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Examining a Coaching Routine to Support Teacher Learning

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Abstract
Mathematics specialists tasked with the responsibility of supporting teacher learning face both the opportunity and the challenge of transforming the organization of the school workplace to support educators' collective, ongoing learning, which is not the norm in most school settings. In this study, we examine a coaching routine called *Teacher Time Out* (TTO), which was organically developed by a school-based mathematics coach and the teachers with whom she worked. Through the routine, coaches and teachers work through complex, in-the-moment pedagogical decision making while collectively facilitating mathematics discussions among students. The routine thus opens opportunities for educators to learn about ambitious teaching alongside their colleagues. We report findings from an analysis of 360 TTOs that occurred over three years of one coach's work supporting a school-wide, multi-year instructional reform effort in mathematics teaching and learning. We found that the coaching routine fostered teachers' collective inquiry into practice, as they engaged with the unpredictability of teaching during real-time instruction with students. We discuss the potential of this routine to support coaching as a lever for organizational reform, reshaping mathematics teaching across many classrooms.

Keywords
Mathematics coaching; professional learning; elementary mathematics

Mathematics specialists who are tasked with the responsibility of supporting teacher learning (whom we will refer to as "coaches") are increasingly asked to think about their work as a lever for system reform to impact mathematics teaching across many classrooms (Mangin & Dunsmore, 2015). Coaches face the challenge of organizing the school workplace to create and sustain opportunities for collective, ongoing learning. Here we report the findings of an analysis that examined a coaching routine called *Teacher Time Out* (TTO), which was organically developed by a school-based mathematics coach and the teachers with whom she worked. The routine is nested in a learning process in which the coach facilitated teachers' collective inquiry during authentic practice, as they engaged with the unpredictability of teaching during real-time instruction with students.

The analysis reported here examined 360 TTOs that occurred over three years of a coach's work supporting a school-wide, multi-year instructional reform effort in mathematics teaching and learning. We are interested in whether and how this routine has the potential to assist coaches as they facilitate teachers' development of instructional practices. To this end, we wanted to understand: What opportunities to learn about ambitious teaching were made possible when teachers and the coach engaged in TTOs? How did these learning opportunities change over time as the coach engaged with teachers in TTOs? Examining the TTO contributes to the field's understanding of routines that coaches can draw on to support teachers' learning *while teaching.*

Theoretical Perspectives

Teaching as a Social Practice

Teaching can be conceptualized as a social practice that develops as individual practitioners interact with one another and build a shared repertoire of tools, perspectives, and practices to accomplish their work (Engeström & Middleton, 1996). Teacher learning – the sensemaking and development involved in this dynamic practice – is situated in the contexts within which educators work. Teachers' settings, activities, and ways of interacting become a fundamental part of what they learn (Borko, 2004; Greeno et al., 1996). A view of teaching as a social practice also assumes that professional knowledge is co-constructed (Kelly, 2006). As teachers interact with others, they have opportunities to learn from "seeing, discussing, and engaging in shared practices" (Levine, 2010, p. 111).

To make sense of this dynamic, distributed work, teachers need regular opportunities to share knowledge and engage in ongoing reflection about practice with others (Collinson, 2008; Dunscombe & Armour, 2004). A skilled facilitator, such as a mathematics coach, can support teachers to collectively grapple with the core elements of their work, pushing them to recognize and leverage their various assumptions, experiences, and ways of reasoning to support each other's learning (Ball & Cohen, 1999). In their collaborations with teachers, coaches can create opportunities to deepen content knowledge and instructional practice (Hopkins et al., 2013, 2018; Munson & Dyer, 2020), challenge deficit frameworks (Battey & Franke, 2015; Ramkellawan & Bell, 2017), and support sensemaking around reform efforts (Huguet et al., 2014; Woulfin, 2018; Woulfin et al., 2018). While coaching structures, such as co-teaching (Poglinco et al., 2003) and side-by-side coaching (Munson & Dyer, 2020), have been proposed to support teachers in the moment of teaching, collaborative reflection on the enactment tends to occur later on. Less is known about coaching routines that enable coaches and teachers to collectively inquire into teaching as it unfolds.

Ambitious Mathematics Teaching

We are concerned with an approach to coaching that assists teachers to develop the judgment, attunement, and decision-making necessary to be responsive to students' thinking in the moment of instruction. A vision for teaching mathematics that honors student thinking, positions students as capable of making sense of mathematical ideas, and supports them to use their understandings to solve complex problems (Anthony et al., 2015; Lampert et al., 2011) has gained growing support over the past decade (e.g., National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). This orientation toward mathematics teaching has been called "ambitious teaching" (Lampert et al., 2011).

Ambitious teaching in mathematics involves (a) establishing rigorous mathematical goals for all children (Aguirre et al., 2013; Lampert et al., 2013; National Council of Teachers of Mathematics, 2014), (b) coming to know children as intellectual and social beings and believing in their brilliance (Aguirre et al., 2013); (c) facilitating meaningful discourse by recognizing the mathematical connections in students' ideas and pursuing their thinking (Chapin et al., 2013), (d) attending carefully to the meaning that students are making and positioning them competently; and (e) building a supportive community by affirming learners' diverse identities and attending to the ways learners respond to each other (National Council of Teachers of Mathematics, 2014; Robertson et al., 2015). Ambitious teaching "places multiple, competing, and often ambiguous demands on the teacher" (Ghousseini, 2017, p. 189) as they attend to the ways students make sense of disciplinary ideas and relate to one another socially.

Coaching to Support Inquiry in Practice

Learning ambitious teaching is not a solo or decontextualized endeavor. It requires settings in which educators can consult and reflect with one another as multiple instructional demands surface. We are interested in how coaches can carve out time and space to facilitate collective inquiry *in* practice, supporting teachers to develop the social, intellectual, and material resources needed to develop ambitious practice on the job (Hopkins & Spillane, 2015; Kazemi, 2008). Such inquiry can occur during the collective enactment of a lesson with students, as educators engage in reflexive cycles of experimentation and reflection while teaching together (Goldsmith et al., 2014). Inquiry in practice provides opportunities to enact critical in-the-moment knowledge and then iteratively link reflection and action as educators work to solve problems that they notice as teaching and learning unfold (Elkjaer, 2004; Jayanti & Singh, 2010; Kelly, 2006). It centers a stance of deliberate uncertainty in and about practice that supports educators, individually and collectively, to try on new approaches to teaching and to see their work as an ongoing inquiry process (Ball & Cohen, 1999).

