NON LINEAR JOINT SOURCE CHANNEL CODING FOR BROADCAST CHANNELS

by

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A thesis submitted to the Faculty of the University of Delaware in partial fulfillment of the requirements for the degree of Master of Science in Electrical and Computer Engineering

Summer 2015

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ACKNOWLEDGMENTS

All Praise be to God the Great.

I would like to thank my mentor and advisor Professor Javier Garcia-Frias for supporting me and encouraging me all the way through my research and for offering me the opportunity to join the amazing small family of the Information Processing and Communications Lab. His passion for research and acute critical mind forces you to think out of the box and I am truly indebted to him for where I am today. Also special thanks to my lab-mates: Bo Lu and Lu Li, who made this journey much more enjoyable.

I would also like to acknowledge the great power of Open Education and the equal opportunity it provides to all of us and the tremendous empowerment effects it can have on the individual, enabling him to truly achieve his potential. In particular, I greatly acknowledge the MIT Open Course Ware which was one of the first initiatives of its kind and I am greatly indebted to the Digital Communication Series, which truly made it clear that this is the right career for me.

Finally, I would like to thank my family for supporting me in taking such a fool-heartedly endeavour as Graduate School. In particular I dedicate this thesis to my Mother, Naglaa. Without her support and encouragement, I would not have made it half way through.

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ABSTRACT

A novel Joint Source Channel Scheme for the two user Gaussian Broadcast channel is developed here. The scheme transmits a Bivariate Gaussian Source over a Discrete Memoryless Additive White Gaussian Noise Channel. The scheme belongs to the family of codes termed Nested Quantization. The ultimate theoretical communication limits of the Gaussian Broadcast channel are given and the different coding schemes to achieve these limits are explained. The scheme developed in this thesis is a Hybrid Digital-Analog System similar to a particular hybrid scheme that was shown to be the *only* scheme to achieve the theoretical limits of the Gaussian Broadcast Channel. The scheme proposed in this thesis, however, is a zero delay mapping with minimal encoding/decoding complexity. We investigate the transmission scheme for different source correlation values and show by simulation the near optimality of the proposed scheme for a wide range of correlation values. In particular, we demonstrate the scheme for independent source pairs as well as low and high correlation values. We show the connection between the space filling curves used for point to point channels and the system proposed here.

Chapter 1

INTRODUCTION

The use of digital communications systems based on the Shannon separation principle between source and channel coding [1] has led to ubiquitous communications in our society. In this framework, continuous signals are first acquired and source encoded. Then, capacity approaching channel codes are utilized. It is well known that this approach is optimal provided that there are no constraints in terms of complexity and delays. However, long block lengths are required, and these separated systems are not very robust to changes in the channel parameters.

Recently, systems based on analog joint source-channel coding have been discussed in the literature [2, 3]. In this approach, the concatenation of the (vector) quantizer, source encoder and channel encoder characteristic of digital systems is substituted by an end-to-end analog encoder. This discrete-time, continuous-amplitude system directly processes the acquired samples using a non-linear transformation, whose output is transmitted directly through the channel after proper modulation. A schematic diagram of canonical digital communication vs analog JSCC systems is depicted in Figure 1.1. For the same performance, these schemes may present more robustness and require less encoding/decoding complexity than traditional digital systems. A more detailed discussion about analog joint source channel coding will be given in Chapter 3.

One might argue that the advantage of analog JSCC systems of having much less encoding/decoding complexity would become less and less valuable as we are able now to operate at near capacity levels of point to point systems with the current encoding/decoding complexity of digital communication systems. Moreover, it is well known by the established Moore's law of semiconductors that the processing power for a unit cost doubles every 18 or 24 months. Even with the apparent slowing down

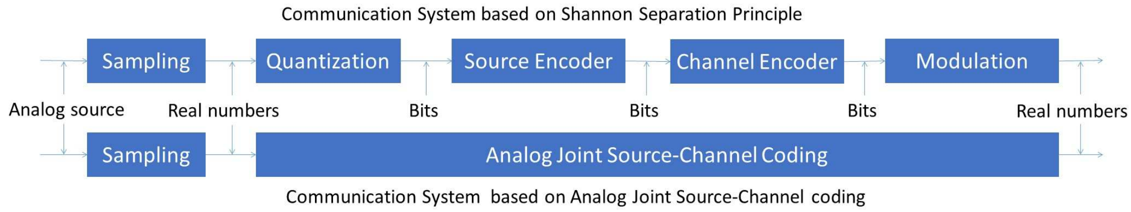


Figure 1.1: Canonical Digital Communications Systems based on the separation principle vs Analog JSCC systems.

of that trend in recent years [4] due to the different walls that the technology has hit, one might still argue that in general, the processing capabilities would certainly increase albeit at a slower rate and this is indeed what everybody is suggesting. As [4] puts it delicately "Nobody will say that Moores Law is over. But its starting to get really complicated". One might be led to conclude (misleadingly) from these observations that we really do not need any substantial reduction in the complexity of our current communication systems as the technology will always be able to catch up with our complicated systems. We will try to show the fallacy of this argument and the relevance/need for joint source channel encoding systems.

It is well established that the biggest applications of communications today was and still is wireless environments. The dream of every wireless communications engineer is to have (practically) unlimited throughput for every user. One can easily argue for the role that communications have played in changing our life in the past 20 years as well as the many improvements that this field can still provide for us. Since we all share the same bandwidth, classical approaches to accommodating multiple users is through separating the users orthogonally and assigning to each user his own degrees of freedom. This separation is done by assigning each user a "chunk" of independent communication channel either in time/frequency/code or a combination thereof. More recently the trend has become to allow the users to share the bandwidth (non-orthogonally) and perform some sort of successive interference cancellation (SIC) on the different users information, which theoretically leads to higher throughput for each user. The design philosophy of SIC also gives more freedom in assigning user rates and performing system design since orthogonal transmissions are known to be optimal only in limited several cases [5]. Unfortunately, that comes with the extra price of scaling the complexity either linearly or exponentially depending on the type of system used.

More recently, another degree of freedom point of view has been explored, namely the *space*. That is, communications systems are able to transmit more data by the added degrees of freedom that multiple transmit and receive antennas provide and by exploiting the channel statistics. Such systems are well known by their name Multiple Input Multiple Output (MIMO) Systems and the coding techniques employed for those systems are called space time codes. When several users are communicating over the channel and each employs multiple antennas, the system is called Multi User MIMO (MU-MIMO). The general achievable rates for any configuration of users of the MU-MIMO case is still an open area of research and indeed a very active one [6]. The added degrees of freedom stemming from the space are indeed significant. It was proved in [7] for example that for a MIMO system with n transmit and receive antennas and when the channel follows a Rayleigh fading model, then the capacity increases linearly with n for the same average transmit power. Indeed this is a very strong result stating that one might simply get extra capacity on the same bandwidth by increasing the number of antennas[1].

MIMO systems are quite popular and widely used in current communication systems because as explained previously they can provide extra system throughput, hence helping us solve the "Bandwidth crunch" problem. Bandwidth crunch describes the observation that we are running out of frequency bands to communicate over and the current bands are already exhausted and almost fully utilized. MIMO systems help communication engineers to transmit more information bits for a fixed bandwidth. Currently, there are two main approaches for the bandwidth crunch phenomenon that

[1] This effect requires the fading to be independent at the different receive antennas, which can be achieved by properly spacing the antennas and insuring an almost "independent" path between the different receive and transmit antennas. For details see [7].

we are experiencing in this age: One approach tries to design suitable communication systems in the empty Multi Gigahertz frequency band (millimeter-wave communication systems). This approach is showing some promise and there are already some initial R&D deployments [8] and uses for Backhaul transmission for wireless base stations (eNodeB[2]) and main serving gateway. There is, however, a big skepticism from the research community about the feasibility of using millimeter-wave systems in normal cellular operations (i.e, from the User Equipment (UE) to the base station). Some complaints about millimeter wave communication is its severe susceptibility to weather conditions as millimeter wave propagation is affected by rain [9]. Raindrops are roughly the same size as the radio wavelengths and therefore cause scattering of the radio signal. In the 60 Ghz Frequency band for instance, and for rain rates comparable to that in the East Coast of the United States (10 mm/hour), the attenuation would increase to 10 dB/KM just due to rain alone. The previous observation and other remarks explain why some people in the research and the industry communities are skeptical about the feasibility of practical communication over the Multi-Ghz spectrum.

The other promising technology that has high potential of solving the existing bandwidth crunch is Massive MIMO [10]. In massive MIMO systems, each transceiver is equipped with several antennas (100 transmit and receive in [11]). It was recently reported by DoCoMo, one of Japan's largest cellular service providers, that they were able to transmit data at a rate in excess of 10 Gbps using their demonstrated system [11]. Of course, such a system was a simple point-to-point system under near ideal conditions, however it requires very high encoding and decoding complexity. Such complexity would be even higher in a real multi-user scenario, and indeed it can be shown that the complexity grows exponentially with the number of users for MU-MIMO systems [10]. This is because optimal precoding needs to be performed on each source in the transmitter, as well as optimal decoding at the receiver. Such schemes might be suitable for certain types of applications. However, they are not well suited for

[2] This is the name of the base station in UMTS LTE systems

the mobile phone, since the decoding complexity would be too prohibitive and would consume a significant amount of battery time. Also it is a general design philosophy to have decoders at the User Equipment that consume as little power as possible, since the RF circuit is the biggest and fastest source of battery drain at the UE (comparable to the power consumed by the screen in most modern smart phones commercially available). There are current research trends that suggest that the energy density of batteries is increasing but still, one would hope to use the extended battery storage capacity in perhaps extending the cell phone active time or perhaps in other aspects more visible to the customer, rather than "wasting" the added battery storage capacity on very expensive decoding and encoding operations.

Moreover, as will be discussed in the next chapter, Canonical Digital Communication Systems are not very suitable for communication over networks, and in fact they are sub-optimal in general. This leads us to ask the question about the feasibility of analog JSCC systems as a viable alternative that can potentially be very suitable.

The original work proposed in this thesis[3] [12] is based on analog joint source-channel coding and successive encoding/decoding techniques, in particular, on a scheme devised in [13] called "Nested Quantization" (NQ), which in [14] was adapted for analog joint source channel coding over the Gaussian Multiple Access Channel (GMAC). We extend this framework to the Gaussian Broadcast Channel by developing novel mappings that are appropriate in this environment.

This thesis is organized as follows: Chapter 2 gives a brief introduction to Multi-Terminal Communications and defines formally the Broadcast channel and the theoretical limits of communication over it. Chapter 3 introduces Analog JSCC systems in general and explains in detail the motivation for their utilization introducing the specific mapping used for the Broadcast Channel when transmitting independent sources. In Chapter 3 we focus on communicating independent sources. This will help us gain intuition about the general correlated case. Chapter 4, which generalizes the

[3] Parts of this thesis was presented in the International Conference on Acoustics, Speech and Signal Processing (ICASSP) in Florence, Italy on May 2014.

proposed scheme to correlated sources, demonstrates how it is different from other schemes and presents Simulation results for several cases of interest. Finally, we provide the concluding remarks of the thesis and summarize the main contributions in Chapter 5.

Chapter 2

MULTI TERMINAL COMMUNICATIONS

The classical communication problem pioneered by Shannon was the point to point communication system with Discrete Memoryless sources and AWGN. It presents the most basic toy problem [1] that strips away most constraints on the problems as well as practical requirements. Yet, this problem truly captures the essence of the underlying concept of modern digital communication systems.

As mentioned in the introduction, point to point communication systems are well studied and understood and we now have practical coding schemes that come as close as desired to the Shannon capacity [16]. It took almost 50 years since Shannon's theory to approach the Shannon predicted theoretical limit via Turbo Codes, LDPC codes and similar code types. However, the general case in which a set of users wish to communicate one or more sources to a given subset of the users, as shown in Figure 2.1, is still very much open to research. For the general case we still do not know the theoretical limit for data transmission, let alone practical coding schemes that achieve those (unknown) rates.

The problems and challenges facing researchers in Network Information Theory are not inherent to the problems of information or communication per se, but rather they are inherent to the fact that we are dealing with a *network*, a collection of elements interacting and affecting each other, usually in a random manner. To give a concrete example of the inherent difficulties in dealing with networks, we borrow an example

[1] Robert Gallager in his book [15] highly emphasizes the importance of simple toy problems that on their own right might not have an application in real life, or perhaps limited applications. For example Shannon's original problem assumed infinite block length and perfect synchronization between the transmitter and receiver in order for his proposed system to work as he proposed. Both conditions are hardly achieved in any communication system (especially the former condition).

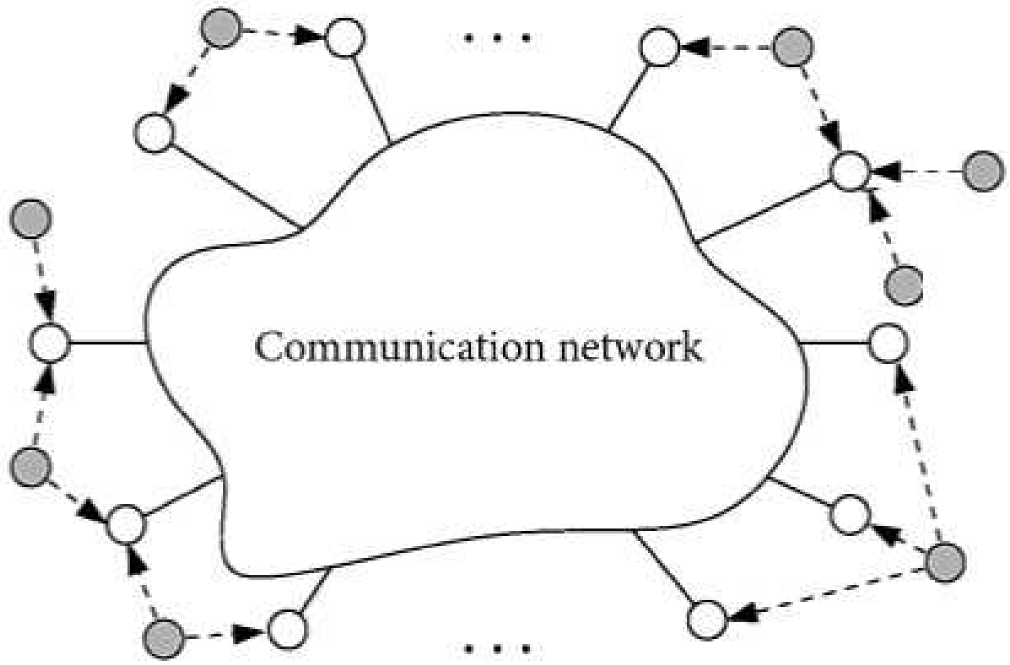


Figure 2.1: The shaded circles are information sources (can be text file, video, audio). The empty circles are observers of this data (also called users). Each user observes one or more sources and tries to communicate the information content of his observed source(s) to one or more of the users. (Figure taken from [17]).

from classical physics, namely the 3 body problem [18] depicted in Figure 2.2 below. In this example, we have 3 point masses moving in free space with certain velocities. Each body exerts gravitational pull forces on the other two. The problem is to find their trajectories over time. Such simple problem has no closed form expression! It must be approximated and solved numerically. Although for all practical purposes it is solved, yet there is no closed form expression that describes the movement trajectories under this simple assumption. Solutions to most current problems in Network Information Theory (such as channel capacities and specific coding techniques) do not have a closed form expression nor a computable asymptotic function to characterize them in polynomial time (i.e with current techniques and methods, most problems in Network Information Theory are not tractable).

