Efficient Solutions to Traffic Congestion Externalities:
More Complicated Than You Might Think

A problem-based learning exercise developed by
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Model Specification and Welfare Calculations

General Model:

Marginal net benefit (MNB): $MNB = 20 - x$

Marginal private congestion cost (MPCC):

$$MPCC = 0.5\max\{0, (x-8)\}$$

$$MPCC = \begin{cases} 0 & if\ x \leq 8 \\ 0.5(x-8) & if\ x > 8 \end{cases}$$

Marginal social congestion cost (MSCC):

$$MSCC = \begin{cases} 0 & if\ x \leq 8 \\ x[0.5(x-8)] & if\ x > 8 \end{cases}$$

Scenario #1: No Congestion; No Toll

Predict the equilibrium number of drivers:

Solve for the equilibrium number of drivers by setting

Marginal Net Benefit (MNB) = Marginal Private Congestion Cost (MPCC)(=0).

$20 - x = 0 \rightarrow x = 20$; therefore, up to 20 drivers will enter the road

Calculate the socially optimal number of drivers:

Assuming that there are 18 students playing the role of drivers,

max $\int(20 - x)$ st. $0 \leq x \leq 18$

$x = 18$

Calculate Social Net Benefit (SNB) with the equilibrium number of drivers:

SNB = $\int_0^{18}(20 - x) = 198$

Which can also be solved with simple geometry by adding the area of a triangle and rectangle such that $\frac{1}{2}(18)(20 - 2) + (2 * 18) = 198$.

<u>Scenario #2: No Congestion; $9 Toll</u>

Predict the equilibrium number of drivers:

Solve for the equilibrium number of drivers by setting MNB = MPCC + toll

$20 - x = 9 \rightarrow x = 11$; therefore, 11 drivers will enter the road

Calculate Social Net Benefit (SNB) with the equilibrium number of drivers:

SNB = $\int_0^{11}(20 - x) = 159.5$ (assuming that the revenue is redistributed)

or,

$\frac{1}{2}(11)(20 - 9) + (9 * 11) = 159.5$

<u>Scenario #3: Congestion; No Toll</u>

Predict the equilibrium number of drivers:

Solve for the equilibrium number of drivers by setting MNB = MPCC

$20 - x = 0.5(x - 8) \rightarrow x = 16$; therefore, 16 drivers will enter the road

Calculate the socially optimal number of drivers:

max $\int(20 - x - x[0.5(x - 8)])$ st. $0 \leq x \leq 18$

$(2 + 0.5x)(10 - x) = 0 \rightarrow x = 10$

Calculate Social Net Benefit (SNB) with the equilibrium number of drivers:

SNB = $\int_0^{16}(20-x) - \int_8^{16} x(0.5(x-8)) = \ [20*16-0.5(16)^2] - \left[\left(\frac{1}{6}(16)^3 - 2(16)^2\right) - \left(\frac{1}{6}(8)^3 - 2(8)^2\right)\right] \ = 192 - (170.67 + 42.67) = 192 - 213.34 = -\ 21.34$

<u>Scenario #4: Congestion; $9 Toll</u>

Predict the equilibrium number of drivers:

Solve for the equilibrium number of drivers by setting MNB = MPCC + toll

$20 - x = 0.5(x-8) + 9 \rightarrow x = 10$; therefore, 10 drivers will enter the road

Calculate Social Net Benefit (SNB) with the equilibrium number of drivers:

SNB = $\int_0^{10}(20-x) - \left[\int_8^{10} x\big(0.5(x-8)\big)\right] = 150 - [-33.33 + 42.67] = 140.66$

assuming that the toll revenue is redistributed.

or,

$\sim \left[\frac{1}{2}(10)(20-10) + (10*10)\right] - \left[\frac{1}{2}(10-8)*10\right] = 150 - 10 = 140$

<u>Scenario #5: Congestion; $10 Toll</u>

Predict the equilibrium number of drivers:

Solve for the equilibrium number of drivers by setting MNB = MPCC + toll

$20 - x = 0.5(x-8) + 10 \rightarrow x = 9.33$; therefore, 9 drivers will enter the road

Calculate Social Net Benefit (SNB) with the equilibrium number of drivers:

SNB = $\int_0^{9}(20-x) - \left[\int_8^{9} x\big(0.5(x-8)\big)\right] = 139.5 - [-40.5 + 42.67] = 137.33$

assuming that the toll revenue is redistributed.