# GREAT LAKES BASIN COMMISSION 

ANN ARBOR, MICHIGAN

## LIMNOLOGICAL

SYSTEMS ANALYSIS
of
THE GREAT LAKES
Phase I

HYDROSCIENCE, INC.
WESTWOOD, NEW JERSEY
LIMNOLOGICAL SYSTEMS ANALYSISOF
THE GREAT LAKES
PHASE I - PRELIMINARY MODEL DESIGNby
Hydroscience, Inc.363 Old Hook Road,Westwood, New Jersey

$$
07675
$$

for theGreat Lakes Basin CommissionAnn Arbor, Michigan48106
Contract Number: DACW-35-71-C0030

# Great Lakes Basin Commission 

Post Office Box 999
3475 Plymouth Road
Ann Arbor, Michigan 48106
Attention: Mr. Frederick O. Rouse

Gentlemen:
We are pleased to transmit herewith, in partial fulfillment of Contract Number DACW-35-71-C0030, our report entitled "Limnological Systems Analysis of the Great Lakes, Phase I Preliminary Model Design."

We conclude that the Limnological Systems Analysis is a feasible and desirable approach to analyzing water resource problems in the Great Lakes. The recommended program provides for the development of analytical capabilities and the application of these capabilities to the examination of water resource planning problems in the Great Lakes.

The significant contributions by Dr. Robert V. Thomann and Dr. Dominic M. Di Toro to all phases of the project are especially acknowledged. In particular, they have consistently provided technical guidance and insight to the study program.

Very truly yours,

HYDROSCIENCE, INC.


Very truly yours,

HYDROSCIENCE, INC.


JLM: DJO/vjw
Page
SECTION I - SUMMARY AND CONCLUSIONS ..... 1
SECTION II - RECOMMENDATIONS ..... 7
SECTION III - INTRODUCTION ..... 9
Purpose and Orientation ..... 9
Methodology ..... 10
Description of Limnological System ..... 13
SECTION IV - WATER RESOURCE PROBLEMS AND VARIABLES ..... 19
Water Resource Uses and Problems ..... 19
Variables and Planning Activities ..... 35
References ..... 47
SECTION V - EXISTING DATA ..... 49
Introduction ..... 49
Physical Data ..... 49
Chemical Data ..... 59
Biological Data ..... 83
Special Data ..... 83
Summary ..... 84
SECTION VI - PROBLEM ORIENTED MODELS ..... 87
Introduction ..... 87
Mathematical Models ..... 89
Administrative Problem Definition ..... 90
Model Development ..... 91
Evaluation Process and Model Status ..... 92
SECTION VII - AVAILABLE MODELS - STATE OF THE ART ..... 99
Hydrological Balance Models ..... 99
References ..... 113
Ice and Lake Wide Temperature Models ..... 115
References ..... 125
Thermal Models ..... 127
References ..... 134

## TABLE OF CONTENTS

PageSECTION VII (continued)
Lake Circulation and Mixing Models ..... 137
References ..... 168
Erosion and Sediment Models ..... 175
References ..... 181
Chemical Models ..... 183
References ..... 200
Eutrophication Models ..... 203
References ..... 225
Dissolved Oxygen Models ..... 229
References ..... 239
Pathogens and Indicator Bacteria Models ..... 241
References ..... 253
Fishery Models ..... 255
References ..... 268
Ecological and Food Chain Models ..... 271
References ..... 293
SECTION VIII - MODEL SYNTHESIS FOR PLANNING NEEDS ..... 297
Summary of Evaluation of Model Status ..... 297
References ..... 311
SECTION IX - DEMONSTRATION MODEL ..... 313
Introduction ..... 313
Model of Chloride and Total Dissolved Solids ..... 318
Lake Erie Western Basin Chlorides and Coliform Models ..... 327
Coliform Model Development and Verification ..... 333
Lake Eutrophication Model ..... 337
Data Sources ..... 354
The Exogeneous Variables ..... 359
A Food Chain Model of Cadmium in Western Lake Erie ..... 405
Summary ..... 427
References ..... 429