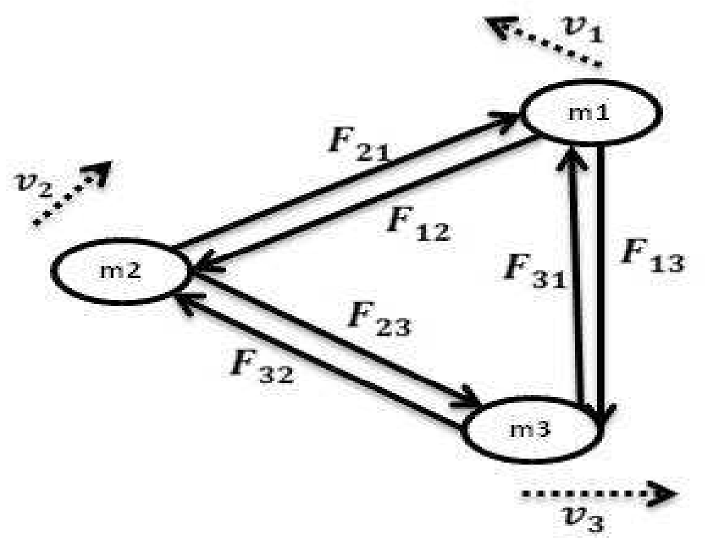


Figure 2.2: The interactions between 3 bodies in free space.

Note that in the 3-body problem, everything is deterministic: the force interaction between the bodies is a deterministic equation and the initial velocities and positions of the 3 bodies are also given. Compare this to the case in Network Information Theory where the sources that we wish to transmit are random by nature (if they were not random and unknown, then why bother communicating them?) Not only are the sources random but the different communication channels are also random and in the general case with different distributions. Taking all this into consideration one begins to appreciate the seriousness of the problem presented to Information Theorists and Communications Engineers.

Current research efforts in Network Information Theory focus on special configuration for the general multi-user network model presented in Figure 2.1. We list the most common ones here

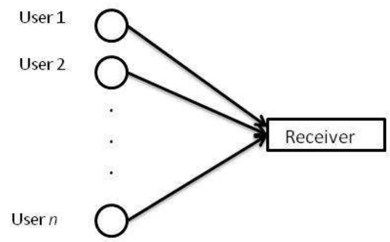
- Multiple Access Channels
- Broadcast Channels
- Relay Channels
- Interference Channel
- Two Way Channel

In the multiple access channel, there are several (2 or more) users wishing to communicate data (possibly correlated) to a central receiver. This is analogous to the *uplink* scenario for cellular networks where different users (UE) communicate and send their data to the base station (eNB). In the broadcast channel, there is one transmitter wishing to communicate data to several receivers. Again in the cellular setting, this corresponds to the *downlink* where the eNB transmits data to several UEs. In the relay channel, there is a relay (or several relays) that receive(s) the data from the transmitter and forward it (after possibly some manipulation) to the receiver. The receiver also receives the original transmitted data of the transmitter as well as the information sent by the relay. In the Interference Channel, there are two or more users wishing to transmit the information only to their respective receiver, that is receiver i is only interested in the information of transmitter i and nothing else. This is like a hybrid between the MAC and broadcast channels. There is also the two way channel in which there are two users wishing to communicate to each other *in both directions*. These different scenarios are depicted in Figure 2.3. Note that in the general setting, the imposed general network would be a combination of any of the above configurations.

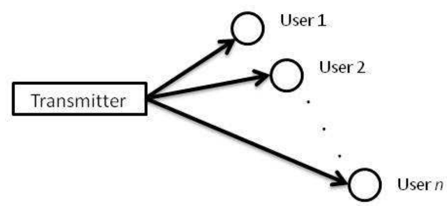
Our focus in this thesis will be the Broadcast Channel, in particular the Gaussian Broadcast Channel as we will define in the next section.

2.1 The Broadcast Channel

The broadcast channel was first considered by Cover in [19]. The problem formulation of the Broadcast channel is the following: Suppose you have a central



(a)



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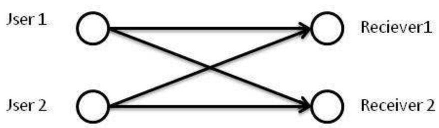
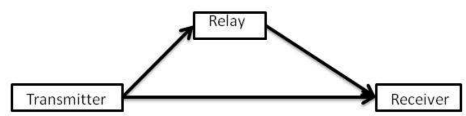
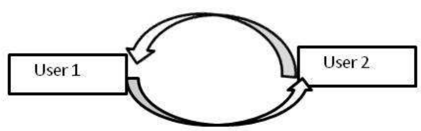


(c)



(d)



(e)

Figure 2.3: Different configurations of the general multi terminal communications problem: (a) Multiple Access Channel; (b) the Broadcast Channel; (c) Interference Channel (d) Relay Channel; and, (e) Two Way Channel.

transmitter and several receivers, the central transmitter wishes to communicate a common message to all the receivers as well as a private message to each receiver[2]. Cover showed in [19] that time sharing is strictly suboptimal for the broadcast channel

[2] One might even consider a more general case where the central transmitter has W messages and each receiver is interested in a particular subset of the messages

in the general case[3]. He showed that the optimal way to achieve the channel capacity is via *superposition coding*. The idea of superposition coding is used frequently in communication systems. It is similar to Successive Interference Cancellation (SIC) in Multiple Access Channels. Next, we give a concrete example about the sub-optimality of time sharing in general for broadcast channels, showing the need to perform some sort of superposition coding. This example is given in [20].

Dutch and Spanish speaker: Suppose you speak both Dutch and Spanish and you wish to communicate to two persons, one who only speaks Dutch and the other person only speaks Spanish. Furthermore, suppose that each language has two 2^{20} words, and assume that there are no common words in Spanish and Dutch. Also assume that the speaker can speak at a rate of one word per second in any language, hence the speaker can speak at rate of 20 bits/second. If the speaker chose to communicate to the Dutch speaker half the time and the Spanish speaker half the times by speaking a word to the Spanish speaker and then a word to the Dutch speaker alternatively, then he would be communicating to each listener at a rate of half a word per second or equivalently 10 bits/second to each listener. Can he do better? In fact, yes. We note that we can communicate information via the *order* in which the speaker interleaves the words to each listener.This is possible because each listener can determine if the spoken word is either Spanish or Dutch (we have assumed that the two language alphabets are disjoint[4]). We can choose any order of the words as shown in Figure 2.4 below. Thus in effect we can communicate that extra information of order. If we look at 4 consecutive time slots, there are 4 possible different orderings. Hence we can communicate 2 extra bits in these 4 time slots(seconds). Thus we added an extra 0.5 bits of information and the overall communication rate would be 20.5 bits/second[5].

[3] Time Sharing or Frequency Sharing or CDMA or any other orthogonal scheme can achieve the capacity of the Broadcast Channel in special cases. For example for the two user Gaussian broadcast with the two receivers having the same noise variance, time sharing is optimal.

[4] I believe that the authors in [20] chose Spanish and Dutch in their example because in reality Spanish and Dutch sound very different from each other, at least they do to the author.

[5] In fact by considering a larger word block length instead of 4 in the above example, it can be

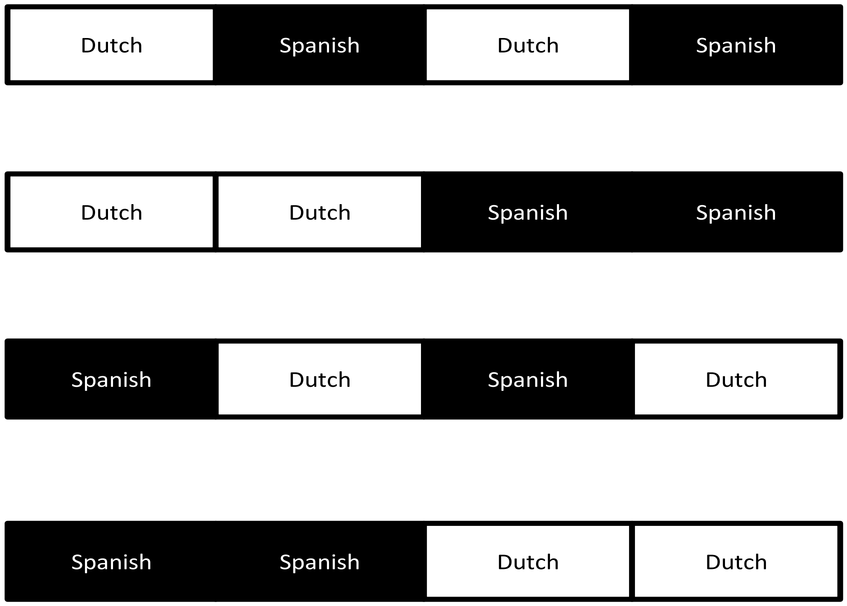


Figure 2.4: Different orderings of 4 messages in a 4 time slot communication system. Each order represents a message from a codebook.

Next we formalize the above argument. However, note that this characterization is the one typically used when working with discrete digital systems as will be shown in the next section. Since we are considering analog joint source channel coding systems here, we will give an alternate characterization of the broadcast channel that is specific to the case in which we are interested in sending analog Gaussian sources. We shall still consider the digital (discrete-values) version of the Broadcast channel because some of the arguments are similar and it is clearer to see the techniques in the digital domain.

2.1.1 Formal Characterization of the Discrete Broadcast Channel

Before we proceed, we discuss the terminology to be used in the rest of the thesis: A random variable will be denoted by capital letters X,Y. Samples will be denoted by

shown that in the limit of taking the block length to ∞, we can communicate at most 1 bit using the described scheme. Thus bringing the total number of transmitted bits to 21 bits/second [20].

lower-case letters x, y. A vector $(x(1), x(2), \cdots, x(n))$ will be denoted by x^n.

A broadcast channel consists of an input alphabet $\mathcal{X}$, two output alphabets $\mathcal{Y}_1$ and $\mathcal{Y}_2$, and a probability transition function $p(y_1, y_2|x)$ for $x \in \mathcal{X}, y_1 \in \mathcal{Y}_1 \quad and \quad y_2 \in \mathcal{Y}_2$. We shall consider memoryless broadcast channels in which

$$p(y_1^n, y_2^n|x^n) = \prod_{i=1}^{n} p(y_1(i), y_2(i|x(i)), \qquad \text{where} \tag{2.1}$$

$i \in \mathbb{N}$ is the time index.

The broadcast channel has one encoding function f and two decoding functions g_1 and g_2 We define the encoding function f as

$$f = \{1, 2, 3, \cdots, 2^{nR_1}\} \times \{1, 2, 3, \cdots, 2^{nR_2}\} \rightarrow \mathcal{X}^n, \tag{2.2}$$

where R_i is user i information rate and it has units of bits[6] per channel use.

We define the decoding functions as

$$g_1 : \mathcal{Y}_1^n \rightarrow \{1, 2, \cdots 2^{nR_1}\} \tag{2.3}$$

$$g_2 : \mathcal{Y}_2^n \rightarrow \{1, 2, \cdots 2^{nR_2}\} \tag{2.4}$$

That is, the encoding function f takes as input a pair of two messages, one intended for user 1 and the other intended for user 2. At the decoders side, decoder i receives $y_i^n \in \mathcal{Y}_i$ and estimates user i transmitted message among the possible choices of $\{1, 2, \cdots 2^{nR_i}\}$

We define the average probability of error as

$$P_e^{(n)} = \{g_1(y_1^n) \neq W_1\}||\{g_2(y_2^n) \neq W_2\}, \tag{2.5}$$

[6] Since we choose the base of the terms in 2.2 to be 2 (i.e. 2^{nR_i}).

where (W_1, W_2) are assumed to be uniformly distributed over $2^{nR_1} \times 2^{nR_2}$

We say that a rate (R_1, R_2) is achievable for the broadcast channel if there exists a sequence of $((2^{nR_1}, 2^{nR_2}), n)$ codes (functions (f, g_1, g_2)) with $\lim_{n\to\infty} P_e^{(n)} = 0$.

A broadcast channel is said to be physically degraded if $p(y_1, y_2, x) = p(y_1|x)p(y_2|y_1)$. The channel is also said to be stochastically degraded if its conditional marginal distributions are the same as that of a physically degraded broadcast channel, that is if there exists a distribution $p'(y_2|y_1)$ such that

$$p(y_2|x) = \sum_{y_1} p(y_1|x)p'(y_2|y_1) \tag{2.6}$$

Since the capacity only depends on the marginal distributions, the capacity of the physically degraded broadcast channel is the same as that of the stochastically degraded channel, hence hereinafter we shall assume physical degradedness[7].

Now we give the capacity region for the physically degraded Broadcast Channel when sending independent information at a rate R_i to user i.

