AERODYNAMICS OF THE PSYCHROMETER

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First Paper: Theory and Experiments Relating to Energy Transformations in a Jet of Air

INDEX TO SECTIONS

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General Introduction

In the year 1792, "The late most ingenious philosopher, Dr. James Hutton of Edinburgh . . . dipped a thermometer in water, both being previously brought to the general temperature; and then exposing the wet bulb to a current of air, he marked how many degrees of mercury sunk in the tube; and he estimated the dryness of the air by the quantity of the depression." [1] Thirty years later, Ivory [2] put forward what seems to have been the first recorded explanation of this observed depression of the wet bulb and its dependence upon atmospheric humidity. August,[3] in 1825, and Apjohn,[4] in 1834, offered precisely the same explanation of the phenomenon, neither writer having previously had access to the work of Ivory. Rapid progress having been made between the years 1822 and 1825 in the investigation of fundamental laws of cooling and vaporization, the ideas under consideration assumed the form with which most of us are familiar under the hands of August.[5] These, stated in the barest possible terms, consist in the hypothesis that the depression of the wet bulb is due to a state of thermal equilibrium subsisting between heat lost from the wet bulb by evaporation and heat imparted to the wet bulb by the atmospheric air at the expense solely of its internal heat energy. Important dynamical and thermodynamical effects due to the motion of the air were not taken into consideration.

Dissatisfied with this ingenious if somewhat naïve explanation of the wet bulb depression and its relation to atmospheric humidity, Clerk Maxwell [6] attacked the problem from an entirely new angle, and one wholly in the spirit of modern Aerodynamics. Having previously derived, by methods of the Kinetic Theory of Gases, differential equations for the motion of a mixture of two gases,[7] he calculated the flux of vapor through perfectly still, dry air, from the wet bulb, and multiplying this quantity by the heat of vaporization of water, equated the result to the rate at which heat was supplied to the wet bulb by heat radiation. For the latter purpose, he made use of what was equivalent to a rough approximation to the well-known Stefan Law.

Maxwell's article was followed by a paper in which Stefan [8] deduced the Maxwell formula (with a correction) from results which he had previously developed in a series of lucid,

[1] Ivory, *Philosophical Magazine*, 1822, **60**, page 81. See also Playfair, *Edinburgh Trans.*, 1805, **5**, Part III, page 67, foot-note, Life of James Hutton.

[2] *Loc. cit.*

[3] *Annalen der Phys. u. Chem.*, 1825, **5**, pages 69 and 335.

[4] *Trans. Roy. Irish Acad.*, 1837, **17**, pages 275 and 283, two papers read in the years 1834 and 1835 respectively.

[5] It is worth while to observe in passing that it was August (*Annalen der Phys. u. Chem.*, 1825, **5**, page 71) who proposed the name *Psychrometer* for the wet bulb hygrometer. This word had previously been employed to denote a thermometer; thus it is defined in Chambers' Cyclopaedia (1727 to 1741) as "An instrument for measuring coldness of the air; more usually called thermometer." See Murray's *New English Dictionary*.

[6] *Encyclopaedia Britannica*, ninth edition, Edinburgh, 1877, Vol. VII, page 218 (article on *Diffusion*).

[7] *Phil. Mag.*, 1860 (4), **19**, page 19; **20**, page 21; also 1868 (4), **35**, pages 129 and 185.

[8] *Zeitschr. Österreich. Gellsch. für Met.*, 1881, **16**, page 177.

brilliant papers on evaporation.[1] Here, as in Maxwell's work, the differential equations of gas diffusion constituted the foundation.

From the standpoint of Pure Mathematics, the method by which the Maxwell-Stefan formula was arrived at is so elegant that it is to be regretted that this formula is not supported by observed facts. Repeated tests have shown conclusively that it does not yield a first approximation. The reason is not far to seek. Air in the neighborhood of the wet bulb is chilled by the presence of the cold evaporating surface; the action of gravity sets up convection currents, and the conditions under which a solution was obtained are therefore not realized. The reason for the suppression of the gravitation terms in the Maxwell-Stefan theory is obvious: their retention led to a system of non-linear, partial differential equations. Solutions for systems of this type have not yet been discovered.

To return to the Ivory-August theory.[2] With a suitable apparatus, embodying essential precautions for the elimination of experimental error, which will be described in a subsequent paper, the psychrometric formula growing out of this theory yields, under favorable conditions, an approximation. But measurements made by the writer show that, under unfavorable conditions,[3] this formula may be in error by as much as 23 per cent.[4]

To the reader, it will doubtless occur at once that a nearer approach to exactitude could hardly be expected of an analysis wholly ignoring effects of viscosity and heat conduction in a turbulent gas mixture, together with the work done by the air against fluid pressures and the influence of turbulence on the mechanically averaged mean temperatures rendered by the thermometers. It may be objected that when, as is usually the case, wet bulb and dry bulb are subjected together to similar air currents, these effects will disappear from a working formula for the psychrometer. But the experiments of the writer show that this objection is invalid. The rough texture of the wick sets up differences between the effects of external work, viscosity, heat conduction and turbulence, at the wet and at the dry bulb, and these differences have to be considered.

It has been the object of the writer, therefore, in preparing the following papers, to investigate the magnitude of the combined effects of external work, viscosity, heat conduction and turbulence on the state of moist air flowing past an obstacle, with a view to an experimental determination of an accurate psychrometric formula. This investigation, taking its departure from the hydrodynamic equations and the Maxwell diffusion equa-

[1] *Sitzber. Akad. der Wiss. Wien.*, 1871, **63** (2), page 63; 1873, **68** (2), page 385; **83** (2), page 943; also 1872, **65** (2), page 323.

[2] Regnault [*Annales de Chim. et de Phys.*, 1845 (3), **15**, page 129, and 1853 (3), **37**, page 257], with some objections and modifications, adopted August's formula, and this formula, with more or less unsatisfactory empirical modifications, has been generally adopted by writers on the psychrometer up to the present time (see Appendix, fifth paper of the present series). It was again discovered independently by a writer in a well-known engineering journal as late as the year 1911!

[3] Low absolute atmospheric humidity.

[4] See third paper of the present series, Table V, No. 26, and the fourth paper, Table I, No. 26. In the case of extremely low absolute humidities, such as are encountered in Antarctic regions, the per cent error would be greatly in excess of the error observed in the case of experiment No. 26.

tions, has fallen into five parts, the main results of which are outlined in the ensuing papers as follows:

I. *Theory and Experiments Relating to Energy Transformations in a Jet of Air.*

II. *Experiments Relating to Energy Transformations in the Region of Eddies Set Up by an Obstacle in a Jet of Air.*

III. *Theory and Experiments Relating to Energy Transformations in the Region of Eddies Set Up by a Wet Obstacle in a Jet of Moist Air.*

IV. *An Aerodynamic Formula for the Psychrometer and its Experimental Verification.*

V. *Verification of the Aerodynamic Theory of the Psychrometer in the Case of a Small-Scale Apparatus.*[1]

[1] These five papers constitute a report submitted by the author to the Secretary of the Research Fellowship Board of the National Research Council, Washington, U.S.A., on March 10, 1925.

PART I

Mean Motion of a Turbulent Gas. Fundamental Mathematical Definitions and Relations

1. *Purpose of the present paper.* Well-known laws governing steady and slow motions of viscous fluids in tubes have been deduced from simple hypotheses concerning stresses and strains in continuous media.[1] These laws have received a satisfactory experimental confirmation.[2] When, however, the velocity of flow of a fluid exceeds a certain critical value, depending upon the density of the fluid, its kinetic viscosity coefficient and the diameter of the tube,[3] then the motion becomes tumultuous and apparently unordered. With the fluid in this condition, it has not been possible to measure the detailed velocities, densities, pressures, etc. of the fluid mass elements, or to trace the mass elements in their progress along the complex and involved paths characteristic of turbulent flow. The laws governing this type of motion are as yet little understood.

Stokes and Saint-Venant deduced the classical hydrodynamic equations for viscous fluids from the hypothesis that the internal stress-components are linear functions of the strain-velocities.[4] But it is possible, on the one hand, that, when a fluid reaches a critical velocity and turbulence sets in, this hypothesis no longer applies, and that the appearance of turbulent motion indicates a stage at which squares and products, etc. of the strain-velocities must be taken into consideration. On the other hand, the Stokes–Saint-Venant hypothesis may be applicable to the detailed motion[5] of the fluid elements under all conditions; in which case, the laws of turbulent motion would find expression in general solutions of the classical differential equations.

The former point of view necessitates the derivation, *ab initio*, of a system of equations for turbulent motion. The latter point of view was adopted by Reynolds[6] and Lorentz,[7] and it appears at the present time to be the more promising. It has accordingly been assumed as the basis of the present study.

With the exception of extremely simple cases of stream-line flow, however, solutions of the hydrodynamical equations have not been obtainable; which is not surprising, for the relative complexity of the mathematical aspect of turbulent motion is to be inferred from the extreme complexity of its physical aspect.

For further progress, therefore, it appears to be necessary to acquire additional information from observation and experiment, under the guidance of such mathematical considera-

[1] Navier, *Mem. Acad. des Sci.*, 1822, **6**, page 389. Poisson, *Journ. de l'Ecole Polytech.*, 1829, **8**, page 1. Saint-Venant, *Compt. Rend.*, 1843, **17**, page 1240. Stokes, *Trans. Camb. Phil. Soc.*, 1845, **8**, page 287.

[2] Poiseuille, *Compt. Rend.*, 1842, **15**, page 1167; *Mém. des Savantes Étrangers*, 1846, **9**.

[3] Stanton and Pannell, *Lond. Roy. Soc. Phil. Trans.*, 1914, **214**, page 199.

[4] Maxwell deduced these equations for a gas by methods of the Kinetic Theory of Gases, with the aid of approximations logically equivalent to the same assumption.

[5] But not to the *average* motion or general drift.

[6] *Lond. Roy. Soc. Phil. Trans.*, 1895, **186**, page 123.

[7] *Abhandlungen über Theoretische Physik*, Leipzig u. Berlin, 1907, third memoir, page 43.

tions as are available. For this purpose, energy relations existing in a turbulent air column are of special interest.

2. *Mean motion of a turbulent fluid. Turbulent motion.* The velocities, pressures and temperatures of a turbulent fluid, as rendered by laboratory instruments, are not the detailed velocities, pressures and temperatures of the fluid elements contemplated in the classical Hydrodynamics. On the contrary, they are mean values of these variables, averaged over more or less indefinite intervals of time depending on the lag and sensitivity of the instruments employed and on the manner in which observations are made. In fact, if φ denote the velocity, temperature, pressure or density of a mass element of a turbulent fluid at an instant t, then a mean value of this variable *at instant t* may be defined by the integral

$$\bar{\varphi} = \frac{1}{\tau}\int_{\tau-\frac{t}{2}}^{\tau+\frac{t}{2}} \varphi \, dt \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

where τ is the interval of time over which the averaging process is extended.[1] The general effect of this averaging process is to smooth out irregularities characteristic of turbulent motion, and thus to yield functions which may be assumed to correspond to observed average drift and observed average thermodynamic states.

Accordingly, if we write

$$\varphi = \bar{\varphi} + \varphi' \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

this equation defines a residual motion, or state, φ', which is superimposed upon the mean motion, or state, and which in the following pages will be referred to explicitly as *turbulence* or *turbulent motion.*

3. *Properties of steady mean motions.* The apparatus used for the ensuing study was designed and operated with a view to yield, as nearly as possible, a steady *mean* motion. When, after the attainment of critical velocities, turbulence appeared, the residual or *turbulent* motion was invariably unsteady in the extreme.

For *steady, mean* motions, the following relations are deducible at once from equations (1) and (2).[2] Let φ and ψ denote any two variables characterizing the motion or thermodynamic state of a fluid element; then

$$\left.\begin{aligned} \bar{\bar{\varphi}} &= \bar{\varphi} \\ \overline{\bar{\varphi}\psi} &= \bar{\varphi}\bar{\psi} \\ \overline{\varphi'} &= 0 \\ \overline{\varphi\psi} &= \bar{\varphi}\bar{\psi} + \overline{\varphi'\psi'} \\ \overline{\frac{\partial\varphi}{\partial x}} &= \frac{\partial\bar{\varphi}}{\partial x} \\ \overline{\frac{\partial\varphi}{\partial t}} &= \frac{\partial\bar{\varphi}}{\partial t} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots \quad (3)$$

[1] *Cf.* Reynolds, *loc. cit.*, page 125, and Lorentz, *loc. cit.*, page 58, equation (30).
[2] See Lorentz, *loc. cit.*

4. *Application of the conceptions of mean and turbulent motion to fundamental aerodynamical relations.* Assuming the validity of the hypothesis that the stress-components in the fluid are linear functions of the strain-velocities, the equations representing the Law of Conservation of Momentum for the fluid have the well-known form [1]

$$\left.\begin{aligned} \frac{du}{dt} &= X - \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{3}\frac{\mu}{\rho}\frac{\partial \theta}{\partial x} + \frac{\mu}{\rho}\nabla^2 u \\ \frac{dv}{dt} &= Y - \frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{1}{3}\frac{\mu}{\rho}\frac{\partial \theta}{\partial y} + \frac{\mu}{\rho}\nabla^2 v \\ \frac{dw}{dt} &= Z - \frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{1}{3}\frac{\mu}{\rho}\frac{\partial \theta}{\partial z} + \frac{\mu}{\rho}\nabla^2 w \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (4)$$

Multiplying the first of these equations through by u, the second by v and the third by w, and adding the results, we obtain the relation [2]

$$\frac{d}{dt}\left(\frac{q^2}{2}\right) = -\frac{1}{\rho}\left(\frac{dp}{dt} - \frac{\partial p}{\partial t}\right) + \left(\frac{\mu}{\rho}\right)W - gw \quad \ldots\ldots\ldots \quad (9)$$

where

$$q^2 = u^2 + v^2 + w^2 \quad \ldots\ldots\ldots \quad (10)$$

and

$$W = \tfrac{1}{3}\left(u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial z}\right) + (u\nabla^2 u + v\nabla^2 v + w\,\nabla^2 w) \quad \ldots\ldots\ldots \quad (11)$$

The term $-\frac{1}{\rho}\left(\frac{dp}{dt} - \frac{\partial p}{\partial t}\right)$ represents the rate at which static pressure, acting inward across the boundary of a unit mass of the fluid, performs work of translation.[3]

The term $-gw$ represents the rate at which work is done by gravity on a unit mass of the

[1] In these equations, the operator d/dt is given by

$$\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z} \quad \ldots\ldots\ldots \quad (5)$$

the velocity of cubical dilatation, θ, is given by

$$\theta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \quad \ldots\ldots\ldots \quad (6)$$

or, by the Law of Conservation of Matter,

$$\theta = \left(-\frac{1}{\rho}\frac{d\rho}{dt}\right) \quad \ldots\ldots\ldots \quad (7)$$

Also

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad \ldots\ldots\ldots \quad (8)$$

ρ = the density of the medium,
μ = the *dynamical* coefficient of viscosity of the medium,
p = static pressure, *viz.*, that portion of the total fluid pressure (exerted by the medium per unit area upon any closed surface in the medium) which does not depend explicitly upon the stresses due to viscosity. This portion of the total pressure is normal to the surface and has the same magnitude in all directions. [See, for example, Appell, *Mécanique Rationnelle*, Paris, 1909, 3, page 554, equation (9)].

[2] For this equation, and in what follows, the positive direction of the z-axis is assumed to be vertically upwards.

[3] The total work, per unit time, of static pressure *on* a unit mass of the fluid is given by the expression

$$-p\frac{d\left(\frac{1}{\rho}\right)}{dt} - \frac{1}{\rho}\left(\frac{dp}{dt} - \frac{\partial p}{\partial t}\right)$$

of which the first term represents rate of work of *dilatation*, and the second term represents rate of work of *translation*.

medium, and the expression $\left(\frac{\mu}{\rho}\right)$ W is equal to that portion of the work of internal friction, performed per second *on* a unit mass of the medium, which (1) sets up changes of kinetic energy, (2) increases or diminishes the *translatory* work of static pressure and (3) contributes to or abates the work of the body forces, but which is not dissipated in the form of heat.

For the present purpose, it is important to observe that $\frac{d}{dt}\left(\frac{q^2}{2}\right)$ is the rate of change of the *total* kinetic energy per unit mass of the fluid.[1]

Making use again of the hypothesis of linear stress-components above referred to, the total work done on a unit mass of the medium by all of the internal stresses was calculated by Stokes. Applied to air flow in a horizontal tube, under conditions of radiation equilibrium to be described later, the Law of Conservation of Energy may, with the aid of Stokes' result, be stated as follows:[2]

$$\frac{d}{dt}\left(\frac{q^2}{2}\right) + \mathrm{J}\cdot c_v\cdot\frac{d\mathrm{T}}{dt} = -\left\{p\frac{d\left(\frac{1}{\rho}\right)}{dt} + \frac{1}{\rho}\left(\frac{dp}{dt} - \frac{\partial p}{\partial t}\right)\right\}$$
$$+\left\{\frac{\mu\mathrm{W}}{\rho} + \frac{\mu\mathrm{S}}{\rho}\right\} - gw + \mathrm{J}\cdot\vartheta\cdot\nabla^2\mathrm{T} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (12)$$

where S is Stokes' dissipative function multiplied by 2 and divided by the viscosity coefficient μ, *viz.*:

$$\mathrm{S} = \frac{4}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{dz}\right)^2$$
$$+\left[\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right]$$
$$-4\left[\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} + \frac{\partial v}{\partial y}\frac{\partial w}{\partial z} + \frac{\partial w}{\partial z}\frac{\partial u}{\partial x}\right] \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (13)$$

Combining relations (9) and (12), we have

$$\mathrm{J}\cdot\vartheta\cdot\nabla^2\mathrm{T} + \frac{\mu\mathrm{S}}{\rho} = p\,\frac{d\left(\frac{1}{\rho}\right)}{dt} + \mathrm{J}\cdot c_v\cdot\frac{d\mathrm{T}}{dt} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (14)$$

Thus the energy relation for a *non*-viscous fluid in motion has the same form as the energy relation for a non-viscous fluid at rest. For a *viscous* fluid, the term $\left(\frac{\mu}{\rho}\right)$ S is important and

[1] So much importance attaches to the significance of this term, that the writer may be pardoned for laying some emphasis upon the well-known theorem, to the effect that if u, v and w denote the velocity components of a fluid mass element, referred to fixed, rectangular coördinate axes, then the velocity q, defined by equation (10), is the resultant of
(*a*) a velocity of translation of the fluid element,
(*b*) a velocity of rigid-body rotation of the fluid element,
(*c*) a velocity due to a deformation of the fluid element.
The kinetic energy under consideration, $q^2/2$, thus includes not only kinetic energy of translation but also the kinetic energy of rotation and the kinetic energy of deformation of the fluid.
See, for example, Lamb, *Hydrodynamics*, Cambridge, 1895, pages 33, 34; Appell, *Mécanique Rationnelle*, Paris, 1909, **3**, pages 261–267; Abraham u. Föppl, *Theorie der Elektrizität*, Leipzig, 1904, **1**, pages 45 to 47.

[2] J = the mechanical equivalent of heat,
c_v = specific heat of the medium at constant volume,
ϑ = the thermal conductivity of the medium,
T = the temperature of the medium, measured on the absolute scale.

is equal to that portion of the work of internal friction, performed per second on a unit mass of the medium, which is dissipated in the form of heat, and which manifests itself in temperature increase and work of dilatation.

With the aid of the characteristic equation of the medium,[1]

$$p = \rho RT \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

the density ρ in equation (9) may be replaced by temperature and static pressure, giving the relation:

$$p \frac{d}{dt}\left(\frac{q^2}{2}\right) = -RT\left(\frac{dp}{dt} - \frac{\partial p}{\partial t}\right) + (\mu RT)\,W - gwp \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

If, now, we calculate the mean value of the left-hand member of this equation, according to the definition expressed by equation (1), and apply equations (3) for a *steady, mean* motion, then the analysis leads to three sets of terms: (*a*) a set involving only the mean motion of the medium, (*b*) a set involving both mean motion and turbulent motion, and (*c*) a set involving turbulent motion alone. In fact, if we define the operator $\bar{d}/dt$ by the relation

$$\frac{\bar{d}}{dt} = \frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x} + \bar{v}\frac{\partial}{\partial y} + \bar{w}\frac{\partial}{\partial z} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

then a precisely similar analysis, applicable to each one of the subsequent terms of equation (16) in succession, leads to the equation [2]

$$\frac{\bar{d}}{dt}\left(\frac{\bar{q}^2}{2}\right) = -\left(\frac{R\bar{T}}{\bar{p}}\right)\left(\frac{\bar{d}\bar{p}}{dt}\right) + (\mu R)\,(\bar{T}\,\bar{\Omega}) + \Psi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

where

$$\bar{\Omega} = \tfrac{1}{3}\left(\bar{u}\frac{\partial\bar{\theta}}{\partial x} + \bar{v}\frac{\partial\bar{\theta}}{\partial y} + \bar{w}\frac{\partial\bar{\theta}}{\partial z}\right) + (\bar{u}\nabla^2\bar{u} + \bar{v}\nabla^2\bar{v} + \bar{w}\nabla^2\bar{w}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18a)$$

The function Ψ is, then, the sum of a set of terms involving both mean and turbulent motion, and of another set of terms involving turbulent motion alone.[3]

[1] R is a constant for the medium under consideration. It should be observed that it does *not* follow from equation (15) that

$$\frac{1}{\bar{\rho}} = \frac{R\bar{T}}{\bar{p}}$$

On the contrary,

$$\frac{1}{\bar{\rho}} = \frac{R\bar{T}}{\bar{p} - R\,\overline{\rho' T'}}$$

[2] Since a steady *mean* state is assumed throughout, the partial derivative with respect to the time of any one of the variables of *mean* motion is zero, e.g.:

$$\frac{\partial}{\partial t}\left(\frac{\bar{q}^2}{2}\right) = 0; \quad \frac{\partial\bar{p}}{\partial t} = 0, \textit{ etc., etc.}$$

For horizontal *mean* flow, which is here under consideration, $\bar{w} = 0$; hence the absence from (18) of the body-force term.

[3] The corresponding function for motion of a *liquid* is a sum of terms involving turbulent motion alone. See Lorentz, *loc. cit.*

A precisely similar analysis of equation (14) gives the relation[1]

$$(-\Phi) + J\vartheta\nabla^2\overline{T} + (\mu R)\left(\frac{\overline{T\Sigma}}{\overline{p}}\right) = (Jc_p - R)\frac{d\overline{T}}{dt}$$
$$+ R\left(\frac{\overline{dT}}{dt} - \frac{\overline{T}}{\overline{p}}\frac{\overline{dp}}{dt}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (19)$$

where $(\mu\Sigma)/2$ is the Stokes dissipative function of the *mean* motion, and Φ is a function analogous to Ψ.

Equations (18) and (19) constitute the basis of the ensuing study.

[1] The two terms in parenthesis on the right-hand side are retained in their present form in order to distinguish the rate of dilatation work and the rate of change of internal heat energy corresponding to the mean motion.

PART II

Apparatus

5. *The jet channel.* In order to obtain conditions corresponding to approximate integrations of the foregoing equations, a small model of M. Eiffel's wind tunnel[1] was constructed of brass and connected with a suitable exhaust-fan (see Figures 2 and 3). Air was drawn by the exhauster through an entrance nozzle ABR (Figure 1) into a six-inch

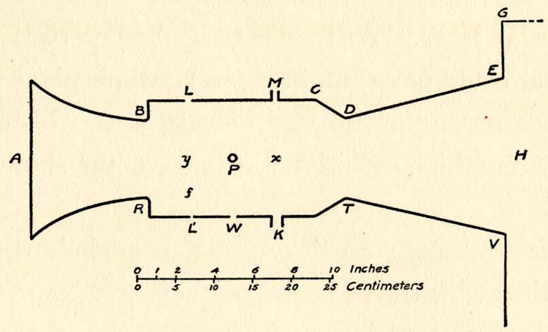

Figure 1. The Jet Channel, Longitudinal Section.

channel BDTR, whence it issued into a small expansion cone DEVT leading into a retention-tank H. The exhauster was connected through another small cone to the opposite side of the tank at a position some 28 inches lower than the opening EV. Entering the jet channel BDTR through the throat BR, the air passed through the channel in a 4-inch jet surrounded by a region of nearly stationary air. Between this region of slack air and the jet was a thin region of extremely unsteady mean motion (see sections 18 and 19 *infra*).

The entire exterior wall of the jet channel, from A to the cone-nozzle DT, was silver plated and burnished, in order to shield the interior from radiation by external objects. The interior wall was likewise silver plated and burnished, to reduce radiation between the wall and material under observation inside of the channel. Acting as a screen also, the inside of the retention-tank H was of bright metal; and an additional screen of silvered glass was mounted on a stand, with its silvered surface towards the channel. This screen was usually placed at a distance of 7.4 centimeters outward from the entrance nozzle and squarely across it. Material inside of the jet channel was thus effectively screened on all sides from exterior objects, while at the same time radiation exchanges between that material and the screening walls were reduced to a minimum.

It might be supposed that the close proximity of the screen to the entrance nozzle would obstruct the air flow. But such was not the case. The air made its way into the channel, not from the front, but around the edges of the entrance nozzle. A fine silk thread would

[1] Eiffel, *Resistance of the Air*, translated by Hunsaker, Boston, 1913.

lie quietly along the glass with the wind in the channel at 35 miles an hour, and a 75-mile wind would not cause a great disturbance at the reflecting surface.

The object of the retention-tank H was to equalize pulsations due to the blades of the rotor of the exhaust-fan when running at low velocities, and also to prevent eddies, formed at the exhauster intake, from propagating backward into the channel.

6. *The power plant.* Power was supplied by a 2 horse-power, 3 phase, 550-volt motor, driving a 10.5-inch exhaust fan.[1] A set of No. 6 Evans friction cones between motor and exhauster made possible a continuous change of wind velocity in the channel through a range from 10 to 75 miles an hour. With an extra pulley on the follower-cone, low velocities up to 10 miles an hour could be maintained. The whole plant (Figure 3) was mounted on a heavy, raft-like base resting on inflated bicycle tires, which reduced vibration to a degree at which an adjacent high-sensitivity galvanometer showed little or no effect of mechanical disturbance.