TABLE OF CONTENTS
Page
SECTION $X$ - RECOMMENDED PHASE II STUDY ..... 433
Problems Proposed for Study in Phase II ..... 433
Alternate Programs ..... 438
Recommended Phase II Study ..... 449
SECTION XI - ACKNOWLEDGEMENTS ..... 459
SECTION XII - NOMENCLATURE ..... 461
Section VII ..... 461
Section IX ..... 471

## LIST OF FIGURES

FigureNumberPageMETHODOLOGY FOR GREAT IAAKES LIMNOLOGICAL SYSTEMS ANALYSIS ..... 11
GREAT LAKES BASIN DRAINAGE BOUNDARIES ..... 14
COMPONENTS OF WATER RESOURCE PROBLEM ..... 20
CHLORIDE CONCENTRATIONS IN LAKE MICHIGAN ..... 23
LOCATION OF PHYTOPLANKTON GROWTH ..... 25
LOCATION OF DISSOLVED OXYGEN PROBLEMS ..... 27
SHORE EROSION AND SILTING ..... 30
LOCATION OF AREAS AFFECTED BY ICE JAMS ..... 32
LOCATION OF WATER LEVEI GAGING STATIONS ..... 51
LOCATION OF GAGED SURFACE RUNOFF AREAS ..... 55
LOCATION OF FIRST AND SECOND ORDER METEOROLOGICAL DATA STATIONS ..... 56
LOCATION OF SOLAR RADIATION AND PAN EVAPORATION DATA STATIONS ..... 57
LOCATION OF UPPER AIR/WIND DATA STATIONS ..... 58
DETERMINATION OF MODEL STATUS ..... 93
ROLE OF HYDRODYNAMIC MODELING OUTPUT ..... 138
COMPARISON OF COMPUTED AND OBSERVED CURRENT ..... 147 MEASUREMENTS
COMPARISON OF COMPUTED AND OBSERVED CURRENT MEASUREMENTS ..... 148
PROGRESS OF THE THERMAL BAR FROM WINTER TO FUL工 SUMMER STRATIFICATION ..... 152
OBSERVED AND COMPUTED BUFFALO MINUS TOLEDO SET-UP ..... 155
COMPARISON OF OBSERVED VERSUS CALCULATED SOLUBILITY PRODUCTS ..... 194

# LIST OF FIGURES <br> (continued) 

Figure Number
21 PHYTOPLANKTON GROWTH RATE INTERACTIONSPage2PHYTOPLANKTON DEATH RATE, ZOOPLANKTONGROWTH AND DEATH RATES, INTERACTIONS213
VERIFICATION OF RILEY'S PHYTOPLANKTON AND ZOOPLANKTON MODELS ..... 216
VERTICAL DISTRIBUTION - VERIFICATION ..... 219
ANTIOCH VERIFICATION, 1966 ..... 222
AREAS WHERE PROBLEMS ASSOCIATED WITH HIGH BACTERIAL CONCENTRATIONS EXIST ..... 242
BACTERIA工 DENSITIES IN LAKE MICHIGAN ..... 243
BACTERIOLOGICAI DATA AT BIG BAY BEACH r ..... 245
COORDINATE SYSTEM FOR DISCHARGE OF BACTERIAINTO A LAKE248
COMPARISON OF OBSERVED AND COMPUTED VALUES FOR BACTERIA IN LaAKE MICHIGAN VICINITY ..... 249INDIANA HARBOR
MAJOR SUB-SYSTEMS OF FISHERY MODELING ..... 256
CATCH AND EFFORT DATA FOR NORTHERN GREEN BAY ..... 258
SIMULATION FOR LAKE TROUT OF LAKE MICHIGAN ..... 261
SARDINE ANCHOVY MODEL ..... 265
A TEN COMPARTMENT MODEL ..... 274
A.TEN COMPARTMENT MODEL WITH SPATIAL DEFINTION ..... 275
A FOOD CHAIN MODEI ..... 276
INTERACTIONS AMONG SIX COMPARTMENTS ..... 278
GEOMETRICAL PRESENTATION OF EQUILIBRIUM ..... 282