Theorem 2.1.1 Capacity of Broadcast Channel *The Capacity Region for sending independent information over the degraded broadcast channel $X \to Y_1 \to Y_2$ is the convex hull of the closure of all (R_1, R_2) satisfying*

$$R_2 \leq I(U; Y_2) \qquad\qquad R_1 \leq I(X; Y_1|U) \tag{2.7}$$

for some joint distribution $p(u)p(x|u)p(y_1, y_2|x)$, where the auxiliary random variable U has cardinality bounded by $|U| \leq \min\{|X|, |\mathcal{Y}_1|, |\mathcal{Y}_2|\}$

Proof See [20] Chapter 15.6.

[7] Physical degradedness means that actually the second received source can be obtained by getting the second source and actually (physically) modifying it via another channel between the first and second receiver. Statistical degradedness means the second source distribution looks like it was obtained by physical degradedness but in fact it may not. The Gaussian channel is always physically degraded.

We are not interested in the details of the proof itself, rather in an observation about the proof technique itself. The coding idea is basically to have a big "cloud" U that both users can see. However, only the strongest user (with less noise) can see what is inside the cloud. The other (worse) user can differentiate the cloud and know the type of the cloud (the specific $u \in U$), but not what is inside. In that case the cloud u is the information that is intended for user 2, and the contents of the cloud is the information intended for user 1. This idea will be used in the actual proposed communication system in Chapter 3. The above argument can not be used directly for analog systems because we cannot completely remove the component of the second source (the cloud) at the first receiver. We are able to do that for digital systems because we can enumerate all possibilities of the second source. However, for analog sources we would always have a residual noise even after MMSE filtering or whatever filtering technique one chooses.

A rather very interesting remark about the capacity region for the Broadcast channel is that feedback indeed *increases* the capacity region [21]. This remark is true for most settings in Multi-Terminal Communications and it should not come as a surprise. On the other hand, we observe that feedback *does not* increase the capacity for point to point communications [8] [20]. This exemplifies the fact that point to point communication is a very special case of the general communication problem and hence many simple schemes that are generally not optimal for the networked case would be optimal for the point to point case.

2.1.2 Broadcasting Gaussian sources over the Gaussian Broadcast Channel

In this section we shall formally define the Gaussian Broadcast channel for transmitting a bivariate Gaussian source.

[8] Although feedback does not increase capacity for point to point communications, it can certainly reduce the error exponent as well as the complexity of the encoding/decoding significantly.

Let (S_1, S_2) be a stationary, memoryless bivariate Gaussian distribution with zero mean and Covariance matrix $\boldsymbol{C}$ given by

$$\boldsymbol{C} = \begin{bmatrix} 1 & \rho^2 \\ \rho^2 & 1 \end{bmatrix}, \tag{2.8}$$

with $0 \leq \rho \leq 1$. In the above presentation we have assumed that each source has unity variance. The encoder function f is defined to take (S_1^n, S_2^n) and produce X^n, the channel input. The output of the broadcast channel Y_i is given by

$$Y_i(k) = X(k) + Z_i(k), \qquad i = 1, 2, \text{ and } \qquad k \in N_+, \tag{2.9}$$

where $Y_i(k)$ is the channel output observed by the $i-th$ receiver at time k and $Z_i(k)$ is the zero mean independent Gaussian noise with variance N_i experienced by user i at time k. Without loss of generality here, we assume $N_2 \geq N_1$.

The decoding functions g_i of the broadcast channel are defined to take Y_i^n and produce an estimate $\hat{S}_i^n$ of the original transmitted source symbol S_i^n. Note that the function g_i is only required to produce an estimate of the source for user i, although it can be helpful in the decoding if one of the users has an estimate about the other user's data (in fact to achieve optimality in the digital case, the stronger user *must* be able to recover the weak user's data to determine the main cloud in which his information lies within). The Gaussian Broadcast channel for Gaussian sources is shown in Figure 2.5.

To define the error between the original source symbol S_i and the decoded source symbol $\hat{S}_i$, we use the Mean Squared Error criteria given by

$$D_i(s_i^n, \hat{s}_i^n) = \frac{1}{n}\sum_{j=1}^{n}(s_i(j) - \hat{s}_i(j))^2 \qquad for \qquad i = 1, 2 \tag{2.10}$$

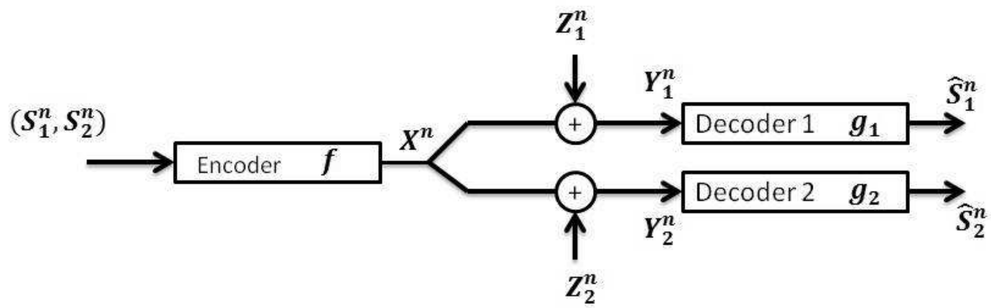


Figure 2.5: Broadcasting a Bivariate Gaussian source over the Gaussian Channel.

We also operate under an average transmit power constraint P defined by

$$\frac{1}{n}\sum_{j=1}^{n} X^2(j) \leq P \tag{2.11}$$

Next we give the definition of achieveability of a given distortion pair (D_1, D_2)

Definition A distortion pair $(D_1, D_2) \in \mathbb{R}_+ \times \mathbb{R}_+$ is achievable under power constraint P if for any $\epsilon > 0$ there exists an $A \in \mathbb{N}_+$ such that $\forall n \geq A$, there exists a broadcast channel code (functions f, g_1, g_2) achieving a distortion pair of (d_1, d_2) such that $D_i \geq d_i + \epsilon$ for $i = 1, 2$.

The collection of all the achievable distortion pairs under power constraint P for a given bivariate source is denoted by $\mathcal{D}(P, \rho, N_1, N_2)$. Such collection can be fully characterized by determining the following function

$$D_2(P, \rho, N_1, N_2, D_1) = \min_{(D_1, d_2) \in \mathcal{D}(P,\rho,N_1,N_2)} d_2 \tag{2.12}$$

The function in (2.12) basically says that to get the region $\mathcal{D}(\cdot)$, we should fix D_1 and search for the minimum achieved d_2 given that D_1. We can simply deduce the extrema of both D_1 and D_2. This can be done by ignoring one of the sources and transmitting only the other source. In this case the Broadcast channel becomes a single point to point channel for the user intended to receive that information. For example, ignoring user 1 and transmitting only S_1, user 2 achieves a distortion of D_2^{min} and user 1 achieves a distortion of D_1^{max}. Similarly we can get D_1^{min} and D_2^{max}, where

$$D_1^{max} = \frac{(1-\rho^2)P + N_1}{P + N_1}, \qquad D_2^{min} = \frac{N_2}{P + N_2} \tag{2.13}$$

$$D_1^{min} = \frac{N_1}{P + N_1} \qquad D_2^{max} = \frac{(1-\rho^2)P + N_2}{P + N_2} \tag{2.14}$$

.

Note that the *max* values of the distortions are obtained by noting that the other receiver whose information is not sent can still receive a noisy version of the other source (that we actually transmitted). Since the sources are correlated, this receiver can recover some information. We note that if the sources are independent ($\rho = 0$) then that receiver would not be able to get any information from the other source and hence the distortion would be the source variance itself. The source variance represents the maximum possible distortion available incurred when sending the Gaussian source in a point to point system. This maximum distortion is attained when we send nothing and simply decode all source symbols to the mean (zero in our case). Hence it is only useful to consider values of D_1 in 2.12 for those $D_1 \in [D_1^{min}, D_1^{max}]$

The following Theorem (taken from [22]) provides complete characterization of the achievable distortion regime $\mathcal{D}(\cdot)$ via characterization of $D_2(\cdot)$.

Theorem 2.1.2 *If $D_1 > D_1^{max}$, then*

$$D_2(P, \rho, N_1, N_2, D_1) = \frac{N_2}{P + N_2} \tag{2.15}$$

if $P \le \frac{2\rho N_1}{1-\rho}$, *then for* $D_1 \in [D_1^{min}, D_1^{max}]$, $D_2(\cdot)$ *is given by*

$$
\begin{aligned}
&D_2(P,\rho,N_1,N_2,D_1) = D_2^u(P,\rho,N_1,N_2,D_1)\\
&= \left(\sqrt{1-\frac{D_1(P+N_1)}{P}+\frac{N_1}{P}} - \sqrt{\frac{\rho^2}{1-\rho^2}\Big(\frac{(P+N_1)D_1}{P}-\frac{N_1}{P}\Big)}\right)^2 \\
&\times \frac{(1-\rho^2)P}{P+N_2}+\frac{N_2}{P+N_2}
\end{aligned} \tag{2.16}
$$

On the other hand if $P > \frac{2\rho N_1}{1-\rho}$, *then* $D_2^u(\cdot)$ *is characterized by*

$$
D_2(P,\rho,N_1,N_2,D_1) = \begin{cases} D_2^u(P,\rho,N_1,N_2,D_1) & \textit{if } D_1 \in [D_1^{min}, D_1^-] \cup (D_1^+, D_1^{max}] \\ D_2^h(P,\rho,N_1,N_2,D_1) & \textit{if } D_1 \in [D_1^-, D_1^+] \end{cases}, \tag{2.17}
$$

where

$$
D_2^h(P,\rho,N_1,N_2,D_1) = \frac{1}{P+N_2}\left(\frac{N_1(1-\rho^2)}{D_1}+N_2-N_1\right), \tag{2.18}
$$

$$
D_1^- = \frac{(P+2N_1)(1-\rho^2)-\sqrt{(P^2-(P+2N_1)^2\rho^2)(1-\rho^2)}}{2(P+N_1)} \tag{2.19}
$$

$$
D_1^+ = \frac{(P+2N_1)(1-\rho^2)+\sqrt{(P^2-(P+2N_1)^2\rho^2)(1-\rho^2)}}{2(P+N_1)} \tag{2.20}
$$

We shall only discuss the coding schemes that achieve such bounds. For a complete proof of Theorem 2.1.2, see [22]. Note that $D_2^u(\cdot)$ means using the uncoded scheme and $D_2^h(\cdot)$ means using the hybrid scheme. We shall explain in some detail what is meant by the uncoded and hybrid schemes in the following discussion.

Let us discuss in some detail the meaning of Theorem 2.1.2. (2.15) is obtained by transmitting source 2, S_2, at the channel using the full power P. Then we perform MMSE decoding at receiver 2 to get the aforementioned distortion (which is the lowest possible achieved distortion for source 2). This basically converts the problem into a point-to-point problem. Receiver 1 would decode his information solely on the information that is originally intended for user 2 and achieves the aforementioned distortion of D_1^{max} (as explained in 2.13). In Appendix A we gave a proof of why user 1 achieves D_1^{max} in that case. The second case of Theorem 2.1.2 given by (2.16) is concerned when the transmission power, P, is below a certain threshold. In that case, uncoded transmission is optimal for all values of D_1. The last part of Theorem 2.1.2 states that when P is greater than a threshold, then you would have regions (depending on D_1) in which uncoded transmission is optimal or a specific hybrid coding scheme is optimal but not the uncoded one. Figure 2.6 depicts these regions visually.

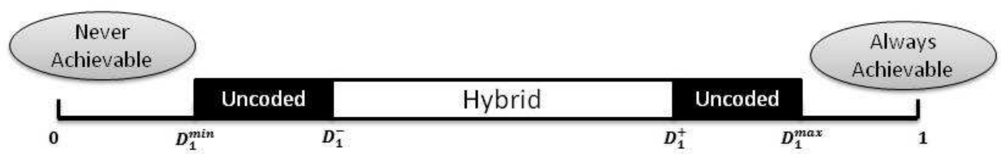


Figure 2.6: Optimal Coding schemes and associated regions as described by (2.17) in Theorem 2.1.2. Note that the maximum distortion incurred can not be larger than 1 because we are working with sources of variance 1.

Let us now describe the coding schemes that achieve these optimal distortion regimes of the Broadcast Channel. By uncoded scheme we mean that scheme that sends

$$X = tS_1 + rS_2 \qquad \text{where} \qquad t, s \in \mathbb{R}. \tag{2.21}$$

The scheme is called uncoded because the block length of the code is 1 and the code has zero delay. We simply scale each user information and send the sum over the channel. The decoder performs MMSE estimation to obtain

$$\hat{S}_i = \mathbb{E}(S_i|Y_i) \qquad \text{for} \qquad i = 1, 2 \qquad , \tag{2.22}$$

where $\mathbb{E}(\cdot)$ is the Expected value operator.

This simple scheme was shown to be optimal for the cases discussed in Theorem 2.1.2 when the parameters (t, r) are chosen properly. This was first proved in [23].

Next we consider the hybrid scheme given by

$$X^n = tS_1^n + rS_2^n + X_{2d}^n \qquad \text{where} \qquad t, s \in \mathbb{R}, \tag{2.23}$$

and X_{2d}^n is a scaled quantized and then coded version of the second source (the source intended to the receiver with more noise). The authors in [22] showed that if the digital code corresponding to user 2 is designed properly (using optimal capacity achieving codes), so that receiver 2 is able to obtain X_{2d}^n error free (this is possible due to the Noisy Channel Coding Theorem [20]) and when choosing the proper scale factors (t, r), then the optimal distortion is achieved. Note that if receiver 2 can decode X_{2d}^n error free then receiver 1 can do that as well because receiver 1 is the stronger receiver (by Physical degradedness of the Broadcast channel). And in fact Receiver 1 needs to recover X_{2d}^n to achieve the stated distortion in Theorem 2.1.2.

The authors in [22] claim that this is the first case in the literature of Network Information Theory in which a hybrid scheme was shown to be required to achieve optimality. They have shown that using optimal digital codes based on the separation principle (analyzed in [24]) is strictly suboptimal in certain regions. Similarly, uncoded (analog) transmission was analyzed thoroughly in [23] and again shown to be strictly suboptimal in certain regions. This is indeed a very interesting result and reinforces the points discussed in the introduction about the challenges facing pure digital communication systems.