7. *Static and dynamic pressures.* *Static pressure,* defined above in a foot-note to section 4, is believed to be identical with the pressure, per unit area, exerted by the fluid on a perfectly smooth material surface coincident with trajectories of the fluid elements. *Mean static pressure,* defined according to relation (1), is understood to be identical with the mean pressure, per unit area, exerted by the fluid on a perfectly smooth material surface coincident with stream-lines defined by steady mean flow.

Dynamic pressure, on the other hand, is defined as wind pressure, per unit area, on a very small surface at rest with respect to the flow and normal to the trajectories of the fluid elements. *Mean dynamic pressure* is the mean pressure on such a surface normal to stream-lines defined by steady mean flow.

8. *The Pitot tube and measurement of mean wind velocity.* For the measurement of mean static pressure and the mean velocity of forced draft in the channel, a double Pitot tube was used as a standard. This tube was essentially a replica of the one in use at the wind tunnel of the Massachusetts Institute of Technology during the academic year 1919–20. The latter tube had previously been tested at the National Physical Laboratory, Teddington, England, and reported to indicate wind velocity in accordance with Bernoulli's Law to a precision within about 0.25 per cent.[2]

The only difference between the tube used by the writer and the tube belonging to the Institute was that the former was 4 millimeters shorter, and, at the shoulder, was constructed of a solid piece of brass in such a manner as to be absolutely air-tight and comparatively rugged.

The annular portion of the Pilot tube, into which led the small holes in the sides (the

[1] This excellent exhauster (American Blower Co., Type V, No. 2) was kindly lent to the writer for the present investigation by officials of the Boston office of the American Blower Company.

[2] Hunsaker, Reports on Wind Tunnel Experiments in Aerodynamics, *Smithsonian Misc. Coll.*, 1916, 62, No. 4, pages 8 and 30.

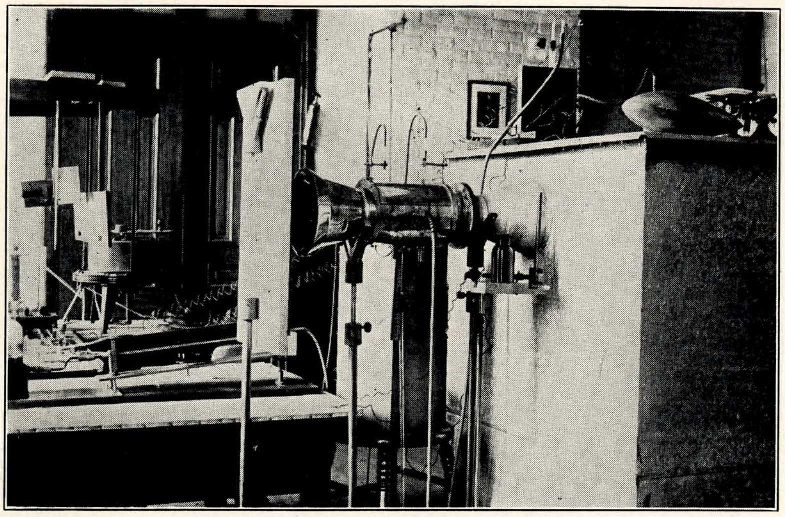

Figure 2. The Jet Channel and Accessories.

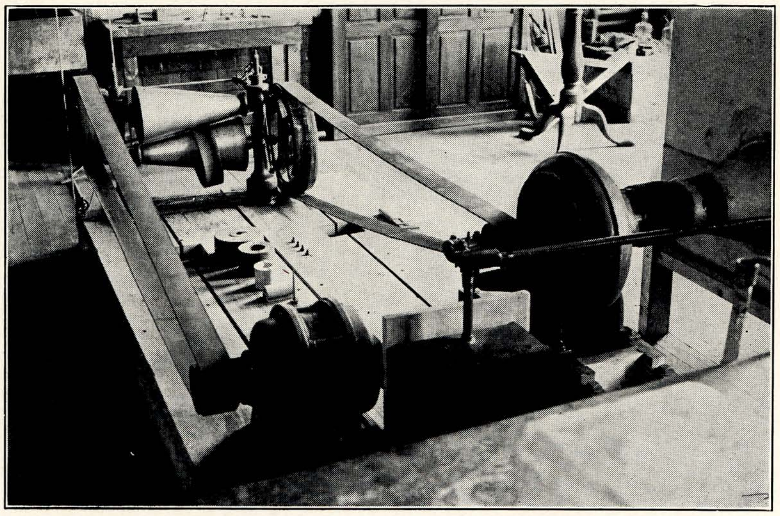

Figure 3. The Power Plant.

static openings), gave a mean static pressure of the air; while the inner portion of the tube yielded a mean dynamic pressure.

To set up the Pitot tube in the channel, it was inserted through the short guide-tube K (Figure 1), which was tightly closed by means of a split rubber stopper bored to hold the two vertical members of the Pitot tube. External clamps with the aid of this stopper held the Pitot tube rigidly in place.

9. *The static opening in the jet channel.* In order to measure static pressure without obstructing the jet channel, a quarter-inch hole, or *static opening* P (Figure 1), four inches from the throat BR, led into a short brass tube, to which was joined vacuum tubing connecting with an inclined alcohol manometer (see section 10). The inner edge of this static opening was carefully polished until all traces of a bur were removed.

10. *Manometers.* The alcohol manometer connected with the static opening of the jet channel was of the Krell type, and identical with the one described by Hunsaker [1] except for increased length and certain minor mechanical improvements.[2] The difference between atmospheric pressure and static pressure indicated by this manometer was proportional to the displacement of the alcohol meniscus in the tube. It was calibrated against a vertical U-tube manometer of 5/8-inch glass tubing containing distilled water. This vertical manometer could easily be read to the nearest half millimeter of water, corresponding to about 0.04 millimeter of mercury. By plotting a large number of observations, it was possible to draw a straight line giving pressure differences to within about 0.01 millimeter of mercury, in terms of readings of the Krell manometer.

For this manometer, the temperature correction was [3] $-R\,\frac{(B - S')\cdot\Delta t}{1 + B\cdot\Delta t}$ and the corrected scale reading R' was very approximately given by $R' = R\,[1 - 0.00113\,(t - 20)]$ where temperature is measured on the centigrade scale.

For the purpose of calibrating the indications of the static opening of the jet channel, the Pitot tube was connected with a second Krell manometer which had previously been calibrated against the first one.

11. *The barometer.* A newly filled, quarter-inch bore, standard barometer by Green was suspended on the wall of the aerodynamic laboratory, as far as possible from windows and steam-pipes. Barometric data contained in the ensuing tables were obtained with this instrument and corrected for temperature, altitude and latitude.

12. *Thermo-couples.* Temperature measurements in the jet channel were made with sets of copper-constantan thermo-couples, making possible a high degree of precision

[1] Hunsaker, *loc. cit.*, page 34.
[2] See Figure 4.
[3] R = reading of manometer,
B = the apparent thermal cubical coefficient of expansion of the alcohol mixture,
S' = the true thermal linear coefficient of expansion of the wooden scale,
Δt = the difference between standard temperature t_0 and the temperature t at which reading R was made.

Each set consisted of two thermo-couples in series (four thermojunctions to a set). To prevent appreciable heat conduction to or from the thermojunctions, wire of small diameter was used, No. 35 (B. & S.) for constantan and No. 40 for copper.[1] The leads were double silk-wound and the thermojunctions soldered with pitch flux. Short circuiting was avoided by staggering the two warm junctions about 5 mm., one above the other, and the two cold junctions were arranged in the same way. The cold junctions were inserted in a glass tube (of 4 mm. outside diameter) the closed end of which was filled with light oil supplied by the U. S. Bureau of Standards. Insulation resistance between the two staggered cold thermojunctions, with an electromotive force of about 2 volts on the circuit, was considerably over 11 megohms, or considerably more than 50,000 times the resistance of the thermo-couple circuit.

Leads from the thermojunctions terminated in a set of neutral clips (providing electrical contact between wide copper strips) arranged in such a manner that four thermo-couples (constituting two thermo-couple sets) could be connected with the potentiometer singly, in series, in parallel or in opposition. The purpose of this arrangement will be discussed in the next section.

13. *The potentiometer.* The potentiometer referred to in the preceding section was provided with an extremely accurate slide-wire, coiled around a marble core in 33 1-meter turns (Figure 4). Mounted over this slide-wire and carrying the sliding contact was a revolving disc 36 cm. in diameter, the limb of which was graduated into 1000 divisions, enabling the observer easily to read revolutions to one one-thousandth of a turn. This arrangement, with the aid of a Leeds and Northrup high-sensitivity d'Arsonval galvanometer, made it possible to render the sensitivity of the apparatus such that one division of the slide-wire scale (0.001 revolution) and a 1-millimeter galvanometer deflection respectively corresponded to 0°.001 C. But it was obviously necessary to render the sensitivity of the thermometric apparatus of the same order as that of the inclined alcohol manometers. Potentiometer resistances were accordingly so adjusted as to give approximately 0°.004 C. per slide-wire division, with a corresponding change in the galvanometer deflection.

14. *Elimination of parasitic electromotive forces.* Two of the thermo-couples described above, when connected in series, with a temperature difference of 20°C. between warm and cold junctions, gave an electromotive force of about 1600 microvolts. In the galvanometer circuit of the potentiometer, parasitics (due to temperature differences along the line at soldered connections, switch contacts, etc.) amounted, under unfavorable conditions, to as much as 3 microvolts, or about 0.2 per cent of the total electromotive force.

[1] Measured diameters of the bare wires were as follows: constantan, 0.13 mm.; copper, 0.08 mm. Tests showed that even under extreme conditions the effect of heat leakage along the leads was very considerably less than 1 in 5000.

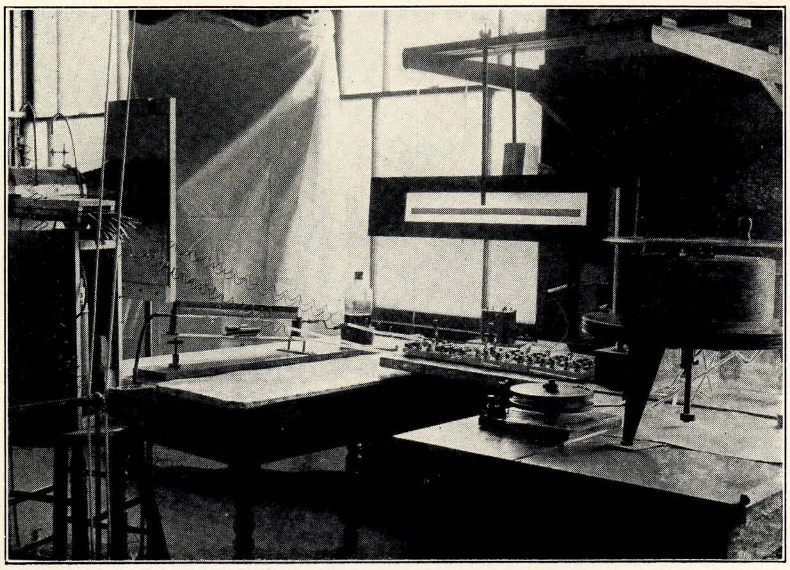

FIGURE 4. The Krell Gage. Parts of Jet Channel and Potentiometer.

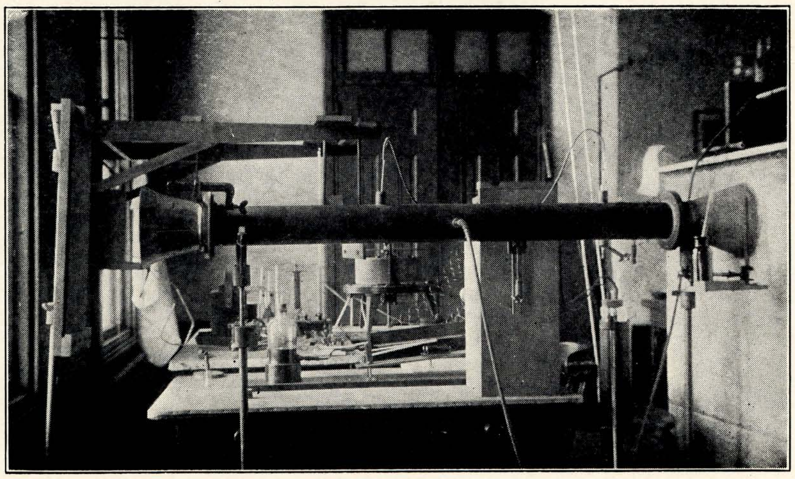

FIGURE 5. The Four-Inch Channel.

To eliminate effects of these parasitics was therefore important. This was accomplished (1) by the use of a White eliminating switch,[1] and (2) by the method of reversals.[2] Each method was entirely effective. For the present purpose, the former was the more convenient.

But the White switch and the method of reversals eliminated only those parasitics arising along the galvanometer circuit as far as the switch connecting the thermo-couple terminals with the potentiometer. Parasitics arising in this switch and in the clips at the thermo-couple terminals were measured by making an observation with a thermo-couple set connected singly to the potentiometer, and then making a second observation after the thermo-couple leads had been reversed at the clips[3] and the current reversed in the battery circuit of the potentiometer. From these observations, after allowing for potentiometer parasitics, the required parasitics were easily calculated. Owing doubtless to precautions taken,[4] the parasitics arising in this part of the circuit were invariably found to be negligible.

15. *Calibration of the thermo-couple sets.* To calibrate this apparatus for temperature, Adams' curve for a copper-constantan thermoelement[5] was taken as a standard. To this curve was applied a deviation curve for each thermo-couple set, obtained from observations at the ice point, the steam point and the transition temperature of sodium sulphate

For the ice point, the glass tubes containing the thermojunctions were immersed in clean, finely shaved, clear ice, which was packed tightly into distilled water contained in a thermos flask. An ice mixture as nearly identical with this as possible was invariably used for the cold junctions in all subsequent tests and experiments recorded in these papers.

A good steam point was difficult to obtain, and determinations with the boiler employed differed by some hundredths of a degree. They were sufficiently consistent however to facilitate construction of the deviation curves, which were important here only in the range between 0° and 33°C.

The transition temperature of sodium sulphate (32°.383 C.), extremely convenient for the present investigation, was obtained by following explicitly the procedure detailed in Richards' and Wells' paper.[6]

Repeated tests, carried out during a period of several weeks, yielded determinations consistent within 0°.004 C. This was sufficient for purposes of the contemplated aerodynamic experiments.

[1] White, *Journ. Am. Chem. Soc.*, 1914, **36**, No. 9, page 1856, Thermoelectric Installations, Potentiometers for Thermoelectric Measurements, Leakage Prevention.

[2] White, *Phys. Rev.*, 1907, **25**, page 344, Potentiometer Installation.

[3] See section 12.

[4] Use of neutral clips, neutral switch, etc., see White, *loc. cit.*

[5] Adams, *Bull. Am. Inst. Mining and Met. Engin.*, September, 1919, No. 153, page 2117.

[6] Richards and Wells, *Proc. Am. Acad.*, 1902, **38**, page 431. Also Richards, *Am. Journ. Sci.*, 1898 (4), **6**, page 201. Convenient dimensions for parts of the apparatus were obtained from Dickinson and Mueller, *Bureau of Standards Bull.*, 1907, **3**, page 656.

PART III

Aerodynamics of the Jet Channel

16. *Effects of radiation exchanges between channel and air jet. Conduction of heat from channel walls.* Dry air is very nearly diathermous. Such exchanges of radiation energy as occurred between air jet and channel walls during the tests and experiments recorded below must have been due to the relatively small quantity of water vapor in the air. Absorption of radiation energy per unit mass of the air jet, in excess of emission, was therefore necessarily small, and the very low emissivity of the burnished silver surfaces of the channel doubtless rendered it a wholly negligible quantity, compared with forms of energy transfer entering into the statement of the Law of Conservation of Energy in section 4. It was unnecessary, therefore, to include radiation exchanges in the statement of this law for the special case under consideration.

Warming of the air during contact with the channel walls may, on the contrary, have been sometimes appreciable. Air thus heated was carried inward towards the channel axis by turbulent mixing; for although, as will be shown later, very little mean flow was observable in the outer layer of slack air surrounding the jet, there was nevertheless considerable erratic eddy motion in that region, the mixing action of which constituted an effective form of heat transfer. Warmed air, arriving in this manner at the region of intense turbulence which, like a hollow, cylindrical sheath, surrounded the air jet, was caught up in the jet proper and hastened out of the channel before it had time to penetrate far into the 4-inch cylindrical jet around the channel axis.

The experimental portion of the jet was thus practically free from thermal influences due to the channel walls.

17. *Effect of radiation on the jet channel thermojunctions.* A thermometer suspended in a wind channel does not, in general, register the temperature of the air. If the bulb of the instrument is in thermodynamic equilibrium, its temperature depends in part upon radiation exchanges with neighboring material other than the air. Thus air temperature, as indicated by the instrument, would depend upon more or less accidental material configurations and upon their exposure, and that of the instrument itself, to direct and reflected sunlight, steam-pipes and other relatively warm or cold objects. Effective shielding of the instrument was therefore essential. The thermojunctions in the jet channel were accordingly shielded, from every direction, by the use of bright metal surfaces, as described in section 5.

How fundamental this precaution proved to be was shown by the fact that it was not until the silvered screen was set up in front of the entrance nozzle [1] that consistent results could be obtained.[2]

18. *Static pressure in the experimental cross-section of the jet channel.* Data contained in the tables given below were obtained in the jet channel cross-section containing the static opening P (Figure 1).[3] For brevity, this cross-section will be referred to as the *experimental cross-section* of the channel. Mean static pressure in the plane of this experimental cross-section was measured by setting up the Pitot tube so that its static openings were contained in that plane. The annular member of the Pitot tube was then connected with a Krell manometer, which then indicated the difference K_A between barometric pressure in the sensibly still air of the large room and the mean static pressure along the Pitot tube.

Another Krell manometer was connected to the static opening (P, Figure 1) of the jet channel. This manometer measured the difference K_S between barometric pressure in the room and the mean static pressure on the channel wall at the opening P.

In the geometrical axis of the channel, it was found that, for velocities up to 33.5 meters per second (75 miles per hour),

$$K_A = 0.951\ K_S$$

Thus the mean pressure *difference* was *greater* at the static opening in the channel wall than it was in the channel axis; and consequently, mean *static pressure* was *less* at the static opening in the wall.

From the foregoing equation, mean static pressure in the channel axis could be ascertained at once, by means of the static opening P, without the aid of the Pitot tube.

Throughout a circle of 7 cm. diameter, about the channel axis and in the experimental cross-section, mean static pressure was found to be sensibly constant.

19. *Dynamic pressure in the experimental cross-section of the jet channel and along the channel axis.* Mean dynamic pressure was measured by means of a small-diameter impact tube, and the results were checked with the dynamic part of the Pitot tube. Constructed of 3 mm. (O.D.) copper tubing and bent in a right angle 3.5 cm. from its impact opening, the small-diameter impact tube was provided with a brass nipple at its lower end, to which vacuum tubing leading to the Krell manometer could be hermetically attached. When adjusted directly against the wind, this tube yielded mean dynamic pressure as defined in section 7.

Tests made in the experimental cross-section of the channel (section 18) with this tube, and verified with the impact part of the Pitot tube, showed that throughout a circular region of about 7 cm. in diameter, mean dynamic pressure was equal to the barometric pres-

[1] See section 5 and Figures 2 and 4.

[2] The warm junctions of two thermo-couple sets were, for example, placed at x and y (Figure 1) respectively. With the screen in place before the entrance nozzle, repeated tests, consisting of temperature readings in a mean flow of 15 m/s made at half-minute intervals and averaged over a period of 10 minutes, gave a practically perfect agreement at the two points. But, with the screen removed, discrepancies between the averages as great as 0°.22 C. were observed.

[3] See section 9.

sure of the practically still air in the room. This region was surrounded by a region a little more than 1 cm. thick, characterized by extreme unsteadiness and irregularity of mean motion as rendered by the manometers. This latter region doubtless had its origin in a surface of discontinuity of the Helmholtz-Kirchhoff type. It is well known that these surfaces are unstable and tend to roll up and break into vortices and swirls.

With the impact tube in this layer, the alcohol column of the Krell manometer oscillated from one end of the scale to the other in an erratic manner, making measurements impossible.

Outside of this sheath of air characterized by unsteadiness of mean motion, and behind the shoulder BR (Figure 1) was a region of nearly slack air. Thus, for example, with the wind at 15 meters per second in the channel axis, the difference between mean dynamic pressure at a point 15 millimeters from the channel wall and mean static pressure on the wall itself was only 0.18 millimeter of mercury, while, at the wall itself, mean dynamic pressure fell to exact equality with mean static pressure.

To investigate variation of mean dynamic pressure along the channel axis, tests were run with the entire jet channel (Figure 1) clamped to the windward end of a 4-foot (1.219 meter) tube shown in Figure 5.[1] Measurements of mean dynamic pressure at various points in the geometric axis of the channel and in the geometric axis of the 4-foot tube showed that, *along this line*, mean dynamic pressure was precisely equal to the barometric pressure in the still air of the room at the open end of the Krell manometer. This result, obtained with the small impact tube, was checked with the dynamic member of the Pitot tube.

Dynamic pressure throughout the central 7-centimeter circle in the experimental section of the channel above referred to was therefore equal to atmospheric pressure in the room.

20. *Measurement of mean velocity of the air jet.* Since mean static pressure was the same all over the 7-centimeter circle in the experimental section of the channel described in the preceding two sections, and since mean dynamic pressure all over this 7-centimeter circle was equal to atmospheric pressure in the room, the calibration of the static opening in the channel against the static member of the Pitot tube (section 18) supplied data for ascertaining the mean velocity of this portion of the air jet without subsequent use of the Pitot tube (section 8).[2]

21. *Critical velocity of the air jet.* For a 4-inch tube of circular cross-section, the Stanton and Pannell critical velocity[3] is 0.37 meters per second. The presence of the shoulder BR (Figure 1) at the rear of the entrance nozzle apparently caused stream-line motion to break down at a lower velocity in the jet channel. Thus all of the measurements recorded in the following pages were made in a turbulent stream of air.

[1] This 4-foot tube was of brass and 4 inches (10.16 cm.) in diameter. In the figure, the entrance nozzle only is clamped to the tube. For the tests here described, the entire jet channel occupied this position.

[2] *Indicated* velocity as obtained from the static opening of the channel was, uniformly for all velocities here used, 2.6 per cent larger than *true* velocity as given by the Pitot tube.

[3] *Lond. Roy. Soc. Phil. Trans.*, 1914, 214, page 199.

22. *Dynamical cooling of the air jet.* A large obstacle like a thermometer bulb, inserted in the air jet, set up an eddy, the temperature of which differed from that of the surrounding air, and thus affected the indications of the instrument. This effect was inappreciable in the case of the minute thermojunctions described in section 12. A pair of these junctions, maintained normal to the jet by means of raw silk fibers, gave sensibly the same indications as another pair held parallel to the mean stream-lines with their ends pointing up the wind.

A thermo-couple set, arranged with warm junctions in the center of the experimental cross-section of the channel and normal to the jet, was accordingly used for making determinations of the mean temperature of the jet in the channel. Simultaneous determinations of mean static pressure and mean velocity were made with the static opening of the channel and a Krell manometer.

For the purpose of investigating the effect of flow on mean air temperature, it was however desirable to obtain air temperature not only in the experimental cross-section, but also at a point outside of the channel in the return current, where the mean flow velocity was negligibly small compared with velocity in the channel. This was, however, rendered impossible by the effect on the thermojunctions of radiation from external material. Reproducible results were obtained only when the thermojunctions were uniformly shielded from outside disturbances (see section 17). The following procedure was accordingly adopted. The wind was started, the potentiometer was then balanced and corrected for parasitics and the wind temperature read and recorded; the potentiometer was then balanced and corrected again, the wind stopped and the still air temperature read and recorded. This entire process was repeated until six readings in the forced draft and five readings in still air had been obtained. The average of the six forced draft readings was then taken as the mean temperature of the forced draft, and the average of the five still air readings was taken as the mean temperature of the still air. Since this still air, when its temperature was observed, had just previously been brought into the channel from the outside at an extremely low velocity by the exhauster while coming to rest immediately after the next preceding run, its temperature was assumed to remain sensibly unchanged during transit, and hence to be equal to the air temperature at the desired external point.

It was essential to eliminate effects due to gradual warming of the air in the room by the action of the exhauster. This seems to have been satisfactorily accomplished by making the foregoing readings at exactly equal intervals of time (in this case, 2 minutes), and by making both the first and the last reading consistently either in the forced draft or in still air.

Directly the wind was stopped, a rapid rise of temperature ensued, followed by a much smaller and much more gradual decrease. When, for example, the power-switch was opened with wind at 15 meters per second, the rotor in the exhauster continued to revolve for about 4 seconds. The above-mentioned rapid and comparatively large temperature increase, occurring immediately upon cutting off the power, was due to replacement in the channel of dynamically cooled air by air drawn into the channel with very little mean velocity and consequently slight dynamical cooling. The subsequent smaller and much more gradual temperature decrease was caused by the cooling of the entire air content of the room which invariably took place after cessation of churning by the blades of the exhauster.

Results of the foregoing measurements are shown in Table I and Figure 6, where forced draft velocity refers to the 7-centimeter working cross-section of the jet (sections 18 and 19).

TABLE I

DYNAMICAL COOLING, AND VARIATIONS OF PRESSURE WITH WIND VELOCITY IN THE AIR JET

No.	Forced draft m/s	$\bar{t}-\bar{t}_0$ (centigrade) °	$\bar{p}_0-\bar{p}$ mm.	No.	Forced draft m/s	$\bar{t}-\bar{t}_0$ (centigrade) °	$\bar{p}_0-\bar{p}$ mm.
1	0.0	0.00	0.00	4	20.0	−0.05	1.83
2	10.0	−0.01	0.46	5	26.3	−0.12	3.15
3	15.0	−0.03	1.03	6	31.4	−0.18	4.48

The quantities $\bar{t}_0$ and $\bar{p}_0$ denote mean air temperature and static pressure at a point exterior to the channel, in the return current, where mean wind velocity was sensibly zero; $\bar{t}$ and $\bar{p}$ denote the corresponding quantities in the experimental cross-section of the channel.