## LIST OF FIGURES

 (continued)FigureNumber40414243
4445
464748
4950
5152
53
UNCERTAINTY IN LINEAR AND NON-LINEAR MODELS ..... 288
Page
SUMMARY OF MODEL STATUS ..... 299
STEPS IN MODEL SYNTHESIS ..... 301
EXAMPLE OF MODEL SYNTHESIS ..... 303
COMPARTMENT CALCULATION TIMES ..... 307
TIME TO COMPUTE ONE YEAR SIMULATION
TIME TO COMPUTE ONE YEAR SIMULATION AT FIVE LEVELS IN DEPTH FOR VARIOUS HORIZONTAL, ..... 310SPACINGSDEMONSTRATION MODEL FRAMEWORK314
PRIMARY INPUT - OUTPUT VARIABLES DEMONSTRATION MODEL ..... 315
HYDROLOGICAL BALANCE ..... 320
COMPUTED VERSUS OBSERVED CHLORIDES ..... 323
SOURCE COMPONEITTS FOR LAKES ERIE AND ONTARIO ..... 324
PROJECTED TOTAL DISSOLVED SOLIDS CONCENTRATIONS FOR GREAT LAKES ..... 325
PROJECTED TOTAL DISSOLVED SOLIDS CONCENTRATIONS FOR LAKE ERIE WITH IN-BASIN CONTROL MEASURES ..... 326
COMPARISON OF MEASURED AND PREDICTED VELOCITIES AT FOUR LOCATIONS IN WESTERN LAKE ERIE ..... 328
VERTICALLY INTEGRATED NET CIRCULATION IN WESTERN LAKE ERIE ..... 329
TWO-DIMENSIONAL 88 COMPARTMENT MODEL OF WESTERN LAKE ERIE ..... 331
CHLORIDE VERIFICATION COMPARISON OF MODEL RESULTS AND OBSERVED DATA ..... 334

> LIST OF FIGURES (continued)
FigureNumberPage
58 COLIFORM BACTERIA VERIFICATION ..... 336
PROJECTED DISTRIBUTION OF COLIFORM BACTERIA UNDER A HYPOTHETICAL TREATMENT POLICY ..... 338
CIRCULATION PATTERN IN WESTERN LAKE ERIE SHOWING PREVAILING CURRENT DIRECTIONS ..... 343
CHLORIDE CONCENTRATIONS IN WESTERN LAKE ERIE ..... 344
STEADY-STATE TRANSPORT FOR SEVEN COMPARTMENT ..... 345 WESTERN LAKE ERTE MODEL
355
KINETIC PATHWAYS OF THE ENDOGENOUS VARIABLES
356
WATER QUALITY MONITORING LOCATIONS IN WESTERN LAKE ERIE
360
WATER TEMPERATURE VARIABLES IN WESTERN LAKE ERIE - APRIL THROUGH OCTOBER, 1970VARIATION OF PHOTOPERIOD AND SOLAR RADIATIONFOR WESTERN LAKE ERIE362
WESTERN BASIN SECCHI DISK - 1970 ..... 363
VARIATION OF EXTINCTION COEFFICIENT IN TWO COMPARTMENTS OF THE WESTERN LAAKE ERIE MODEL ..... 364
LATERAL DISTRIBUTION OF NUTRIENTS AT THE MOUTH OF THE DETROIT RIVER ..... 365
CHLOROPHYL工 CONCENTRATIONS OF TRIBUTARY STREAMS ..... 366
INORGANIC PHOSPHORUS CONCENTRATIONS OF TRIBUTARY STREAMS ..... 367
ORGANIC PHOSPHORUS CONCENTRATIONS OF TRIBUTARY STREAMS ..... 368
ORGANIC NITROGEN CONCENTRATIONS OF TRIBUTARY STREAMS ..... 369
AMMONIA NITROGEN CONCENTRATIONS OF TRIBUTARY STREAMS ..... 370