The discussion above does not suggest replacing all digital communication systems merely because they are sub-optimal in specific cases. This is simply not how the world around us works. Take the example of the celebrated Internet Protocol (IP) and the ongoing debate about migrating to IPv6 and the inherent current deficits on the existing IPv4. Existing systems utilizing IPv4 break one of the most powerful features that the Internet was founded on. Namely, a unique network address per machine. Yet we hesitated to abandon IPv4 and tended for an easier solution with Network Address Traversal (NAT). However hybrid systems that combine both digital and analog codes seem to fit perfectly into the current paradigm of new technology inception and how things work in today's world. That is, one can design devices communicating using a hybrid analog-digital communication system and yet remain backward compatible with older digital systems (since older purely digital systems can be considered a hybrid system with no analog subcomponent). This principle was used before in many particular cases such as stereo FM radio[9]. Before stereo FM, the FM signal just contained one audio channel. Then, stereo speakers became popular and there had to be a way to broadcast two audio channels and still the old receiver should get the standard one audio channel as before. So on the primary frequency band, you would transmit the sum of the Right(R) and Left(L) Speakers. The old receivers with just one audio channel would receive this and would play the R+L as usual, you would also broadcast the difference R-L on the neighboring secondary frequency band and only stereo receivers would get this R-L sub channel. Hence, by combining the primary band and the neighboring secondary channel, you can get an independent R channel and L channel.

Hybrid systems also have another interesting property for multiple access channels as discussed in [26]. In [26], the authors showed that for the 2-user MAC channel, only one digital code in combination of two orthogonal (possibly analog) is required to achieve any point in the capacity region. It was known that orthogonal systems are optimum for the MAC only for a certain rate pair where the rates are allocated proportionately to the powers of each user. To achieve any other optimal rate pair in the capacity region, digital codes and techniques (successive interference cancellation) must be used. Indeed the proposed communication scheme that we propose for the Gaussian Broadcast channel is a hybrid one and will be explained in Chapter 3.

So as we saw above, (analog) uncoded transmission achieves the theoretical limit (is optimal) for the GBC for a given region. This is not the only case in which analog uncoded systems are known to achieve optimality. Other examples include the case

[9] This is discussed in greater detail in [25].

discussed by [27] in 1965 in which the author considers transmitting a Gaussian source over a bandwidth matched Gaussian Channel. He showed that in this case simply sending the source (after scaling to meet the average power constraint) and performing MMSE estimation achieves the theoretical limit. This result was extended for the Gaussian Multiple Access Channel (GMAC) for an arbitrary number of users and it was shown in [5] that uncoded transmission of the user's source samples (after scaling by a Matrix $\mathbf{H}$) and performing MMSE estimation on the joint received vector does in fact achieve the optimal performance[10] when imposing symmetry conditions (that is equal power to each user and the expected source distortion for each user is the same)

2.2 Optimal Distortion Region for independent Gaussian sources

We shall first consider sending independent Gaussian sources over the channel and then generalize the proposed scheme for the general correlated case. Note that in most source coding problems considering independent sources is considered trivial. However, the problem is not trivial in this setting because simple time-sharing is not optimal and sophisticated coding techniques are required to achieve optimality. Moreover, considering the independent case will shed light into the correlated case and the argument would be much clearer.

The Optimal distortion pair attainable when transmitting independent sources is readily obtained by plugging $\rho = 0$ in Theorem 2.1.2. However, we notice that when the two Gaussian source components are independent, then source-channel separation is optimal in that case. We shall give a general case that even considers source and channel mismatched bandwidths. Let us first define what we mean by bandwidth match or mismatch, we define the source-channel use factor κ as the number of source symbols M that each user wishes to transmit using the channel B times, that is $\kappa = \frac{M}{B}$. The following derivation is based on the optimality of the separation principle for independent sources for the Broadcast Channel [24]. We assume that both users wish

[10] This general idea of this paper is to generalize scalars to vectors and matrices and optimum performance is still achieved and proven under symmetry conditions as explained in [5]

to transmit at the same bandwidth expansion/reduction factor κ. Note if $\kappa = 1$ means that each user transmits $\kappa = 1$ source symbol using the channel once. This is equivalent to the Gaussian Bivariate case, whose optimal distortion region was derived in the previous section. In that case we can write

$$\kappa R(D_1) < \frac{1}{2}\log_2(1 + \frac{(1-\gamma)P}{\gamma P + N_1}) \tag{2.24}$$

$$\kappa R(D_2) < \frac{1}{2}\log_2(1 + \frac{\gamma P}{N_2}), \tag{2.25}$$

where $\gamma \in [0, 1]$ and $R(D_i)$ is the Rate Distortion for each source given by $R(D_i) = \frac{1}{2}\log_2(\frac{1}{D_i})$. γ can be thought of as the "power sharing" factor i.e., it determines the percentage of power each user source component gets. (2.24) states that for each user to send his data at a rate κ then user i performs source coding on his group of κ source symbols using a source code of a rate at most $R(D_i)$ per source symbol. Then it applies a channel code on the encoded data at a rate of 1[11]

Simplifying 2.24 above, we get the two achieved distortions parametrized by γ as follows

$$D_1 = (1 + \frac{(1-\gamma)P}{\gamma P + N_1})^{-\kappa} \tag{2.26}$$

$$D_2 = (1 + \frac{\gamma P}{N_2})^{-\kappa}, \tag{2.27}$$

where $\gamma \in [0, 1]$. γ is a parameter that controls the power allocated to each user's data.

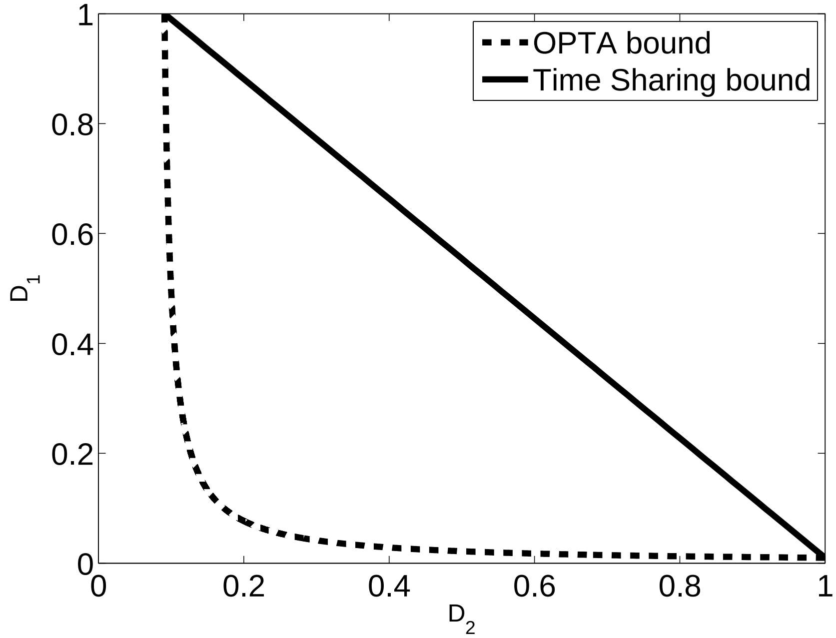
Figure 2.7 shows the plot of D_2 against D_1 both on a linear and logarithmic scale. For the logarithmic scale we define the Source to Distortion Ratio (SDR) as

$$SDR_i = 10\log_{10}(1/D_i) \tag{2.28}$$

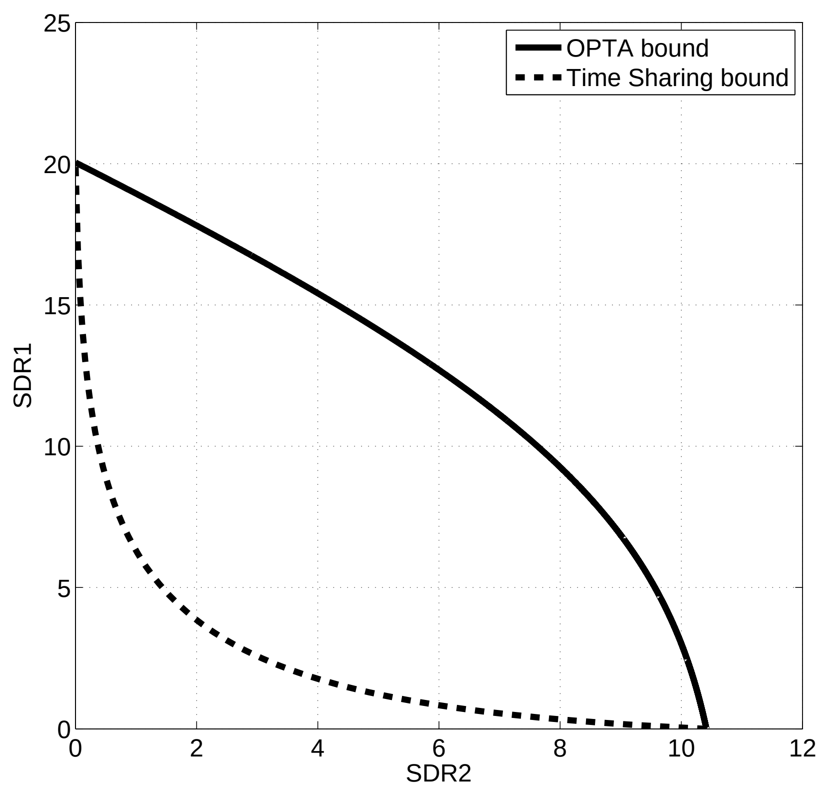
[11] Note that we can use any source code of rate $x > 0$ together with any channel code of rate $\frac{\kappa}{x}$. This will gave the desired rate of κ source symbols per channel use.

The extreme points of the graph are obtained by ignoring one of the two sources and utilizing the full power to only transmit to the other user. In that case the broadcast problem degrades to a point-to-point communication problem. A time sharing solution of the extreme points is also plotted in Figure 2.7. The time sharing performance is obtained by utilizing the channel ζ of the time to transmit user 1 data (using the point to point scheme) and $1-\zeta$ of the time to transmit user 2 data, where $0 \leq \zeta \leq 1$. Time sharing techniques are explained in detail in the Appendix as their simplicity merits some deep understanding.

Restricting ourselves to the bivariate case again, $\kappa = 1$, we remark that by plugging $\rho = 0$ in Theorem 2.1.2, then the hybrid scheme (along with the uncoded scheme as stated above) is the only optimal coding scheme over the whole Range of $[D_1^{min}, D_1^{max}]$ in that case. The uncoded scheme will only be optimal at the extreme points where $D_1 = \{D_1^{min}, D_1^{max}\}$.



(a)



(b)

Figure 2.7: OPTA curves for $P = 1$, $N_1 = 0.01$, $N_2 = 10N_1$ and $\rho = 0$: (a) the Linear Scale; and, (b) the Log Scale.

Chapter 3

ANALOG JOINT SOURCE CHANNEL CODING SYSTEMS

As discussed briefly in the Introduction, analog JSCC systems were discussed at times when information theory was in its infancy. This occurred around the fifties of the previous century and was quite hot until researchers lost their interest sometimes in the sixties. The probable reason for that is at that time analog (continuous) systems such as Amplitude and Frequency Modulation (AM and FM) were the norm at that time, hence it would seem natural to consider analog JSCC systems at that time. However, the amount of IEEE papers talking about analog JSCC is quite small (as shown in Figure 3.1). This is perhaps explained by the fact that Shannon in his seminal paper proved the optimality of separate source-channel coding and also because of the development of Rate Distortion Theory and Vector Quantization, hence the need for analog JSCC was probably not obvious at that time.

One would still recall the arguments of researchers and communications engineers at that time who thought that digital communications would never take off because of the enormous bandwidth that digital signaling required at that time (sampling, quantization), as well as the much higher complexity at that time (that was the dawn of the transistor and the Microelectronics revolution). Researchers regained interest in analog JSCC systems around the nineties. This is highly correlated to the revived interest in Network Information Theory when point-to-point communication systems were to a large degree understood and deemed to be a solved problem for all practical uses. Researchers from Network Information Theory found that uncoded (analog) transmission is optimal for several cases, for example as discussed in Section 2.2, while its complexity and delay is much less than for digital systems. Analog JSCC

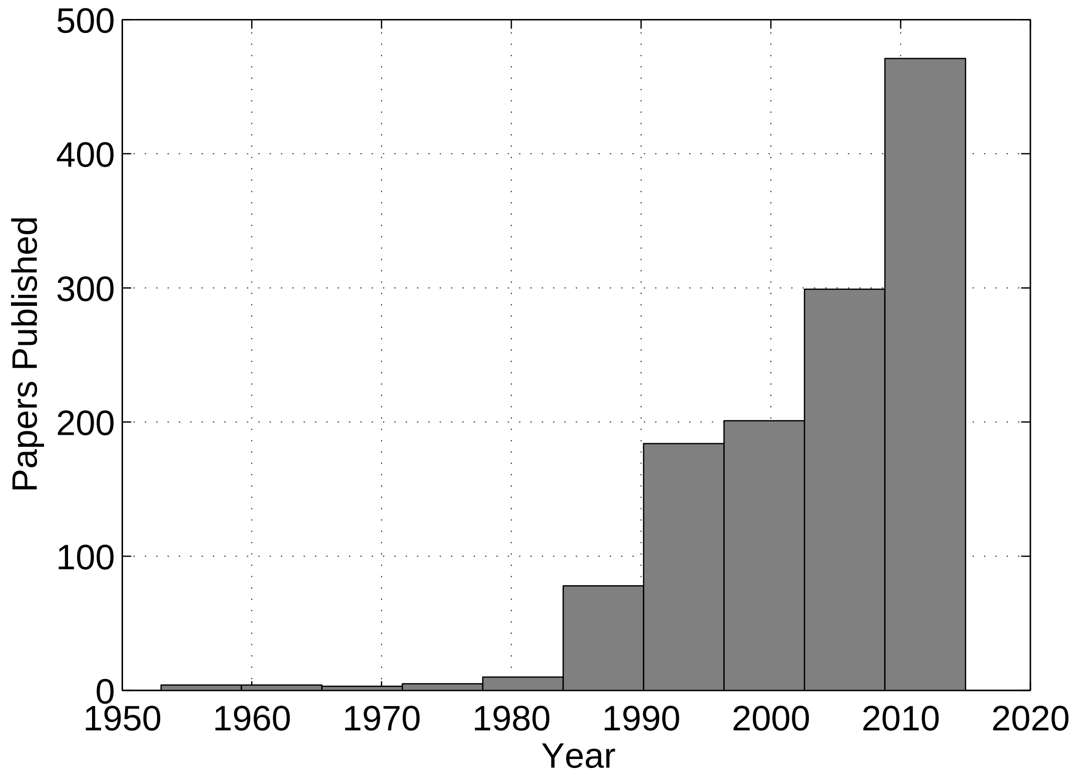


Figure 3.1: Frequency of Research Papers published on IEEE since 1950. Notice the almost exponential growth in Research in the past few years. (obtained via search term "Analog Joint" in IEEE Xplore and obtained on October 15 2014.).

systems have the nice property that it is easier to preserve correlation among different sources when designing the codebooks, since it easier to exploit correlation in the original source space. It becomes much more difficult to exploit correlation after performing quantization because the correlation model becomes much more complicated and one would have to look at bit-levels and different quantization levels jointly and so forth. Moreover, one of the strongest benefits to digital systems, namely that performing optimum source coding followed by optimum channel coding can still achieve optimality is no longer valid in networks in general as discussed in section 2.1.2. Surprisingly, even in point to point communications, separation still may not hold and in fact it does not in general. Consider the example given in [28]. Separation for point to be point communication systems can however be shown to hold for “well behaved systems” such when the channel is i.i.d and the noise is ergodic and stationary [28].