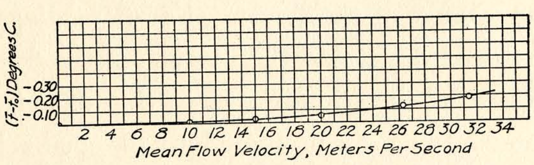

FIGURE 6. Dynamical Cooling in the Air Jet.

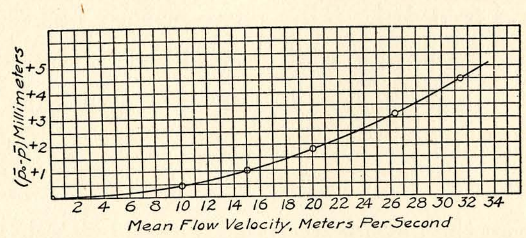

FIGURE 7. Pressure Variation in the Air Jet.

23. *Pressure variations in the air jet.* The difference between mean static pressure in the return current and mean static pressure in the experimental cross-section of the 7-centimeter jet (sections 18 and 19) was at all times correctly obtainable by means of a Krell manometer and the static opening of the channel, with the aid of the calibration described in section 18. This difference $(\bar{p}_0 - \bar{p})$ is shown in Table I and Figure 7.

24. *Velocity variations in the air jet.* The exhauster discharged directly into the room which was, however, sufficiently large to reduce the mean velocity of the return current enough to have no visible effect upon the sensitive Krell gages in their usual position alongside the channel and connected with the Pitot tube. The change of mean flow velocity,

between a point in this quiet region of the return current and a point in the experimental section of the channel, was therefore very approximately equal to the mean flow velocity at the latter.

25. *Energy of turbulent motion.* In equation (18), the function Ψ represents energy due to the turbulent motion of the air (see section 4). This energy arises from the following causes: (*a*) turbulent changes of the kinetic energy of the fluid, (*b*) translation work performed by the air and due to turbulent changes of static pressure, (*c*) work performed by the body forces through turbulent vertical displacements of the fluid elements, and (*d*) the nondissipative work of internal friction which is brought about by turbulent changes of the deformation velocities of the medium.

The function Ψ, therefore, may be regarded as defining the *mechanical energy of turbulent motion* per unit mass of the air.

Analogous considerations indicate that the function $(-\Phi)$ of equation (19) may properly be regarded as defining the *heat energy of turbulent motion* per unit mass of air.[1]

26. *Mechanical energy of turbulent motion in the central region of the air jet.* We have seen that throughout a central region of the air jet, about 7 cm. in diameter, dynamic pressure $\bar{p}_i$ was constant relative to distance from the entrance nozzle, and equal to atmospheric pressure $\bar{p}_o$ at a point outside the channel, in the return current, where the air was practically at rest. But the Pitot tube indicated mean wind velocity, to an accuracy of 0.25 per cent, in accordance with Bernoulli's Law (section 8), which, with the aid of equation (15), may be written, for the mean motion, in the form

$$\frac{\bar{v}^2}{2} = \frac{R\bar{T}}{\bar{p}}(\bar{p}_i - \bar{p}) \quad \ldots \ldots \ldots \ldots \quad (20)$$

where $\bar{p}$ is static pressure at the lateral openings of the Pitot tube.

But since, in the central region, it was shown (section 19) that

$$\bar{p}_i = \bar{p}_o$$

then

$$\frac{\bar{v}^2}{2} = \frac{R\bar{T}}{\bar{p}}(\bar{p}_o - \bar{p}) \quad \ldots \ldots \ldots \ldots \quad (21)$$

thus, in this central region, the mean motion of the jet obeyed Bernoulli's Law.[2]

Let us compare this experimental result with the law expressed in equation (18). Integrating along a stream-line of the mean flow,[3] from a point A in the return current out-

[1] It should be emphasized that turbulent motion under consideration in this paper is explicitly defined by equation (2) as *residual* or *relative to the mean motion* defined by equation (1). The averaging interval τ, in the case of the mean motion data of Table I, was 22 minutes.

[2] This was *not* the case in the region of unsteady mean motion surrounding the 7-centimeter cylindrical portion of the jet under consideration.

[3] It is here assumed that, at the point where the fluid element under consideration attains a critical velocity, stream-lines, velocities, pressures, etc. of the nonturbulent motion pass *continuously* into the respective stream-lines, velocities, pressures, etc., of the mean motion.

side the channel, where the air is sensibly quiet, to a point B in the central region of the jet abreast of the static opening of the channel, we have

$$\frac{\bar{q}^2}{2} = -\int \frac{R\bar{T}}{\bar{p}} d\bar{p} + \mathcal{E}_1 \qquad (22)$$

where

$$\mathcal{E}_1 = \int \{(\mu R)(\overline{T\Omega}) + \Psi\} dt \qquad (23)$$

Under extreme conditions corresponding to the highest velocity investigated (31.4 meters per second), the quotient $(R\bar{T}/\bar{p})$ varied between the limits of integration by only about seven-tenths of one per cent of its initial value. With a velocity of 15 meters per second, the variation was only about two-tenths of one per cent. Moreover the appreciable variation of this quotient was confined to a relatively short arc beginning at the initial point of the path of integration. Its average value over the entire path of integration was therefore very nearly equal to its value at the upper limit of integration. If, then, $\bar{T}$ and $\bar{p}$ denote the respective values of mean air temperature and pressure at the upper limit of integration, we have, to a precision about equal to that quoted for the Pitot tube (section 8)

$$-\int \frac{R\bar{T}}{\bar{p}} d\bar{p} = \frac{R\bar{T}}{\bar{p}} (\bar{p}_o - \bar{p}) \qquad (24)$$

Consequently, throughout the range of velocities investigated,

$$\mathcal{E}_1 = o \qquad (25)$$

to a degree of approximation very nearly equal to the assigned accuracy of the Pitot tube.[1]

Thus, in the inner portion of the jet (and there only) the gain of *mechanical* energy of turbulent motion, per unit air mass, was very nearly equal to that part of the non-dissipative work of the viscous stresses which was due to the mean motion; or else these two quantities were both so minute as to lie within the range of experimental error.

27. *Combined effects of heat energy of turbulent motion, heat conduction and viscosity, in the central region of the air jet.* Turning now to equation (19), and integrating approximately, as in the last section, we have

$$\mathcal{E} = (J \cdot c_p - R)(\bar{T} - \bar{T}_o) + \left[R(\bar{T} - \bar{T}_o) - \frac{R\bar{T}}{\bar{p}}(\bar{p} - \bar{p}_o)\right] \qquad (26)$$

where

$$\mathcal{E} = \int (-\Phi) dt + \int \left\{J \cdot \vartheta \cdot \nabla^2 \bar{T} + (\mu R)\left(\frac{\overline{T\Sigma}}{\bar{p}}\right)\right\} dt \qquad (27)$$

[1] It should be recalled that data and results here given apply only to that portion of the air jet characterized by sensibly constant dynamic pressure, not only throughout its cross-section, but also along stream-lines of mean flow contained in that cross-section, from points outside of the entrance nozzle to the experimental cross-section under observation. These data and results do not apply to other portions of the jet.

The function $\mathcal{E}$ is equal to the algebraic sum of changes in the following quantities, per unit mass of air during transit from A to B:

(1) Heat energy of turbulent motion (see section 25).

(2) Heat conducted into the unit air mass by virtue of space variations of *mean* air temperature.

(3) Energy dissipated by the internal stresses in raising *mean* air temperature and performing dilatation work against *mean* static pressure.

This function, therefore, represents the combined effects of energy transformations due to turbulence, heat conduction and viscosity upon the air column under consideration.

28. *Variation of the energy integral $\mathcal{E}$ with mean velocity of flow.* To calculate the energy integral $\mathcal{E}$ from the data of Table I, we have

$$\left.\begin{aligned} J &= 4.183 \ \frac{joules^{[1]}}{calorie} \\ c_p &= 0.2417^{[2]} \\ R &= 0.287 \ \frac{joules^{[3]}}{gram\ degree} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (28)$$

The average barometric pressure in the laboratory while data for Table I were being obtained was very nearly 759 mm. and the average temperature was 17°.4 C. The ratio $\overline{T}/\overline{p}$ was accordingly very nearly equal to 290.5/759, which corresponded nearly to standard atmospheric conditions. Equation (26) then became

$$\mathcal{E} = 0.725\,(\bar{t} - \bar{t}_0) + [0.287\,(\bar{t} - \bar{t}_0) + 0.1098\,(\bar{p}_0 - \bar{p})] \quad \ldots\ldots \quad (29)$$

where the symbol $\bar{t}$ now indicates not *time* but *air temperature* measured on the Centigrade scale. Since the last term of equation (26) was homogenous in $\bar{p}$, it was convenient to express static pressure there and in equation (29) in terms of millimeters of mercury.

The first term on the right-hand side of the latter equation (referred to as H in Table II) denotes, if negative, the heat-loss, in joules per gram of air, corresponding to a decrease of mean temperature. The term in square brackets (referred to as W in Table II) denotes, if positive, dilatation work, in joules per gram of air, performed by the air against mean static pressure. $\mathcal{E}$, if positive, is equal to the gain in heat energy of turbulent motion, in joules per gram of air, plus conductivity and viscosity effects noted in section 27. The three quantities H, W and $\mathcal{E}$ thus denote changes in these respective forms of energy, that took place in any given mass element of air while passing along a line of mean flow into the 7-centimeter air jet between the above-described limits of integration.

[1] Marks and Davis, *Steam Tables and Diagrams*, New York, 1919.
[2] Swann, *Lond. Roy. Soc. Phil. Trans.*, 1910, A, 210, page 230.
[3] Calculated from standard air density, as given by *Smithson. Met. Tables*, Washington, 1918, Table 98. The reader will observe that the small Arabic numerals in equations (28) refer to foot-notes, and are not to be interpreted as exponents.

TABLE II

ENERGY TRANSFORMATIONS IN THE AIR JET. VARIATION OF THE ENERGY INTEGRAL $\mathcal{E}$ WITH FLOW VELOCITY

No.	Forced draft q m/s	H joules per gram	W joules per gram	$\mathcal{E}$ joules per gram Observed	Calculated from (30)
1	0.0	0.0000	0.0000	0.0000	0.0000
2	10.0	−0.0072	0.0476	0.0404	0.0410
3	15.0	−0.0218	0.1045	0.0827	0.0839
4	20.0	−0.0362	0.1865	0.1503	0.1400
5	26.3	−0.0870	0.3115	0.2245	0.2275
6	31.4	−0.1305	0.4402	0.3097	0.3097

Consideration of dimensional relations in equation (26) suggests a functional dependence of the energy integral $\mathcal{E}$ upon the square of the mean flow velocity, $\bar{q}^2$. The energy integral $\mathcal{E}$, calculated from the data of Table I with the aid of equation (29), is shown in Table II; and the functional dependence referred to is shown in Figure 8.

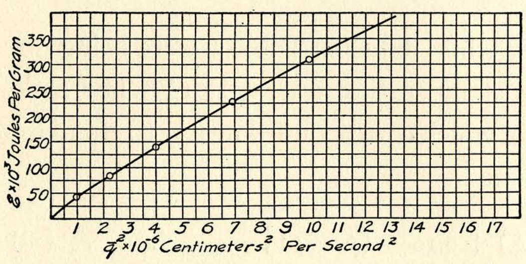

FIGURE 8. The Energy Integral $\mathcal{E}$ as a Function of the Square of the Mean Velocity.

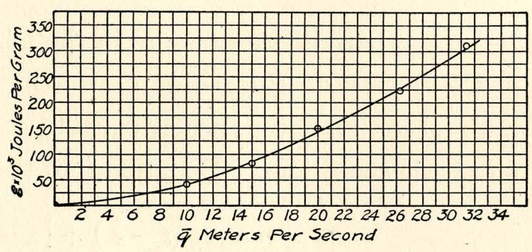

FIGURE 9. The Energy Integral $\mathcal{E}$ as a Function of the Mean Velocity.

When $\mathcal{E}$ is plotted in logarithmic coördinates as a function of the first power of the velocity, $\bar{q}$, it is observed to obey the empirical law

$$\mathcal{E} = 6.89 \times 10^{-4} \times \bar{q}^{1.773} \qquad (30)$$

Figure 9 shows relation (30) plotted in Cartesian coördinates, together with the experimental points recorded in Table I.

29. *Thermodynamical significance of the experimental data and results.* The foregoing data and results, then, indicate that, in the central region of the jet, dilatation work by the air against mean static pressure was performed (1) at the expense of heat loss corresponding to a fall of mean temperature and (2) at the expense of the combined effects of heat energy of turbulent motion, of thermal conductivity and of viscosity represented by the function $\mathcal{E}$. Thus all of the heat energy represented by this function may be regarded as having been transformed into dilatation work against mean static pressure. When the inner portion

of the jet was moving with a mean flow velocity between the critical velocity and 15 meters per second, about 80 per cent of the dilatation work against mean static pressure was performed at the expense of the combined thermal effects $\mathcal{E}$ under consideration. With increase of flow velocity up to 31.4 meters per second, the contribution of the $\mathcal{E}$ effects to dilatation work against mean static pressure fell to about 73 per cent, the remaining 27 per cent having been performed at the expense of the internal heat of the air.

PART IV

Summary

Steady mean motion of a turbulent gas. Fundamental aerodynamic relations. An extension of the Reynolds-Lorentz analysis of mean and turbulent fluid motion to the case of a gas, leads to two fundamental energy relations analytically defining (1) *mechanical energy of turbulent motion* (non-dissipative) and (2) *heat energy of turbulent motion* (dissipative). Mechanical energy of turbulent motion is shown to be due (*a*) to turbulent changes of the kinetic energy of the fluid, (*b*) to work of translation performed by the fluid and due to turbulent changes of static pressure, (*c*) to work performed by the body forces through turbulent vertical displacements of the fluid elements, and (*d*) to the non-dissipative work of internal stresses which are brought about by turbulent changes of deformation velocities. Heat energy of turbulent motion is analyzed in an analogous manner.

The object of the investigation has been to measure, in a horizontal air current, (1) the magnitude of the mechanical energy of turbulent motion plus that part of the non-dissipative work of the viscous stresses which is due to the mean fluid motion; and (2) the magnitude of the heat energy of turbulent motion, plus heat conduction due to space variations of the mean air temperature, plus the dissipative part of the work of the viscous stresses due to the mean fluid motion.

The wind channel and thermoelectric methods of air temperature measurement. These measurements were made in a small brass replica of M. Eiffel's wind tunnel. Air was drawn through this small channel by an exhauster at velocities up to 31.4 meters per second. The jet thus produced was characterized by uniform *mean* static and dynamic pressures over a central cross-sectional area of about 7 cm. in diameter. Results obtained refer to this central region and to this region only. Air temperatures were measured by means of sets of minute copper-constantan thermo-couples; and accuracy was attained by effective screening of air current and thermojunctions from radiation effects, by suitable methods of averaging observations, and by means of other fundamental and essential precautions detailed in the text.

Measurements of energy transformations in a current of air. Air entering the channel at room temperature undergoes a distinct cooling by the time it reaches the experimental cross-section of the channel. This mean cooling effect bears a definite relation to mean wind velocity.

Measured values of variations of mean pressure and temperature establish the fact that a mathematical expression occurring in the fundamental energy relations is nearly constant under the given conditions, thus permitting simple approximate integrations.

These integrations, in conjunction with observed data, show that the cooling effect above referred to is due to the performance of dilatation work by the air against mean static pressure at the expense (1) of heat loss corresponding to a fall of mean air temperature and (2) of forms of energy denoted in the text by the function $\mathcal{E}$.

This function $\mathcal{E}$ is the algebraic sum of the following quantities: (1) gain of heat energy of turbulent motion, (2) heat conducted into the moving air mass by virtue of space variations of mean air temperature, (3) energy dissipated by the internal stresses in raising mean air temperature and in performing dilatation work against mean static pressure.

Of appreciable relative magnitude, this function, under standard atmospheric conditions, obeys the empirical law:

$$\mathcal{E} = 6.89 \times 10^{-4} \times \bar{q}^{1.773}$$

where $\mathcal{E}$ is in joules per gram of air, and $\bar{q}$ is mean flow velocity, in meters per second.

The algebraic sum of *mechanical energy of turbulent motion* and that part of the non-dissipative work of the viscous stresses which is due to the mean motion is so small as to lie beneath the range of accuracy assigned to the Pitot tube. Hence, very approximately, for the range of flow velocities investigated, the air current, in the restricted portion of the channel under consideration, obeys Bernoulli's Law.

AERODYNAMICS OF THE PSYCHROMETER

By George Porter Paine

Second Paper. Experiments Relating to Energy Transformations in the Region of Eddies Set Up by an Obstacle in a Jet of Air

INDEX TO SECTIONS

PART I

Introduction

1. *Heat energy due to the turbulent motion of a fluid.* In the first paper of the present series,[1] the *mean motion* of a gas was defined analytically in a manner which was believed to conform closely to the motion of a gas as rendered by aerodynamic instruments. The turbulent flow of a fluid is characterized by an undulating, eddying, apparently unordered motion superimposed on the mean flow, and this superimposed motion, the *turbulent motion* proper, was also defined analytically in that paper. By means of an extension of the Reynolds-Lorentz analysis of mean and turbulent motions to the motion of a compressible fluid, it was possible to define mathematically the energy of turbulent motion of the fluid; and it was then shown that, in general, part of the energy of turbulent motion is dissipated (*a*) in raising the mean temperature of the fluid, and (*b*) in causing the fluid to perform dilatation work against mean static pressure. For brevity, this part of the energy of turbulent motion was referred to as the *heat energy of turbulent motion.* The remainder of the energy of turbulent motion, which could not be dissipated, was referred to as the *mechanical energy of turbulent motion.*

2. *Object of the present paper.* The object of the present paper is to present the results of an investigation of the combined effects of heat energy of turbulent motion, thermal conductivity and viscosity, in the wake of an obstacle situated in a turbulent air jet, throughout a range of mean flow velocities from 3 to 32 meters per second.

[1] Theory and Experiments Relating to Energy Transformations in a Jet of Air. References to this paper will be indicated by the Roman numeral I.

PART II

Apparatus

3. *Equalization of the averaging intervals of different parts of the apparatus. Thermo-couple bulbs.* The data contained in the following pages were obtained in the jet channel described in the earlier paper above referred to. Measurements of temperature were made with the aid of sensitive thermoelectric apparatus, and velocities were measured by Krell manometers in conjunction with a Pitot tube and the static opening in the channel. Pressures were measured by Krell manometers with the aid of additional devices, an account of which will presently be given.[1]

Throughout the measurements discussed in the previous paper, the averaging interval[2] τ was maintained uniformly the same by the fact that each datum was obtained by taking averages of observations made at regular intervals over a period of 22 minutes. Mean motion defined by this value of τ was practically steady.

At the start, the lag of the thermometric apparatus was very much less than that of the Krell manometers. For the measurements contained in the present paper, it was therefore deemed advisable to increase temperature lag without decreasing sensitivity, thus rendering the averages, which were mechanically integrated by the thermoelectric apparatus, of the same order as the averages mechanically integrated by the manometers.[3] The warm junctions were accordingly placed in thin glass tubing closed at one end and having an outside diameter of 2.67 millimeters. The closed end was filled with oil to a depth of 3.2 centimeters, and the thermojunctions were immersed in this oil, the insulation resistance of which was then tested and found to be of the same order as in the case of the cold junctions (I, section 12).

With this arrangement, which will be referred to as a *thermo-couple bulb*, observed temperature fluctuations seemed to be of the same order as fluctuations of observed pressure, while, at the same time, repeated tests with the transition temperature of sodium sulphate indicated an accuracy within $0^\circ.004$ C., and a sensitivity corresponding very nearly to $0^\circ.001$ C. per division of the slide-wire scale of the potentiometer.[4]

4. *Description of the obstacle.* The obstacle used for the present purpose was a brass rod, one-half inch in diameter and six inches long, extending through the slack air (I, sections 18 and 19) completely across the channel from one wall to the other and normal to

[1] This apparatus was described in I, except for modifications outlined in the present paper.
[2] See I, equation (1).
[3] Thus making the value of τ (I, equation (1)), for *single* observations with the two kinds of instruments, sensibly the same.
[4] It should be added that additional tests, made with thermojunctions in glass without oil, were unsatisfactory; doubtless because of the low thermal conductivity of air and the mechanical instability of the comparatively long column of air inside the tube.

the channel axis. Its surface was brightly burnished, and its ends were provided with attachments, so that it could be clamped firmly in place in the channel, while at the same time the various openings in the channel wall could be made air-tight. The position of the rod was up the wind, immediately in front of the openings M and K (I, Figure 1).

The following pages contain a study of some of the properties of the wake or eddy set up by this obstacle in the jet of air. This eddy is represented diagrammatically in Figure 3. With variations of wind velocity, the eddy changed shape and size, but apparently not to such an extent as to necessitate changes of position of the thermal and manometric instruments in the eddy.

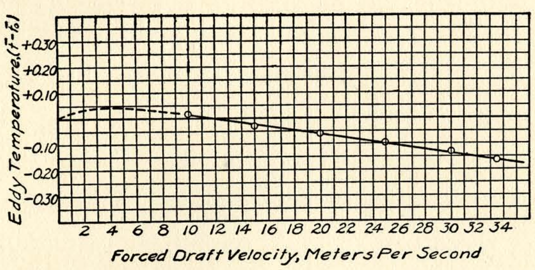

FIGURE 1. Eddy Temperatures.

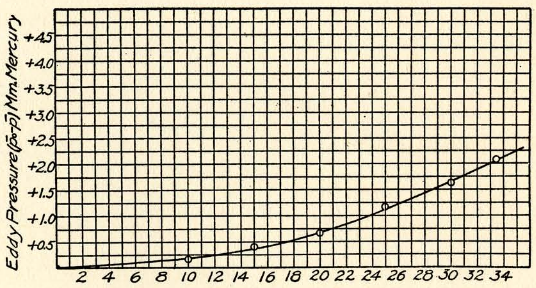

FIGURE 2. Eddy Pressures.

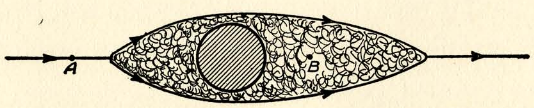

FIGURE 3. Schematic Diagram of the Eddy.

5. *Method of obtaining temperatures.* A thin, wooden, stream-lined collar was fastened to the rod as near the upper end as possible. A thermo-couple bulb was then passed down through the opening M (I, Figure 1) and also through a hole in the wooden collar, and held fast by one of the exterior universal clamps shown on the outside of the channel in I, Figure 2. The thermojunctions were thus held rigidly in position at a point x (I, Figure 1), 5 millimeters from the rod and exactly in its rear. Preliminary tests indicated this as the region of maximum intensity of the eddy set up by the obstacle.

To measure the mean temperature of the jet at a point removed from the influence of the eddy, a second thermo-couple bulb was suspended in a horizontal position by No. 40 (B. & S.) copper wires led through fine holes in the channel wall, with the extremity of the bulb about 4 centimeters in advance of the brass rod. The object of the horizontal position was to eliminate, as much as possible, any impact or eddy effects which might be due to its own presence in the jet.

The difference between the readings of the two thermo-couple sets thus installed evidently gave approximately the depression of mean temperature in the region of maximum eddy intensity, relative to the mean temperature of the main portion of the air jet.

It should be emphasized that reproducible results were obtainable only when the thermocouples were completely screened from radiation effects due to outside material (I, sections 5, 16 and 17). Consistent use of the silvered screen in front of the entrance nozzle (I, sections 5 and 17) proved to be essential.

6. *Method of obtaining static pressures.* In order to measure differences of mean static pressure corresponding to the eddy temperatures discussed in the last section, a brass obstacle was used which was an exact replica of the first one, except for details of attachment. A thin, stream-lined, wooden collar, in this case fastened to the rod near its base, firmly supported two thin, copper pressure tubes. These tubes had an outside diameter of a little over 3 millimeters. One end of each tube was cut off in a section normal to the axis of the tube, and the cut edge was carefully smoothed and polished, thus removing all traces of a bur. This end of the tube was used as a static opening. The other end of the tube was soldered into a brass nipple, to which vacuum tubing was hermetically joined.

It is possible that the static opening of each of these tubes should have been provided with a thin lip, or guide-plate, extending on both sides of the opening along the lines of mean flow. But the presence of a rigid surface of considerable area would have appreciably modified the eddy, and it was consequently abandoned, reliance being placed on the smooth edges of the static openings and the true alignment of the tubes.

One of these tubes was fastened, parallel to the brass obstacle, with its static opening in the same position in the eddy previously occupied by the thermo-couple bulb. The opening of the other tube was directly to the windward of the brass rod, and its axis was distant about 8 millimeters from the surface of the rod and parallel to it. Both tubes were adjusted as nearly as possible normal to the mean air flow, with their openings in the geometric axis of the channel.

Outside the channel, the projecting nipples of the 3-millimeter tubes were connected by vacuum tubing to the respective arms of a Krell alcohol manometer, at 3° inclination for velocities up to 30 meters per second, and, for higher velocities, at 6° inclination.

Before each test, all rubber joints were wired, and the tightness of the joints was tested with the aid of an aspirator pump.

7. *Method of obtaining velocities.* The mean velocity of the 7-centimeter working cross-section of the jet, over which static and dynamic pressures were sensibly uniform in the unobstructed channel,[1] was obtained with the aid of a Krell manometer from the static opening of the channel, which had previously been calibrated against the Pitot tube.[1]

[1] See I, sections 8–10 inclusive, 18, 19, 20, and 24.