## LIST OF FIGURES (continued)

FigureNumberPage75 NITRATE NITROGEN CONCENTRATIONS OF TRIBUTARY STREAMS ..... 371
RELATIONSHIP BETWEEN TOTAL AND ALGAL CELL COUNTS AND TOTAL CHLOROPHYLI MEASUREMENTS ..... 374IN WESTERN LAKE ERIE
77
CHLOROPHYLI VERIFICATION ..... 37878
798081828384
858687
88
8990
91
9293
CHLOROPHYLL RELATIONSHIPS IN COMPARTMENT NUMBER 7 ..... 379
THE ZOOPLANKTON SYSTEM MODET RESULTS ..... 381
COMPARISON OF HISTORICAL ZOOPLANTON COUNTS OBSERVED IN WESTERN LAKE ERIE ..... 382
ORGANIC NITROGEN VERIFICATION ..... 383
AMMONIA NITROGEN VERIFICATION ..... 384
NITRATE NITROGEN VERIFICATION ..... 385
TOTAL NITROGEN AND PHOSPHORUS IN WESTERN ..... 387 LAKE ERIE
388
INORGANIC PHOSPHORUS VERIFICATION
389
VERIFICATION OF THE TOTAL PHOSPHORUS SYSTEM
391
CHLOROPHYL工 HINDCAST TO 1930
392
ZOOPLANKTON HINDCAST TO 1930
393
ORGANIC NITROGEN HINDCAST TO 1930
394
AMMONIA NITROGEN HINDCAST TO 1930395
TOTAL PHOSPHORUS HINDCAST TO 1930 ..... 396
INORGANIC PHOSPHORUS HINDCAST TO 1930 ..... 397
HISTORICAL TRENDS IN WESTERN BASIN EUTROPHICATION ..... 399

# LIST OF FIGURES (continued) 

Figure
Number
Page
95 INFLUENCE OF PHYTOPLANKTON GROWTH ON LAKE 402
96
97

98
99

101 COMPUTED CADMIUM CONCENTRATION ALONG SOUTH SHORE OF WESTERN LAKE ERIE 425 PHYTOPLANKTON CONCENTRATIONS VERSUS TIME
FOR PHOSPHORUS REMOVAL POLICIES
PHYTOPLANKTON CONCENTRATIONS VERSUS TIME FOR NITROGEN AND PHOSPHORUS REMOVAL POLICY INTERACTIONS OF EUTROPHICATION AND FOOD
CHAIN MODELS CHAIN MODELS
FOOD CHAIN MODEL CONFIGURATION 419
COMPARISON OF FOOD CHATN MODEL OUTPUT WITH 422 SOME OBSERVED DATA IN WESTERN LAKE ERIE425
COMPUTED CADMIUM CONCENTRATION FACTORS ALONG SOUTH SHORE OF WESTERN LAKE ERIE ..... 426
102

103
WATER RESOURCE PROBLEMS AND MATHEMATICAL MODELS INÇLUDED IN PHASE II PROGRAM451TableNumberPage
1 GENERAL GREAT LAKES INFORMATION ..... 15
2 WATER RESOURCE PROBLEMS ..... 3634
KINDRED VARIABLE GROUPING ..... 40
PLANNING AND MANAGEMENT FUNCTIONS RELATED TO MODEL INPUT VARIABLE GROUPINGS ..... 45
PLANNING AND MANAGEMENT FUNCTIONS RELATED TO MODEL OUTPUT VARIABLE GROUPINGS ..... 46
SUMMARY OF WATER LEVEL GAGING STATIONS IN ..... 50 THE GREAT LAKES BASIN
LAKE CIRCULATION DATA ..... 53
GREAT LAKES WATER SAMPLING DATA SUMMARY ..... 60
SUMMARY OF PUBLISHED WASTE INPUTS TO THE GREAT LAKES ..... 76
SUMMARY OF AVAILABLE STATE AND PROVINCIAL WASTE INPUT DATA ..... 77
CHARACTERISTICS OF THERMAL MODELS ..... 131
COMPUTED AND OBSERVED PERIODS OF THE FIRST FIVE MODES OF LONGITUDINAL FREE OSCILIAATIONS ..... 156OF LAKE MICHIGAN
SUMMARY OF SOME HORIZONTAL DISPERSION ..... 164 COEFFICIENTS IN THE GREAT LAKESMINERAL DISSOLUTION192
ILLUSTRATIVE RECONCENTRATION FACTORS ..... 285
RANKING OF MODELING FRAMEWORKS ..... 298
PROBLEM CATEGORIES AND RELATED MODEITNG ..... 304
FRAMEWORKS
FRAMEWORKS ..... 321