More recently analog JSCC systems were proposed and used for the AWGN channel. The systems proposed in [2, 3] are based on space filing curves and the idea is to go from the source space to the channel space via a projection onto a curve. If we restrict ourselves to linear systems, then except for the bandwidth matched case, the system performance degrades at high SNR and the performance saturates at a rather low SNR [29]. Hence to achieve better performance in the high SNR regimes, Non-Linear mappings must be considered. We shall discuss a particular type of these Non-Linear mappings, namely the space filling curves in section 3.3 and show the connection to the new mapping that we propose in this thesis.

3.1 Nested Quantization

The proposed communication scheme for the Broadcast Channel is based on Nested Quantization (NQ). NQ first appeared in [13] and was used for the Multiple Access Channel (MAC). NQ is used to transmit two digital indices (both from finite alphabets) on the same channel symbol.

A variant of NQ called *Scalar Quantizer Linear Coder* (SQLC) was used in [14], again for the MAC channel and in that scheme one digital index (from a finite alphabet) and an analog (continuous) symbol are mapped to one channel symbol. The proposed scheme in this thesis is based on SQLC.

In SQLC, the two users source symbols are encoded into one channel symbol. The first symbol, x_1, is passed through a uniform quantizer of step Δ to produce y_1. The second symbol, x_2, is scaled by α and clipped to force it to lie in the interval $[-\frac{\Delta}{2}, \frac{\Delta}{2}]$ to produce y_2. The sum of $y_1 + y_2$ is sent through the channel after scaling it by a factor, β, that controls the power of the transmission system. That is

$$y_1 = \Delta\lceil \frac{x_1}{\Delta} \rfloor \tag{3.1}$$

$$y_2 = \mathcal{L}_{\pm\frac{\Delta}{2}}(\alpha x_2) \tag{3.2}$$

$$\eta = y_1 + y_2 \tag{3.3}$$

$$y = \beta\, \eta, \tag{3.4}$$

where $\lceil \cdot \rfloor$ rounds its argument to the nearest integer, α controls the spread of the second symbol and $\mathcal{L}_{\pm\frac{\Delta}{2}}[\cdot]$ forces its output to lie within the interval $[-\frac{\Delta}{2}, \frac{\Delta}{2}]$. That is

$$\mathcal{L}_{\pm\frac{\Delta}{2}}(\lambda) = \begin{cases} \lambda & \text{if } -\frac{\Delta}{2} \leq \lambda \leq \frac{\Delta}{2} \\ \frac{\Delta}{2} & \text{if } \lambda > \frac{\Delta}{2} \\ -\frac{\Delta}{2} & \text{if } \lambda < -\frac{\Delta}{2} \end{cases} \tag{3.5}$$

Note that $\lceil \frac{x_1}{\Delta} \rfloor$ in (3.1) produces an *integer* (positive, negative or zero). That integer is then scaled by Δ to produce y_1.

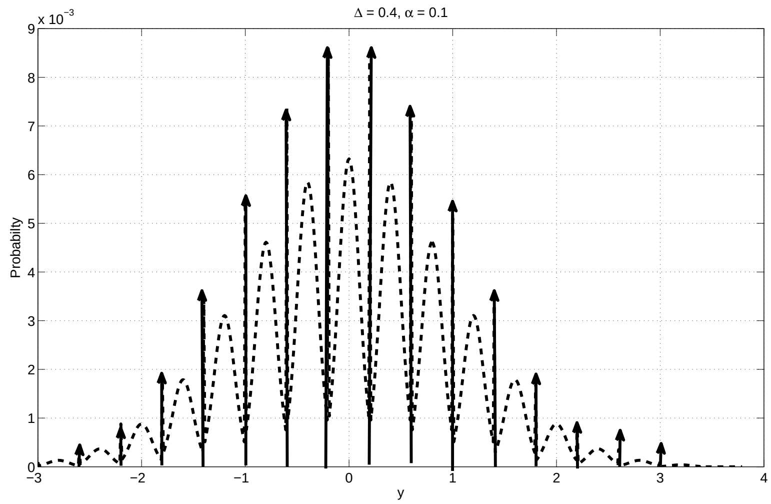
Figure 3.2 shows the distribution of y symbols that are sent over the channel for different values of interest. Notice that in Figure 3.2 the "arrows" represent the tails of the x_2 distribution that get clipped and become mass points. As we decrease Δ, the clipping effect becomes more pronounced for x_2 as shown in Figure 3.2(a).

The above system can be applied for the MAC because for fixed Δ and β, the x_1 data can be encoded separately from the x_2 data and transmitted to the channel (notice that in the MAC the users do not have access to each other information). A schematic of the proposed encoder is presented in Fig. 3.3.

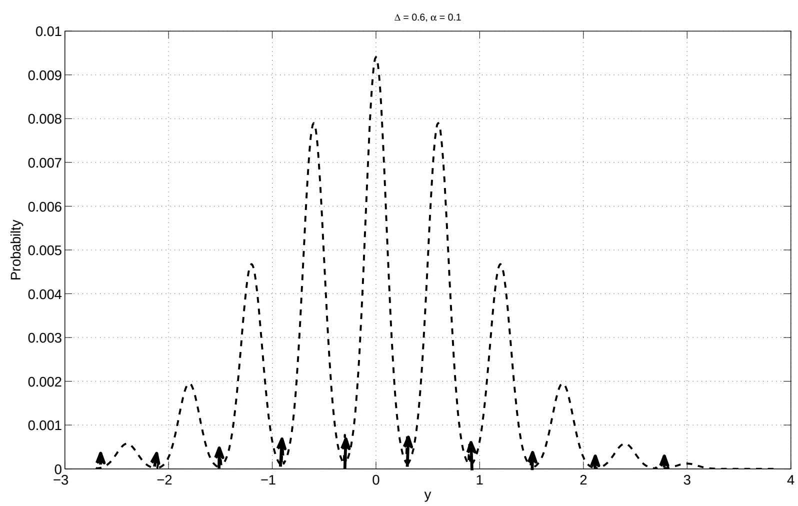
The decoder receives $z = y + n$ and performs MMSE decoding on z to obtain the MMSE estimate of η as

$$\hat{\eta} = \frac{\beta}{\sigma_n^2 + \beta^2} z. \tag{3.6}$$

Note that this derivation assumes that the distribution of z (or equivalently



(a)



(b)

Figure 3.2: The y distribution when: (a) $\Delta = 0.4, \alpha = 0.1$ and $\beta = 1$, (b) $\Delta = 0.6, \alpha = 0.1$ and $\beta = 1$.

y because the noise is Gaussian) is Gaussian. However, as shown in Figure 3.2 the distribution of y is not Gaussian but a mixture of continuous and discrete random variables. Moreover, the process that generates y is strongly non-linear. The optimal decoding rule can be obtained via calculus of variation methods and this is studied in [30].

Afterwards the decoder quantizes the obtained $\hat{\eta}$ according to (3.1) to obtain an

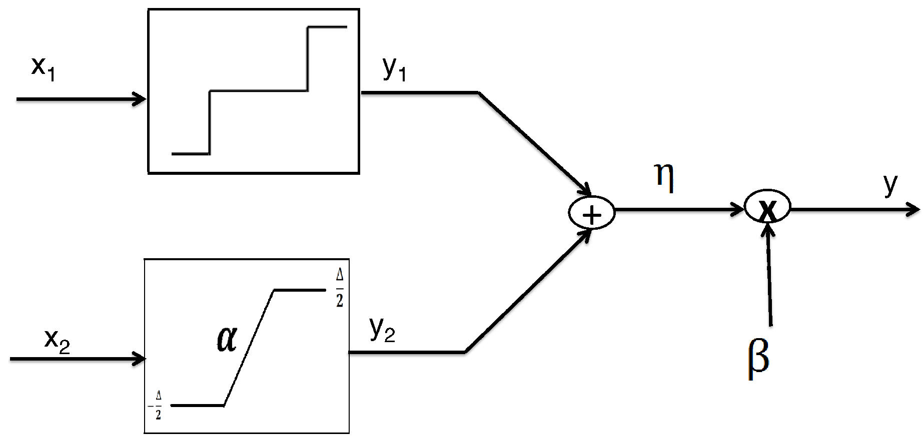


Figure 3.3: Block diagram showing the SQLC encoder. x_1 is passed through a uniform scalar quantizer of step Δ and x_2 is scaled and clipped to force it to lie within $[-\frac{\Delta}{2}, \frac{\Delta}{2}]$.

estimate of the quantized first symbol, $\hat{y_1}$, as $\hat{y_1} = \Delta\lceil\frac{\hat{\eta}}{\Delta}\rfloor$. The second symbol estimate is obtained by subtracting $\hat{y_1}$ from $\hat{\eta}$, that is $\hat{y_2} = \hat{\eta} - \hat{y_1}$. The transmitted symbol pair is estimated from $(\hat{y_1}, \hat{y_2})$ as

$$\hat{x_1} = \hat{y_1} \tag{3.7}$$

$$\hat{x_2} = \frac{1}{\alpha}\hat{y_2} \tag{3.8}$$

The decoder structure of the SQLC system is shown in Fig. 3.4. The proposed scheme can be thought as equivalent to projecting the source pair (x_1, x_2) onto a space filling curve of dimension 1 to produce (y_1, y_2), as shown in Fig. 3.5.

Observe the discontinuity of the curve shown in Fig. 3.5. If y_2 is very close to the edge of the lines shown in the figure and if the noise value is large so that the decoder moves the received pair from one branch to the next, then the distortion for x_2 will be severe. This is the well known threshold effect in analog joint source channel coding,

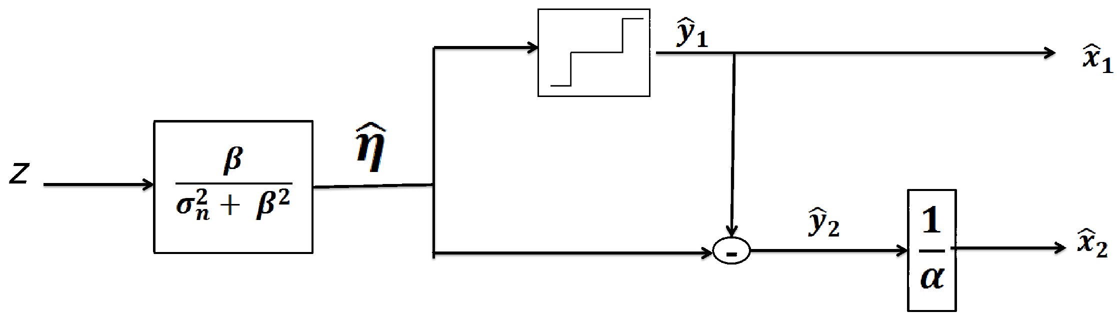


Figure 3.4: Decoder of the SQLC scheme. Notice that the quantizer is the same as the one used in the encoder.

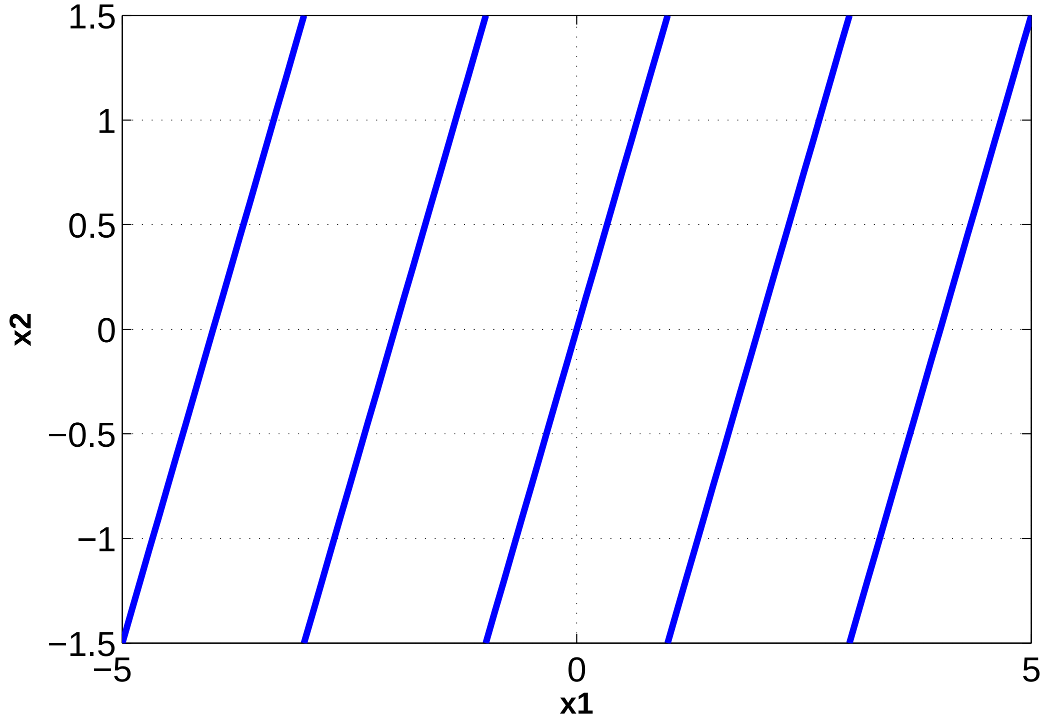


Figure 3.5: Space filing curve utilized in the proposed system. Source symbols (x_1, x_2) are projected onto the curve to produce (y_1, y_2).

which results from the discontinuity of the curve where the source points (x_1, x_2) are projected onto. To alleviate this problem we propose a modified version of SQLC that we call *Alternating-Sign* SQLC (AS-SQLC). The proposed scheme better suits the broadcast channel, taking advantage of the fact that in the broadcast channel the transmitter has access to *both* users data and can encode them jointly.