Identifying organizational routines that enable such collective inquiry is critical to the work of coaches who seek to support teacher learning across a school. Feldman and Pentland (2003) defined organizational routines as "repetitive, recognizable patterns of interdependent actions, carried out by multiple actors" (p. 94). This manuscript examines a coaching routine that emerged as a mathematics coach supported groups of teachers to collectively enact teaching together and was later used in one-on-one coaching when the coach supported individual teachers in their own classrooms. We examine how the coaching interactions within the routine changed over time because we were curious whether and how the routine continued teacher learning as teachers become more experienced with ambitious teaching and deepened their knowledge of teaching mathematics.

Context of Case Study

The TTO coaching routine examined in this analysis was developed by the coach and teachers at Hilltop Elementary school (all names have been changed to pseudonyms). The routine was used in two professional learning contexts. One was a full-day, job-embedded professional learning structure called "Math Labs." The second was one-on-one coaching, in which the coach provided individualized support to teachers back in their classrooms. Because the Math Lab structure is relatively new, we describe it at length.

Math Labs

Each Math Lab spanned one school day (about 6 hours). Math Labs were facilitated by the mathematics coach and involved teams of same grade-level teachers, specialists (e.g., English language support teachers), and the principal.[1] The Math Lab follows a typical learning cycle structure (see, Figure 1; see, Kazemi et al., 2018); we briefly comment on the four-part structure.

At the start of a typical Math Lab, the coach leads the team in doing mathematics together, examining relevant content and practice standards, and discussing research on children's thinking – an opportunity to learn content connected to a particular mathematical focus. Then, the team co-plans an instructional activity (e.g., see *choral counting*, Franke et al., 2018). Such activities make explicit the teaching moves that structure teacher-student interaction, allowing teachers to routinely enact the principles that undergird high-quality mathematics teaching (Kazemi et al., 2009). During co-planning, educators identify the goals of the task, consider strategies students might use, generate questions to elicit students' ideas, and anticipate how students might respond to those questions. The resulting lesson plan is treated as a rough guide, which is flexible enough to allow teachers to follow students' ideas during enactment (see also, Gibbons et al., under review).

Central to Math Labs are two classroom visits, which take place in participating teachers' classrooms, with students present. The teachers and coach enact the planned instructional activity, collectively listening and responding to student thinking around the focal content. The intent is for the team to co-construct the lesson *together*. Although one or two educators assume a "lead teacher" role, this person is not thought to be modeling instruction, but rather is a volunteer willing to lead as the group enacts the teaching. Emerging from this expectation for shared participation, the coach and teachers at Hilltop established a cue for pausing instruction during the classroom visits so that they

[1]Elsewhere, we discuss the role of the principal in the Math Learning Lab professional learning structure (see, Kazemi et al., n.d.), the coach's role in facilitating Math Learning Labs (Gibbons et al., 2017b), and the history of the university-school partnership between university-based educators and Hilltop educators (Kazemi & Resnick, 2019). While teacher educators from the university were involved in supporting the design and enactment of multiple Math Learning Labs for the first three years of the partnership, this analysis focuses on the coach's participation in and use of TTOs with the teachers at Hilltop.

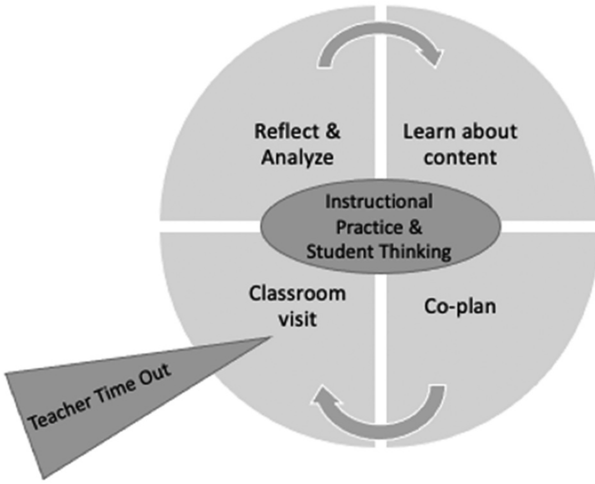

Figure 1. Math lab learning cycle.

could consult with one another as a lesson unfolded. This cue indicates when one member wants to think aloud, raise a question for the team, or engage directly with students. Because the team plans together, anyone is welcome to call a "*Teacher Time Out" (TTO)* – a term coined by the coach – to share an idea, express curiosity, consider instructional decisions in the moment, or engage directly with students. Following the classroom visit, the coach facilitates a reflection and analysis of the visit in a conversation focused on what the team learned about students' thinking, the focal mathematical content, and the team's pedagogical choices.

Importantly, as the TTO coaching routine developed at Hilltop, it extended beyond Math Labs into individual classrooms. The coach and teachers engaged in the routine during one-on-one coaching, as the coach supported individual teachers' classroom instruction. In this context, either the coach or teacher could call a TTO to problem solve together about how to respond to students.

School Context

Our case study draws on data from a partnership between Elham Kazemi, a university-based teacher educator, and school leaders and teachers at Hilltop Elementary, an urban school located in the northwestern United States, serving over 400 Kindergarten through fifth-grade students from diverse racial, ethnic, and linguistic communities. Hilltop steadily improved learning outcomes for students, transitioning from performing in the bottom 5% of schools in the state to becoming a school of distinction in the state. Coaches worked alongside teachers to center instruction on students' sense-making and create classroom communities where students felt known and seen.

Hilltop had approximately 18–20 classroom teachers each school year. There was some teacher turnover from year to year. From Year 1 to Year 2, three new teachers joined the Hilltop community; from Year 2 to Year 3 four new teachers joined. Additionally, teachers could elect to change grade levels within the school from year to year. Thus, as a collective, Hilltop teachers needed ongoing support to deepen their understanding of the mathematics they taught, how children learned that mathematics, and the goals of ambitious pedagogy.