PART III

Data and Results

8. *Eddy temperatures.* Each datum contained in Table I is the average of 10 consecutive observations which were made in as rapid succession as possible. If we denote by $\bar{t}$ the mean air temperature in the region of maximum intensity of the eddy, and by $\bar{t}_o$ the mean air temperature to windward of the obstacle at the horizontal thermo-couple bulb, then the difference $(\bar{t} - \bar{t}_o)$ will, for convenience, be referred to as *eddy temperature.*

TABLE I

EDDY TEMPERATURES

No.	Forced draft m/s	Eddy temperature C.°	No.	Forced draft m/s	Eddy temperature C.°
1	0.0	0.00	5	25.0	−0.09
2	10.0	+0.02	6	30.0	−0.13
3	15.0	−0.03	7	33.5	−0.17
4	20.0	−0.06			

To eliminate effects, upon observed eddy temperature, of temperature changes in the room due to progressive warming of the air by the exhauster and other fortuitous causes, the observer read the temperature of the horizontal bulb first, when making the first pair of observations; and in making the second pair, he read the temperature of the second bulb first; and so on alternately.

Special attention was given to the eddy temperature corresponding to a mean wind velocity of 10.0 meters per second, which was reproduced and checked, with uniform agreement to the nearest hundredth of a degree.

In the range of velocities between 0 and 10 meters per second, eddy temperature was unquestionably positive, but satisfactory numerical values were not obtained. The dotted part of the curve shown in Figure 1 is therefore extrapolated. This warming effect at low velocities, which was unmistakable, probably corresponded to a stream-line *régime* between velocities of 0 and about 0.37 meters per second (I, section 21), and, between velocities of 0.37 and about 2 meters per second, to the *régime* of channel turbulence preceding the appearance of the eddy in the rear of the obstacle.

9. *Eddy pressures.* Each entry of Table II shows the average of 10 pressure observations made in rapid succession (see also Figure 2). The *eddy pressure* there given is defined as the difference between mean static pressure $\bar{p}$ in the region of maximum intensity of the eddy and mean static pressure $\bar{p}_o$ at the pressure tube to the windward of the obstacle. Mean static pressure was thus found to be less in the eddy than in other parts of the 7-centimeter, central portion of the jet.

Eddy pressure varied greatly from one point to another along the channel axis in the region of the eddy, and the technique of ascertaining the region of maximum intensity of the eddy, which was very small, was difficult. Moreover, the plane of the opening of each pressure tube had to be adjusted with great accuracy until it lay parallel to the mean flow. This adjustment was also difficult in the case of the very small tubes necessitated by the small size of the channel. Curves obtained at different times were all of precisely the same character as the curve shown in Figure 2, which was plotted from the data of Table II. The eddy pressures here shown were numerically the largest of any set obtained.

TABLE II

EDDY PRESSURES

No.	Forced draft m/s	Eddy pressure mm	No.	Forced draft m/s	Eddy pressure mm
1	0.0	0.00	5	25.0	1.16
2	10.0	0.16	6	30.0	1.63
3	15.0	0.40	7	33.5	2.06
4	20.0	0.68			

10. *Combined effects of heat energy of turbulent motion, heat conduction and viscosity, in the region of maximum intensity of the eddy.* Approximations which were utilized in the previous paper[1] are applicable to mean flow in and near the eddy under consideration. The ratio $\overline{T}/\overline{p}$ during the experiments here described was within one-tenth of one per cent of 0.3858 throughout the entire velocity range, thus permitting the use of this value in the last term of I, equation (19). With the aid of these approximations, a simple integration of that equation led to a relation having a precision of the same order as that of the aerodynamic data, and identical in form with that already obtained, *viz.*:

$$\mathcal{E} = 0.725\,(\bar{t} - \bar{t}_o) + [0.287\,(\bar{t} - \bar{t}_o) + 0.1106\,(\bar{p}_o - \bar{p})] \quad \ldots\ldots\ldots \quad (1)$$

but, in this case, the approximate integration was performed along a stream-line defined in terms of mean velocity, from the point A to the point B (Figure 3), the boundary conditions having been obtained by the successive insertion of thermo-couples and pressure tubes into the channel at points A and B respectively.

11. *Variation of the energy integral $\mathcal{E}$ with velocity of flow.* Referring to Table III, which was calculated from the data contained in Tables I and II by means of equation (1), H denotes the first term of the right-hand member of equation (1), and is the heat gain corresponding to an increase of mean temperature. W denotes the term in square brackets and, if positive, is the dilatation work performed by the air against mean static pressure. Finally, $\mathcal{E}$, if positive, is equal to the gain in heat energy of turbulent motion, plus heat conducted into the mass under consideration by virtue of space variations of mean tempera-

[1] See I, section 26.

ture, plus that part of the heat dissipated by the internal stresses which are due to mean deformation velocities. These quantities, here given in *joules per gram of air*, are equal to the *changes* in the respective forms of energy that took place in a given fluid element of the air while passing along a line of mean flow between the above-described limits of integration, A and B, Figure 3.

TABLE III

TRANSFORMATIONS OF ENERGY IN THE EDDY

No.	q m/s	H joules per gram	W joules per gram	$\mathcal{E}$ joules per gram
1	0.0	0.0000	0.0000	0.0000
2	10.0	+0.0145	0.0234	0.0379
3	15.0	−0.0218	0.0356	0.0139
4	20.0	−0.0435	0.0580	0.0145
5	25.0	−0.0652	0.1027	0.0375
6	30.0	−0.0942	0.1432	0.0488
7	33.5	−0.1232	0.1792	0.0560

Results shown in Table III indicate that, up to some velocity between one and two meters per second, the combined energy denoted by $\mathcal{E}$ was dissipated in increasing the mean air temperature and in performing dilatation work against mean static pressure. For velocities above this value, dilatation work against mean static pressure was performed by the air, not only at the expense of heat energy of turbulent motion, etc., but also at the expense of the internal heat energy of the air, as manifested by a fall of mean temperature. Furthermore, the effects denoted by $\mathcal{E}$ reached a maximum at about 4.2 meters per second, and dropped to a minimum at about 18.3 meters per second, from which minimum they increased with mean flow velocity throughout the remainder of the velocity range investigated.

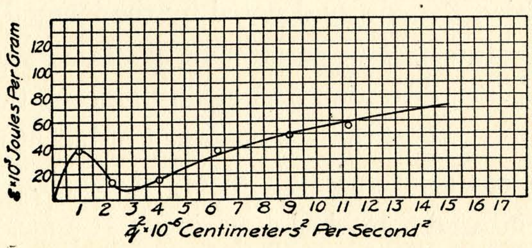

FIGURE 4. The Energy Integral $\mathcal{E}$ as a Function of the Square of the Mean Velocity.

12. *The energy integral $\mathcal{E}$ as a function of the square of the mean velocity.* Dimensional considerations suggest that the energy integral $\mathcal{E}$ should be a function of $\bar{q}^2$, the square of the mean flow velocity. Figure 4, plotted from results contained in Table III, exhibits this functional dependence.

PART IV

Summary

Air temperatures and pressures in the wake of an obstacle in a turbulent air current. For the present investigation, a brass rod having a burnished surface was fastened in the wind channel described in an earlier paper.[1] Mean temperatures and pressures were measured in the wake of this obstacle. The mean values under consideration were defined in the paper referred to. In that paper also were described methods by which temperature measurements were obtained. By these methods, slightly modified, it is found that, when mean flow velocity is increased from zero up to about four meters per second, eddy temperature[2] increases to a maximum. When mean flow velocity is further increased, the eddy temperature undergoes a systematic decrease from zero, at a velocity of 12 meters per second, to $-0.^\circ 17$ C. at a velocity of 33.5 meters per second. The warming effect at low velocities probably corresponds to a stream-line *régime* between velocities of 0 and about 0.37 meters per second (I, section 21), and between velocities of 0.37 and about two meters per second, to a *régime* of channel turbulence preceding the appearance of the eddy in the rear of the obstacle.

Eddy pressures[3] increased systematically with velocity.

Measurements of energy transformations in the eddy set up by an obstacle in a current of air. Heat energy of turbulent motion was defined for compressible fluids in the paper referred to in the preceding section. Measurements were made, in the wake of the obstacle, of the energy integral denoted in the text by $\mathcal{E}$. This integral is equal to the algebraic sum of (1) the gain of heat energy of turbulent motion, (2) heat conducted into the air current by virtue of space variations of mean temperature, and (3) heat dissipated by those internal stresses in the air which are due to mean deformation velocities. These measurements show that, for low velocities, the energy denoted by the integral $\mathcal{E}$ was dissipated in increasing mean air temperature and in performing dilatation work against mean static pressure. For higher velocities, dilatation work against mean static pressure was performed by the air, not only at the expense of the heat energy of turbulent motion, etc., denoted by $\mathcal{E}$, but also at the expense of the internal heat energy of the air. The energy transformations represented by the integral $\mathcal{E}$ reached a maximum at a mean flow velocity of about 4.2 meters per second, and dropped to a minimum at about 18.3 meters per second; from which minimum they increased with mean flow velocity throughout the remainder of the range investigated.

[1] The first paper of the present series.
[2] Air temperature in the eddy, minus air temperature in the surrounding current.
[3] Static pressure in the surrounding current minus static pressure in the eddy.

AERODYNAMICS OF THE PSYCHROMETER

By George Porter Paine

Third Paper: Theory and Experiments Relating to Energy Transformations in the Region of Eddies Set Up by a Wet Obstacle in a Jet of Moist Air

INDEX TO SECTIONS

PART I

Introduction

1. *Purpose of the present paper.* In the preceding papers of the present series, we have considered energy transformations in an unobstructed jet of air, and also in the region of eddies surrounding an obstacle in a jet of air. We proceed now to certain transformations of energy in the region of eddies surrounding an obstacle with a wet surface diffusing water vapor into a jet of moist air.

PART II

Apparatus

2. *The jet channel and the wet obstacles used for the present investigation. Wet and dry bulbs.* For obvious reasons, it was desirable to carry out the present study in a portion of a current of air, of such a character that, throughout any cross-section normal to the mean flow, mean temperature and mean static and dynamic pressures were sensibly invariable from one point to another. Such an air column was closely approximated by the 7-centimeter experimental portion of the cross-section of the jet channel described in the first paper of the present series.[1]

Into this jet were inserted the two thermo-couple bulbs described in the second paper.[2] The universal clamps shown on the top of the jet channel, in I, Figure 2, maintained the bulbs rigidly in a position normal to the mean flow of the air jet.[3] The closed end of one of the thermo-couple bulbs, which contained a thermo-couple set immersed in oil, was covered with a wick described in the next section. For convenience, we shall call this wick-covered thermo-couple bulb the *wet bulb*, and the other thermo-couple bulb, which was bare, the *dry bulb*. The wet bulb, with one of several types of wick used, is shown in Figure 1.

The dry bulb was inserted through the channel wall through the hole L (I, Figure 1) and its closed extremity was centered at point y (I, Figure 1). The wet bulb was maintained at the point x (I, Figure 1) in the same manner, and was exactly 4 inches down the wind from the dry bulb. Both bulbs were in the geometric axis of the channel and normal to the mean air flow.

Mathematical considerations, which will be taken up later, presuppose that the bulbs were installed side by side. This arrangement was not practicable in the small working cross-section of the jet. The dry bulb was therefore set in the channel axis ahead of the wet bulb. Some doubt arose as to whether the eddy set up by the dry bulb was sufficiently

[1] References to earlier papers of the present series will be indicated by Roman numerals.
[2] II, section 3.
[3] See I, sections 12 to 15 incl.

near the wet bulb to affect its temperature. Tests were accordingly run *without the wick*, both bulbs being perfectly dry, and both aligned in the channel axis. Averages of twenty observations for each bulb, taken alternately at regular intervals, gave respectively an agreement in temperature, to within about 0°.006 C. This arrangement was accordingly considered satisfactory.[1]

Radiation exchanges between wet and dry bulbs at the given distance were assumed to be small, on account of the small temperature differences and the small diameter of the bulbs (II, section 3).

3. *Wicks.* Tests described later in these pages show that the rate of evaporation from the wet bulb may under certain conditions be profoundly influenced by length, thickness, texture and immersion[2] of the wick. For wet bulbs of small diameter, a thick, linen, double-braided, tubular wick, closed at the lower end and saturated by plunging in water, gave the best results. For wet bulbs of larger diameter, a wick, made by taking about one and one-eighth turn around the glass bulb with a strip of fine linen lawn, gave good results. The lower end of this wick was drawn together with a thread, and the wick was immersed by plunging in water. Details of wick construction are contained below in the account of the tests.

4. *Preparation of wicks and chemical precautions.* In order that all traces of grease might be removed from the wicks, they were soaked, before using, for 15 minutes in a 1/100 solution of hydrochloric acid, hot; rinsed in tap water; soaked 15 minutes in a 1/100 solution of ammonia, hot, and then rinsed by allowing tap water to run over them for about 5 minutes, after which they were rinsed four times in distilled water and then placed in a glass-stoppered bottle of distilled water where they were kept until used.

All glass parts pertaining to the wet and dry bulbs were made chemically clean before using by soaking over night in cleaning solution, rinsing in tap water and rinsing again in distilled water. Such parts of the apparatus as were required to be dry were rendered so by means of an aspirator pump.

5. *Other precautions.* Each thermo-couple bulb was adjusted in position so that the center of the oil column in the bulb (II, section 3) lay in the geometric axis of the channel. It was essential for the wicks to extend some distance above the oil level in the thermo-couple tube[3] (II, section 3) in order to prevent heat leakage by conduction down the glass into the effective portion of the wick.[4] At the same time, it was necessary that the upper end of the wick should not come within 3 or 4 millimeters of the channel wall, on

[1] When, in a preliminary test, the *wet* bulb was clamped *ahead* of the dry bulb, the dry bulb was appreciably affected.

[2] For the sake of brevity, we shall refer to the mechanism by which water is supplied to the wick as the *immersion*. In addition to the properties mentioned in the text, wick performance is also influenced by magnitude, shape and chemical condition.

[3] This oil column in the thermo-couple tube was analogous to a thermometer bulb.

[4] This effect, however, would be extremely small.

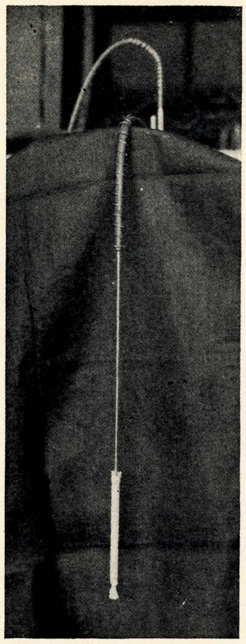

FIGURE 1. The Wet Bulb.

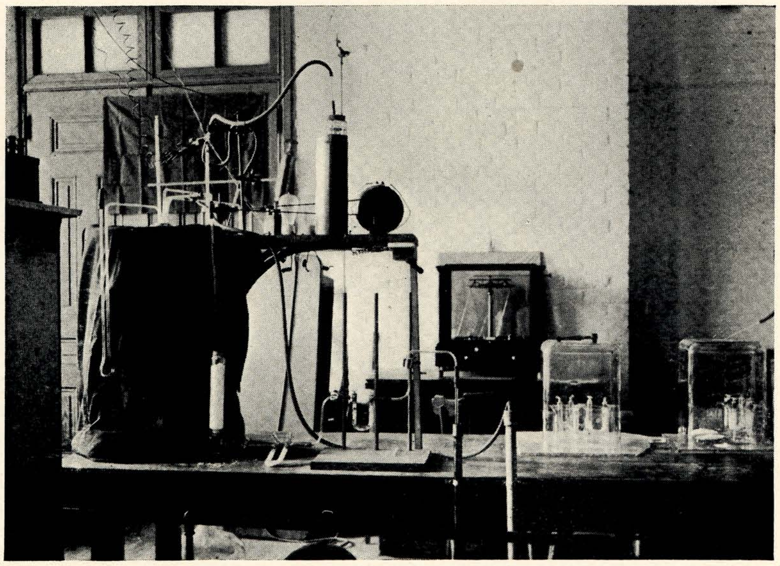

FIGURE 2. The Chemical Hygrometer.

account of heat leakage from the metal of the wall to the wick. The thermo-couple tube was thus covered by the wick to a distance of about 4.3 centimeters above the level of the oil in the tube, and the entire wet bulb was thus in a state of thermal insulation with its metallic surroundings.[1]

6. *Immersion.* All the experiments in the ensuing pages, with a few exceptions, which will be considered in detail, were performed with a wet bulb which was saturated by raising a glass tube, closed at one end and filled with water, up through the guide-tube K (I,

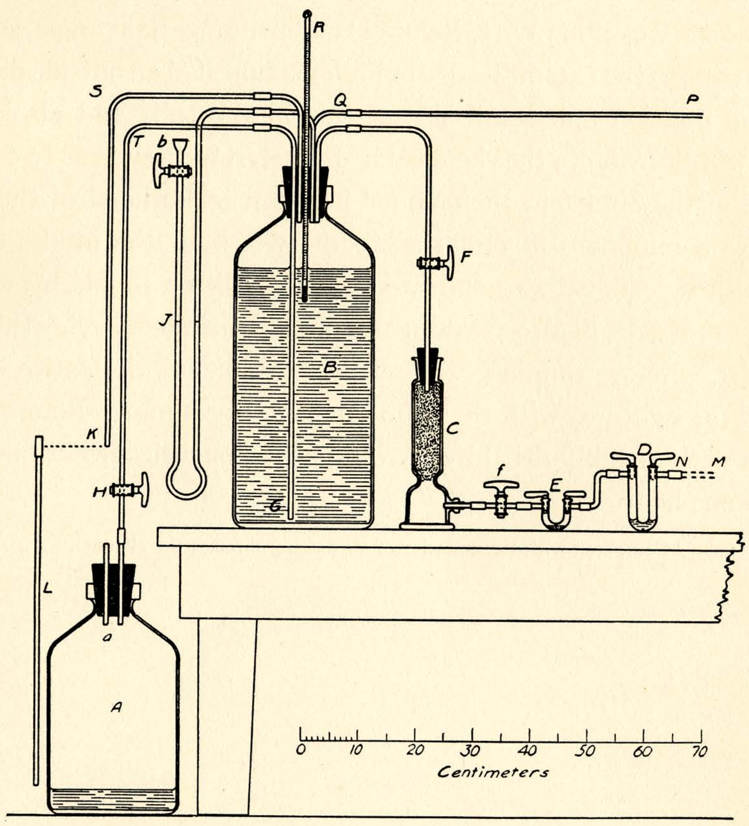

FIGURE 3. Diagram of the chemical hygrometer.

Figure 1) until the wick was completely immersed in water. The tube was then held in this position a specified length of time until the wick was completely saturated. At the end of this time, the tube was withdrawn and the guide-tube K closed tightly with a rubber stopper.

[1] There was, of course, some heat conduction taking place along the thermo-couple leads into the wet bulb, for there was at times a considerable temperature gradient from near the thermojunctions to a point about 7 centimeters along the leads. But that this leakage was extremely small was shown by a simple calculation. In section 38, Appendix III, *infra*, it is shown that, with a wet bulb depression of 20°C., under standard conditions there set forth, the moist air passing the wet bulb gives off about 4.93 joules of heat energy per centimeter length of wet bulb per second. Under the same conditions, it is easy to show that heat would be conducted inwards along four No. 40 (B. & S.) copper leads at the rate of 0.00108 joules per second. The order of the effect of thermal leakage along the leads was therefore less than 1 in 5000.

7. *The chemical hygrometer.* In order to ascertain the humidity of the air near the dry bulb in the jet channel, a relatively small specimen of air was drawn from the channel through a chemical hygrometer, a description of which will be found below in Appendix I, together with a list of references useful in this connection.

During a run, lasting, say, 90 minutes, about 30 liters of air were withdrawn from the channel in this way. Air thus taken for analysis was sucked into a brass tube at a point about 3 centimeters below the tip of the dry bulb. Suction was obtained by means of the aspirator of the hygrometer (Figure 3, A, T, G, B). The brass tube was inserted through the small opening W (I, Figure 1) and bent at a right angle, so as to bring the intake 1.6 centimeters to the rear of the dry bulb. This tube had an outside diameter of only 3 millimeters. The position and small size of this intake tube and also the very small quantity of air [1] withdrawn into the chemical hygrometer, were such as to cause no appreciable disturbance in the air jet in the channel in the neighborhood of the thermo-couple bulbs. To insure a minimum of radiation to the wet bulb, the intake tube was silver-plated and burnished. Its exterior end was provided with a nipple by which it was attached to a glass tube (MN, Figure 3) leading to the sulphuric acid drying tube (D, Figure 3) of the hygrometer. Before running each set of experiments, the entire hygrometer was invariably tested for tightness with the aid of an aspirator pump. Before each individual experiment, it was thoroughly dried by drawing air through it with a powerful aspirator for not less than one hour.

[1] Compared with the quantity of air passing through the channel.

PART III

Wick Characteristics

8. *Characteristics of a double-braided linen wick, with capillary feed.* The wick under investigation consisted of a bulb-covering, or *wick* proper, provided with a capillary upward *feed*, consisting of several strands of wicking, the ends of which were immersed in a glass tube of distilled water at K (I, Figure 1).

The *wick* proper was a segment 7.5 centimeters in length, cut from a long linen tube which was made on a 16-carrier braider, with 32 ends of Barbour's No. 45 F.D.A. bleached linen thread. Two of these ends were wound on each bobbin, or carrier, giving a total number of 32 ends, or 16 double ends. In order to make the wick of sufficiently small inside diameter (2.67 millimeters),[1] giving a close fit for the glass thermo-couple tube, the take-up gear ratio was figured out to put in all of the material that the braid would take up, thus allowing for very little longitudinal expansion of the braided tubing when in its loosest position. When the glass tube was inserted, pulling the braid out very slightly gave a tight fit of the wicking to the glass tube, while, at the same time, the threads of the wick came into contact with each other so closely that all interstices were practically closed.[2]

TABLE I

CHARACTERISTICS OF A DOUBLE-BRAIDED LINEN WICK, WITH CAPILLARY FEED
4 CENTIMETERS OF FEED EXPOSED TO WIND
WIND VELOCITY = 15.0 M/S

No.	Time m s	Wet bulb C.°	No.	Time m s	Wet bulb C.°
1	0 0	20.59	10	3 0	10.66
2	0 15	13.16	11	4 0	10.64
3	0 30	10.77	12	5 0	10.71
4	0 45	10.70	13	6 0	10.70
5	1 0	10.65	14	7 0	10.72
6	1 15	10.63	15	8 0	10.75
7	1 30	10.63	16	9 0	10.78
8	1 45	10.65	17	10 0	10.78
9	2 0	10.58			

One end of the 7.5 centimeter length was drawn together tightly with a clove hitch of Barbour's No. 18 F.D.A. bleached linen thread.

The *feed* in this case consisted of 16 strands of Barbour's No. 18 F.D.A. linen thread.

[1] The diameter of the bare cylindrical thermo-couple bulb used in the ensuing tests was 2.67 millimeters. The drying-up effect, described below, was much less pronounced in the case of bulbs having a diameter of 6.5 millimeters; as will be shown in the fifth paper of the present series. It follows at once from Stefan's analysis, referred to in the first paper, that the rate of evaporation should be appreciably greater from a cylinder of small diameter than it is from a cylinder having a diameter that is comparatively large.

[2] These excellent wicks, of substantial aid in the present research, were designed and made for the writer by Mr. Moore, of Chadbourne and Moore, Inc., Boston and Chelsea, Mass., U.S.A.

Immersion was effected as follows. At the beginning of each test, the water-tube, described above in section 6, was raised through the guide-tube K (I, Figure 1) sufficiently to immerse the capillary feed and the entire bulb-covering. Then, at the zero instant, after having remained in this position for 10 seconds, the water-tube was lowered until 4 centimeters of the feed, between bulb-covering and water-tube, were left exposed to the action of the air jet. The water-tube was then clamped in this position, with about 12 centimeters of the feed remaining under water in the tube. The exposed 4-centimeter portion of the feed here mentioned was necessary in order to cool the water supply as much as possible before it reached the bulb-covering.

The temperature of the wet bulb was observed at the zero instant above referred to, and at subsequent intervals, as indicated in the upper curve (curve A) of Figure 4.

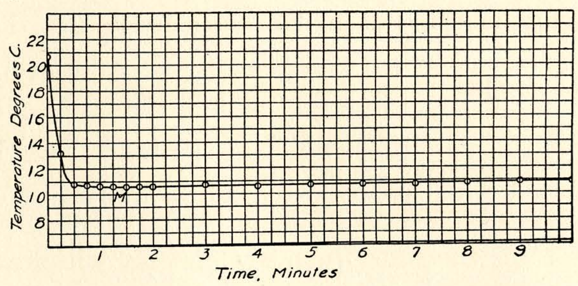

FIGURE 4. Characteristics of a double-braided linen wick, with capillary feed; 4 centimeters of feed exposed to wind. Wind velocity: 15.0 meters per second.

From the initial instant to the instant when the minimum M of this curve was attained, rapid cooling by evaporation was indicated. During this stage the wick was practically saturated with water. At the same time, evaporation tended to dry up the wick; a tendency which was offset, to some extent, by the capillary action of the feed. But, in order that the latter action should become operative, it was necessary for the bulb-covering to become partly dry; for, if saturated, the wick would fail to draw water from the tube. This drying-up effect is clearly shown by the curve. During a short time after the attainment of the minimum M, however, the drying-up effect was practically inoperative, because water arrived at the outer surface of the bulb-covering from its interior as fast as evaporated. Only after about 30 seconds following the occurrence of minimum M, did the drying-up effect become sensible, and it increased gradually during the subsequent 10 minutes during which the experiment was run. In some cases, this effect was observed to increase in magnitude for as much as 1 hour and 20 minutes, before equilibrium between capillary feed and evaporation was attained. It should be added that, especially under conditions of low relative atmospheric humidity, the windward side of the bulb-covering became almost dry at the end of the hour, with a fairly good water supply on the opposite side, where the bulb-covering was protected. The lower curve (curve B) of Figure 4, has for its ordinates, on the right-hand side of point M, the difference between the ordinates of curve A and the minimum M. To the left of point M, the curve B is extrapolated. This curve accordingly exhibits the drying-up effect under consideration.

