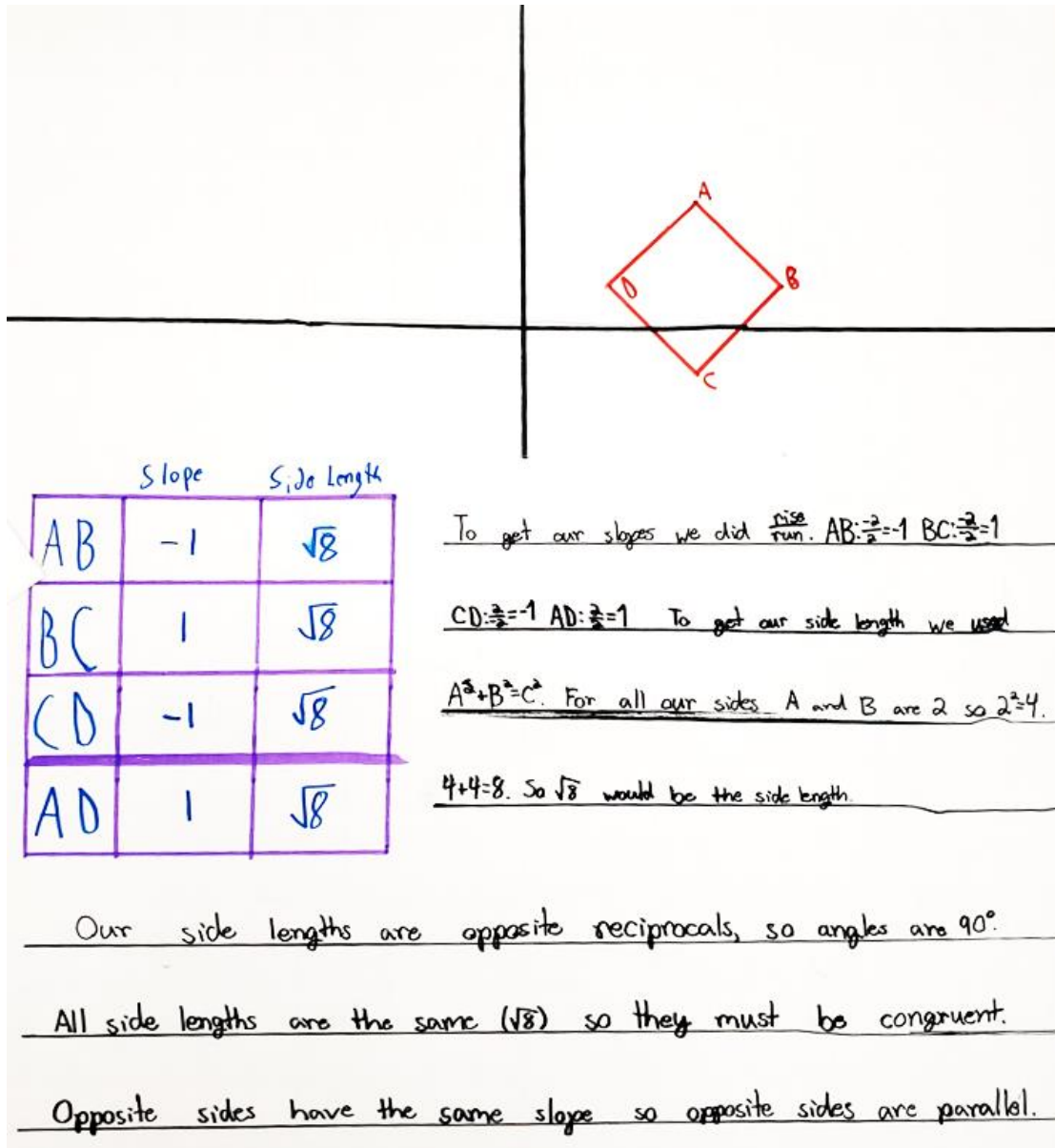


Figure D.11: A group poster proving that a quadrilateral is a square from Mrs. Taylor's class

Appendix E
IRB/HUMAN SUBJECTS APPROVAL



Institutional Review Board
210H Hulihan Hall
Newark, DE 19716
Phone: 302-831-2137
Fax: 302-831-2828

DATE: March 22, 2022

TO: Ethan Smith
FROM: University of Delaware IRB

STUDY TITLE: [1883156-1] Grappling with Mathematical Ambiguity: Teachers' Efforts to Support Students' Writing
SUBMISSION TYPE: New Project

ACTION: DETERMINATION OF EXEMPT STATUS
EFFECTIVE DATE: March 22, 2022

REVIEW CATEGORY: Exemption category # (1)

Thank you for your New Project submission to the University of Delaware Institutional Review Board (UD IRB). According to the pertinent regulations, the UD IRB has determined this project is EXEMPT from most federal policy requirements for the protection of human subjects. The privacy of subjects and the confidentiality of participants must be safeguarded as prescribed in the reviewed protocol form.

This exempt determination is valid for the research study as described by the documents in this submission. Proposed revisions to previously approved procedures and documents that may affect this exempt determination must be reviewed and approved by this office prior to initiation. The UD amendment form must be used to request the review of changes that may substantially change the study design or data collected.

Unanticipated problems and serious adverse events involving risk to participants must be reported to this office in a timely fashion according with the UD requirements for reportable events.

A copy of this correspondence will be kept on file by our office. If you have any questions, please contact the UD IRB Office at (302) 831-2137 or via email at hsrb-research@udel.edu. Please include the study title and reference number in all correspondence with this office.

INSTITUTIONAL REVIEW BOARD

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