Methods

In this analysis, we sought to understand how the TTO coaching routine supported teachers' professional learning. Thus, we asked the following questions: (1) What opportunities to learn about

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ambitious teaching were made possible when teachers and the coach engaged in TTOs? (2) How did these learning opportunities change over time as the coach engaged with teachers in TTOs?

Data Collection and Analysis

We were able to study the TTO routine because the collective enactment of teaching during the classroom visit portion of the Math Labs was routinely videotaped so that the coach, whom we call Tara, could utilize the video of the enactment when facilitating teacher reflection. We examined video data to study the TTOs taken across three years of Math Labs among teams of K-5 teachers, the coach, and the principal at Hilltop. Data sources included 48 video recordings of the classroom visit portion of Math Labs, captured on small cameras by one of the educators (approximately 38 hours of video footage).

The authors first reviewed each video recording to identify its component TTO episodes. We defined a TTO as an opportunity for any educator to pause instruction to consult with each other about teaching. A TTO episode spanned from the point an educator paused instruction, through the ensuing deliberation, to the moment instruction with students resumed. To characterize the 360 TTO episodes, for each episode we identified: 1) who called the TTO, 2) why the TTO was called, and 3) what happened as a result of the TTO. We found that each classroom visit included between two and 17 TTO pauses, with an average of nine TTOs called per visit. Each TTO interaction lasted between two and 88 seconds (mean 15.32 s, median 9 s, mode 3 s). The person who volunteered to lead the teaching initiated 29% of the TTOs; the rest were called by another adult participant.

We drew on the construct of *opportunities to learn*, defined as the "affordances for changing participation and practice . . . that are available to the learner to participate in particular ways" (Greeno & Gresalfi, 2008, p. 172). In this view, learning and knowledge are "situated in the interactions among members of a particular community engaged with the material world" (Jordan & Henderson, 1995, p. 41). To understand what opportunities teachers had to learn about ambitious teaching when the teachers and the coach engaged in TTOs (Research Question 1), we developed a deductive coding scheme informed by literature on ambitious teaching (e.g., Ghousseini, 2017; Kazemi et al., 2009; Lampert et al., 2013; see Appendix A). Example codes include eliciting student thinking; make sense of student thinking; orient students to each other's thinking; manage space, materials, board work; use representation to support student reasoning; attend to content goals.

The authors individually coded each video along with a third coder, collectively met to discuss any discrepancies and came to consensus on the final codes. The authors wrote memos to document who initiated the TTOs, what occurred immediately after each TTO episode, and the content of discussion during the TTOs. Because we were interested in understanding how the learning opportunities resulting from the TTO interactions changed over time (Research Question 2), our memos included changes we noted in each of these aspects of the TTO routine over the three years. We also examined changes in who called the TTO, why the TTO was called, and what happened as a result by examining percentages for each of the three years and comparing them over time.

We felt it important to discuss the potential of the coaching routine outside of the Math Lab setting in order to better understand how it functioned in multiple settings to support teacher learning. Thus, we examined the potential of the coaching routine outside of the Math Lab setting. In order to examine how the coach supported teachers across multiple professional learning settings, the first author shadowed the coach and captured 40 hours of observation, examining the coach at work in over 60 events, including in-classroom coaching support (during Year 3 and Year 4 of the larger study). The first author audio or video-recorded the events, took field notes, and collected artifacts from each event. While we have analyzed recordings of the coach's one-on-one work in teachers' classrooms, this data is not the primary focus of the analysis that follows. We use data from a single one-on-one classroom visit to share an example of the coaching routine in this setting. This part of the analysis allowed us to highlight aspects of the coaching routine that cut across one-on-one coaching and Math Lab contexts, as both offered opportunities for the coach and teacher to collectively experience and contemplate aspects of ambitious teaching. The aim of our analyses is to contribute to conceptualizing how a coaching routine can assist coaches in supporting teacher learning of ambitious teaching practices and deepen those practices over time.

Results

This section is organized into three parts. First, to offer a sense of what TTO interactions sound like, we provide examples of classroom interaction from two different contexts of the coach's work: one-on-one support in a teacher's classroom and a Math Lab classroom visit, both of which involved multiple TTOs. We then look across the corpus of data from Math Labs, reporting the learning opportunities that arose in relation to ambitious teaching as teachers and the coach engaged in TTOs. Next, we discuss how these learning opportunities changed over time during the Math Labs as the coaching routine was established and educators developed their ambitious teaching practice, individually and as a professional community, over the three years. Thus, we focus our findings on *what* was being learned as well as on *how* learning was taking place (Elkjaer, 2004) to understand how the TTO coaching routine has the potential to support collective inquiry *in* practice and support teachers to develop the conceptual resources and orientations needed to enact ambitious teaching.

Examples of TTO Interactions

In this section, we aim to give the reader an image of how TTOs unfolded as the coach worked with teachers – individually and as a team – to enact instructional activities with students.

During One-on-One Coaching

The TTO coaching routine became part of how the coach, Tara, supported teachers to deepen their instructional practices in their own classrooms. Tara worked alongside individual teachers as lessons played out, frequently checking in about what they were learning about students' mathematical thinking. They conferred with one another as students worked with partners on mathematics problems, and when teachers facilitated class discussions, they used TTOs to consider particular moments of instruction together. Teachers might ask for assistance, or Tara might call a TTO to suggest a particular action or pose questions directly to students. Based on data from the larger corpus of observations and teacher interviews, it was clear that teachers saw Tara as a knowledgeable other who could think through instructional decisions alongside them – related to what to do next with students, why they might try a particular instructional move, or how their actions could impact students' thinking.

To illustrate how the TTO coaching routine was used during one-on-one coaching, we report on a classroom visit from Year 4 during which Tara supported a 4th grade teacher, Anna. The students were sitting together on a rug, engaged in a choral count[2] by fractions on a number line (see, Lind & Lomax, 2018). This lesson came approximately two weeks after a Math Lab that had focused on deepening Anna and her colleagues' understanding of learning and teaching about fractional equivalence. During the classroom visit, Anna and the coach took a total of seven TTOs; the enactment of the choral count lasted approximately 14 minutes.