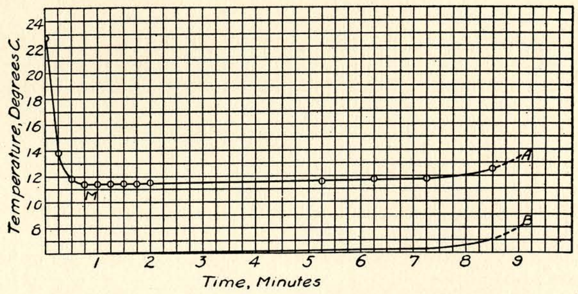

FIGURE 5. Characteristics of a double-braided linen wick, without feed. Wind velocity: 15.0 meters per second.

9. *Characteristics of a double-braided linen wick, without feed.* In Figure 5 are shown characteristics of the same double-braided linen wick, consisting of the bulb-covering but without continuous feed of any kind. This wick, in place on the thermo-couple tube, is shown in Figure 1. It was saturated by means of the water-tube as described in the last section. At the zero instant, the water-tube was entirely withdrawn from the channel, and the temperature read thereafter at suitable intervals.[1]

TABLE II

CHARACTERISTICS OF A DOUBLE-BRAIDED LINEN WICK, WITHOUT FEED
WIND VELOCITY = 15.0 M/S

No.	Time m s	Wet bulb C.°	No.	Time m s	Wet bulb C.°
1	0 0	22.69	7	1 30	11.43
2	0 15	13.81	8	1 45	11.45
3	0 30	11.77	9	2 0	11.47
4	0 45	11.38	10	5 15	11.57
5	1 0	11.41	11	6 15	11.66
6	1 15	11.39	12	7 15	11.72
			13	8 30	12.54

Under these conditions, without continuous feed, the minimum temperature M occurred about $1^m\ 15^s$ after withdrawal of the water-tube, and the drying-up effect (curve B) was inoperative until after the minimum had been clearly established. This effect was appreciably more marked than it was with the feed, after the end of the second minute; and the drying-up curve B rose rapidly after the seventh minute.

[1] See *supra*, section 6.

10. *Characteristics of a thin linen wick, with capillary feed.* The next wick to be considered consisted of a tight-fitting bulb-covering of extremely fine, pure linen lawn, provided with a capillary feed made of 4 strands of 3-thread No. 2 knitting cotton. Immersion was effected as described above in section 8. The characteristic curves are shown in Figure 6. In this case, cooling was so rapid during the first 15 seconds as to make it difficult

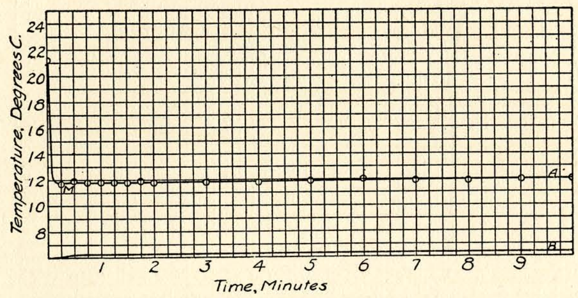

FIGURE 6. Characteristics of a thin linen wick, with capillary feed; 4 centimeters of feed exposed to wind. Wind velocity: 15.0 meters per second.

accurately to follow it with the slide-wire of the potentiometer. A very distinct minimum M was, however, clearly indicated, with a rapid rise of the drying-up curve B immediately following. This rapid drying-up effect was partly due to the thinness of the bulb-covering, and partly to the fact that sufficient capillary flow to keep the bulb-covering moist was available only after considerable drying-up had taken place. A slower, progressive drying-up effect was also evident here, as in the preceding cases.

TABLE III

CHARACTERISTICS OF A THIN LINEN WICK, WITH CAPILLARY FEED
4 CENTIMETERS OF FEED EXPOSED TO WIND.
WIND VELOCITY = 15.0 M/S

No.	Time m s	Wet bulb C.°	No.	Time m s	Wet bulb C.°
1	0 0	21.22	10	3 0	11.81
2	0 15	11.66	11	4 0	11.81
3	0 30	11.86	12	5 0	11.90
4	0 45	11.83	13	6 0	11.95
5	1 0	11.83	14	7 0	11.89
6	1 15	11.84	15	8 0	11.92
7	1 30	11.82	16	9 0	12.03
8	1 45	11.87	17	10 0	12.04
9	2 0	11.81			

11. *Characteristics of a thin, loose-fitting, linen wick without feed.* A wick similar to the one described in the last section but without continuous feed was tested, with the results shown in Figure 7. The bulb-covering was sewed into the form of a cylinder and applied

loosely to the thermo-couple tube, so that it enclosed the tube but did not at all points touch it. The minimum M, Figure 7, occurred considerably later than in the preceding test; a phenomenon probably due to the thermal capacity of the water contained in the loose portions of the wick. The minimum M continued long enough to indicate that the drying-up effect (curve B) during the first 75 seconds was sensibly inoperative; after which, drying up of the wick was rapid and pronounced. This last effect was exaggerated by the portions of the bulb-covering not in contact with the glass.

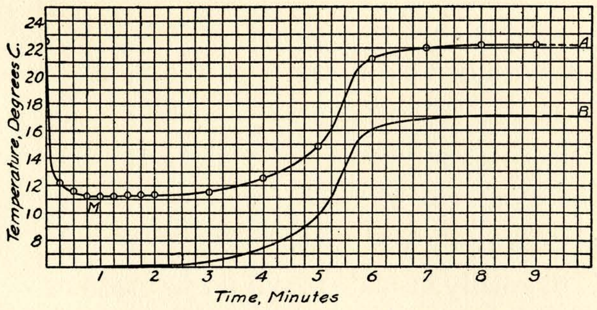

FIGURE 7. Characteristics of a thin, loose-fitting, linen wick, without feed. Wind velocity: 15.0 meters per second.

12. *Additional tests.* Tests run with a bulb-covering of fine linen lawn, applied as described in section 6, so as to be *at all points closely in contact with the tube*, and entirely without continuous feed, yielded curves substantially equivalent to those shown above in Figure 5.

TABLE IV

CHARACTERISTICS OF A THIN LOOSE-FITTING LINEN WICK, WITHOUT FEED
WIND VELOCITY = 15.0 M/S

No.	Time *m s*	Wet bulb C.°	No.	Time *m s*	Wet bulb C.°
1	0 0	22.48	9	2 0	11.28
2	0 15	12.08	10	3 0	11.53
3	0 30	11.16	11	4 0	12.54
4	0 45	11.16	12	5 0	14.81
5	1 0	11.22	13	6 0	21.17
6	1 15	11.23	14	7 0	21.96
7	1 30	11.25	15	8 0	22.17
8	1 45	11.28	16	9 0	22.16

An extended series of tests with various types of gravity feed and combinations of gravity with capillary feeds demonstrated that it was impracticable to supply water to the bulb-covering at the minimum temperature M, defined by the foregoing curves, by allowing water to flow *down* to the wet bulb. In the tests referred to, the descending fluid was subjected to the forced draft, but some of the heat of the liquid invariably became available for vaporization, which at once rendered the minimum wet bulb temperature M dependent upon the temperature of the liquid in the water-tube and upon the rate of feed; which for psychrometric purposes was most undesirable.[1] Moreover, water fed to the bulb-covering wholly or partly by the action of gravity, invariably was driven by the wind to flow down the leeward part of the bulb-covering, thus leaving the windward part nearly dry.

13. *General conclusions.* Since definite boundary conditions, experimentally attainable, were desired for purposes of the ensuing measurements, it was essential that wet bulb temperature should be rendered independent of the initial temperature of the water supplied to the wick and of the relation between rate of water supply and rate of evaporation, which depends upon various experimentally indeterminate variables.

It was evident, therefore, that continuous feed of all types should be abandoned. It was further evident that the *minimum* M, *defined by the wick-characteristic curves*, is independent of adventitious causes provided that:

(1) The wet bulb is thermally insulated from its surroundings.

(2) The wet bulb is shielded from radiation exchanges with neighboring material.[2]

(3) The wick consists of bulb-covering only, without feed of any kind.

(4) The bulb-covering is in close contact with the glass bulb at all points, so that a *continuous* mutual film of water wets both cloth and glass.

(5) The initial wetting is effected by complete immersion of the bulb-covering in distilled water until complete saturation of the material of the covering is attained.

The causes which, under the foregoing definite conditions, were operative in maintaining, for a short interval of time, a sensibly steady state of the wet bulb at the minimum temperature M, will be investigated in detail in the ensuing pages.[3]

[1] This effect was conclusively demonstrated later by tests made with the aid of IV, equation (5).

[2] See I, sections 16 and 17.

[3] It should be added that the rapidity with which a wet bulb minimum M is attained was observed to depend upon forced draft velocity and also upon the relative humidity of the air; and such a minimum can, therefore, be ascertained only from continuous observation of the wet bulb, and not from a single observation at the end of a fixed time after the initial wetting.

PART IV

Aerodynamics of a Wet Cylinder in a Jet of Moist Air

14. *Atmospheric vapor and wick vapor.* Let us now turn to certain aspects of the behavior of a jet of moist air in the channel described in the first paper of the present series.[1] This moist air is a mechanical mixture of perfectly dry air and of water vapor. For the present, we shall consider what takes place in the channel while the air jet is flowing past a perfectly dry obstacle. Later, we shall consider the effect of the presence of a wet bulb. The water vapor, contained in the air constituting the jet, will be referred to in these pages as *atmospheric vapor*, as distinguished from *wick vapor*, *i.e.*, the water vapor evaporated from the bulb-covering, which will be investigated later. The mixture of *dry air* and *atmospheric vapor* flowing into the entrance nozzle [2] will be called *atmospheric air*.

15. *General laws governing the flow of moist air.* For the *dry air* entering the jet channel, we have, in accordance with the Law of Conservation of Momentum,[3]

$$\left.\begin{aligned} \frac{du_a}{dt} &= X_a - \frac{1}{\rho_a}\frac{\partial p_a}{\partial x} + \frac{1}{3}\frac{\mu_a}{\rho_a}\frac{\partial \theta_a}{\partial x} + \frac{\mu_a}{\rho_a}\nabla^2 u_a - A\rho_w(u_a - u_w) \\ \frac{dv_a}{dt} &= \dots\dots\dots\dots\dots\dots \\ \frac{dw_a}{dt} &= \dots\dots\dots\dots\dots\dots \end{aligned}\right\} \quad \dots\dots \quad (1)$$

The subscript a refers to the *dry air*, and the subscript w refers to the *atmospheric vapor*.

Multiplying the first of these equations by u_a, the second by v_a and the third by w_a, adding and reducing, we have[4]

$$\begin{aligned} \frac{d}{dt}\left(\frac{q^2_a}{2}\right) &= (u_a X_a + v_a Y_a + w_a Z_a) \\ &\quad - \frac{1}{\rho_a}\left(\frac{dp_a}{dt} - \frac{\partial p_a}{\partial t}\right) + \frac{\mu_a}{\rho_a} W_a \\ &\quad - A\rho_w\,[u_a(u_a - u_w) + v_a(v_a - v_w) + w_a(w_a - w_w)] \quad \dots\dots\dots \quad (2) \end{aligned}$$

The last term of this equation evidently denotes the rate at which work is done on the dry air by the atmospheric vapor, *per unit mass of dry air*.

An exactly analogous equation holds for the atmospheric vapor; the two members of

[1] I, section 5.
[2] Ibid.
[3] See I, section 4, equation (4). For the forces of diffusion, see Maxwell, *Phil. Mag.*, 1860 (4), **19**, page 19; **20**, page 21; also 1868 (4), **35**, pages 129 and 185.
[4] For notation, see I, section 4. The symbol A denotes a resistance coefficient (not the Maxwell coefficient of diffusion) constant for any two gases, but variable from one pair of gases to another.

which, if added respectively to the two members of equation (2) and the results multiplied by $d\tau$ and equated, give the relation

$$\left[\rho_a \frac{d}{dt}\left(\frac{q^2_a}{2}\right) + \rho_w \frac{d}{dt}\left(\frac{q^2_w}{2}\right)\right] d\tau$$
$$= \Big\{ - g\left[\rho_a w_a + \rho_w w_w\right] - \left[\frac{db}{dt} - \frac{\partial b}{\partial t}\right] + \left[\mu_a W_a + \mu_w W_w\right] - A\rho_a\rho_w\left[(u_a - u_w)^2 + (v_a - v_w)^2 + (w_a - w_w)^2\right]\Big\} d\tau \quad (3)$$

where

$$b = p_a + p_w \quad (4)$$

or the total static atmospheric pressure,[1] and $d\tau$ is an infinitesimal element of volume.

The Law of Conservation of Energy may be written for this mass $(\rho_a + \rho_w)\, d\tau$ of atmospheric air as follows, if we assume the two constituents of the mixture to be at the same temperature: [2]

$$\left\{\left[\rho_a \frac{d}{dt}\left(\frac{q^2_a}{2}\right) + \rho_w \frac{d}{dt}\left(\frac{q^2_w}{2}\right)\right] + J\left[\rho_a C_v + \rho_w c_v\right]\frac{dT}{dt}\right\} d\tau$$
$$= \Bigg\{ - \left[\rho_a p_a \frac{d\left(\frac{1}{\rho_a}\right)}{dt} + \rho_w p_w \frac{d\left(\frac{1}{\rho_w}\right)}{dt}\right] - \left[\frac{db}{dt} - \frac{\partial b}{\partial t}\right] + \left[\mu_a W_a + \mu_w W_w\right] + \left[\mu_a S_a + \mu_w S_w\right] - g\left[\rho_a w_a + \rho_w w_w\right] + J\left[(\rho_a \vartheta_a + \rho_w \vartheta_w)\, \nabla^2 T\right] - A\, \rho_a\rho_w\left[(u_a - u_w)^2 + (v_a - v_w)^2 + (w_a - w_w)^2\right]\Bigg\} d\tau \quad (5)$$

For convenience in this and the following pages, the symbol C denotes specific heat of dry air, and c denotes specific heat of water vapor.

Combining equations (3) and (5), we have

$$J\left[\rho_a C_v + \rho_w c_v\right]\frac{dT}{dt} + \left[\rho_a p_a \frac{d\left(\frac{1}{\rho_a}\right)}{dt} + \rho_w p_w \frac{d\left(\frac{1}{\rho_w}\right)}{dt}\right]$$
$$= \left[\mu_a S_a + \mu_w S_w\right] + J\left[(\rho_a \vartheta_a + \rho_w \vartheta_w)\nabla^2 T\right] \quad (6)^{3}$

[1] See I, section 7.
[2] For notation, the reader is again referred to I, section 4.
[3] Thus the work done by the forces of diffusion is not dissipated in increasing the temperature of the *mixture*, nor in causing the mixture to perform dilatation work against static pressure.

The characteristic equation for dry air may be written [1]

$$p_a = \rho_a \mathrm{RT} \quad \ldots\ldots\ldots\ldots\ldots \quad (7)$$

and the characteristic equation for water vapor, under atmospheric conditions of temperature and pressure, is [2]

$$p_w = \rho_w \mathrm{BT} \quad \ldots\ldots\ldots\ldots\ldots \quad (8)$$

Let V denote the *specific volume* of the atmospheric air under consideration (*i.e.*, the volume occupied by one unit mass) and let ν denote the *specific humidity* of the atmospheric air (the mass of *atmospheric vapor* in one unit mass of atmospheric air). Then

$$\rho_a = \frac{1 - \nu}{\mathrm{V}}; \quad \rho_w = \frac{\nu}{\mathrm{V}} \quad \ldots\ldots\ldots\ldots\ldots \quad (9)$$

Making use of these relations, together with a simple well-known thermodynamic transformation, we have from equation (6)

$$(1 - \nu)\, \mathrm{JC}_p \frac{d\mathrm{T}}{dt} + \nu \mathrm{J} c_p \frac{d\mathrm{T}}{dt} - \frac{\mathrm{GT}}{b} \frac{db}{dt}$$
$$= [(1 - \nu)\, \mathrm{S}_a + \nu \mathrm{S}_w] + \mathrm{J}\, [(1 - \nu)\, \vartheta_a + \nu \vartheta_w]\, \nabla^2 \mathrm{T} \quad \ldots\ldots\ldots \quad (10)$$

in which equation, G is the variable gas constant for moist air, given by the relations [3]

$$b\mathrm{V} = \mathrm{GT}$$
$$\mathrm{G} = \mathrm{R} + (\mathrm{B} - \mathrm{R})\nu \quad \ldots\ldots\ldots\ldots\ldots \quad (11)$$

Equation (10) represents energy changes per second, *per unit mass of atmospheric air*, in the dry channel.

16. *Energy transformations near the wet bulb.* Let us now turn to a consideration of the action, on a unit mass of atmospheric air, of the wet bulb in the channel. We shall confine our attention to the interval of time (something over one minute) during which wet bulb temperature remains steadily at its minimum value M defined by the wick characteristic curves of section 13. During this interval of time, the entire wet bulb (*i.e.*, the thermocouple bulb and its bulb-covering) has reached a uniform temperature, and consequently there is no change taking place in the heat of the liquid. Therefore no part of the heat of the liquid can serve as a heat supply for the purpose of evaporation. At the same time, the quantity of water on the wick is sufficient to form a continuous film, and thus to enable evaporation to take place from a continuous liquid surface. The wet bulb used with the present apparatus was, as we have seen, thermally insulated from its surroundings; consequently no appreciable part of the heat of vaporization was due to thermal conductivity

[1] See I, equation (15).
[2] See *infra*, Appendix II.
[3] With regard to dimensions, it should be observed that

$$\mathrm{V} = \frac{[cm.]^3}{[gram]}$$

and that ν is a non-dimensional number.

along any part of the apparatus. Precautions, moreover, in the design of the apparatus [1] reduced radiation effects inside the channel to a minimum which was practically negligible.[2] Under these very special conditions, all of the *wick vapor* [3] was produced by heat supplied by the jet of *atmospheric air.*

Let ds/dt denote, accordingly, the mass of wick vapor evaporated per second by a unit mass of atmospheric air.[4] Then the differential equation (10) for the jet becomes (after transposing)

$$J[(1-\nu)\vartheta_a + \nu\vartheta_w]\nabla^2 T + [(1-\nu)S_a + \nu S_w] = (1-\nu)JC_p\frac{dT}{dt} + \nu Jc_p\frac{dT}{dt} - JL'\frac{ds}{dt} - \frac{GT}{b}\frac{db}{dt} \quad \ldots\ldots\ldots \quad (12)$$

in which equation L′ is the heat of vaporization of water at the minimum wet bulb temperature M.

Applying now, to equation (12), the mean value analysis of section 4 of the first paper of this series, for *steady* states of *mean* motion, we have [5]

$$J[(1-\bar{\nu})\vartheta_a + \bar{\nu}\vartheta_w]\nabla^2\bar{T} + [(1-\bar{\nu})\bar{\Sigma}_a + \bar{\nu}\bar{\Sigma}_w] - \Xi = (1-\bar{\nu})JC_p\frac{d\bar{T}}{dt} + \bar{\nu}Jc_p\frac{d\bar{T}}{dt} - JL'\frac{d\bar{s}}{dt} - \left(\overline{\frac{GT}{b}}\right)\left(\overline{\frac{db}{dt}}\right) \quad \ldots\ldots\ldots \quad (13)$$

The function $(-\Xi)$ is the energy of turbulent motion which is dissipated per unit mass of atmospheric air per second (*a*) in raising the mean temperature of the atmospheric air, (*b*) in performing dilatation work against mean static pressure and (*c*) in supplying heat of vaporization to the wet bulb.

Referring to the definition of the specific humidity ν, on a previous page, it is important to observe that this quantity is the mass of *atmospheric* vapor in one gram of *atmospheric air.*[6] It does *not* denote the sum of the masses of *atmospheric vapor* and *wick vapor* in a unit mass of *atmospheric air.*

It is possible that, during the transit of a mass element of atmospheric air down the wind channel, this specific *atmospheric* humidity ν may undergo some change; but this change is obviously a relatively small quantity, and the mean value $\bar{\nu}$ may be assumed to be approximately constant during the progress of the air along the mean stream-line contemplated in the next section.[7]

[1] See I, sections 16 and 17.
[2] See observed radiation data and results *infra*, in Appendix III.
[3] See definition *supra*, section 14.
[4] s is therefore a non-dimensional number.
[5] For the definition of $\bar{\Sigma}$, see I, section 4, after equation (19).
[6] *Cf.* definitions of *atmospheric vapor*, *wick vapor*, etc., *supra*, section 14.
[7] The quantity $\bar{\nu}$ does however frequently change appreciably from one atmospheric state to another. The variation under consideration in the text is not that due to changes in humidity, from one time to another, of the outside air which is drawn into the channel, but to changes in a given air mass caused by its passage down the channel past the wet bulb. Relative to the ensuing integration along a stream-line, $\bar{\nu}$ is assumed to be approximately a constant.

An integration of equation (13) along a stream-line defined by the steady *mean* motion of the atmospheric air, from a point A_0 in the sensibly still air outside of the jet channel, to the *wet* bulb at point A_1, leads accordingly to the equations [1]

$$E_1 = (1 - \bar{\nu})JC_p(\bar{t_0} - \bar{t'}) + \bar{\nu}Jc_p(\bar{t_0} - \bar{t'}) - JL'\bar{s} \quad \text{(14)}$$

$$E_1 = \int_{A_0}^{A_1} \Big\{ J[(1 - \bar{\nu})\vartheta_a + \bar{\nu}\vartheta_w]\nabla^2\bar{T} + [(1 - \bar{\nu})\bar{\Sigma}_a + \bar{\nu}\bar{\Sigma}_w]$$

$$- \Xi + \left(\frac{\overline{GT}}{\bar{b}}\right)\left(\frac{d\bar{b}}{dt}\right)\Big\} dt \quad \text{(15)}$$

17. *Significance of the integral* E_1. The following considerations are important, *viz.*:

(*a*) The first term in square brackets, of the right-hand member of equation (15), is equal to the heat energy conducted to (or from) the unit air mass under consideration during its transit from point A_0 to the wet bulb. This is by no means the total thermal conduction taking place in the unit air mass, but only that part of the thermal conduction which is due to space rates of change of *mean* air temperature $\bar{T}$.

(*b*) Similarly, the second term in square brackets is not equal to the total energy dissipated by the internal stresses of the fluid; but it is that part of the energy dissipated by the viscous stresses brought into existence by the space rates of change of *mean* deformation velocities.

(*c*) The integral

$$- \int_{A_0}^{A_1} \Xi \, dt$$

is the heat energy of turbulent motion [2] dissipated per unit mass of atmospheric air during transit from A_0 to A_1.

(*d*) The last term of the left-hand member of equation (15) is equal to the *translatory* work [3] of the atmospheric air against mean static atmospheric pressure per unit mass while in transit along the mean stream-line from A_0 to A_1.

18. *Energy transformations near the dry bulb.* Assuming that the *dry* bulb is located in the air jet in a position beside the wet bulb such that no appreciable influence is exerted by one bulb upon the other, then, if we integrate from a point A_2 in still air outside the channel to the *dry* bulb at A_3, we have

$$E_2 = (1 - \bar{\nu})JC_p(\bar{t_0} - \bar{t}) + \bar{\nu}Jc_p(\bar{t_0} - \bar{t}) \quad \text{(16)}$$

where E_2 is a function analogous to E_1 of equation (15).

[1] The t's with bars over them denote mean temperatures measured on the C. scale.
[2] See I, section 25 for definition. See also *supra*, section 16, after equation (13).
[3] Since $\frac{\partial \bar{b}}{\partial t} = 0$. See I, sections 3 and 4.

19. *Energy formula.* Subtracting the respective members of equation (16) from those of equation (15), and equating results, we have, *after dropping the bar notation*[1]

$$E = J(1 - \nu)[C_p(t - t') + \left(\frac{\nu}{1 - \nu}\right) c_p (t - t') - \left(\frac{s}{1 - \nu}\right) L'] \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where

$$E = E_1 - E_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

This equation refers explicitly to the *atmospheric air.* An analogous energy equation can be written for the *wick vapor.* The latter equation is superfluous for the present purpose.

20. *Significance of the integral* E.[2] It might be supposed that the eddy set up by the wet bulb was so nearly identical with the eddy set up by the dry bulb, that the integral E would have been wholly negligible compared with the other terms of equation (17). But measurements contained in the following pages show that such was not at all the case. Experimental results in fact indicated that the comparative roughness of the bulb covering and perhaps the slightly larger diameter of the wet bulb, caused an excess of energy to be dissipated at the wet bulb by forces of viscosity and by turbulent motion.

21. *Boundary value hypotheses.* During the continuance of the temperature minimum M investigated in sections 8–13, it is assumed that the *total* water vapor content of the air at the wet bulb surface is saturated at wet bulb temperature. These two conditions are attained only when the wick is of such a nature that evaporation takes place from a continuous film of liquid.

When the foregoing conditions are attained, $p_a = b' - P'$ where b' is the total atmospheric pressure at the wet bulb, and P' is the pressure of saturated water vapor at wet bulb temperature. Accordingly, for the *dry air* at the wet bulb, the relation

$$V = (1 - \nu) \frac{RT'}{b - P'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

follows at once from equations (7) and (9).

Similarly, for the *atmospheric vapor* at the wet bulb, we have from equations (8) and (9)

$$V = \nu \frac{BT'}{p'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

in which equation, and in what follows, the symbol p, without subscript, will denote the partial pressure of the *atmospheric vapor.* The symbol p' accordingly denotes the partial pressure of the atmospheric vapor at the wet-bulb surface.

In the case of the *wick vapor,* it is further assumed[3] that, during the steady mean state,

[1] It should be clearly understood that, the bar notation having for convenience been dropped, equation (17) refers to the *mean* motion and state of the gas mixture, which is believed to correspond to its *observed* motion and state. See I, section 2.

[2] See also *supra*, section 17.

[3] This hypothesis was tacitly assumed by August in his derivation of the psychrometric formula contained in his classical memoir, in the *Annalen der Phys. u. Chem.*, 1825, 5, pages 69 and 335. In August's memoir, the wet bulb minimum was however assumed to be permanent.

which we now know to correspond to minimum wet bulb temperature M, the mass of wick vapor, per unit mass of atmospheric air, at the wet bulb, is equal to the mass of wick vapor which has been evaporated by the energy supplied to the wet bulb, per unit mass of atmospheric air during its transit from A_0 to A_1. Accordingly the wick vapor density at the wet bulb is s/V. But the partial pressure of the wick vapor at the wet bulb is $(P' - p')$; consequently

$$V = s\frac{BT'}{P' - p'} \qquad (21)$$

To an accuracy of about 1 per cent, $b' = b$ and since ν is practically invariable along the stream-line under consideration $p' = p$.

