DYNAMIC RAIL OVERTURNING: MODELLING AND APPLICATION

By

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INTRODUCTION

The problem of rail overturning, which constitutes one of the many causes of derailments, is of growing concern to the railroad industry. A better understanding of the rail overturning mechanism can prove to be an important step towards the development of safe and adequate railroad track systems.

An analytical model for examining the dynamic behavior of rail overturning is developed in Ref. [1]. The mathematical model of the rail-track system and dynamic equilibrium utilized to determine the lateral deflection and rotation of the rail subjected to time-dependent lateral and vertical forces and constant axial force. The analysis is based on linear elastic theory.

Referring to Fig. (1), the xyz system of rectangular coordinates and various external loads acting on the rail are shown. The wheel loads in the vertical and lateral direction are considered time-dependent and are noted by $P_v g(t)$ and $P_h f(t)$ respectively, where g(t) and f(t) represent functions of time. These wheel loads are applied at the position shown in Fig. (1). The axial load P_A, assumed constant, is

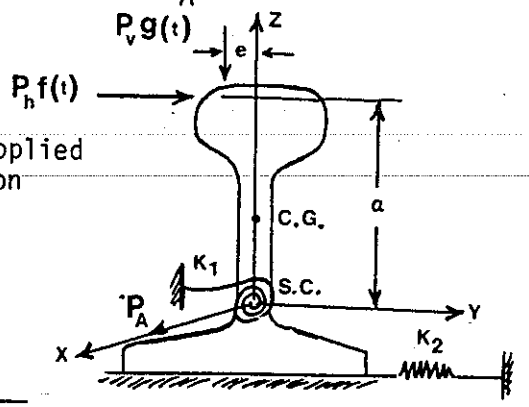

Figure 1. Spring Constants and Applied Loads at the Loaded Cross-section (x=0)

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$$+ K_1\phi - K_2$$

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Boundary cond

$$\left.\frac{\partial y}{\partial x}\right|_{x=0} = 0$$

$$\left.\frac{\partial \phi}{\partial x}\right|_{x=0} = 0$$

$$EI\left.\frac{\partial^3\phi}{\partial x^3}\right|_{x=}$$

$$EI_z\left.\frac{\partial^3 y}{\partial x^3}\right|_{x}$$

and

$$Y(\infty,t) =$$

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$$Y''(\infty,t) =$$

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considered acting at the shear center. The resistance to torsional movement and lateral translation are modelled as elastic linear springs with spring constants K_1 and K_2 respectively.

Equations of rotational and lateral equilibriums and boundary conditions derived in Ref. [1] are presented here as

$$E\Gamma \frac{\partial^4 \phi}{\partial x^4} - GJ \frac{\partial^2 \phi}{\partial x^2} + \rho(I_p + 2Ah_o^2) \frac{\partial^2 \phi}{\partial x^2} + \rho Ah_o \frac{\partial^2 y}{\partial x^2}$$

$$+ K_1 \phi - K_2 \beta (Y - \beta\phi) = 0 \tag{1}$$

$$EI_z \frac{\partial^4 y}{\partial x^4} + P_A \frac{\partial^2 y}{\partial x^2} + \rho A \frac{\partial^2 y}{\partial t^2} + \rho Ah_o \frac{\partial^2 \phi}{\partial t^2}$$

$$+ K_2 (y - \beta\phi) = 0 \tag{2}$$

Boundary conditions

$$\left.\frac{\partial y}{\partial x}\right|_{x=0} = 0 \tag{3a}$$

$$\left.\frac{\partial \phi}{\partial x}\right|_{x=0} = 0 \tag{3b}$$

$$\left.E\Gamma \frac{\partial^3 \phi}{\partial x^3}\right|_{x=0} = \frac{\alpha P_h f(t) - P_v g(t) e}{2} \tag{3c}$$

$$\left.EI_z \frac{\partial^3 y}{\partial x^3}\right|_{x=0} = \frac{P_h f(t)}{2} \tag{3d}$$

and

$$Y(\infty, t) = 0 \tag{4a}$$

$$\phi(\infty, t) = 0 \tag{4b}$$

$$Y''(\infty, t) = 0 \tag{4c}$$

$$\phi''(\infty, t) = 0 \tag{4d}$$

In order to solve the partial differential equations of motion (1) and (2) together with the boundary conditions (3) and (4), the Galerkin's method applied. The method resulted as a system of coupled, ordinary differential equations in the time domain. It was assumed

$$Y(x,t) = \sum_{n=1}^{\infty} a_n(t) e^{-\frac{n\pi x}{L}} \left(\cos \frac{n\pi x}{L} + \sin \frac{n\pi x}{L}\right) \quad (n = 1, 2, \ldots \infty) \tag{5a}$$

$$\phi(x,t) = \frac{1}{e}\sum_{n=1}^{\infty} b_n(t)e^{-\frac{n\pi x}{L}}\left(\text{Cos}\ \frac{n\pi x}{L} + \text{Sin}\ \frac{n\pi x}{L}\right)\ (n = 1, 2, \ldots\infty) \quad (5b)$$

where e_o is an arbitrary length included so that the a's and b's all have the same dimensions. The resulted system of coupled ordinary differential equations are as follows:

$$\sum_{n=1}^{\infty}\frac{1}{(m+n)(m^2+n^2)}\left[\frac{\rho A L e_o}{\pi}(m^2+n^2+mn)\ddot{a}_n(t)\right.$$