Tara entered the classroom as Anna started counting by thirds with the 4th grade students. Tara took a seat on the floor among students, as she often did so that she could interact with students and better listen to their mathematical reasoning. The class counted by thirds up to six-thirds and discussed how that amount is equivalent to two wholes. The students then started a new choral count with Anna, this time by sixths. The transcript below takes place after they had written the count on the board from 0/6 to 9/6 (see, Figure 2). Students counted chorally, "zero-sixths, one-sixths, two-

[2]The intent of this instructional activity is to support students to "examine number relationships that enable them to identify, to discuss, and to use patterns and the structure of the number system" (Franke et al., 2018, p. 3).

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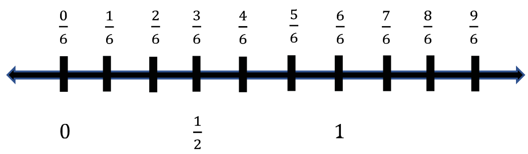

Figure 2. Representation of the choral count by sixths used by Anna.

sixths, three-sixths, four-sixths . . . " Anna stopped the students after they said "nine-sixths" and led a discussion, in which she asked the students: "How many sixths are in two-wholes?" and "How do you know?" Students made connections between the number of wholes and the relationship between the numerator and denominator (i.e., that multiplying the number in the denominator by the number of wholes would show how many sixth- or third-size pieces were equivalent to that number of wholes). The transcript below starts at a moment of student excitement around connections between counting by sixths and multiplication.

(1) Amy (student): Are you teaching us how to multiply [fractions]?
(2) Other students: [*laughter and crosstalk*]
(3) Anna: Am I? [*laughing with the students*]
(4) Students: Yes.
(5) Anna: So what did you notice about how to multiply?
(6) Students: [*crosstalk*]
(7) Anna: Five, four.. [*counting down to get students' attention*] Does anyone notice anything about multiplying a fraction?
(8) Students: It's the same.. It's almost the same.
(9) Anna: Turn and tell your neighbor what you just said.
(10) Students: [*crosstalk for approximately 1 min and 20s*]
(11) Anna: [*calls students back together partner talk*] Ms. Tara did you hear anyone?
(12) Tara: We're just talking about what changed and what didn't [change].
(13) Anna: Ah. That's also what we were talking about too. So, let's see. Naila, can you tell the class what you noticed and said?
(14) Naila: So three-thirds times three equals nine-thirds because I can . . .just add in the three plus three plus three and then that equals nine. And then denominator stays the same.
(15) Anna: Hm. The denominator stays the same.
(16) Tara: Can someone add on to that? [*motioning her hands in a "T" to indicate she wanted to take a TTO*]
(17) Hakim: Um, and also you, um.. The [inaudible] the equal can also- is the same as, um, like the numer- the numerator, if you split it in half by the, uh, denominator, and- 'cause you have [inaudible] there are.
(18) Anna: The numerator, if you.. Say that again?
(19) Hakim: If you split, um.. If you get the numina- numerator and you take a denominator and you see how many you can put into the numerator, then you can see how many wholes there are.
(20) Anna: Oh. So if I take 12 divided by six, what does that equal?
(21) Students: Two.
(22) Anna: And how many wholes is that?
(23) Students: Two wholes.
(24) Anna: That's pretty cool (laughs), isn't it? . . .Fractions are pretty neat. Awesome.
(25) Tara: [*to* students] So why does the numerator change and the denominator didn't?

During this series of exchanges, Anna and Tara worked with the class to explore a student's comment connecting the number of sixths in a given number of wholes with the multiplicative relationship between numerator and denominator. The first TTO occurred in line 11, after the students engaged in partner talk, discussing their initial connections between counting by sixths and multiplication to understand how many sixths are in a different number of wholes. Because Tara positioned herself on the ground with students, she was able to listen and interact with pairs of students. Thus, she was well situated to nominate a pair of students to share their thinking when Anna called the whole group back together. In her response, Tara indicated to Anna what the group had discussed to better assist Anna as she considered whom to call on to develop the key mathematical ideas in this discussion. They were able to collectively explore how to return from the turn and talk, using the students' ideas they both overheard to consider how to move forward.

The second TTO occurred in line 16 when Tara motioned to Anna that she would like to ask the students a question directly. With her question, Tara wanted to both highlight a key idea in Naila's contribution – that the denominator stays the same when multiplying a fraction by a whole number – and involve multiple students in making sense of what Naila had shared. The third TTO, taken in line 25, similarly intended to invite more students to unpack some of the class's initial noticings about the multiplication of fractions by a whole number.

Approximately 30 turns of talk later, Anna called another TTO after a student, Amara, described the numerator as "the number of pieces" and the denominator as "how big the pieces are" and explained why the size of the piece does not change while the number of pieces does.

(64) Anna: The number of pieces. I love it. Teacher Time Out. I'm like, where do I go from here? . . .
(65) Tara: The main thing is checking in to make sure, are there lots of kids who understand [the idea]. Get lots of students to say it back.

In line 64, Anna paused to acknowledge the strong explanation offered by a student and wonder aloud about her next instructional move. In response to Anna's question, Tara again suggested that Anna consider facilitating the discussion in a way that would encourage more students to build on and share their understandings of the big mathematical ideas.

In this one-on-one example, the TTO routine allowed Tara to assist Anna with an important facet of ambitious teaching: eliciting and responding to students' ideas such that children come to see themselves as valued members of the classroom intellectual community while engaging deeply with ideas about fractions (Anthony et al., 2015). Anna and Tara jointly inquired into Anna's facilitation of a mathematical discussion by checking in, as the lesson unfolded, about which students to invite to share their thinking with the whole class and how to ensure students had multiple opportunities to make sense of the discussion's key ideas related to multiplication of fractions. As demonstrated earlier in the vignette, TTOs also allowed the coach to model asking questions that aimed to illuminate student thinking and engage students with one another's ideas.

Importantly, the TTO coaching routine allows for joint reflection to continue after the lesson is over. Later that day, Anna and Tara met together to deconstruct parts of the lesson and discuss how the teaching moves they tried influenced student thinking.