Hence, approximately,

$$\frac{\nu}{1 - \nu} = \left(\frac{R}{B}\right)\left(\frac{p}{b - P'}\right)$$

$$\frac{s}{1 - \nu} = \left(\frac{R}{B}\right)\left(\frac{P' - p}{b - P'}\right)$$

and, from (7), (8) and (9), without approximation,

$$1 - \nu = \frac{1}{1 + \left(\frac{R}{B}\right)\left(\frac{p}{b - p}\right)} \qquad (22)$$

22. *Energy formula applied to jet channel apparatus.* With the aid of the last three relations, equation (17) becomes

$$E = \left[\frac{J}{1 + \left(\frac{R}{B}\right)\left(\frac{p}{b - p}\right)}\right]\left[C_p(t - t') + \left(\frac{R}{B}\right)\left(\frac{p}{b - P'}\right)c_p(t - t') - \left(\frac{R}{B}\right)\left(\frac{P' - p}{b - P'}\right)L'\right] \qquad (23)$$

Since R and B occur only in a ratio, we may utilize the following values. For dry air containing 0.0004 of its weight of carbonic acid,[1]

$$R = 0.002\,1520 \frac{\textit{millimeters merc.}}{\left(\frac{\textit{grams}}{\textit{meter}^3}\right) \times \textit{degree abs.}}$$

and for superheated water vapor, up to the saturation line,[2]

$$B = 0.003\,464.$$

For the specific heat of dry air at constant pressure, we have Swann's value [3] $C_p = 0.2417$.

The specific heat of superheated water vapor under constant pressure is very nearly given[4] by $c_p = 0.453$. Furthermore,

$$J = 4.183 \frac{\textit{joules}}{\textit{calorie}}$$

[1] R was calculated from standard air density as given by *Smithsonian Meteorological Tables*, Washington, 1918, Table 98.
[2] See *infra*, Appendix II.
[3] *Lond. Roy. Soc. Phil. Trans.*, 1910, A, **210**, page 230.
[4] See Marks and Davis, *Steam Tables and Diagrams*, New York, 1919, page 97, Fig. 5.

The values of L′, the heat of vaporization of water at wet bulb temperature, and of P′, the pressure of saturated water vapor at wet bulb temperature, were taken from standard tables.[1]

Equation (23) may thus be written

$$E = J(1 - \nu)\left\{0.2417\,(t - t') + 0.281\left[\frac{p\,(t - t')}{b - P'}\right] - 0.621\left[\frac{(P' - p)L'}{b - P'}\right]\right\} \qquad (24)$$

$$J(1 - \nu) = \frac{4.183}{1 + 0.621\left(\dfrac{p}{b - p}\right)} \qquad (25)$$

which relations give E in terms of *joules per gram of atmospheric air.*

[1] P′ was taken from the *Smithsonian Physical Tables*, Washington, 1916; and L′ was taken from Peabody's *Steam and Entropy Tables*, New York, 1912.

PART V

Experimental Evaluation of the Energy Integral E

23. *Experimental procedure.* Hygrometric determinations, which supplied data for the ensuing evaluation of the energy integral E, were made as follows. The apparatus was set up as described in I, sections 5–15 inclusive, and III, sections 2–7 inclusive and section 13.[1] Preliminary to a run of the apparatus, the drying tubes of the chemical hygrometer were cleaned, filled, treated with ether, suspended in the dessicator and finally weighed. The long, glass intake tube of the hygrometer (NM, Figure 3) was thoroughly dried by connecting to a strong aspirator pump for not less than an hour. Meanwhile, the exhauster was started and run at a rate sufficient to draw through it, in about twenty minutes, the entire air content of the room, thus stirring the air into a more homogeneous condition than usually existed at the start. A fresh ice bath was prepared for the cold thermojunctions, and a new, clean wick was installed on the wet bulb. Windows and doors were closed, and steam-pipes shut off. Upon completion of these preparations, the drying tubes were fastened into the chemical hygrometer, which was then tested to ascertain whether it was air-tight.

If this test proved satisfactory, the chemical hygrometer was started and allowed to run for 1 hour and 50 minutes, during which time the flow was maintained very nearly at a prescribed rate by means of the brass stop-cock H (Figure 3).

Meanwhile, the friction cones had been set to give the desired velocity of the air jet, and the wick had been given a thorough preliminary saturation with the water tube just before starting the chemical hygrometer. Precisely five minutes after the chemical hygrometer was started, the wet bulb was again immersed and the minimum temperature M observed; immediately after which the dry bulb temperature was observed. These readings were repeated at regular intervals of ten minutes until five minutes before the chemical hygrometer was closed.

Thus the chemical hygrometer yielded the mean value of the vapor pressure p, mechanically integrated over an interval of 110 minutes; while the mean of the ten readings each of the wet and dry bulb gave approximate mean values integrated over an interval of 100 minutes. The middle points of the two averaging intervals were coincident, and the intervals τ[2] were both long enough and approximated each other sufficiently in magnitude to render the results yielded by the chemical hygrometer comparable with those rendered by the jet channel apparatus.

The barometer (and attached thermometer) were read at intervals of twenty minutes.

[1] The reader is again referred to Appendix I for description and terminology of the chemical hygrometer, with a brief account of precautions essential to its successful operation.

[2] See I, section 2, equation (1).

It should be recalled that static pressure in the jet channel at the dry bulb and at the intake of the chemical hygrometer differed slightly from static pressure in the relatively still air of the room. The value b for both of these instruments was consequently obtained by applying to the barometric pressure a correction obtained from the Krell manometer and a previously prepared calibration curve.

At the end of the run, the chemical hygrometer was again tested for tightness, observations having previously been made of the thermometer R, Figure 3, and the water manometer J. If this test was satisfactory, the drying tubes were removed, treated with ether, suspended in the dessicator for a suitable time and weighed once more.

Data were then averaged, and instrumental corrections applied.

24. *Hygrometric data.* Resulting data for thirty consecutive runs of the apparatus are contained in Table V. Notation is explained in the foregoing pages and in Appendix I on the chemical hygrometer. In the present place, it should however be pointed out that b'' denotes the barometric pressure in the room at the instant when the chemical hygrometer is closed at the end of the run; and b denotes the *mean* value, during the run, of static pressure at the dry bulb and the mouth of the intake of the chemical hygrometer. The volume V_1 of the collecting bottle of the chemical hygrometer is given in Appendix I, and this datum completes the data required to carry out the calculation of results contained in the subsequent tables.

25. *Experimental evaluation of the integral* E. Values of E, calculated from equations (24) and (25) of section 22, from the data of Table V, are contained in Table VI. Dimensional considerations, together with the observed character of the phenomena, suggest a functional dependence expressed by the relation

$$E = F(x) \quad \ldots \ldots \ldots \ldots \ldots \quad (26)$$

where

$$x = \left[\left(\frac{P' - p}{b - P'}\right)\left(\frac{t - t'}{T}\right)\left(q^2 \times 10^{-6}\right)\right] \quad \ldots \ldots \ldots \ldots \quad (27)$$

This relation, plotted from observed data, is shown in Figure 8, where $x \times 10^5$ is set off on the axis of abcissas and E, on the axis of ordinates.[1] From this figure, it is evident that E, for wind velocities up to fifteen meters per second, is independent of wind velocity and also of atmospheric temperature, pressure and humidity. Experiments detailed in a subsequent paper show that E varies from one configuration of material boundaries to another, and is therefore an instrumental constant.

For the apparatus under consideration in the present paper, the most probable value of E is 0.84 joule per gram. The probable error of this determination is ±0.035 joule per gram, or 4 per cent of the most probable value of E.[2]

[1] The velocity q of the air jet is expressed in centimeters per second.
[2] Subsequent determinations yielded, for a different type of apparatus, the value 0.727 to an accuracy of 4 per cent. (See the fifth paper of the present series.)

26. E *defined throughout a restricted range of air jet velocities.* In connection with this evaluation of the energy integral, it should be emphasized in accordance with definitions laid down in I, section 2 and in III, sections 17 and 20, that E is not defined except when an eddy exists around both the wet bulb and the dry bulb. On account of differences in the nature of the surfaces of these two bulbs and their different diameters, the critical

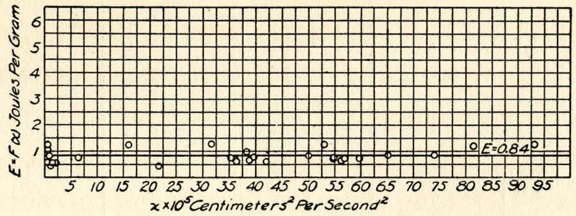

FIGURE 8. The energy integral E as a function of

$$\left[\left(\frac{P'-p}{b-P'}\right)\left(\frac{t-t'}{T}\right)(q^2\times 10^{-6})\right]\times 10^5.$$

velocities of these eddies were probably not the same. Let q_c denote the larger of these two critical velocities; then E is defined when and only when[1] the mean velocity of the working portion of the air jet satisfies the condition

$$q_c < q \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (28)$$

27. *The function* E *is determinate over restricted ranges of wet bulb depression and of absolute atmospheric humidity. Dynamical cooling without evaporation.* As an immediate consequence of results set forth in section 25, it follows that the function E is undefined in atmospheric air near saturation, as will presently be shown. Let us suppose a case where atmospheric vapor is so nearly saturated that it saturates when cooled to wet bulb temperature; that is to say, let us suppose that

$$p = P'$$

If, for example, under these conditions,

$$t' = 15^\circ \text{ C.} \qquad \text{and} \qquad b = 760 \text{ mm}$$

then

$$p = P' = 12.78 \text{ mm}$$

and dry bulb temperature t would be 15°.8, showing a wet bulb depression of 0°.8. Thus, the atmospheric vapor lacks a little of saturation,[2] and there is a small wet bulb depression, while at the same time there is no evaporation at the wet bulb.

This effect, evidently due to turbulence and viscosity (and referred to, for brevity, in these pages as *dynamical cooling*) is probably caused by the difference between the rough surface of the wet bulb, and the smooth, polished surface of the dry bulb.

[1] Accordingly, the straight line shown in Figure 8 ends abruptly before reaching the E-axis.
[2] The pressure of saturated water vapor at 15°.8 C. is about 13.46 mm.

TABLE V

HYGROMETRIC DATA

No.	Chemical Hygrometer				Psychrometer			
	b'' mm	t'' C.°	h mm merc.	m mgr	Forced draft m/s	Dry bulb C.°	Wet bulb C.°	b mm
1	757.52	17.81	7.74	129.93	3.8	19.77	12.03	757.60
2	764.10	17.75	9.26	127.58	3.8	17.91	11.20	763.98
3	759.73	24.21	4.70	306.27	3.8	25.99	21.02	759.80
4	761.21	23.39	4.26	265.42	3.8	24.75	19.26	761.15
5	759.41	23.50	5.59	287.97	3.8	25.44	20.28	759.41
6	754.77	18.90	4.78	234.01	3.8	21.01	16.51	754.69
7	759.92	17.83	4.93	197.08	3.8	17.85	14.11	760.12
8	762.30	17.21	5.29	210.15	3.8	17.13	14.35	762.93
9	766.49	20.80	6.03	102.41	13.4	27.66	13.96	765.79
10	767.14	20.20	8.38	102.00	13.4	20.84	11.03	766.63
11	757.98	17.04	4.93	158.18	13.4	19.13	12.68	757.89
12	755.98	15.65	5.07	80.48	15.0	16.36	7.97	754.87
13	755.06	19.27	5.66	82.75	15.0	19.00	9.36	754.21
14	761.40	20.41	5.59	81.51	15.0	20.29	9.97	760.50
15	750.68	22.53	7.87	86.32	15.0	21.09	10.54	749.37
16	750.61	21.38	3.68	87.48	15.0	20.44	10.29	749.54
17	751.42	18.36	6.11	76.06	15.0	17.82	8.44	749.76
18	756.36	18.13	5.44	73.68	15.0	17.52	8.22	754.78
19	764.08	21.52	4.56	72.18	15.0	21.17	9.96	763.45
20	770.29	18.36	5.14	68.72	15.0	17.71	8.18	768.39
21	775.52	15.91	5.29	54.41	15.0	15.11	6.16	775.06
22	768.09	16.20	5.07	91.19	15.0	15.08	8.13	766.43
23	779.87	20.89	3.24	63.50	15.0	24.40	10.81	778.70
24	755.22	21.85	2.65	61.91	15.0	23.68	10.29	753.50
25	757.94	28.61	2.72	80.16	15.0	30.99	14.18	757.17
26	766.24	20.66	7.72	33.47	15.0	18.34	6.62	764.76
27	760.62	18.50	4.19	67.40	15.0	17.61	7.96	759.87
28	754.06	17.95	3.82	84.03	15.0	17.64	8.82	752.36
29	770.84	17.53	3.16	50.75	15.0	18.21	7.45	769.27
30	767.81	20.10	4.48	58.02	15.0	19.75	8.62	766.71

When atmospheric vapor in the air is saturated, dynamical cooling still takes place; as a consequence of which, instead of formation of wick vapor at the wet bulb, atmospheric vapor is condensed out of the air. Under these circumstances, boundary conditions[1] leading to the deduction of equations (24) and (25) are not satisfied. These equations are therefore valid only when

$$p \leqq P' \qquad (29)$$

[1] See *supra*, section 21.

TABLE VI

THE ENERGY INTEGRAL E AS A FUNCTION OF x, WHERE

$$x = \left[\left(\frac{P' - p}{b - P'}\right)\left(\frac{t - t'}{(273.1 + t)}\right)(q^2 \times 10^{-6})\right]$$

No.	q m/s	$x \times 10^5$	E joules per gram	No.	q m/s	$x \times 10^5$	E joules per gram
1	3.8	1.49	0.57	16	15.0	41.9	0.60
2	"	1.28	0.47	17	"	74.2	0.83
3	"	0.63	1.04	18	"	38.5	0.99
4	"	1.04	0.81	19	"	57.0	0.72
5	"	0.65	0.76	20	"	38.8	0.65
6	"	0.58	1.24	21	"	36.4	0.67
7	"	0.48	0.53	22	"	21.6	0.41
8	"	1.78	0.55	23	"	93.4	1.27
9	13.4	65.2	0.86	24	"	81.6	1.20
10	"	31.7	1.28	25	"	133.5	1.49
11	"	16.0	1.23	26	"	59.8	0.73
12	15.0	35.4	0.77	27	"	39.7	0.79
13	"	49.8	0.85	28	"	36.3	0.64
14	"	52.8	1.26	29	"	54.7	0.71
15	"	54.6	0.75	30	"	56.4	0.67

Most probable value of E.. 0.84
Probable error of this determination.. ±0.03

This condition, in turn, imposes a restriction [1] also upon wet bulb depression, namely: [2]

$$(t - t') \geqq \left(\frac{E}{1.011}\right)\left[\frac{1 + 0.621\left(\dfrac{P'}{b - P'}\right)}{1 + 1.162\left(\dfrac{P'}{b - P'}\right)}\right] \quad \ldots\ldots\ldots\ldots\ldots \quad (29a)$$

28. *Experimental ranges of the hygrometric variables.* Experimental data leading to the foregoing conclusions were representative of ordinary atmospheric conditions at sea level. The experimental ranges of the hygrometric variables are shown in Table VII.

TABLE VII

EXPERIMENTAL RANGES OF THE HYGROMETRIC VARIABLES

Velocity of air jet	3.8 to 15.0 m/s
Atmospheric vapor pressure	1.83 to 16.66 mm merc.
Relative humidity	12 to 76 per cent
Dry bulb temperature	15.08 to 30.99 C.°
Wet bulb temperature	6.16 to 21.02 C.°
Static pressure in the channel	749.37 to 778.70 mm

[1] Derived from relations (24) and (25).

[2] Direct experimental verification of restrictions (29) and (29*a*) with saturated atmospheric vapor is not essential, in view of the very complete experimental confirmation of the practical validity of formulas (24) and (25) contained in the foregoing pages and in the fourth and fifth papers of the present series. Professor Alexander McAdie, however, has kindly made some tests for the writer with his whirling psychrometer at the Blue Hill Observatory, under conditions of fog formation taking place on the spot, which seemed clearly to indicate a saturated condition of the vapor in the atmosphere. Repeated tests invariably yielded an appreciable wet bulb depression, indicating that heat of condensation was insufficient to cause wet bulb depression, due to dynamical cooling, to disappear.

PART VI

Summary

Boundary conditions on a wet obstacle in a turbulent air current. Observed wick characteristics. Apparatus described in the two preceding papers of this series was used for the present investigation. Thermo-couples were enclosed in small-diameter, thin-walled glass tubes partly filled with light oil, and inserted in the wind channel. One of these tubes was covered with a wick, and tests were run with wicks of various types, for the purpose of finding a means of maintaining experimentally determinate boundary conditions at the wet bulb surface. These tests demonstrate that the mean temperature of the wet bulb, when the latter is exposed to a current of air immediately after wetting, falls to a minimum M, and remains there for an appreciable length of time, with a degree of steadiness proportional to the steadiness of the mean state of the atmospheric air current to which it is exposed. Additional tests, with a wet and a dry bulb simultaneously exposed to a uniform turbulent air current, show that (under important restrictions which will be set forth under the next heading) the difference between the readings of the two instruments (wet bulb depression) depends (*a*) on atmospheric temperature, pressure and humidity, and (*b*) on the configuration of solid boundaries of the apparatus.

Experimentally determinate boundary conditions. Boundary conditions, however, are experimentally determinate, and wet bulb depression M depends only upon the foregoing factors, when the following conditions are realized:

(1) The wet bulb must be thermally insulated from its surroundings.

(2) It must be shielded from radiation exchanges with neighboring material at a different temperature.

(3) The wick must consist of bulb-covering only, without continuous feed of any kind. When water, for example, is supplied to the bulb-covering from a cistern, by the capillary action of a wick, the temperature minimum M depends upon a relation between dryness of bulb-covering and rate of capillary flow, and this relation is experimentally indeterminate.

(4) The wick must be completely saturated by immersion in water immediately before exposure to the air current.

(5) The material, texture and shape of the wick must be such as to maintain, exposed to the air, a continuous film of liquid on the enclosed glass tube for an appreciable length of time.

A typical set of wick characteristic curves is shown in the figures.

Energy transformations at the wet bulb and at the dry bulb in a turbulent current of moist air. Differential equations, set up for the air current in the wind channel containing the thermocouple bulbs, show that the work done by forces of diffusion cannot be dissipated in heat nor in causing the moist air to perform dilatation work against mean static pressure. Observed conditions admit of a simple, approximate integration of these equations, thus making possible measurements of the algebraic sum of the following energy transformations taking place along any mean stream-line between two points at which boundary conditions can be ascertained: (1) heat energy conducted to (or from) the air mass under consideration by virtue of space rates of change of mean air temperature, (2) energy dissipated by the internal stresses due to space rates of change of mean deformation velocities, (3) heat energy of turbulent motion dissipated by the air, and (4) *translatory* work of the air against mean static pressure.

Measurements show that this algebraic sum is greater when the integration is performed along a mean stream-line from still air to the wet bulb, than it is when the integration is performed along a mean stream-line from still air to the dry bulb. This may be due *in part* to larger space variations of mean temperature near the wet bulb. But since this difference persists when atmospheric conditions prevent evaporation from the bulb-covering, it is to be inferred that the comparatively rough surface of the bulb-covering intensifies turbulent motion, and increases space rates of change of mean deformation velocities of the air at the wet bulb, thus increasing there the work dissipated by the viscous stresses in the air, and consequently augmenting the algebraic sum under consideration.

Effect of wind velocity and atmospheric conditions on energy transformations at the wet bulb. Measurements made under a variety of atmospheric conditions show that the excess E, at the wet bulb, of the magnitude of the sum of the energy transformations above described, over its magnitude at the dry bulb, is (within certain limits) independent of mean flow velocity and atmospheric conditions.[1] The energy integral E is, in fact, very approximately an instrumental constant.

Effective range of forced draft velocities. This observed constancy of E holds throughout a range of mean flow velocities greater than the critical velocity of the forced draft or of the bulb eddies, and less than 15 meters per second. At velocities above 15 meters per second, fine spray is carried away from the bulb-covering by the forced draft, thus rendering boundary conditions experimentally indeterminate.

A necessary condition restricting the effective range of wet bulb depressions. Dynamical cooling, an effect of turbulence and viscosity analyzed in the text, gives rise, in general, to part of the observed depression of the wet bulb. When the water vapor in the air is sufficiently near saturation, the entire observed wet bulb depression is due to this effect. Bound-

[1] Further confirmation of this result is contained in the fifth paper above referred to.

ary conditions at the wet bulb are accordingly shown to be experimentally determinate only when the following necessary condition is imposed upon wet bulb depression, namely:

$$(t - t') \geqq \left(\frac{E}{1.011}\right)\left[\frac{1 + 0.621\left(\frac{P'}{b - P'}\right)}{1 + 1.162\left(\frac{P'}{b - P'}\right)}\right]$$

where

t = dry bulb temperature,
t' = wet bulb temperature,
P' = pressure of saturated water vapor at wet bulb temperature t',
b = barometric pressure,
E = an energy integral evaluated by the writer and shown to be an instrumental constant.[1]

A description of the chemical hygrometer used in this investigation, together with a list of references relating to that instrument, is contained in Appendix I.

APPENDIX I

The Chemical Hygrometer

The essential parts of the chemical hygrometer used for the present purpose are shown in Figure 3 (section 6). Air was drawn from the jet channel through the drying tubes D and E by means of an aspirator B, A. A sufficient number of drying tubes was used to absorb the entire moisture content of the air passing through them. To ascertain the mass of moisture thus absorbed, the drying tubes were weighed before and after each run.

29. *Equation of the chemical hygrometer.* A definite mass of moist air, drawn through the drying tubes during a run, was obtained by means of tube MN (Figure 3) from a point f (I, Figure 1) in the jet channel near the dry bulb of the psychrometer. The chemical hygrometer thus rendered the mean hygrometric state of the moist air, at the psychrometer dry bulb, during a run.

This mean hygrometric state may be denoted by the following variables:

M_d = weight, in grams, of the dry air contained in the mass of moist air under consideration,
v = the volume, in cubic meters, which this mass of moist air, drawn into the drying tubes, would occupy in its average state at point f during the run.

The average volume of the M_d grams of dry air in this mass at point f was therefore v, as was also the average volume at point f of the water vapor contained in the given mass of moist air.

[1] Temperatures in the above formulas are measured on the Centigrade scale, and pressures are measured in millimeters of mercury. E is in joules per gram of air.

p = the average pressure of water vapor at point f during the run. It is here expressed in millimeters of mercury.
t = average air temperature, in Centigrade degrees, at point f during the run. This quantity was given by the dry bulb of the psychrometer.
b = average static pressure at point f during the run. This pressure, which was what we have called static pressure in the wind channel, was obtained by subtracting from the barometric pressure in the room the reduction of pressure in the channel axis as given by the Krell manometer. Observed values are given above in the ninth column of Table V.

At the instant when the water level in collecting bottle A (Figure 3) had reached an engraved mark on the small part of the neck, which indicated that a volume of V_1 cubic meters of water had been drawn off from the aspirator bottle B, all stop-cocks (except b, Figure 3) were closed. To insure saturation of the air which had accumulated in B, final readings of the thermometer R and manometer J were not made until ten minutes had elapsed after closing the aspirator, at which time:

t'' = the temperature of the saturated mixture in the bottle, as given by thermometer R,
h = the pressure difference obtained with the aid of the *water* manometer J, but reduced to terms of millimeters of *mercury*,
b'' = barometric pressure in the room,
P'' = pressure of saturated water vapor in the aspirator bottle,
V_1 = volume, in cubic meters, of water drawn into the collecting bottle; and hence the volume of the air in bottle B, which had been drawn through the drying tubes.

The mean state of the dry air at point f during the run can thus be expressed by the characteristic equation of dry air:

$$(b - p)\,\frac{v}{M_d} = 273.1\,R(1 + \alpha t)$$

where

$$\alpha = \frac{1}{273.1}$$

Ten minutes after the aspirator had been closed, the state of the dry air in the aspirator was expressed by the equation

$$[(b'' - h) - P'']\,\frac{V_1}{M_d} = 273.1\,R\,(1 + \alpha t'')$$

whence

$$\frac{V_1}{v} = \left[\frac{b - p}{b'' - (h + P'')}\right]\left[\frac{1 + \alpha t''}{1 + \alpha t}\right]$$

But if ρ grams per cubic meter was the average density of the water vapor at point f during the run, and m grams of water vapor were absorbed in the drying tubes, then, since the air was completely dried in passing through the absorbing media,

$$\rho = \frac{m}{v}$$

hence, from the characteristic equation of water vapor (see Appendix II), we have

$$p = b\left(\frac{Q_1}{1 + Q_1}\right) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (30)$$

where

$$Q_1 = \left[\frac{(273.1)\,B}{V_1}\right]\left[\frac{1 + \alpha t''}{b'' - (h + P'')}\right] \cdot m \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (31)$$

For purposes of calculation with a 20-inch slide rule, equation (30) may be written in the very convenient form [1]

$$\left.\begin{aligned} p_1 &= Q_1 b \\ p &= p_1 - p_1 Q_1 \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (32)$$

The last term of the second of relations (32) is of the nature of a small correction.

30. *The aspirator.* The aspirator bottle B (Figure 3) had a capacity of about 30 liters. A fine mark was etched on the narrowest part of the neck of the collecting bottle A, which when filled to the level of this mark was found to contain 0.019371 cubic meters at 20°C. Hence the volume V_1 was given by

$$V_1 = 0.019371\,[1 + 0.000025\,(t - 20)] \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (33)$$

cubic meters, where t denotes room temperature, and 0.000025 is the cubical expansion coefficient of the glass.

This equation refers to the collecting bottle used in Hygrometric Tests 46–89. For Tests 1–45, another collecting bottle was used, for which

$$V_1 = 0.019259\,[1 + 0.000025\,(t - 20)] \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (34)$$

cubic meters.