3.2 Alternating sign SQLC (AS-SQLC)

AS-SQLC works as SQLC with the difference in how y_2 is generated. Rather than using (3.2), in AS-SQLC the second user symbol is generated according to

$$y_2 = \kappa\, \mathcal{L}_{\pm\frac{\Delta}{2}}(\alpha x_2) \tag{3.9}$$

where

$$\kappa = \begin{cases} +1 & \text{when } \lceil \frac{y_1}{\Delta} \rfloor \text{ is even} \\ -1 & \text{when } \lceil \frac{y_1}{\Delta} \rfloor \text{ is odd .} \end{cases} \tag{3.10}$$

Notice that in order to generate the second user symbol, we need knowledge of the first user symbol. This is not possible for the MAC.

At the decoder, we use the same procedure as SQLC with the difference on how $\hat{y_2}$ is obtained. Specifically, $\hat{y_2}$ is obtained by multiplying the estimate by the factor κ as follows

$$\hat{y_2} = \kappa\,(\hat{\eta} - \hat{y_1}) \tag{3.11}$$

where

$$\kappa = \begin{cases} +1 & \text{when } \lceil \frac{\hat{y_1}}{\Delta} \rfloor \text{ is even} \\ -1 & \text{when } \lceil \frac{\hat{y_1}}{\Delta} \rfloor \text{ is odd.} \end{cases} \tag{3.12}$$

This is equivalent to projecting the source pair (x_1, x_2) onto the space filling curve shown below in Fig. 3.6. Note that the space filling curve is *continuous* and x_2 does not experience any threshold effect.

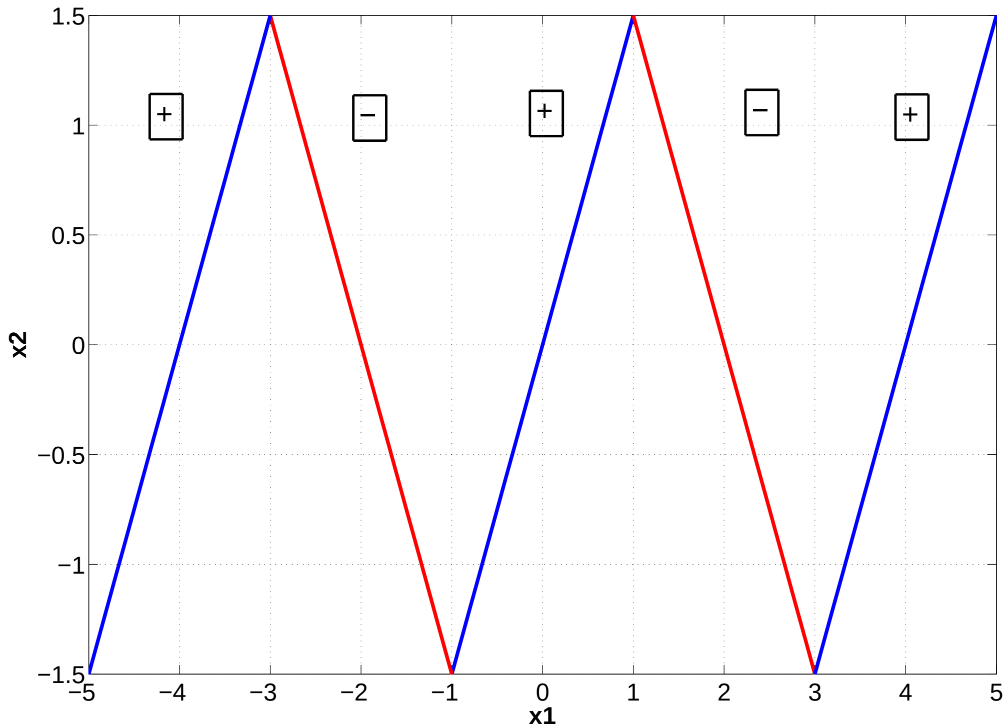


Figure 3.6: Space filling curve for AS-SQLC. The two sources (x_1, x_2) are projected onto the curve and the resulting pair is transmitted as indicated in the text. Notice the *continuity* of the curve eliminating the threshold effect.

Note that decoding the quantized source first and subtracting its contribution and then performing the analog decoding is called *Sequential Decoding*.

It is interesting to remark the difference between the space filling curve in Fig. 3.6 and standard space filling curves designed for point to point communication, such as Shannon-Kotelnikov mappings that will be discussed briefly in the next section (see [31] and [32] for a detailed discussion of such systems). Standard space filling curves are designed for a *symmetric* distortion case. That is, the distortion incurred by user

1, D_1, is usually the same as the distortion incurred by user 2, D_2. Moreover, the distortion measure used in standard designs is the *average* distortion of both sources defined as $D = \frac{D_1+D_2}{2}$. However, the proposed space filling curve for the broadcast channel shown in Fig. 3.6 has the advantage of being able to control the distortions incurred by each user. This is achieved by changing the quantization step, Δ. If Δ is small, source 1, x_1, will be finely quantized and the resulting distortion, D_1, will be small. At the same time, since Δ is small, source 2, x_2 will be "squeezed" and fit within the small interval $[-\frac{\Delta}{2}, \frac{\Delta}{2}]$ so that D_2 will be large. On the other hand, if Δ is large source 1 is quantized coarsely and incurs in a large distortion. At the same time, x_2 is spread over a longer interval and D_2 will be smaller.

It is interesting to note that the distribution of the symbols sent over the channel y for the AS-SQLC is *exactly* the same for the SQLC (i.e. the scheme without flipping the distributions). This observation follows from the fact that we should flip the inner distributions of Figure 3.2 about their mean. Since the inner distribution is a clipped Gaussian, it is symmetric about its mean. Hence, the two distributions are the same. However, we have demonstrated that one scheme suffers from the threshold effect while the other is "continuous" in a certain sense. Hence, we expect the performance to be different. The two schemes will be analyzed in detail in [30].

3.3 Space Filling Curves

As mentioned in the previous section, Space filling curves are designed mainly for AWGN point to point communications. We shall give a brief description of such systems to draw the similarities between AS-SQLC systems.

Systems based on the use of space filling curves were proposed independently by Shannon and Kotelnikov [33, 34]. The basic idea is to project a source vector, a K-tuple, into an N-dimensional surface. Given K source samples and number of channel uses N, a bandwidth reduction system is one in which $K > N$, while in a bandwidth expansion system $N > K$. In this thesis, the proposed AS-SQLC system is equivalent to a 2:1 compression system for point to point communications. The mapping function typically used for point to point systems in that case is the non-linear Archimedean spiral defined parametrically as

$$\begin{cases} x = \dfrac{\Delta}{\pi}\theta \sin\theta \\ y = \dfrac{\Delta}{\pi}\theta \cos\theta \end{cases} for\ \theta \geq 0, \quad \begin{cases} x = -\dfrac{\Delta}{\pi}\theta \sin\theta \\ y = \dfrac{\Delta}{\pi}\theta \cos\theta \end{cases} for\ \theta < 0, \tag{3.13}$$

where Δ is the distance between two neighboring spiral arms and θ is the angle from the origin to the point (x, y) on the curve. The Archimedean spiral is studied in detail in [32]. Notice that if Δ is fixed, then there is a one to one mapping between the angle, θ, and the pair on the curve (x, y). The mapping function $M_\Delta(x', y')$ takes any source pair (x', y') and projects it to the closest point on the spiral, that is

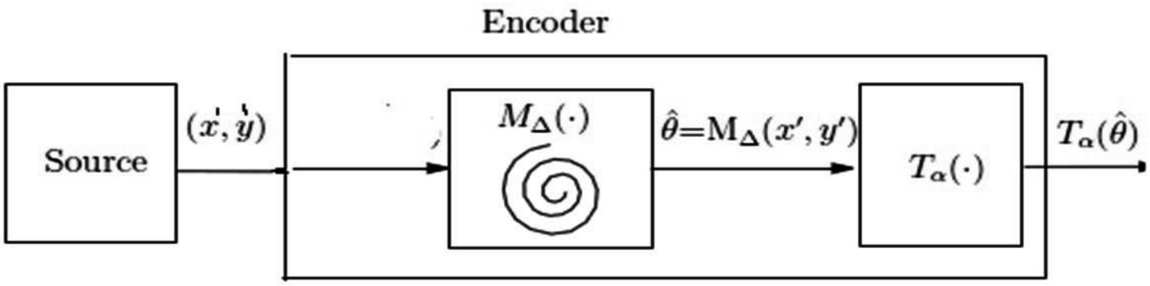
$$\hat{\theta} = M_\Delta(x', y') = \arg\min_\theta\{(x' \pm \frac{\Delta}{\pi}\theta \sin\theta)^2 + (y' - \frac{\Delta}{\pi}\theta \cos\theta)^2\}. \tag{3.14}$$

After the mapping, $\hat{\theta}$ is processed by the function $T_\alpha(\hat{\theta}) = \hat{\theta}^\alpha$. Both parameters (Δ, α) are optimized according to the channel signal to noise ratio, CSNR [31]. The procedure is depicted in Figure 3.7.

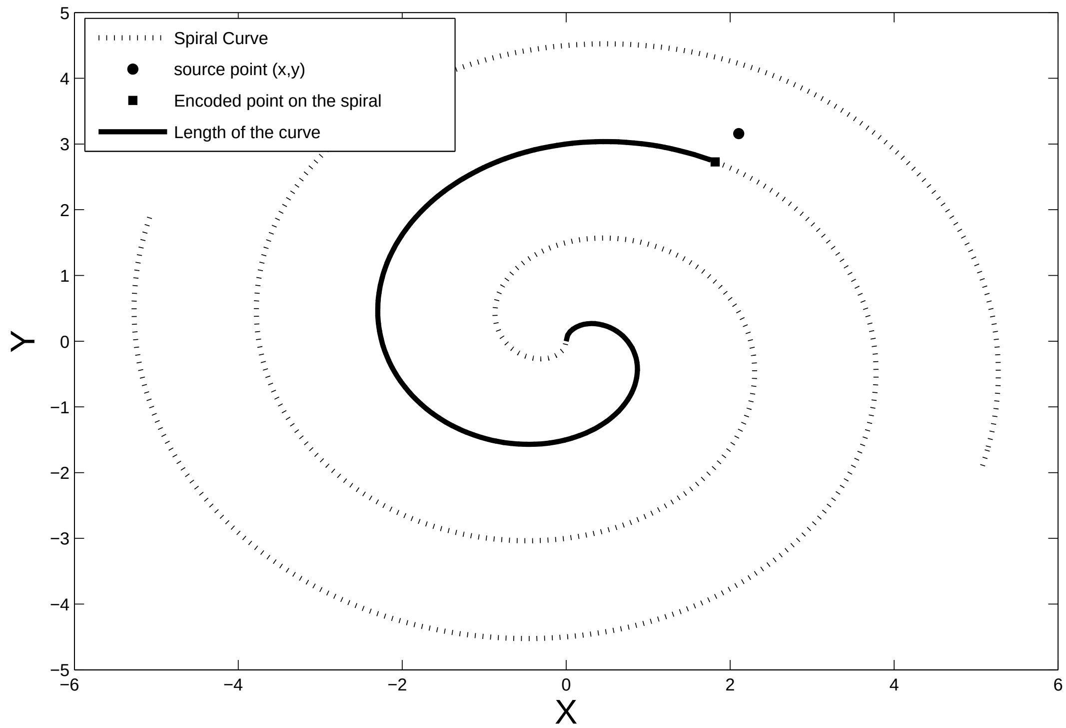
Note that the distance between the spiral arms is called Δ here as well, One can think of the space between the spiral arms in Figure 3.7(b) similarly to the space in which the analog source of the AS-SQLC lives on (which is spanned from $-\Delta$ to Δ as shown in Figure 3.6)

3.4 Proposed Communication System

In this section, we present the complete communication system. We use AS-SQLC to encode the data of user 1, x_1 (the user with larger noise variance) and the data of the second user, x_2 (the user with lower noise variance). We then transmit the resulting y through the channel. We employ the AS-SQLC decoding technique at the decoders of both users. At the second user's decoder (the one with less noise), if we properly choose the different encoding parameters (Δ, α, β), we will be able to extract *both* the discrete (quantized) component and the analog component (with some error).



(a)



(b)

Figure 3.7: (a) depicts the overall system based on Shannon-Kotelnikov mappings, first the source pair is projected onto the spiral curve and then passed to the non-linear function T_α. T_α is a one to one mapping on the length of the curve. (b) depicts a specific space filling curve with $\Delta = 1.4$.