During Classroom Visits in Math Labs

We now describe how the TTO coaching routine was used with groups of teachers. The following example comes from a 2nd grade Math Lab from Year 3 of the study. At the time, the coach had been working with teachers across the school to support student understanding of a particular mathematical idea related to fractions: that the value of a fractional piece is connected to its name in a way that communicates a relationship between equal-sized parts and the whole (Empson & Levi, 2011; Lewis, 2016). During this classroom visit, the coach wanted to support the 2nd grade team to try out equal sharing problems with students, giving herself and the teachers opportunities to notice how students would reason through these problems. The excerpt below took place during a classroom visit in which the coach, principal, and three 2nd grade teachers enacted a co-planned lesson that posed a series of problems to students. The problems in focus were: two children share one brownie; four children share one brownie; and four children share two brownies. When working with equal sharing problems, there is an assumption that "sharing" involved creating *equal-sized pieces*.

Craig, a 2nd grade teacher, took the "lead teacher" role during this classroom visit: he would represent student thinking at the board as the group collectively engaged in teaching. The excerpt below (totaling 2 minutes, 15 seconds) began 5 minutes, 30 seconds into the classroom visit after Craig posed the 2nd equal sharing problem: *If four children share one brownie equally, how much brownie will each child get?* The students[3] had time to consider the problem and share ideas with another student sitting close to them. Craig then pulled the students back together as a whole class to discuss the problem. Just before the transcript excerpt begins, Craig had called a student, Mustafa, up to the board to show how he thought about cutting the brownie (a drawing of a square) into four pieces. Mustafa took the marker and drew two lines to partition the square roughly into quadrants. After Mustafa stepped away from his drawing, other students started to mutter, "Not equal" and, "Not the same size." The coach, sitting among students on the rug, exclaimed: "Now this is really interesting!" and called a TTO to ask the class a question.

(1) Tara (coach): Now, this is really interesting .. [*crosstalk*]. So, teacher time out, I'm going to ask a question. [*To students*] Does it matter where you draw the lines?
(2) Kids respond: No .. Yes .. [*crosstalk*]
(3) Julie (principal): I think we should turn and talk about why it's equal or not equal.
(4) All Kids: [*crosstalk, 40 seconds go by as students engage in discussion with a peer*]
(5) Craig: Five, four, three, two, one. [*Calling students back together as a whole group*]
(6) Craig: So Mustafa—[while you all were talking,] he said, "Oh, can I draw it again?" because he noticed that these were, this one is a little bigger than that one, so he didn't like that. So I'm going to cross that one out [*crossing out the first square that Mustafa drew*]. And then he drew this one [*pointing to a second square that Mustafa drew and partitioned*] and, you know, it's hard. . .so he really tried. He wanted them all to be the same size.
(7) Markos (student): That's not the same size.
(8) Craig: It's as close as he could get. It's okay.
(9) Kris (teacher): [*To teachers*] Can I do a time out? [*To students*] Uh, it's almost like a sketch, right? When we're doing our drawing. We're not doing – we're trying our best, and if we're really trying our best to make it equal, and in our minds it's supposed to be equal, and we can tell somebody that it's equal, and it looks pretty close to being equal, then we'll say that it is.
(10) Craig: [*Repeating Kris*] Then, we'll go on and say that it is [equal].
(11) Julie: And it does look more equal than his first drawing, right?
(12) All Kids: Yes.
(13) Julie: Because he was trying to make them equal-size pieces.
(14) Tara: [*Makes a TTO signal with her hands*] So, Mr. C, I'm really curious, what do we call those pieces?
(15) Craig: That's what I was just going to ask.

[3]Educators who hear about TTO for the first time often wonder what the impact of TTO is on students and their learning. Teachers have reported that students enjoy the experience of having many teachers listen carefully to their thinking and work together within TTOs to make sense of their thinking. Many teachers have commented about how the TTOs do not interfere too much with the pacing of the lesson and appreciate that students get to see their teachers positioned as learners. Before the lesson begins, the team thanks the students in advance for their assistance on this special day and positions them as the true teachers since they will help the team learn.

Four TTOs occurred during this segment of the classroom visit. The first occurred in line 1 when the coach, Tara, decided to ask the students whether it mattered where they drew the lines on the brownie representation. In their planning conversations, Tara and the teachers had talked about the importance of attending to equal-size pieces when partitioning an object. In the moment, the coach wanted to give students an opportunity to discuss the connection between equality and the positioning of the partitioning line segments. The second TTO, closely following the first, was a quick suggestion made by the principal to do a "turn and talk" (line 3); she noticed that kids were responding to the coach's questions in different ways. She suggested a re-phrasing of the idea they wanted students to grapple with related to equality (a decision the team could later reflect) and that they might benefit from talking with a classmate. In the third TTO (line 9), another 2nd grade teacher encouraged students to consider that hand-drawn representations, though inexact, could symbolize equally-sized pieces if the class community agreed to share this understanding. In this moment, the teacher may also have wanted to publicly affirm Mustafa's attempt to make his drawing look more equal. The fourth TTO (line 14) was again called by the coach, to suggest to the lead teacher and others that it might be useful to ask the students to name the fractional pieces. The question posed by the coach was an attempt to support students to connect the size of a piece and its name, to deepen understanding of the relationship between fractional quantities and the whole of which they are a part.

An important principle of ambitious teaching is using learning goals to guide instructional decisions (National Council of Teachers of Mathematics, 2014). In this series of TTO episodes, two of the TTOs were called by the coach to orient students and teachers toward the mathematical goals the team had identified for students (related to fraction equivalence and using a measurement interpretation to describe the size of each part). Furthermore, when the principal suggested that the students engage in a "turn and talk," the team had an opportunity to experience how using this classroom discussion technique could build a shared understanding of mathematical ideas as students compared their ideas with one another and looked back at Mustafa's representation.

Embedded Opportunities to Learn about Ambitious Teaching

We now shift to describing the findings from analyzing the larger corpus of 360 TTOs taken during the classroom visit portion of the Math Labs. In our analysis, we were concerned with the opportunities teachers and the coach had, during TTOs, to deepen their understanding of and ability to enact ambitious mathematics teaching. Table 1 includes moments from the data in which the coach and teachers grappled with ambitious mathematics teaching through TTO interactions. The aspects of ambitious teaching they considered included: making sense of student thinking, eliciting student responses, teaching toward a mathematical goal, orienting students to one another's ideas, facilitating and extending student reasoning, assessing student thinking, representing mathematical ideas, and managing the board work.