Before the weighings were made to ascertain the volume of the collecting bottle, its inner surface was made thoroughly wet. The bottle was then inverted and set to drain for 10 minutes. This precaution was repeated before every run of the hygrometer, in order to insure as nearly as possible a uniform mass of water adhering to the inner surface of the collecting bottle at the beginning of the run.[2]

A 4-millimeter glass tube *a* (Figure 3) was inserted in the stopper of the collecting bottle to allow the escape of displaced air without appreciable evaporation. To refill the aspirator after a run, glass tube L (Figure 3) was inserted in the collecting bottle and attached to tube SK at K. The rubber tube QP was then connected with an aspirator pump run by

[1] Tables for $(1 + \alpha t)$ and its logarithm are contained in the *Smithsonian Physical Tables*, Washington, 1916. The value of B in terms of the units under consideration is given in section 37, *infra.*

[2] At the end of a run when the aspirator bottle B was nearly empty, it was assumed that its content of moist air was at a uniform temperature. But this was not precisely the case. In order to render the temperature more uniform, the aspirator bottle was immersed in a large water bath (Figure 2) provided with an efficient stirrer. This water bath was usually at nearly room temperature. Reduced pressure in the aspirator bottle was however accompanied by cooling; so much so that the temperature rendered by thermometer R was usually from 0°.2 to 0°.3 C. lower than that of the bath. Whether the water bath appreciably increased the accuracy of the apparatus has not at the present time been satisfactorily ascertained.

For some of the tests, a thermo-couple set replaced thermometer R.

the city water, and all stop-cocks were closed. In this way the aspirator could be refilled without disturbing the apparatus. After refilling the aspirator, rubber tube QP was removed, and openings Q and K were closed by means of rubber connections plugged with short lengths of glass rod.

It should be added that the rate of discharge from the aspirator bottle could be governed by the brass stop-cock H adjusted in accordance with a discharge table.

31. *Absorbing media.* During preliminary experiments with the chemical hygrometer, phosphorus pentoxide was used in all of the drying tubes. On account of difficulty in obtaining the pure substance, a quantity was sublimed and sealed up in small bottles for subsequent use. The process of subliming phosphorus pentoxide is exceedingly laborious and consumes much time. Chemically pure sulphuric acid was consequently used in the first tube, and pure phosphorus pentoxide was used in the second one. Repeated tests with a train consisting of one sulphuric acid tube followed by two pentoxide tubes showed that the amount of water vapor absorbed by the second pentoxide tube was so small that it could not be detected by weighing; in fact the sulphuric acid tube alone took up 99 per cent or more of the total mass of water vapor absorbed.

32. *Drying tubes.* Small stop-cock U-tubes of standard pattern were used to hold the absorbing media. When filled, one of these tubes would weigh from 50 to 60 grams. Ramsay's stop-cock grease was used to render the ground glass surfaces air-tight. Sulphuric acid tubes were filled with chemically clean glass beads about 5 millimeters outside diameter and 5 millimeters long. Smaller beads seriously obstructed the air flow. Enough acid was then poured over the beads to form a meniscus in the tube, and a little more; in fact, bubbling the air through a head of about 3 millimeters of the liquid in either arm gave the best results, the remainder of the beads being only moistened with a film of the acid. Before each run, the beads were thoroughly moistened, care being taken not to bring the acid into contact with the stop-cock grease.

Phosphorus pentoxide tubes were prepared by filling about one third full with alternate layers of chemically clean, dry, glass beads and phosphorus pentoxide.

The sulphuric acid tubes were suspended in a vertical position during a run (Figures 2 and 3), but the pentoxide tubes were laid nearly horizontal. In this position, the powder was spread out evenly to allow the air to pass over it. The glass beads were of material assistance in thus spreading it out. A sulphuric acid tube could be used eight or nine times without refilling; a pentoxide tube, on account of the small amount of vapor absorbed, could be used with advantage twenty to thirty times.

It might be of interest to record that a few tests were made by drawing moist air, from a saturator, through spiral tubes immersed in liquid air. The spirals were made of 5-millimeter (o.d.) "Pyrex" glass tubing, and the coils had an outside diameter of 3.5 centimeters. Moist air was drawn through the first spiral, which had fourteen turns, into a second one with five turns. The air flow was adjusted to about 17 liters per hour. A marked accumulation of ice occurred in the first tube at the liquid air level. No trace whatever of a deposit was observable in any other part of the coil. But a large part of the vapor passed through the fourteen turns of the first coil, probably in the form of fine ice particles invisible to the naked eye. For, upon emerging from the first tube above the liquid level, the air was warmed and gave up an additional deposit at the liquid air level on the wall of the second tube. Apparently, therefore, upon emerging into the warm connecting tube, enough ice particles were melted to form the deposit in the second tube referred to. The remainder of the vapor passed through the five turns of the second tube without leaving a perceptible deposit.

Had the air been completely dried, about 87 milligrams of vapor would have been deposited in the spirals. As a matter of fact, 16 milligrams were deposited at the entrance of the first spiral, and 2 milligrams were deposited at the entrance of the second spiral. Thus 69 milligrams passed through the entire nineteen turns of the spirals immersed in liquid air without leaving a trace.

33. *Precautions.* All rubber connections were made as short as possible by bringing the ends of the enclosed glass tubing into contact with each other. Glass was used throughout except for the tube QP (Figure 3), which was removed when the apparatus was in operation. Before installation, all rubber connections were boiled in a 10 per cent solution of sodium hydroxide and washed in distilled water, and all glass parts were soaked in cleaning solution and rinsed in distilled water. All rubber stoppers were hermetically sealed. The entire apparatus was tested before and after each run by means of a jet pump and the manometer J, in order to ascertain whether all joints were air-tight. The long glass feed tube NM was thoroughly dried before each run by means of an aspirator pump.

34. *Standardization of weights.* A 50-gram set of analytical weights was used for weighing the drying tubes. For calibrating the collecting bottle, a good set of kilogram weights was used. These two sets were, of course, cross-standardized. But it was not necessary to refer the result to an absolute standard. For, referring to equation (31), it is clear that the *weighings* enter only through the ratio m/V_1. Accordingly, let

M_1 = weight of water contained in volume V_1 and expressed in terms of the arbitrary standard used for the cross-standardization.

Let

M_2 = weight of absorbed water vapor expressed in the same units. Then, if we denote by [W] the true weight of the arbitrary standard, we have

$$\frac{m}{V_1} = \frac{M_1\,[W]}{\dfrac{M_2\,[W]}{\delta}} = \frac{M_1\delta}{M_2}$$

where δ is the density of water.

35. *The weighings.* As was stated in section 32, the drying tubes when filled weighed from 50 to 60 grams. But the mass of vapor absorbed was relatively small, sometimes not exceeding 0.05 gram. If the ordinary method of weighing by substitution had been used, corrections for the buoyancy of air would have been of the order of from 0.0030 to 0.0170 gram, representing from 6 to 34 per cent of the mass of vapor absorbed. It is well known that the per cent error to which these corrections are liable is considerable, on account of uncertainty as to the exact density of tubes, beads, the reagents and the air. The weighings were therefore made by the counterpoise method, as follows:

A counterpoise tube was made for each drying tube, as nearly a replica of the latter as possible, except that it contained no reagent but was given the proper weight (about 0.9 gram more than its drying tube) by loading with clean, dry, glass beads.

A weighing was begun by accurately balancing a counterpoise tube by means of tare weights and rider. The counterpoise was then replaced on the balance by its drying tube, and the excess weight of the former ascertained by means of small weights in the pan under the drying tube, together with the standardized rider. Buoyancy of the air on the small weights was negligible. The excess was again found after absorption, and the difference between the two excesses gave the true mass of the vapor absorbed.

The use of a larger apparatus, absorbing greater masses of water vapor, was impracticable, for, on account of spraying and dust effects caused by too rapid a flow of air, it would require an extremely long time for a run, and this would be incompatible with the requirements of psychrometric determinations.

All of the tubes were invariably weighed with stop-cocks open; because, if weighed with stop-cocks closed, a change of atmospheric density would cause the result of the weighing to contain the difference between the mass of the enclosed air and that of an equal volume of atmospheric air.

Since a heavy load decreases the sensitiveness of the balance, a limit was imposed upon the accuracy of the weighings by the relatively large load. Under a load of 50 grams, the Troemner analytical balance employed was sensitive to 0.25 milligram per division of the ivory pointer scale. By using a sufficient number of vibrations, the indication of the pointer could be estimated accurately to the nearest tenth of a division.

Preliminary tests showed that a 50-gram load had a tendency slightly to distort the beam of the Troemner balance. The result was a variable sensitivity. This difficulty was overcome by loading both pans of the balance with 50-gram weights and allowing the beam to support the full load for not less than 2 hours before each weighing.

Before every weighing, the drying tubes and their counterpoise tubes were rubbed with a chemically clean cloth wet with ether, which removed all traces of dust, finger marks and stop-cock grease from the outside surfaces. The tubes were then suspended for at least one hour in a dessicator to render their outside surfaces dry.

36. *Bibliography of the chemical hygrometer.* The following is a list in chronological order of some additional references to the literature of this interesting and important apparatus.

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APPENDIX II

THE EQUATION OF STATE FOR SUPERHEATED WATER VAPOR

37. *Elastic properties of superheated water vapor.* Knoblauch, Linde and Klebe[1] give for the characteristic equation of superheated water vapor the relation

$$pv = \mathrm{B}\mathrm{T} - p(1 + 0.000\,002p)\left[\frac{1609}{\mathrm{T}^3} - 0.000\,0052\right]$$

where

$$\mathrm{B} = 0.003\,464$$

provided that p is measured in millimeters of mercury; v, in cubic meters per gram; and T, on the absolute scale. This equation purports to hold for superheated water vapor up to and upon the saturation line.

We are here concerned only with the range of temperatures from 0°C. up to, say, 35°C., and we proceed therefore to compare the term

$$\Psi_1 = p(1 + 0.000\,002p)\left[\frac{1609}{\mathrm{T}^3} - 0.000\,0052\right]$$

with the term BT. Evidently, at any given temperature, Ψ_1 will attain its greatest relative values on the saturation line. The accompanying table indicates the largest possible error that can be incurred in this range of temperatures by the suppression of the term Ψ_1.

[1] *Forscharbeitungen*, Berlin, 1905, **21**, pages 33–55. See also Marks and Davis, *Steam Tables and Diagrams*, New York, 1919, page 98.

$t°$ C.	BT	Ψ_1	*Maximum error*
0	0.9461	0.00034	0.04 per cent of BT
10	0.9807	0.00060	0.06 " " " "
20	1.001	0.00103	0.1 " " " "
30	1.023	0.00148	0.1 " " " "
35	1.067	0.00209	0.2 " " " "

For the study of atmospheric humidities, the superheated water vapor in the atmosphere may accordingly be regarded as a perfect gas, whose characteristic equation, to a precision within two-tenths of one per cent is

$$p = \rho\, BT$$

APPENDIX III

Influence of Heat Radiation upon the Energy Integral E

38. *Numerical considerations.* The very effective screening of wet and dry bulb consistently adhered to throughout the present investigation[1] doubtless reduced greatly the effects of radiation upon the numerical determination of the energy integral E.

In order to estimate the magnitude of errors due to absorption by the wet bulb of radiation energy in excess of the radiation energy which it emitted, we may calculate approximately the error which would be incurred in the most unfavorable possible case, *viz.*: if the walls of the channel and the screens were emitting black-body radiation. Now a segment of the wet bulb one centimeter long had a lateral area of about 0.840 square centimeter, and the area of its longitudinal cross-section was 0.267 square centimeter. Assuming a standard air density of 0.00120 gram per cubic centimeter, it is easy to show that a slab-shaped air mass, having this cross-section and weighing one gram, would be carried past the wet bulb by a wind velocity of 15 meters per second in about 2 seconds.

If, at the same time, the channel walls and screens were at 20°C. and the wet bulb was at 10°C., then calculation from the Stefan Law of Black-Body Radiation shows that the radiation absorbed by the wet bulb during this time in excess of radiation emitted would have been 0.0096 joule.

If this slab-shaped mass of air, as it passed the wet bulb, was cooled to wet bulb temperature t', then its loss of internal energy due to cooling was, very roughly,

$$JC_v\,(t - t') = (JC_p - R)\,(t - t')$$
$$= 0.287 \times 10 = 2.87 \textit{ joules}$$

Thus the radiation effect would be only about three-tenths of one per cent of loss of heat energy due to fall of temperature at constant volume.

[1] I, sections 16 and 17.

But the radiation emitted by the burnished silver surface of the channel was of notably smaller intensity than the radiation of a black body. This very roughly approximate calculation therefore indicates sufficiently well that radiation effects did not enter appreciably into the observed phenomena. That such was the case is further evidenced by the fact that the experimental errors incurred in evaluating E (see Figure 8, section 25) did not diminish with wind velocity, as would surely have been the case had radiation been a cause of systematic error. The errors observable in the thirty determinations shown in Table VI are therefore strictly *non-systematic* and due to the enormous technical difficulty of measuring the extremely small differences of pressure and temperature characterizing a current of air in steady mean motion, and also to mathematical approximations utilized in deriving equation (24).

AERODYNAMICS OF THE PSYCHROMETER

By George Porter Paine

Fourth Paper: An Aerodynamic Formula for the Psychrometer, and Its Experimental Verification

INDEX TO SECTIONS

PART I

Derivation of a Psychrometric Formula

1. *Consequences of the energy formula derived in the preceding paper.* If equation (23) of section 22 of the third paper of the present series be solved for p, the mean pressure of the atmospheric vapor (the vapor existing in the air exclusive of wick vapor) is found to be given by the relations

$$p = \frac{1 + C_p\left(\frac{B}{R}\right)\left(\frac{t-t'}{L'}\right)}{1 + c_p\left(\frac{t-t'}{L'}\right)} P' - \frac{C_p\left(\frac{B}{R}\right)\left(\frac{t-t'}{L'}\right)}{1 + c_p\left(\frac{t-t'}{L'}\right)} b$$

$$+ \left[\frac{\left(\frac{B}{R}\right)E}{JL'}\right]\left[\frac{1}{1-\nu}\right]\left[\frac{b - P'}{1 + c_p\left(\frac{t-t'}{L'}\right)}\right] \quad \ldots\ldots\ldots\ldots \quad (1)$$

$$1 - \nu = \frac{1}{1 + \left(\frac{R}{B}\right)\left(\frac{p}{b-p}\right)} \quad \ldots\ldots\ldots\ldots \quad (2)$$

The coefficient of P′ is, in practice, very nearly equal to unity. Since the last term of the right-hand member is small compared with the first term, it is sufficiently accurate to take for L′ the heat of vaporization of water at 15°C., or 588 calories per gram. Inserting the values of the constants which are given in the paper above referred to, we have

$$p = P' - \frac{0.3891\left(\frac{t-t'}{L'}\right)}{1 + 0.453\left(\frac{t-t'}{L'}\right)} b$$

$$+ (6.55 \times 10^{-4} E)\left(\frac{1}{1-\nu}\right)\left[\frac{b - P'}{1 + 0.453\left(\frac{t-t'}{L'}\right)}\right] \quad \ldots\ldots\ldots \quad (3)$$

and

$$\frac{1}{1-\nu} = 1 + 0.621\left(\frac{p}{b-p}\right) \quad \ldots\ldots\ldots\ldots \quad (4)$$

2. *Practical form of the psychrometric formula.* A very rough approximation can be obtained by omitting the last term of (3), but this approximation is sufficiently accurate

for the calculation of $1/(1-\nu)$ from equation (4). The psychrometric formula may accordingly be written in the following convenient form:

$$\left.\begin{aligned} p_1 &= P' - \frac{0.3891\left(\frac{t-t'}{L'}\right)}{1+0.453\left(\frac{t-t'}{L'}\right)}b \\ \frac{1}{1-\nu} &= 1+0.621\left(\frac{p_1}{b-p_1}\right) \\ Q &= (6.55\times 10^{-4}\,E)\left(\frac{1}{1-\nu}\right)\left[\frac{b-P'}{1+0.453\left(\frac{t-t'}{L'}\right)}\right] \\ p &= p_1 + Q \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (5)$$

This formula is general. The quantity E is, as has been demonstrated,[1] an instrumental constant, which must be determined with the aid of a chemical hygrometer.

The above formulas are greatly simplified by observing that the product

$$\left(\frac{1}{1-\nu}\right)\left[\frac{1}{1+0.453\left(\frac{t-t'}{L'}\right)}\right]$$

is, under all ordinary atmospheric conditions, very nearly equal to unity. With this approximation, formulas (5) reduce to the following:

$$\left.\begin{aligned} p_1 &= P' - \frac{0.3891\left(\frac{t-t'}{L'}\right)}{1+0.453\left(\frac{t-t'}{L'}\right)}b \\ Q &= (6.55\times 10^{-4}\,E)\,(b-P') \\ p &= p_1 + Q \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (6)$$

and, if P denotes the pressure of saturated water vapor at the temperature of the atmospheric air,

$$(\textit{Relative humidity}) = \frac{p}{P} \quad \ldots\ldots\ldots\ldots\ldots \quad (7)$$

Mathematical approximations leading to formulas (6) should be expected to yield relative humidity to a precision of about one per cent.

3. *Psychrometric results experimentally indeterminate in air containing water vapor in a state near saturation. Formula for restricted range of wet bulb depression.* In the third paper of this series, it was shown that dynamical cooling of the wet bulb, due to turbulence

[1] See also the next paper, V, section 11.

and viscosity of the air, can take place without evaporation, and that, in consequence, certain fundamental boundary conditions at the wet bulb are satisfied only when wet bulb depression occurs within the range defined by the relation

$$(t - t') \geqq \left(\frac{E}{1.011}\right)\left[\frac{1 + 0.621\left(\frac{P'}{b - P'}\right)}{1 + 1.162\left(\frac{P'}{b - P'}\right)}\right] \quad \ldots\ldots\ldots\ldots \quad (8)$$

Atmospheric humidity is calculable from psychrometric data only under conditions admitting of a wet bulb depression satisfying relation (8).

PART II

Practical Validity of the Psychrometric Formulas

4. *Experimental results.* In the third paper of this series, it was shown how the instrumental constant E was determined for the jet channel apparatus and found to be equal to 0.84 joule per gram.[1]

Results calculated with the aid of formulas (6) and (7), from data contained in the third paper of the series,[2] are shown in Table I, in the column entitled Calculated Relative

TABLE I

HYGROMETRIC RESULTS FROM THE JET CHANNEL

No.	q m/s	Q	Calculated relative humidity (per cent)	Observed relative humidity (per cent)	Difference	
					positive	negative
1	3.8	0.41	41.1	40.3	0.8	...
2	"	0.42	45.8	44.4	1.4	...
3	"	0.41	65.8	66.1	...	0.3
4	"	0.41	61.2	61.2	0.0	...
5	"	0.41	64.2	64.0	0.2	...
6	"	0.41	65.7	66.7	...	1.0
7	"	0.41	69.2	68.3	0.9	...
8	"	0.41	76.9	76.0	0.9	...
9	13.4	0.42	19.8	20.0	...	0.2
10	"	0.42	29.0	30.1	...	1.1
11	"	0.41	49.3	50.4	...	1.1
12	15.0	0.41	30.8	30.5	0.3	...
13	"	0.41	27.1	27.1	0.0	...
14	"	0.41	24.9	24.7	0.2	...
15	"	0.42	25.6	25.2	0.4	...
16	"	0.41	27.0	26.4	0.6	...
17	"	0.41	26.8	26.6	0.2	...
18	"	0.41	26.5	26.3	0.2	...
19	"	0.42	21.2	20.8	0.4	...
20	"	0.42	24.8	24.2	0.6	...
21	"	0.42	23.1	22.4	0.6	...
22	"	0.42	39.3	37.6	1.7	...
23	"	0.42	14.1	15.0	...	0.9
24	"	0.41	14.6	15.3	...	0.7
25	"	0.41	12.6	13.4	...	0.8
26	"	0.42	11.9	11.6	0.3	...
27	"	0.41	24.2	24.0	0.2	...
28	"	0.41	30.2	29.5	0.7	...
29	"	0.42	17.6	17.2	0.4	...
30	"	0.42	18.7	18.1	0.6	...

[1] See III, section 25.
[2] See III, Table V.

Humidity. The column entitled Observed Relative Humidity was obtained with the chemical hygrometer.

Inspection of the column of Differences shows that the psychrometric formulas deduced in these pages, in conjunction with the apparatus here described, yielded relative humidity to a precision of about one per cent.

PART III

SUMMARY

A practical psychrometric formula, derived from the investigation of energy transformations of which an account is given in the previous papers of the present series, and confirmed by tests with the chemical hygrometer. A practical psychrometric formula, derived from the investigation of energy transformations of which an account is given in the previous papers of this series, has received experimental confirmation based upon a large number of tests with the chemical hygrometer. These tests, made with the large scale apparatus previously described, and carried out under a wide range of the atmospheric variables under consideration, yielded relative humidity to a precision of about one per cent.

The psychrometric formula referred to may be written as follows:

$$\left.\begin{aligned} p_1 &= P' - \frac{0.3891\left(\dfrac{t-t'}{L'}\right)}{1+0.453\left(\dfrac{t-t'}{L'}\right)}\,b \\ Q &= (6.55 \times 10^{-4}\,E)\,(b - P') \\ p &= p_1 + Q \end{aligned}\right\}$$

where E is an instrumental constant, determined with the aid of the chemical hygrometer.[1]

Necessary condition under which psychrometric results are experimentally determinate. Wet bulb phenomena become experimentally indeterminate when the water vapor in the air is near saturation. These phenomena are in fact experimentally determinate only when wet bulb depression satisfies the following relation:

$$(t - t') \geqq \left(\frac{E}{1.011}\right)\left[\frac{1 + 0.621\left(\dfrac{P'}{b - P'}\right)}{1 + 1.162\left(\dfrac{P'}{b - P'}\right)}\right]$$

This restricted range of wet bulb depression is due to the effect referred to in the preceding papers as *dynamical cooling.* An account of the causes and magnitude of this effect is contained in the third paper of the present series.

[1] *Notation:*
t = dry bulb temperature,
t' = wet bulb temperature,
P' = pressure of saturated water vapor at wet bulb temperature,
p = pressure of water vapor in the air,
b = barometric pressure,
L' = heat of vaporization of water at wet bulb temperature,
E = an instrumental constant evaluated by the writer.
Temperatures are measured on the Centigrade scale, and E is in joules per gram. Heat of vaporization is in calories per gram. Pressures are in millimeters of mercury.

AERODYNAMICS OF THE PSYCHROMETER

By George Porter Paine

Fifth Paper: Verification of the Aerodynamic Theory of the Psychrometer in the Case of a Small-Scale Apparatus

INDEX TO SECTIONS

PART I

Introduction

The results set forth in the four preceding papers on the Aerodynamics of the Psychrometer were obtained with the aid of apparatus so large as to be unsuited for ordinary exigencies of humidity determination. A small, portable apparatus was therefore constructed with a view to testing the practical validity of the psychrometric formula derived in the foregoing papers, and verified for the larger apparatus.

PART II

Apparatus

1. *General plan of the apparatus.* The type of apparatus employed was a modification of the Aspiration Psychrometer of Assmann.[1] As in the Assmann instrument, wet bulb and dry bulb were partly enclosed within double shield-tubes, and air was sucked through these tubes into a central channel-tube. The object of the shield-tubes was to create suitable air jets and at the same time to protect the thermometer bulbs from the influence of radiation due to exterior solid objects.

For purposes of the present study it was necessary to devise some satisfactory way of producing a strong forced draft through the shield-tubes and of controlling and measuring its velocity. A very satisfactory forced draft was obtained by means of a small rotary exhauster consisting of a 12-blade rotor in a suitable casing and driven by a small 110-volt universal motor. Forced draft in the shield-tubes was regulated by a relief-valve in the horizontal portion of the central channel-tube, near the junction of the latter with the exhauster. A water manometer, attached to a nipple in the channel-tube, and on the windward side of the relief-valve, made it possible to adjust static pressure in the system with great uniformity and precision.

The foregoing apparatus constituted the aerodynamic psychrometer proper. In order to estimate forced draft velocity in the shield-tubes, the rotary fan exhausted directly into a horizontal 5.08-centimeter (2-inch outside diameter) brass calibrating-tube 72.39 centimeters (2 ft. 4.5 inches) long. At a point 54.61 centimeters (1 ft. 9.5 inches) from the inlet of this tube, a joint containing a suitable opening allowed the insertion and adjustment of the Pitot tube.[2] With this Pitot tube, which was connected to a Krell gage,[3] it was possible closely to estimate the average velocity along the cross-section of the calibrating-tube.

[1] *Zeitschrift zur Instrumentenkunde*, 1892, **12**, page 1.
[2] See I, section 8.
[3] See I, section 10.

Assuming a uniform mean flow in both annular portions of the double shield-tubes, the forced draft velocity past the thermometer bulbs was readily calculated from the known geometrical dimensions of the shield-tubes.

2. *Shield-tubes and the central channel-tube.* The shield-tubes, into which the bulbs and lower portions of the stems of the thermometers were inserted, were, in general respects, the same as those described by Assmann, to whose paper the reader has been referred, but they were modified in detail as follows. Both outer and inner shield-tubes were silver-plated and burnished. Their dimensions were as follows:

Length	8.2 cm
External diameter of outer tube	2.06 cm ($\frac{13}{16}$ inch)
External diameter of inner tube	1.59 cm ($\frac{5}{8}$ inch)
Thickness of brass tubing	0.08 cm ($\frac{1}{32}$ inch)
Diameter of entrance nozzle of outer tube (to outer edge of bell)	2.66 cm
Length of shield-tube below base of thermometer bulb	3.30 cm

As has already been stated, an important function of the shield-tube was to eliminate as far as possible radiation exchanges between the thermometer bulb and surrounding material. A large part of solar and other external radiation impinging on the outer shield-tube was reflected by the burnished silver surface. A small part of such radiation as was absorbed by the outer tube was radiated inward to the inner shield-tube, where, again meeting a burnished silver surface, a large per cent was reflected outward. Thus no appreciable radiation arrived at the thermometer bulb through the shield-tubes. The low thermal conductivity of air and its rapid motion prevented heat transfer by conduction and convection from the outer shield-tube to the inner shield-tube and also from the inner shield-tube to the thermometer bulb.