Hence, we will be able to recover both x_2 and x_1. Note that user 2 data is only x_2, however it needs to know x_1 in order to invert the modulation, enabling the recovery of x_2. At the first user's decoder (the user with more noise), we apply the same decoding procedure. However, only the discrete component of the signal (user 1 data) is recovered in this case. Note that these ideas are very similar to the arguments presented in 2.1.1, where the clouds are the different intervals $[\frac{i\Delta}{2}, \frac{(i+1)\Delta}{2}]$ since user 1 can determine the interval itself (the cloud) and the value within the cloud (the cloud contents), whereas the worst user can only determine the interval itself with confidence but not the values inside the interval.

The system diagram of the complete system is shown in Fig. 3.8. Note that the design philosophy of the communication system here is the same of that of the Dutch and Spanish speaker example discussed in Section 2.1. Here we are able to transmit (not error free) which branch the second source comes from. That is, we are able to say whether it was the positive branch or negative branch. Hence, we are able to transmit 1 extra bit of information at most without sacrificing any data rate or power spent at the encoder function. This is exactly similar to the case of the Dutch and Spanish communicator, if the speaker is able to cleverly arrange the data, he can send extra information. We say that at most one bit of extra information can be transmitted this way. One can in fact characterize the exact maximal data rate that can be communicated this way via mutual information arguments and in fact this is currently under investigation [30]

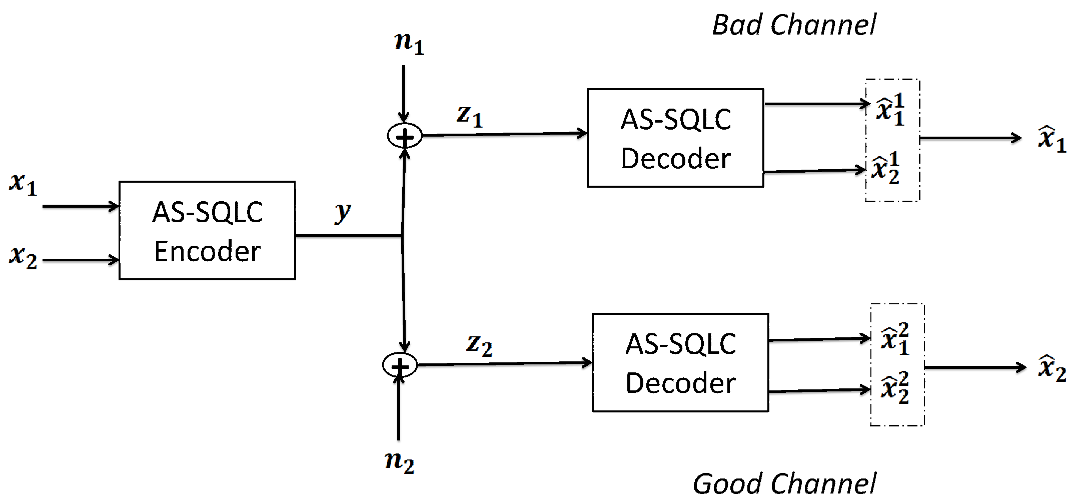


Figure 3.8: Complete system diagram.

Chapter 4

GENERALIZED SYSTEM AND SIMULATION RESULTS

So far we have focused on two independent Gaussian sources. In this section we extend that framework to the case in which the two sources are correlated. We will develop the general scheme for transmitting correlated sources and show that the proposed system in section 3.4 for transmitting independent sources is in fact a special case of the general system developed in this chapter. We will then give Monte-Carlo simulation results of the system performance for several cases of interest

4.1 Transmitting Correlated Sources

In order to transmit correlated sources, we use the scheme described in section 3.2 with only two modification at the transmitter and the two receivers (the rest of the system is exactly as described before). First, equations (3.1), (3.2) at the encoder are modified to become:

$$y_1 = \Delta \lceil \frac{ax_1 + bx_2}{\Delta} \rfloor = \Delta \lceil \frac{w_1}{\Delta} \rfloor \tag{4.1}$$

$$y_2 = \mathcal{L}_{\pm \frac{\Delta}{2}}(\alpha(cx_1 + dx_2)) = \mathcal{L}_{\pm \frac{\Delta}{2}}(\alpha(w_2)) \tag{4.2}$$

Note that we have introduced a linear transformation on the users data $\mathbf{x} = [x_1 x_2]$ to produce $\mathbf{w} = [w_1 w_2]$. That is

$$\mathbf{w} = \mathbf{H}\mathbf{x}, \qquad \text{where} \qquad \mathbf{H} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \tag{4.3}$$

Second, the equations describing the decoding in (3.7), (3.8) at the two receivers are also modified to become

$$\hat{\mathbf{w}} = \mathbf{H}^{-1}\hat{\mathbf{y}} \tag{4.4}$$

$$\hat{x_1} = \hat{w_1} \tag{4.5}$$

$$\hat{x_2} = \frac{1}{\alpha}\hat{w_2} \tag{4.6}$$

Note that the scheme used for the transmission of uncorrelated sources can be considered a special case of the scheme just described with $a = 1,\ b = 0,\ c = 0,\ d = 1$.

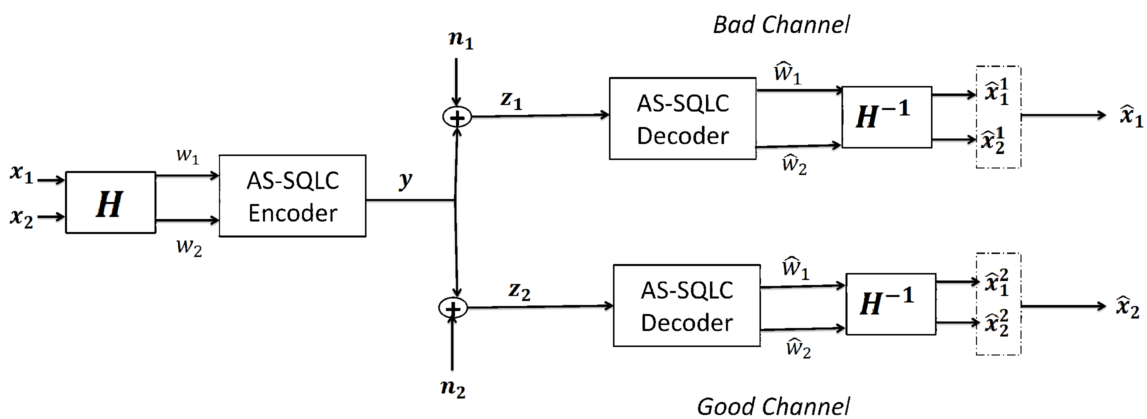


Figure 4.1: Complete system diagram for the correlated case.

We will try to give an explanation on the intuition behind this linear transformation on the source symbols and its importance. We assumed that the source pair $(x_1, x_2) \sim \mathbb{N}(\mathbf{0}, \boldsymbol{C})$, where $\boldsymbol{C}$ is defined in (2.8).

This can be expressed as

$$x_1 = \rho\chi + \sqrt{1-\rho^2}e_1 \tag{4.7}$$

$$x_2 = \rho\chi + \sqrt{1-\rho^2}e_2 \tag{4.8}$$

where e_1, e_2, χ are all independent scalar Normal Gaussian variables ($\mathbb{N}(0, 1)$). χ is the common information and e_1, e_2 are the private information of each source.

Let us assume the following precoding matrix H

$$\boldsymbol{H} = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}, \tag{4.9}$$

Then, in that case and from (4.3), $\mathbf{w}$ is given by

$$w_1 = \rho\chi + \sqrt{1-\rho^2}e_1 \tag{4.10}$$

$$w_2 = \sqrt{1-\rho^2}e_1 - \sqrt{1-\rho^2}e_2 \tag{4.11}$$

Note that if ρ is very large and the sources are highly correlated, then $\sqrt{1-\rho^2}$ is very small and w_2 would have an energy of $\mathbb{E}(w_2^2) = 2(1-\rho^2)$. This is because e_1, e_2 are independent and each has unity variance. Hence, instead of transmitting a source of power 1 (the original x_2), we transmitted a symbol of power $2(1-\rho^2)$ and yet we are able to reconstruct both x_1 and x_2 from these w_1, w_2 reliably. We could have scaled x_2 original by a factor so that its power is $2(1-\rho^2)$ or any smaller if we wanted. But we would be scaling also the common information χ and we would be sending redundant information and using a small amount of power to send e_2. We note that this argument is not very rigorous (a complete rigorous analysis of the correlated problem will be provided in [30]). At the decoder side we are able to combine noisy versions of w_1, w_2 , that is $\hat{w}_1, \hat{w}_2$ to obtain the original sources. In a side note the factor $1-\rho^2$ found frequently in Theorem 2.1.2 is obtained by decomposing the source as in (4.7).

We now give a comparison between the scheme presented here in this thesis, AS-SQLC, and the hybrid coding scheme provided by [22], which was explained in Section 2.1.2. We restate the hybrid scheme formula (2.23) here for convenience

$$X^n = tS_1^n + rS_2^n + X_{2d}^n \qquad \text{where} \qquad t, s \in \mathbb{R}, \tag{4.12}$$

We note that the digital component in AS-SQLC comes from quantizing the first source (the source of the worst user) exactly as in (4.12), where X^n_{2d} is a quantized and *Channel Coded* version of S_2. In AS-SQLC, S_2[1] is only quantized and no channel coding is applied. Also, the analog component of (4.12) is a linear combination of S_1 and S_2. However, in AS-SQLC we have a *clipped* linear combination of S_1, S_2. The extra restriction of clipping in AS-SQLC stems from the fact that we want to remove the digital component completely. In (4.12) no clipping is applied and still we are able to remove the noise completely as defined by the Noisy Channel Coding Theorem by using larger block lengths to spread the noise removing the noise "statistically". However, in AS-SQLC we cannot invoke the Noisy Channel Coding Theorem since we have a block length of 1. Hence, clipping is an "algebraic" way of separating the two sources.

4.2 Simulation Results

We now present the simulation results for the proposed scheme for several cases of interest. First, we present in Fig. 4.2 the results for the transmission of independent Gaussian sources when $P = 1,\ N_2 = 0.002$ and $N_1 = 10N_2$, and compare them with a time-sharing strategy. The results are obtained through Monte-Carlo optimization of the parameters Δ and α in (3.1) and (3.2) so that SDR is maximized. The optimum value of Δ for different values of D_2 is shown in Fig. 4.3. The SDR upperbound shown in Fig. 4.2 is obtained from (2.26) and (2.27). The power, P, is given by $P = \mathbb{E}(y^2)$. The time sharing performance is obtained by utilizing the channel ζ of the time to transmit user 1 data and $1 - \zeta$ of the time to transmit user 2 data, where $0 \leq \zeta \leq 1$.

We now give the performance of the system when transmitting correlated sources. We perform the simulation of the system explained in Section 4 for a source correlation value of $\rho = 0.95$. We perform Monte Carlo optimization of the parameters Δ, α as well as the matrix $\mathbf{H}$. Fig. 4.4 shows the resulting performance for the correlated case when $P = 1,\ N_2 = 0.002,\ N_1 = 5N_2$ and source correlation $\rho = 0.95$. The SDR upperbound in Fig. 4.4 shows the best achieved theoretical distortion pair and

[1] Or a linear combination of S_1 and S_2 as we have the freedom to optimize over the matrix $\mathbf{H}$

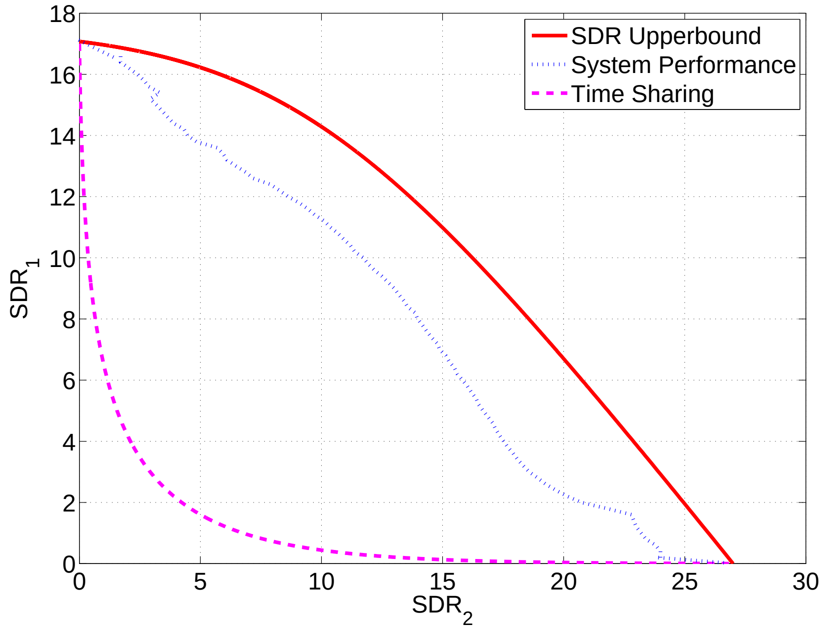


Figure 4.2: System performance for $P = 1$, $N_2 = 0.002$ and $N_1 = 10N_2$ for the optimal values of parameters Δ and α.