Our analysis revealed multiple purposes for why a TTO was called, including: requesting assistance ("Am I missing anything?"), suggesting what to do next ("Could you circle the 1?"), posing a question/comment directly to students ("Who can repeat what Lea said?"), asking the lead teacher a clarifying question about their decision-making ("I'm wondering why you wrote 8.10"), inviting other adults to share their thinking ("What are you wondering at this point?"), asking to take on the lead teaching role ("Can I step in for a second? Because I think I know why you did that, Russell."), and reminding the team about something they had planned to do ("Remember the representation we had discussed?"). This range of purposes illuminates the complex nature of educators' collaboration and the various ways TTOs opened opportunities for teachers to engage in collective inquiry into practice and shared instructional decision-making.

When analyzing the TTOs initiated by the coach, we found that 47% of those she initiated attended to mathematical content, 23% attended to unpacking student thinking (e.g., pressing for student explanation or responding to student confusion or error), 12% attended to managing the instructional activity (often to create a new problem), 10% related to eliciting student thinking or suggesting

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Table 1. Examples of opportunities to learn about ambitious teaching embedded in the TTO Interactions.

Aspects of ambitious mathematics instruction	Components of each aspect that the coach and teachers worked on in the TTOs	Examples from Data[a]
Make sense of student thinking	Listen carefully to and work to understand student thinking	"Is that important? What is important here?" "He saw groups!"
Eliciting student responses	Decide whether to elicit additional student responses or move on	"Should I write a different way, or should we talk about this [Mateo's] way first?" "Is everyone far enough along so we can do a little share?"
	Encourage students to share	"Abdi and Muna had an idea that the class should hear."
Teaching toward a mathematical goal	Sequence elicited ideas for discussion	"Which one do we start with?" (Referring to which to unpack first from the four strategies the team had elicited from students)
	Orient students to the mathematics	"Samuel and Pablo made an easier problem that we should think about." "Just to ask the kids if they made sense between this [8.10] and this [8.1]." "Is there a math term we could use for that, that means the same size?"
Orienting students toward one another's ideas	Direct students' attention toward a classmate's ideas	"Did you hear something you wanted us to [listen to]?" "Oh, were you doing something back to fractions? That would be helpful for us to hear; tell us about that." (directed to student)
	Use of Talk Moves	"Should we ask someone to say it in their own words?" "Wow, do you think we have to have another turn and talk about that?"
Facilitating and extending student reasoning[a]	Press for student explanation	"So are they the same?" (directed toward students)
	Create a new problem	"Mr. Kim, do you have a problem you'd like us to see if we can have them solve?" "Could we do, on the next page . . . 3 divided by 4 equals 4 divided by 3?"
Assessing student understanding	Respond to student confusion/error	"I wonder if we need to separate the two sides of the equation." "Actually-[Kai], I think I may have heard you say something different. Can I jump in?"
Representing mathematical ideas (visually, numerically, and/or verbally)	Attend to verbal or symbolic notation	"The connection in the piece around recording the fraction, the decimal number, and the *words*. T-e-n-t-h-s. Can you write the words out on the board?"
	Use representation to show/ support student reasoning	"I'm thinking about what Ian just told us and I'm thinking about the representation that we had planned. And how we can make that connection. What are you thinking?" "What would that look like if we did it on a number line?"
Managing Instructional Activity	Introducing/Closing the Instructional Activity	"And also try to write an equation – [to educators] what's my phrasing here?" "I think this is a good stopping point. Is that okay with everybody?"
	Manage space, materials, board work[c]	"Can you write that underneath?" "Can you see?" (to students) "I'm not sure they can see." (to educators)

[a]Often a TTO was signaled with an educator forming a "T" with their hands, or "Teacher Time Out" was said aloud. The example shows what came after the TTO was signaled or called. Unless otherwise noted, what was said was directed toward other teachers and/or the coach.
Note: The naming for each aspect comes from Kazemi et al. (2009) unless otherwise noted.
[b]The name for this aspect comes from Ellis et al. (2019). An important distinction is that we include the move "press for student explanation" within this aspect, while Ellis et al. (2019) consider pressing a form of eliciting student ideas.
[c]Lampert et al. (2013)

students turn and talk to a partner, and 8% related to managing materials or board work. When not in the "lead teacher" role, the coach often initiated TTOs to suggest a next move that teachers could try (45% of the TTOs) or pose a question or comment directly to the class (39% of the TTOs). When she was in the "lead teacher" role, which was much less often because she wanted teachers to have

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opportunities to teach together during the classroom visits, she took the role of welcoming teachers to think alongside her (e.g., "Should we pose a new problem?"). She initiated TTOs to invite other teachers to share a question or thought, or to "think aloud" to narrate her thinking about teaching. Most often, she used the TTO coaching routine to model asking questions that could be generative for student discussion or to direct teachers to try out a particular question or move themselves.

Across the data set, TTO conversations highlighted the multifaceted, situational nature of ambitious teaching (Lampert et al., 2013). By working to refine their mathematics instruction together *in* practice, teachers, with the support of the coach, had opportunities to manage the complexities of ambitious teaching collectively and raise questions or issues for further reflection.

Evolution in Learning Opportunities from TTOs

Since the teachers at Hilltop Elementary were engaged in professional learning with the coach across multiple years, we became interested in how the nature of the TTOs changed as teachers became more experienced with ambitious teaching and the coach deepened her practice supporting teachers.

Foci of TTOs over Time

Over time, the most common purposes for TTOs shifted. There was an increase in the number of TTOs taken related to inviting others to consider the next instructional move (8% of the TTOs in Year 1, 14% in Year 2, and 29% in Year 3). We also found that, over time, the TTOs taken during the classroom visits of the Math Labs focused less on the logistics of managing the instructional activity and more on 1) the mathematical content of instruction and 2) facilitating the course of the discussion (see, Figure 3). TTOs that focused on managing the logistics of the instructional activity included those related to launching or closing the instructional activity (e.g., "I think this is a good stopping point. Is that okay?"), managing board work and materials (e.g., "Should we hand out the problem now or launch it first?"), and discussing the sequence of the instructional activity (e.g., "Should I flash the image one more time?"). As can be seen in Figure 3, twenty-one percent of the TTOs called in Year 1 focused on managing the logistics of the instructional activity (Managing IA Logistics). This proportion decreased over time, comprising 12% of the TTOs called in Year 2 and 6% of the TTOs called in Year 3.