In order to prevent appreciable transfer of heat from the outer shield-tube to the inner shield-tube along the metal screws supporting the latter, these screws were imbedded in small boxwood washers, let into the wall of the outer shield-tube. Thus heat could only be conducted from the outer tube to the inner tube along its supporting screws by way of the boxwood washers, which were regarded as effective insulators.[1] This arrangement with washers, which is theoretically less desirable than the ivory ring used by Assmann, on account of the nonreflecting inner surfaces of the washers, was adopted on account of the fragility of an ivory or wooden ring.[2]

With this arrangement, the thermometer bulbs were adequately protected from external disturbances on all sides except at the bottom. A reflecting metal surface, placed horizontally under the entrance nozzles, would obviously be most undesirable; and it was deemed that the best results would be obtained by limiting the attempt at complete screening of

[1] The ratio of the thermal conductivity of boxwood to the thermal conductivity of brass is about 1 to 600. See *Smithsonian Physical Tables*, Washington, 1916, page 207.

[2] A commercial type of the Assmann instrument has the ivory ring replaced by a *brass* one *covered with black lacquer*, thus exaggerating as much as possible the conductivity effect so skilfully eliminated by the inventor.

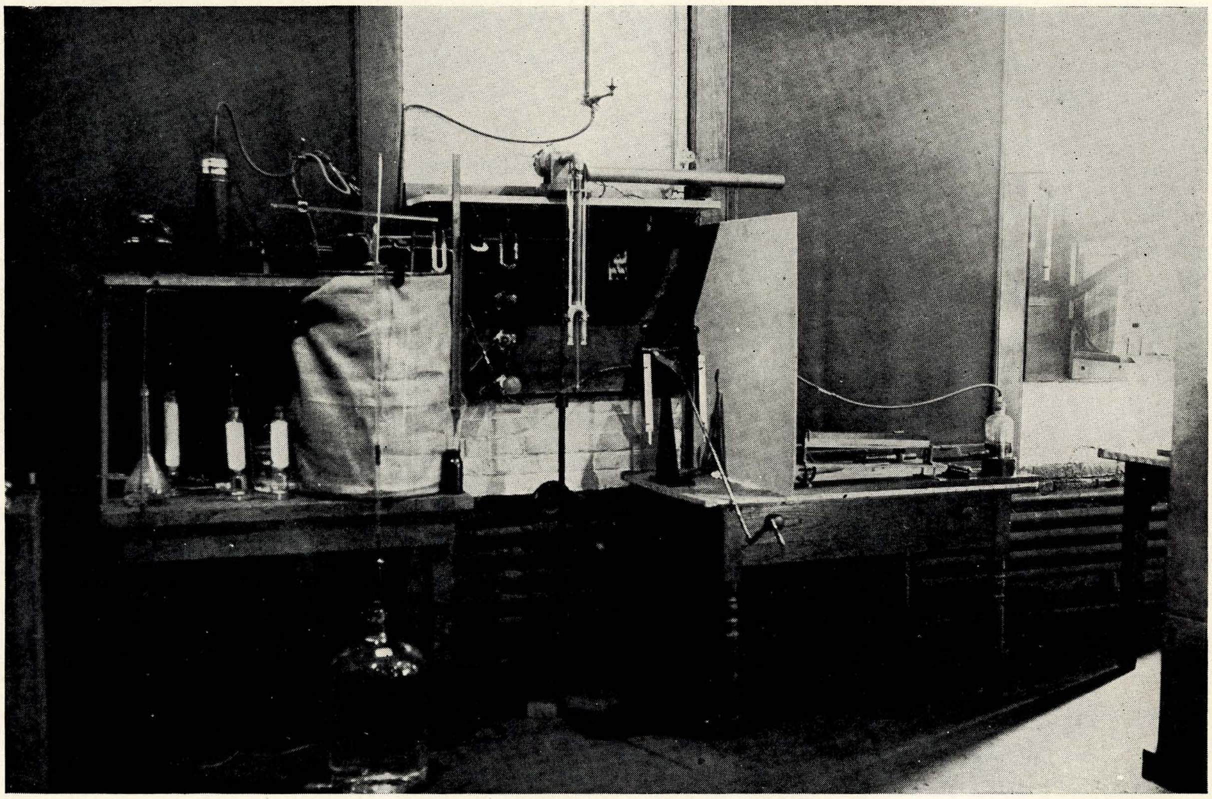

FIGURE 1. General View of the Apparatus.

the thermometer bulbs to making the shield-tubes long enough to insure as small a solid angle as possible, through which radiation would be able to reach the thermometer bulb from the open base of the shield-tube. In the instrument designed by the writer, half the apical angle, measured from axis to element of cone,[1] was 20 degrees. When care was taken not to suspend the psychrometer over a hot or reflecting surface, this arrangement proved very satisfactory indeed.[2]

When forced draft velocity in the shield-tubes exceeded 3 meters a second, the mixing effect of the eddy at the top of the inner shield-tubes near their junction was sufficiently intense to cause cold air from the wet bulb to propagate backward and downward into the shield-tube of the dry bulb, thus giving rise to a very appreciable lowering of dry bulb temperature. This effect was eliminated by a brass partition, inserted in the lower part of the central channel-tube, with its edges hermetically cemented to the latter. The design would be improved by extending separate channel tubes, one for each bulb, all the way up to the horizontal member containing the manometer.

By means of a lock nut, the vertical part of the psychrometer could be turned at any angle, for convenience in reading, and the channel at the same time made air-tight.

3. *Forced draft.* The 4-inch, 12-blade rotor of the exhauster was set eccentrically in the casing in order to give increasing clearance for the blades as they approached the outlet. This arrangement was necessary in order to reduce the strong drag on the rotor, which was set up when the blade ends revolved close to the casing throughout a whole turn. Air was sucked into the exhauster casing to the axial region of the rotor and exhausted by the paddles into a 1-inch tube let horizontally into the casing bottom. A single shaft serving for both motor and rotor was supported by the two journal bearings of the motor.

The basis for wind velocity measurement was the discharge rate through the 2-inch calibrating-tube receiving the discharge from the exhauster (Figure 2). Flow velocity in this tube was not, of course, uniform over its cross-section. When, for example, the velocity in the axis of the calibrating-tube was 7 meters per second (and the velocity in the shield-tubes was 15 meters per second) the velocity indicated by the Pitot tube, when lying along the wall of the calibration-tube and in contact with it, was 6.42 meters per second. The discharge rate of the calibration-tube was accordingly calculated from the average of the velocities observed by means of the Pitot tube along the cross-section. For velocities in the calibrating-tube of 5 meters per second and less, the difference between velocity in the tube axis and velocity near the tube wall could not be detected with the very sensitive Krell manometer at three degree inclination.

[1] Defined with apex at bottom of thermometer bulb and base coincident with that of the tube.

[2] The use of curved tubes was deemed inadvisable for, while offering excellent protection from radiation, they subjected the thermometer bulbs to a nonuniform air jet. By making the shield-tubes longer, the solid angle was reduced, but, again, it was thought inadvisable to draw the air through long metal tubes before it reached the thermometers, on account of possible heating of the air in transit.

The relief-valve of the psychrometer was closed during calibration. Forced draft velocity was then varied by inserting resistances in series with the motor. A curve was thus obtained showing the relation between average forced draft velocity over the cross-section of the calibration-tube and the indications of the water manometer.

Calibration data for the water manometer, relative to forced draft velocities in the shield-tubes, were obtained from the foregoing considerations together with the geometrical dimensions of the shield-tubes, and are shown in Table I. This table was used as the basis of the ensuing tests.

4. *Thermometers.* The thermometers by Köhler, were graduated to tenths of a degree Centigrade, on a scale sufficiently open to admit a good estimate of the nearest hundredth of a degree, and were compared throughout the working temperature range with a thermometer calibrated at the *Physicalisch-Technische Reichsanstalt* and supplied with certificate No. 48024 of that institution. Calibration curves were accordingly constructed and the corrections applied. The ensuing tables contain corrected temperatures only.

The emergent mercury column of the wet bulb thermometer implied a stem correction, but in the case of this instrument, throughout the range between 0° and 30°, this correction was not greater than 0°.02 C.

TABLE I

VELOCITY OF FORCED DRAFT ON THERMOMETER BULBS

Water manometer cm	Forced draft velocity m/s
0.00	0.0
0.25	3.0
0.85	6.0
1.10	7.0
1.80	9.0
2.65	11.0
3.20	12.0
5.00	14.6
5.20	15.0

5. *Immersion.* The wicks used for the tests consisted of bulb-covering only, without continuous feed of any kind.[1] This bulb-covering was a 3 × 5.5-centimeter, rectangular piece of all-linen lawn of the finest weave obtainable.[2] It was wound *wet* once and a half times around the wet bulb, tied tightly at the top and drawn tightly together and tied at the bottom, giving an absolutely tight fit throughout. The upper part of the bulb-covering extended 2.2 centimeters above the mercury bulb. Immediately before each reading, the wick was saturated by inserting a chemically clean glass water-tube nearly full of distilled

[1] See III, sections 3 to 6 and section 13.
[2] This material was not subjected to preliminary chemical treatment (see III, section 4).

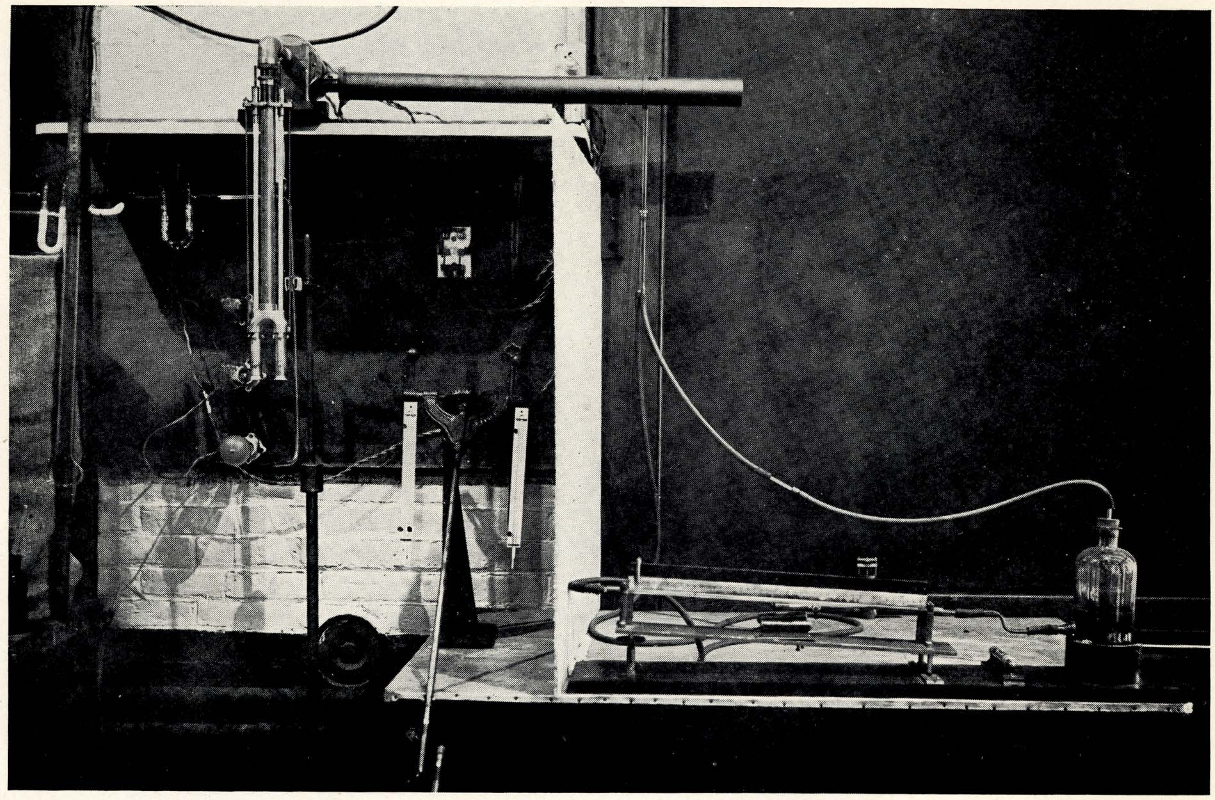

FIGURE 2. Front View of Psychrometer. Calibration Tube, Pitot Tube Connections, and Krell Gage.

water up into the shield-tube containing the wet bulb. By making a preliminary trial with shield-tubes removed, a mark was inscribed on the water-tube, showing a water level at which complete immersion of the wick was effected without overflow from the water-tube, since, for obvious reasons, it was extremely desirable to keep the inner shield-tube dry.

6. *Installation of the chemical hygrometer.* The chemical hygrometer was placed at one side of the psychrometer (Figure 1) with the nozzle of a short intake tube 15.8 centimeters below the entrance nozzles of the psychrometer, in the axis of symmetry of the latter instrument.

In order to reduce to a minimum the volume of air in the hygrometer when the aspirator bottle was full, the calcium chloride tower (see III, Figure 3, C) was replaced by a drying tube containing calcium chloride, and all glass tubing was shortened as much as possible.

7. *Installation of the psychrometer.* The psychrometer was set up on a shelf between two north windows, the chemical hygrometer on one side and an asbestos screen on the other; calibrating tube, Pitot tube and Krell manometer were arranged as shown in Figure 2. The distance of the psychrometer entrance nozzles from the floor was 1.20 meters. Hot or reflecting surfaces were excluded from the region within the solid angles defined by thermometer bulbs and inner shield tubes.[1]

[1] The steam-pipes running along the wall for heating the room were at room temperature during the tests.

PART III

Data and Results

8. *Wick characteristics.* Wick characteristic data shown in Figure 3 indicate that equilibrium at the wet bulb minimum M[1] was not attained until about 3.5 minutes after removal of the water-tube. Equilibrium was then attained, and this state continued for an interval of about 9 minutes, during which wet bulb temperature was very steady. Forced draft in the shield-tubes was meanwhile extremely steady at 15 meters per second, with a relative atmospheric humidity of around 0.70.

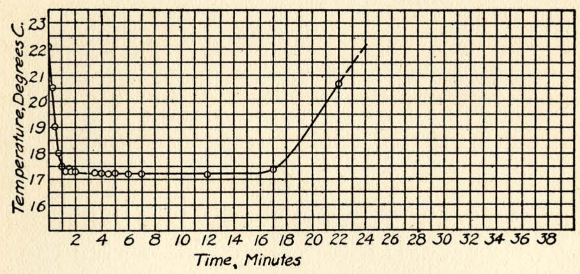

Figure 3. Characteristics of a thin linen wick, without feed. Wind velocity: 15.0 meters per second. Bulb diameter (bare): 0.65 centimeter.

TABLE II

CHARACTERISTICS OF A THIN LINEN WICK, WITHOUT FEED
WIND VELOCITY = 15 M/S

No.	Time *m s*	Wet bulb C.°	No.	Time *m s*	Wet bulb C.°
1	0 0	22.10	10	2 30	....
2	0 15	20.5	11	3 0	....
3	0 30	19.1	12	3 30	17.25
4	0 45	18.0	13	4 0	17.21
5	1 0	17.5	14	4 30	17.20
6	1 15	17.3	15	5 0	17.21
7	1 30	17.4	16	6 0	17.20
8	1 45	17.3	17	7 0	17.21
9	2 0	17.32	18	12 0	17.20
			19	17 0	17.32
			20	22 0	20.70

Data contained in Figure 4 show effects of comparatively low wind velocity. In this case, forced draft in the shield-tubes was only 4.7 meters per second. Under this low velocity, equilibrium was attained after about 4 minutes and it continued for 27 minutes. During this comparatively long interval, wet bulb temperature was not quite so steady as it was under a forced draft of 15 meters per second.

[1] See III, sections 8 and 13.

TABLE III

CHARACTERISTICS OF A THIN LINEN WICK, WITHOUT FEED
WIND VELOCITY = 4.7 M/S

No.	Time *m*	Time *s*	Wet bulb C.°	No.	Time *m*	Time *s*	Wet bulb C.°
1	0	0	21.34	10	3	0	18.60
2	0	15	21.0	11	4	0	18.55
3	0	30	19.8	12	5	0	18.59
4	0	45	19.2	13	10	0	18.52
5	1	0	18.9	14	15	0	18.60
6	1	15	18.6	15	20	0	18.60
7	1	30	18.6	16	25	0	18.60
8	1	45	18.6	17	30	0	18.60
9	2	0	18.58	18	35	0	18.50

The minimum wet bulb temperature M, continuing during the state of equilibrium existing between the various forms of energy transfer expressed by relation (17) of the third paper of the present series is here, as in the preceding papers, defined as the wet bulb temperature t' of formulas (17) and (23) of the paper just referred to.

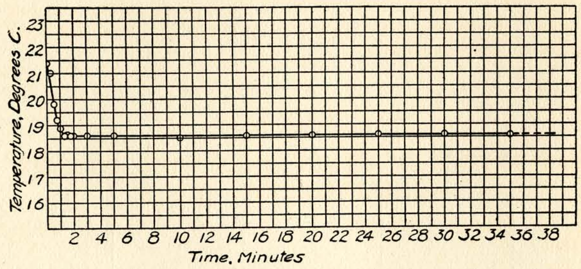

FIGURE 4. Characteristics of a thin linen wick, without feed. Wind velocity: 4.7 meters per second. Bulb diameter (bare): 0.65 centimeter.

It should be added that repeated tests showed that forced draft velocities above 15 meters per second could not properly be employed, on account of water's being carried by the wind away from the wet bulb in liquid form; an effect not contemplated in the derivation of the formulas. This spraying effect was also observed when the wind was run at velocities over 15 meters per second in the jet channel, where the spray accumulated on the wall of the retention chamber opposite the jet channel outlet.

It is important to observe that comparison of the wet bulb minimum (Figure 3), obtained by means of a wet bulb having a diameter of 6.5 millimeters (bare), with that obtained by means of a wet bulb having a diameter of 2.7 millimeters, shows the remarkable degree to which wet bulb minimum M can be made to persist by the use of a wet bulb of sufficiently large diameter. From Stefan's analysis referred to in the first paper of this series, it follows at once that evaporation rate, per unit area of a wet cylinder, increases with decreasing diameter, in still air. Inspection of Figure 5, section 9, of the third paper, in connection with Figure 3 of the present paper, indicates that this effect is very pronounced in forced draft also.

Evidently, therefore, the practical advantage of cylindrical psychrometer bulbs not less than 6.5 millimeters in diameter, over smaller bulbs, is considerable.

9. *Experimental procedure.* The ensuing hygrometric results were obtained as follows. Windows and doors of the room were closed, and the air was stirred by the large jet channel exhauster for at least 10 minutes previously to the run. Meanwhile, the short hygrometer intake tube was thoroughly dried by aspirating. A new wick was used for each test, and this was thoroughly saturated with water during the preliminary proceedings. Before and after each run, the wet bulb shield-tube was removed and the wick inspected, it having once been found partly displaced at the end of a test.[1] At the zero instant, the forced draft having previously been accurately adjusted, the water-tube was pushed up over the wet bulb, held there for 5 seconds, and then promptly withdrawn. Wet bulb temperature was then observed continuously through a small telescope until the minimum M had clearly been attained. This telescope was essential, because warmth, from the body of the ob-

TABLE IV

HYGROMETRIC DATA. THE AERODYNAMIC PSYCHROMETER

No.	CHEMICAL HYGROMETER				PSYCHROMETER			
	b'' mm	t'' C.°	h mm merc.	m mgr	Forced draft m/s	Dry bulb C.°	Wet bulb C.°	b mm
1	759.78	21.42	3.97	169.15	3.0	20.90	14.14	759.57
2	762.13	22.69	5.07	201.33	"	21.96	15.90	762.16
3	757.07	22.64	7.14	245.8	"	22.63	17.88	757.22
4	766.91	21.60	5.51	230.6	"	21.32	16.91	766.79
5	758.38	23.63	2.57	266.6	7.0	23.71	18.97	758.10
6	759.38	22.79	2.87	175.3	"	22.54	15.02	759.17
7	754.77	23.52	3.38	248.4	"	23.64	18.30	754.41
8	765.11	21.40	3.68	137.6	"	20.98	12.84	764.74
9	766.66	20.23	5.96	214.6	11.0	20.35	15.84	766.19
10	758.95	21.01	3.82	235.2	"	20.97	16.86	758.43
11	757.23	22.52	4.93	287.8	"	22.73	19.35	756.78
12	758.56	23.21	4.71	260.1	"	23.18	18.56	759.09
13	763.51	21.50	3.90	179.8	15.0	21.72	14.99	762.70
14	761.04	19.56	1.98	135.5	"	18.72	11.68	760.14
15	759.65	20.26	3.09	208.7	"	20.49	15.57	758.22
16	763.65	19.96	2.50	182.9	"	20.20	14.43	762.50

[1] This mishap can be avoided by using for immersion a water-tube of sufficiently large diameter to fit closely into the inner shield-tube.

server approaching the psychrometer, heated the air about to enter the shield-tubes sufficiently to cause an appreciable change in the indications of both thermometers.

The length of the run for the psychrometer was 90 minutes, during which time readings were made at regular intervals of 10 minutes. Thus the averaging interval τ[1] for the psychrometer was 90 minutes. The chemical hygrometer was run 100 minutes in such a way that the middle points of the averaging intervals for the two instruments coincided. Since barometric pressure did not change greatly during any one run, the barometer was read only three times per run, at the beginning, the middle and the end of the 100-minute interval. The data shown consist of *mean* corrected temperatures, pressures, etc.

10. *Hygrometric data.* Data obtained from sixteen consecutive runs of the apparatus are contained in Table IV. The notation relative to the chemical hygrometer was defined in III, Appendix I, section 29.

TABLE V

THE ENERGY INTEGRAL E AS A FUNCTION OF x, WHERE

$$x = \left[\left(\frac{P' - p}{b - P'}\right) \left(\frac{t - t'}{273.1 + t}\right) (q^2 \times 10^{-6}) \right]$$

No.	q m/s	$x \times 10^5$	E joules per gram	No.	q m/s	$x \times 10^5$	E joules per gram
1	3.0	0.819	1.043	9	11.0	4.88	0.627
2	"	0.644	0.916	10	"	4.07	0.572
3	"	0.396	0.724	11	"	2.52	0.719
4	"	0.364	0.412	12	"	4.88	0.472
5	7.0	2.15	0.712	13	15.0	21.1	0.623
6	"	5.51	0.947	14	"	22.2	0.948
7	"	2.78	0.700	15	"	10.9	0.637
8	"	6.61	0.870	16	"	15.1	0.706

Most probable value of E.......... 0.727
Probable error of this determination.......... ±0.030

11. *Determination of the energy integral* E. From these data, it was possible to calculate the energy integral E defined for the psychrometer in the third paper of the present series.[2] Results of this calculation[3] are shown in Table V. As was stated in the paper referred to, dimensional considerations suggested that E would be a function of x, where

$$x = \left[\left(\frac{P' - p}{b - P'}\right) \left(\frac{t - t'}{273.1 + t}\right) (q^2 \times 10^{-6}) \right] \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

The latter quantity was accordingly calculated from the data of Table IV and the results inserted in Table V. From Figure 5, where E is plotted as a function of x, it is evident that E is an instrumental constant, the small deviations from the mean being due to erratic

[1] I, section 2.
[2] See III, equation (23)
[3] From III, equations (24) and (25).

instrumental errors. E is thus independent of atmospheric humidity and forced draft velocity, and is defined for forced draft velocities between 3 and 15 meters per second, and for relative humidities up to near the saturation line, subject to conditions set forth in III, section 27.

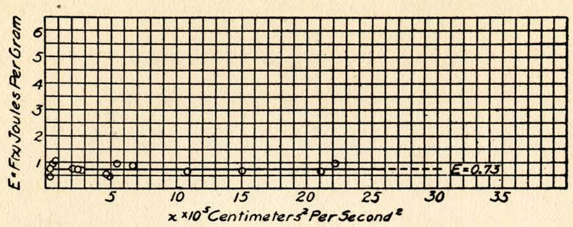

FIGURE 5. The energy integral E as a function of

$$\left[\left(\frac{P' - p}{b - P'}\right)\left(\frac{t - t'}{273.1 + t}\right)(q^2 \times 10^{-6})\right].$$

For the aerodynamic psychrometer here described

$$E = 0.727\,\frac{joule}{gram} \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

with a probable error of ± 0.030 in the determination.

12. *Evaluation of the function Q*. Referring to IV, formula (6), we have

$$\begin{aligned} Q &= (6.55 \times 10^{-4} \times 0.727)\,(b - P') \\ &= 4.76 \times 10^{-4}\,(b - P') \quad \ldots\ldots\ldots\ldots\ldots \quad (3) \end{aligned}$$

13. *Measurement of atmospheric humidity*. This result gives the psychrometric formula[1]

$$p = P' - \frac{0.3891\left(\frac{t - t'}{L'}\right)}{1 + 0.453\left(\frac{t - t'}{L'}\right)}\,b + 4.76 \times 10^{-4}\,(b - P') \quad \ldots\ldots \quad (4)$$

By utilizing in this formula the data of Table IV, the pressure p of the water vapor in the air (absolute humidity) and also the relative humidity were calculated. The results are shown in Table VI. In Test No. 14 the correction Q amounted to 5 per cent of the absolute humidity. In Test No. 26 (of III, Table V, and of IV, Table I) the correction Q amounted to 23 per cent of the absolute humidity. The percentage is greater in the case of lower absolute humidities. Determinations of absolute humidity with the aerodynamic psychrometer were accurate to 1.3 per cent. Relative humidity was rendered to within 0.8 per cent.

[1] See IV, formula (6). It should be remembered that the last term of this equation (4) contains a factor which is an instrumental constant for the psychrometer described in these pages. For convenience of reference, we bring together the following definitions:

p = pressure of the water vapor in the air,
P' = pressure of saturated water vapor at the temperature of the wet bulb,
t = temperature of the dry bulb,
t' = temperature of the wet bulb,
L' = heat of vaporization of water at temperature t', in calories per gram,
b = barometric pressure.

Pressures are measured in millimeters of mercury; temperatures are measured on the Centigrade scale.