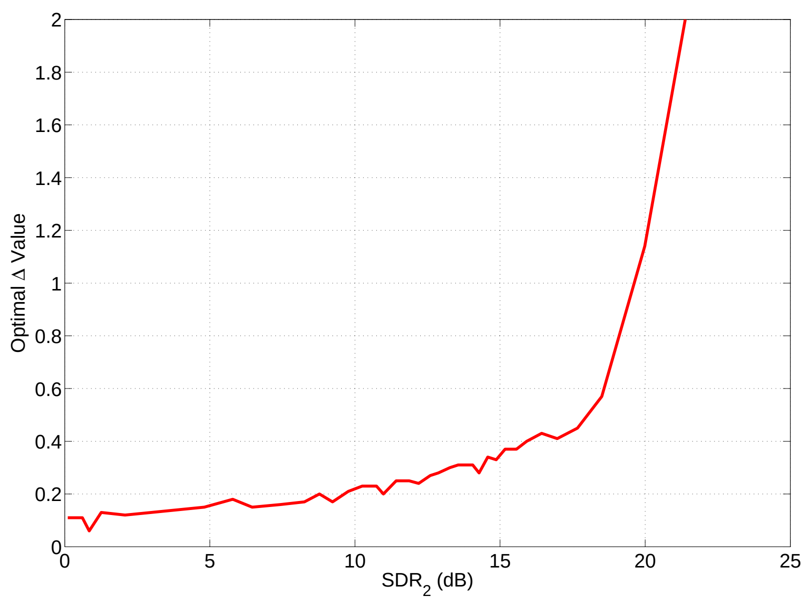


Figure 4.3: Optimal Δ value to achieve the performance shown in Fig. 4.2 for different values of D_2.

is described by the equations given in section III of [22]. Similarly, Fig. 4.5 shows the resulting performance for the correlated case when $P = 1,\ N_1 = 0.002,\ N_2 = 2N_1$ and source correlation $\rho = 0.5$

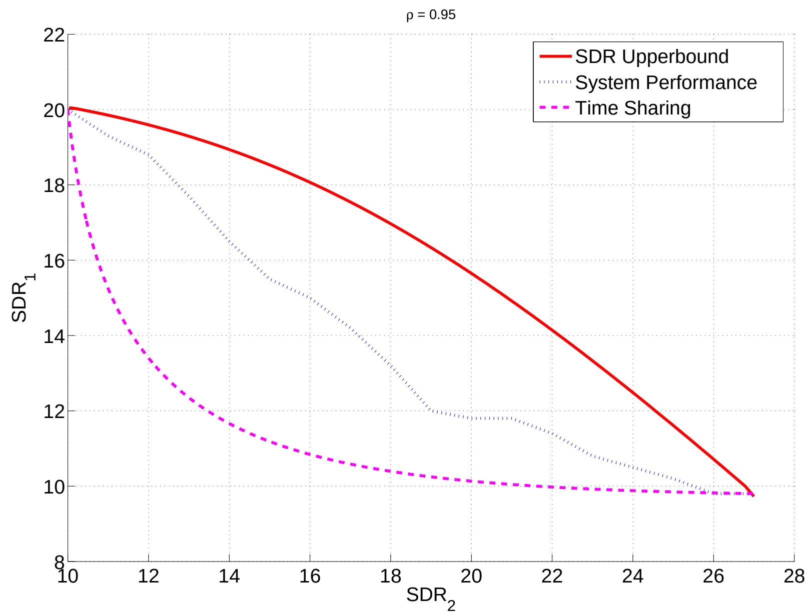


Figure 4.4: System performance for $P = 1,\ N_2 = 0.002$ and $N_1 = 5N_2$ with correlation $\rho = 0.95$.

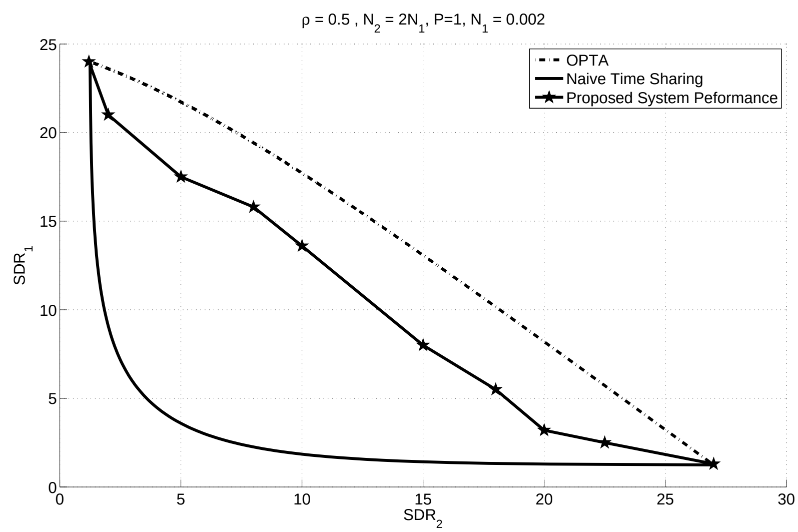


Figure 4.5: System performance for $P = 1$, $N_1 = 0.002$ and $N_2 = 2N_1$ with correlation $\rho = 0.5$.

Chapter 5

CONCLUSION

This thesis dealt with the problem of Broadcasting a Bivariate Gaussian Source over a memoryless Additive White Gaussian Noise Channel. The scheme developed here was a zero delay hybrid digital-analog scheme. The motivations behind using analog and hybrid Systems were discussed. It was shown that such need arises mainly when considering communications over networks due to the sub-optimality of classical digital systems based on the separation principle. The low complexity and delay of analog systems are of great practical interest as well.

Hybrid digital-analog coding systems were shown to have a potential effect on reducing the complexity of the design significantly without sacrificing performance. Their use also gives the system designer great flexibility in system design, as some aspects are easy to implement in the original source domain while other digital solutions may be already readily available. Interestingly, it was shown that for the Gaussian broadcast channel, a hybrid scheme is *required* to achieve the theoretical limits. We adopt a zero delay hybrid scheme based on Nested Quantization Techniques. The proposed zero-delay scheme in this thesis bears similarities to the (infinite length) optimal hybrid scheme that was described in [22]. The proposed technique, AS-SQLC, utilizes a novel space-filling curve that is well suited to the Broadcast Channel. Simulation results show that the resulting performance when independent and correlated Gaussian data is transmitted to each user is close to the theoretical limits for a wide range of distortion pairs.

5.1 Future Work

We have used heuristic empirical arguments throughout the thesis. Surprisingly, these intuitive techniques lead to performance very close to the theoretical limits. As a matter of fact it is better than the "Smart" time sharing explained in the appendix where a complete (totally new) system design is required to achieve every optimal distortion pair. These heuristics arguments shall be made more rigorous and are currently under investigation. Some future work includes:

- We have assumed that the channel symbols are Gaussian distributed and performed MMSE estimation based on that assumption. The optimal decoding rule can be readily obtained by Calculus of variation by solving for the function that minimizes the MMSE distortion of the transmitted source pair.

- Both the proposed scheme, AS-SQLC, and the original SQLC technique generate the same output distribution. Hence, a simple information theory mutual information upperbound would imply that the achieved performance of both systems must be the same. However, we showed how AS-SQLC is able to "squeeze" at most one information bit (namely) the sign of the AS-SQLC branch. Exact maximum information rates for both systems shall be obtained.

- The correlated case needs more analysis. An interesting case that we are currently investigating is performing the Karhunen Love Transform, KLT, on the source pair and transmitting the independent components of the source.

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Appendix A

SIMPLE SCHEMES FOR THE BROADCAST CHANNEL

In this appendix we demonstrate in detail simple achievable schemes for the Broadcast channel and derive their equivalent performance. First of all the broadcast channel can be converted to a point to point communication system by ignoring one of the sources and transmitting only the other source. In this case the Broadcast channel becomes a single point-to-point channel for the user intended to receive that information. For example, ignoring user 1 and transmitting only S_1, user 2 achieves a distortion of D_2^{min} and user 1 achieves a distortion of D_1^{max}. Similarly we can get D_1^{min} and D_2^{max}, where

$$D_1^{max} = \frac{(1-\rho^2)P + N_1}{P + N_1}, \qquad D_2^{min} = \frac{N_2}{P + N_2} \tag{A.1}$$

$$D_1^{min} = \frac{N_1}{P + N_1} \qquad D_2^{max} = \frac{(1-\rho^2)P + N_2}{P + N_2} \tag{A.2}$$

.

Then we can use time sharing as described in previous chapters to obtain “linear combinations” of the extremal distortions. In fact such time sharing solution is called *Naive* Time Sharing as it will be evident next. We know that the uncoded scheme achieves the extreme points. Hence we can simply transmit source 1 ζ of the time and achieve D_1^{min} and source 2 would be achieving D_2^{max} [23]. Also, for the $1-\zeta$ fraction of time that source 1 is NOT transmitting, source 2 would be transmitting his data on the channel and would achieve D_2^{min} and source 1 distortion would be D_1^{max}. Hence the set of time sharing solutions can be expressed parametrically with respect to ζ as

$$D_1(\zeta) = \zeta D_1^{min} + (1-\zeta) D_1^{max} \tag{A.3}$$

$$D_2(\zeta) = (1-\zeta) D_2^{max} + \zeta D_2^{min} \tag{A.4}$$

We note that this is called *Naive* time sharing because at the times where one source is transmitting, the other source is experiencing *maximal* distortion. We note that we can actually devise a smarter time sharing solution whose performance is comparably much better, but the cost is a complicated system design as we shall see next.

In the "smart" time sharing solution, we perform time sharing on the extreme rates of the broadcast channel C_1 and C_2[1]. Using similar arguments to (A.3). The time sharing rates that can be achieved are given by:

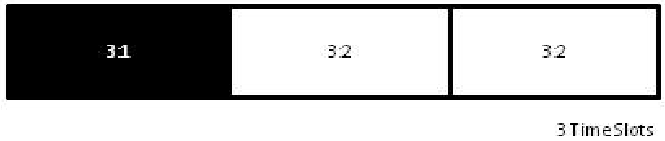
$$R_1(\zeta) = \zeta C_1 \tag{A.5}$$

$$R_2(\zeta) = (1-\zeta) C_2 \tag{A.6}$$

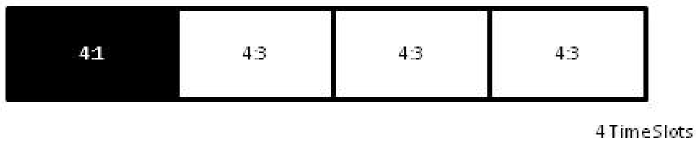
Since in this thesis we focus on transmitting one Gaussian source to each user using the channel once ($\kappa = 1$), we must perform source compression (bandwidth reduction) on the original sources to keep the source/channel bandwidth at unity. For example if $\zeta = \frac{1}{3}$, then in 3 time slots, user 1 will transmit just once and user 2 will transmit the remaining 2 times. Hence, for source 1 we must use a 3 : 1 compression system that takes 3 Gaussian source symbols and produces 1 symbol. Similarly user 2 uses a 3:2 compression system that produces 2 symbols and then transmits these 2 symbols on his allocated 2 channel slots. This is demonstrated in Figure A.1.

The Performance of the "smart" time sharing system is shown below in Figure A.2. Note that the "smart" time sharing system requires re-design of the complete system at every optimal distortion pair, a task that is not to be taken lightly. Again

[1] For an AWGN channel, $C_i = \frac{1}{2}\log_2(1 + \frac{P}{N_i})$



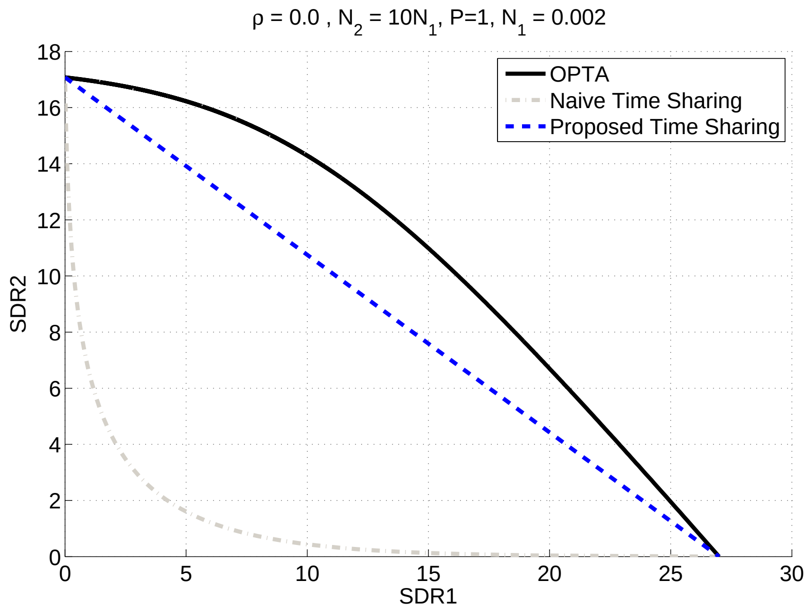
(a)



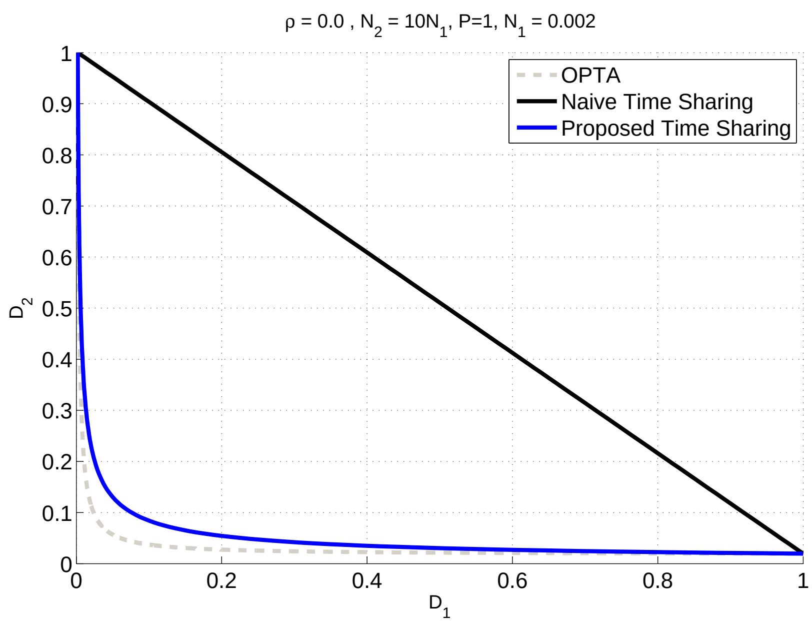
(b)

Figure A.1: Smart Time Sharing Demonstration: (a) shows the case when $\zeta = \frac{1}{3}$; and, (b) shows the case when $\zeta = \frac{1}{4}$.

note the sub-optimality of the smart time sharing scheme: although it requires complete system re-design at each point, it is still suboptimal in general. In fact smart time sharing is optimal for a *symmetric* Broadcast channel. That is when the two user's receiver noise variances are equal, $N_1 = N_2$.



(a)



(b)

Figure A.2: OPTA curves for $P = 1$, $N_1 = 0.002$, $N_2 = 10N_1$ and $\rho = 0$: (a) depicts the Log Scale; and, (b) depicts the Linear Scale.