We hypothesize that, as teachers became more familiar with instructional activities – in Math Labs, their individual practice, and one-on-one work with the coach – they grew more adept at managing the logistics of those activities and needed less assistance from their colleagues. Teachers continued to learn new instructional activities through Math Labs across the three years. However, questions around how to present activities to students, and whether to dwell on a particular aspect of an activity or to move on to the next part, became less pressing as issues of practice to address during their collective work in classroom visits.

In TTOs that focused on mathematical content, educators considered how to: orient students toward the key mathematics in a problem or representation (e.g., "How many halves are in that brownie?"), use representations to illustrate and facilitate student reasoning (e.g., "Does it matter where you draw the lines?"), attend to the details of verbal and symbolic mathematical notation (e.g., "I'm really curious, what do we call those pieces?"), and focus an activity around a particular content goal or mathematical practice (e.g., "I saw students use mostly area models, should we include a linear model?"). The percentage of TTOs in which instructional decision making focused on mathematical content increased from 25% in Year 1 to 40% in Year 2 and constituted 34% of the TTOs called in Year 3. We hypothesize that, as teachers became more comfortable with the routine aspects of enacting instructional activities, they were freed up, individually and collectively, to devote deeper attention to the mathematical goals underlying those activities. As a result, more of their co-problem-solving related to highlighting important mathematical ideas and supporting students' mathematical sensemaking.

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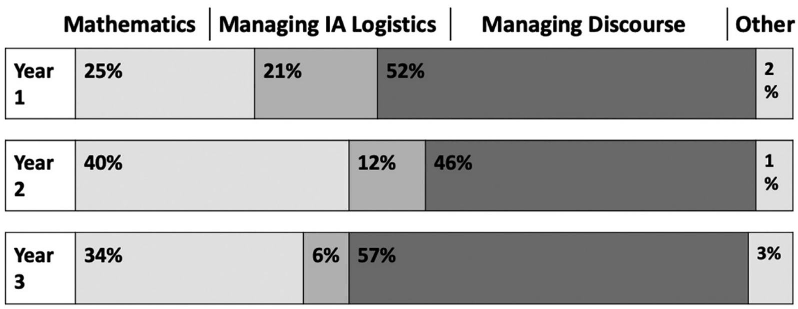

Figure 3. Categories and related percentages of TTOsacross years 1–3. Note: These TTOs were taken during K-5 Math Lab learning events at Hilltop Elementary.

In TTOs focused on facilitating discourse, educators addressed when and how to: elicit student thinking (e.g., deciding which students to call on or inviting additional students to share ideas), respond to student thinking (e.g., pressing for explanation or responding to confusion), and orchestrate conversation among students (e.g., which talk moves to use). The percentage of TTOs related to facilitating discourse was 52% in year 1, 46% in Year 2, and 57% in Year 3. On one hand, discussion facilitation seemed to remain a central concern in TTO conversations across the three years. This aligns with the emphasis ambitious teaching places on classroom discussion (Chapin et al., 2013). At the same time, we found that the specific content of these TTO interactions changed. Initially, the coach and teachers worked primarily on eliciting student thinking, calling TTOs to discuss how to support students to know what and how to share. Over time they worked more on orienting student talk around key mathematical ideas, supporting students to respond to each other, and considering issues of positioning and power in the classroom discourse community.

One might expect that, over time, teachers would call fewer TTOs as they enact lessons together. To the contrary, our data shows that there was an increase of TTOs taken from Year 1 to Year 2 (an average of 8 TTOs taken per classroom visit to an average of 13 TTOs taken), and Year 3 was similar to Year 2. Since ambitious teaching is so complex, it may be that as teachers grew more adept with particular aspects of ambitious practice, new questions arose and further nuances came into view through their continued inquiry.

Coaching Routine Shapes Schoolwide Orientations toward Improvement of Practice

The TTO coaching routine became a regular practice in the coach's one-on-one work with teachers. Teachers reported that during one-on-one coaching, they saw Tara as someone who could think through instructional decisions alongside them. In an e-mail communication with Tara, Anna expressed her perceptions of their work together, "It's nice to have an extra adult brain in the room to bounce ideas off of. You always ask what seem like simple questions but really end up pushing my own thinking past the obvious, and as a result, my instruction changes" (Year 4). From the larger corpus of data, teachers expressed a similar sentiment about the ways collaborative inquiry during Math Labs shaped their individual instruction. Another teacher shared, "In the midst of my classroom, I'm giving myself permission to take a TTO [and ask] what would . . . Linda (5th grade colleague) do here?" (5th grade teacher, Saira, shared during Math Lab in Year 2). When back alone in her classroom with students, this teacher engages in the TTO routine as a mental habit, asking what her colleague or the coach would do to guide her own instruction in the moment. These teachers' reflections suggest that collective inquiry into practice reverberated beyond the in-person shared learning experience, as educators across the school continued to draw on conceptual resources and orientations developed through their work with Tara and colleagues.

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Discussion and Implications

The TTO coaching routine can support coaches and teachers to construct and deepen professional knowledge for mathematics instruction as they work together to conceptualize and addresses problems of practice while teaching (Kelly, 2006). Through TTOs during Math Labs and one-on-one work with Tara, teachers at Hilltop had opportunities to learn about features of ambitious mathematics teaching by enacting and reflecting on this instruction within a collective setting. These features included how to facilitate discussions in which students share reasoning, orient students toward each other's ideas and toward the mathematics, and position them as authors of mathematical knowledge. Additionally, teachers had opportunities to consider how to respond to confusion or error, use representations to support students' reasoning, and teach toward a mathematical goal.

Over time, we found that the content of TTOs shifted in focus from managing the logistics of instructional activities to the professional judgment and pedagogical content knowledge that are crucial to facilitating robust classroom mathematics discussions. Importantly, just under half of the coach-initiated TTOs focused on the mathematical content, suggesting that the coach's participation helped anchor TTO interactions in the mathematical goals of the activity. This may suggest a broader implication for the role of coaches in professional learning routines like the TTO: As teachers navigate multiple demands on their attention, coaches can direct collective focus toward the mathematical knowledge for teaching that underlies instruction, an important aspect of supporting teachers to take up ambitious mathematics teaching (Ball et al., 2009).