$$+ e_o\left\{\frac{K_2 L}{\pi}(m^2+n^2+mn) - \frac{2m^2n^2\pi}{L}P_A + \frac{4m^3n^3\pi^3}{L^3}EI_z\right\}a_n(t)$$

$$\left.+\frac{\rho A h_o L}{\pi}(m^2+n^2+mn)\ddot{b}_n(t) - \frac{K_2\beta L}{\pi}(m^2+n^2+mn)b_n(t)\right]$$

$$= \frac{P_h e_o}{2}\psi_m(0)\ f(t) \qquad (m = 1, 2, \ldots\infty) \quad (6)$$

$$\sum_{n=1}^{\infty}\frac{1}{(m+n)(m^2+n^2)}\left[\frac{\rho A h_o L}{\pi}(m^2+n^2+mn)\ddot{a}_n(t)\right.$$

$$-\frac{K_2\beta L}{\pi}(m^2+n^2+mn)a_n(t) + \frac{\rho}{e_o}(I_p + 2Ah_o^2)\frac{L}{\pi}(m^2+n^2+mn)\ddot{b}_n(t)$$

$$\left.+\frac{1}{e_o}\left\{(K_1 + K_2\beta^2)\frac{L}{\pi}(m^2+n^2+mn) + \frac{2m^2n^2\pi}{L}GJ + \frac{4m^3n^3\pi^3}{L^3}E\Gamma\right\}b_n(t)\right]$$

$$= \frac{P_h\ \alpha\ f(t) - P_v\ e\ g(t)}{2}\eta_m(0) \qquad (m = 1, 2, \ldots\infty) \quad (7)$$

In order to determine the dynamic response of the rail subjected to lateral, vertical, and axial loads, it is necessary that the coefficients $a_n(t)$ and $b_n(t)$ in Eqs. 5 be known. The coefficients can be determined by solving simultaneously the ordinary differential Eqs. (6) and (7) which can be put into the matrix form as:

$$\begin{bmatrix} M^{(11)} & M^{(12)} \\ M^{(12)} & M^{(22)} \end{bmatrix}\begin{Bmatrix} a(t) \\ b(t) \end{Bmatrix} + \begin{bmatrix} K^{(11)} & K^{(12)} \\ K^{(12)} & K^{(22)} \end{bmatrix}\begin{Bmatrix} a(t) \\ b(t) \end{Bmatrix} = \begin{Bmatrix} R^{(1)} \\ R^{(2)} \end{Bmatrix} + \begin{Bmatrix} R^{(3)} \\ R^{(4)} \end{Bmatrix} \quad (8)$$

For all practical purposes, only a finite number of terms, N, can be considered in the infinite series Eqs. (5) so that Eqs. (8) represent 2N unknowns ($a_1, a_2, \ldots, a_N, b_1, b_2, \ldots, b_N$).

The coupled ordinary differential Eqs. (8) and normal mode method of dynamic analysis, together with the other define parameters was utilized to study the different cases of dynamic behavior of the rail.

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References

1. Bhatti,
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3. Chu, K.
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Example

When the lateral force acts on the rail for a short interval of time T_h, the response of the rail can be divided into distinct phases Ref. [1]. During phase I, both P_h and P_v are acting while only P_v acts during phase II. Thus the load-time functions are

$$f(t) = \begin{cases} 1 & 0 < t \leq T_h \\ 0 & 0 < \bar{t} < \infty (\bar{t} = t - T_h) \end{cases}$$

$$g(t) = 1 \quad 0 < t < \infty$$

The initial conditions for phase I are $q_i(0) = \dot{q}_i(0) = 0$, while those for phase II are the ones existing at the end of phase I, i.e. $q_i(\bar{t}=0) = q_i(t=T_h)$ and $\dot{q}_i(\bar{t}=0) = \dot{q}_i(t=T_h)$. The solution to the differention Eqs. (8) is calculated. Finally, the dynamic response at the rail head at the loaded cross-section is calculated using the above equations together with Eqs.

$$y_H(0, t) = \sum_{i=1}^{N} [a_i(t) + \alpha\, b_i(t)] \tag{9a}$$

$$y_B(0, t) = \sum_{j=1}^{N} [a_j(t) + \beta\, b_j(t)] \tag{9b}$$

the response time history of this example is calculated [1]. An important observation made from the response history is that the rail, when subjected to an impulse lateral load, not only deflects outwards as asserted by static analyses [2,3], but also bounces back and deflects inwards by a significant amount at some instants of time. This oscillatory pattern about the mean (undeflected) position may be a cause of loosening of the rail fastenings due to stress reversal and fatigue, unless they are specially designed for the purpose. Another interesting feature of the time-deflection history is that, in the case of 10% modal damping, the response quickly approaches a constant value after the removal of impulse lateral load, which signifies that the system is heavily damped so that the vibrations die down in a short interval of time.

References

1. Bhatti, Majeed H., Torkamani, M.A.M., and Zarembski, A. M., "Dynamic Analysis of Rail Overturning," Structural Series 78-2003, Dept. of Civil Engineering, Illinois Institute of Technology, December 1978.
2. Chu, K. H. and Wang, Y. S., "Investigation of Rail Overturning," Interim Report to AAR, Sept. 1976.
3. Chu, K. H. and Wang, Y. S., "Investigation of Rail Overturning," Final Report to AAR, May 1978.