The TTO coaching routine also provided opportunities for educators to communicate the reasons for professional decisions to others, both in real time as instruction unfolded and in extended reflections after the classroom visits. The experience of teaching together, guided by a coach, encouraged educators to notice and publicize problems of practice, which became the subject of shared inquiry. Coaches can play a vital role in supporting teachers to engage with one another's interpretations and decisions during these conversations, by both modeling and inviting teachers into collaborative inquiry. Our analysis points to the potential of such inquiry to expand educators' conceptual resources and practices for teaching mathematics – including their habits of noticing, ways of interpreting, and repertoires of responses to students' mathematical thinking (Lampert et al., 2013; Munson, 2020).

As coaches work toward systemic change across classrooms (Mangin & Dunsmore, 2015), they consider how to support teachers to deprivatize their instructional practice and engage in collective inquiry, which requires changing norms of interaction among teachers (Elkjaer, 2004). We found that the TTO coaching routine at Hilltop, and the professional learning structures in which the routine was embedded, was a site for developing these collective norms across classrooms and teaching teams within a school. To build on the present analysis, which focuses on the content that participants discussed during TTO interactions, future studies might examine the finer-grained discourse patterns among educators that enable and evidence these broader shifts in norms of professional inquiry.

Additionally, while our analysis sheds light on what teachers might learn about teaching mathematics through engaging in a coaching routine, future research could examine coaches' professional learning as a teacher educator participating in TTOs. How might coaches who are unfamiliar with the TTO routine, and possibly inquiry in practice more broadly, come to incorporate the routine into their coaching? How do coaches develop the judgment to know when and how to draw on the collective experience of teachers during TTOs? What are the recurring decisions, dilemmas, and opportunities that coaches experience as they use TTOs in their classroom-based work with teachers?

Along with our colleagues, we have developed a set of tools to support coaches to learn about and enact the TTO coaching routine (on tedd.org, hosted by the University of Washington). These include sample TTO norms, videos of TTO enactment, protocols for how to introduce TTO to students, and a guide for debriefing the TTO after enacting them in a classroom visit. As coaches and teacher leaders explore these tools and experiment with the TTO routine in their own settings, future research might examine the ways the TTO routine, and coaches' role within it, vary based on context.

We contend that the TTO coaching routine can support coaches to facilitate teachers' collective inquiry into practice, organizing the school workplace around opportunities to learn about teaching together *in* teaching. The TTO coaching routine has the potential to focus coaching interactions on important aspects of ambitious teaching. Over time, as educators become more experienced collaboratively inquiring into and enacting ambitious teaching, new – and, as our study suggests, more complex – questions of practice arise for coaches and teachers to consider. Furthermore, TTOs position coaches as co-explorers of mathematics teaching and learning alongside the teachers with whom they collaborate (Baldinger, 2018). In this way, the routine can expand our perspectives of how pedagogical expertise develops in professional learning settings organized around collective inquiry.

Disclosure Statement

No potential conflict of interest was reported by the author(s).

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Appendix A

Parent and Child Codes	Description of Codes
Elicit Student Thinking	
Elicit student ideas	Elicit students' initial ideas about a problem or question; may involve when to elicit, how to elicit, and/or what to elicit
Encourage students to share	Encourage a range of students to share their ideas
Decide which student to call on	Decide which student(s) to call on next
Responding to Student Thinking	
Press for student explanation	Press for elaboration or justification from students
Make sense of student thinking	Listen carefully to and work to understand and/or describe the details of student thinking
Check for student understanding	Check for student understanding or name what students understand about the mathematics at this point in the IA
Respond to student confusion/error	Respond to student confusion or an error in a student's response
Managing Discourse	
Orient students toward each other's ideas	Encourage students to consider each other's ideas and to direct their responses to each other
Talk moves	Invite students to revoice, restate, add on, disagree, elaborate, revise or clarify as a way to further discussion, give think time to others.
Turn and Talk	Direct students to turn to a partner and exchanges responses before returning to a whole group discussion
Direct the Course of the Instructional Activity	
Decide on next question or problem	Decide on next question or problem to pose to the class
Decide whether to elicit additional student responses or move on	Decide whether to elicit additional student ideas or strategies in response to a particular problem
Sequence ideas for discussion	Decide which student ideas ("answers") to open up for elaboration and discussion
Create a new problem	Someone creates a new problem to pose to students as a way to build on student thinking at this point in the IA
Manage the Instructional Activity	
Introduce IA	Introduce and begin student engagement with the IA
Close IA	Bring the IA to an end
Attend to process	Discuss logistical steps of the IA
Manage space, materials, and board work	Attend to issues of space (physical, materials, board work) while engaging students
Mathematics	
Make sense of the mathematics	Adults discuss the mathematics underlying a problem, big idea, strategy, or representation
Attend to verbal or symbolic notation	Attends to one or more of the three aspects of number: verbal, symbolic, or quantitative (could be discussing the connections among mathematical ideas expressed in words, pictorial representations, and numbers/symbols)
Orient students to the mathematics	Direct students' attention to the core mathematics of a statement, question, problem, or representation. E.g., connect the quantities to the context of the problem, redirect conversation toward important mathematical ideas
Attend to content goal	Attend to the specific mathematical content goal of the lesson
Attend to mathematics practice standards	Explicitly discuss how to support students to: develop mathematical arguments supported by evidence, notice mathematical patterns and structures, communicate ideas using precise mathematical language, reason abstractly and quantitatively, etc.
Use representation to show/support student reasoning	Use a representation to illustrate student thinking, make a student's thinking accessible to others, support student reasoning about a mathematical idea, attend to the mathematics, etc.
Attend to the precision/clarity of a representation	Focus on the labeling, scale, or other visual feature of a representation so that it precisely reflects a student's strategy or a mathematical idea

Coding scheme developed to examine the content of the TTO interactions informed by literature on ambitious teaching (e.g., Ghousseini, 2017; Kazemi et al., 2009; Lampert et al., 2013).