## IIST OF TABLES

(continued)
Table
Number
Page
19 EUTROPHICATION SUB-MODEL VARIABLES 340
THE NITROGEN SYSTEM 352
THE PHOSPHORUS SYSTEM 353

22

DATA SOURCES 357
MAUMEE RIVER MASS DISCHARGES 373
KINETIC PARAMETERS 376
POPULATION PROJECTIONS FOR LAAKE ERIE BASIN 400
ILLUSTRATIVE APPLICATION OF THE PHASE I LIMNOLOGICAI SYSTEMS ANALYSIS DEMONSTRATION 406 MODEL

SUMMARY OF OBSERVED CADMIUM DATA FOR WESTERN
LAKE ERIE
ASSUMED AVERAGE BIOMASS CONCENTRATIONS FOR
FOUR TROPHIC LEVELS
PERCENT OF MASS OF CADMIUM BY SPATIAL SEGMENT AND SYSTEM LEVEL AS CALCULATION FOR FOOD424

CHAIN MODEL

RELATION OF MODEL PROBLEMS 435
SUMMARY OF FUNDING LEVELS 447 PROBLEM TIME AND SPACE SCALE FOR RECOMMENDED 454
APPLICATIONS
ESTIMATED COSTS OF RECOMMENDED APPLICATIONS 455

## SECTION I

## SUMMARY AND CONCLUSIONS

SECTION I
SUMMARY AND CONCLUSIONS

The purpose of this report is to present an assessment of the feasibility of applying a Limnological Systems Analysis (LSA) to the water resource problems of the Great Lakes. A methodology that proceeds along two parallel lines in order to evaluate the feasibility of the Limnological Systems Analysis is established. The first line of analysis evaluates the present and future water resource problems and water use interferences with their associated water resource variables. The second line of analysis evaluates presently available data, problem oriented mathematical models, and present state of the art of models and model building which are required for a Limnological Systems Analysis. The two lines of analysis are synthesized into a problem and model ranking of priority from which feasibility recommendations are drawn. In order to illustrate the Limnological Systems Analysis in several problem contexts, a demonstration modeling framework was constructed.

Water resource problems in the Great Lakes were identified and grouped into seven problem categories as follows:

1. Monthly Lake Water Levels and Flows
2. Erosion, Sediment
3. Ice
4. Toxic and Harmful Substances
5. Water Quality
6. Eutrophication, Fishery
7. Public Health

For each problem category a detailed review was made of associated water uses and water variables to provide the link to the available models. In order to address these problems a number of disciplines and specialties are required and are brought together in a systems context in the modeling framework. A central requirement for framework modeling is the data available for its development and use.

The review of water resource data in the Great Lakes region followed five broad classifications: physical, chemical, biological, and specialized data types. Contact was made with all of the major data storage and retrieval centers in the Great Lakes area and the data were then generally reviewed for geographical and variable coverage incorporating in the review the data needs of the available models. From the large amount of information uncovered during the study, it is concluded that sufficient data presently exists for preliminary model development for many of the water resource problems of the Great Lakes.

A review was also made of the available models that may be useful for a Limnological Systems Analysis and a convenient grouping of eleven modeling frameworks was obtained. The frameworks are:

1. Hydrological balance
2. Ice and lake wide temperature
3. Thermal
4. Lake circulation and mixing
5. Erosion and sediment
6. Chemical
7. Eutrophication
8. Dissolved oxygen
9. Pathogens and Indicator bacteria.
10. Fishery
11. Ecological and food chain

Each of the modeling frameworks was reviewed and analyzed in depth following an evaluation process to determine the present status of the framework as applied to water resource problems. The major considerations in determining model status are: 1) basic understanding and knowledge, 2) data availability, 3) degree of model verification, and 4) degree of model application. Numerical weights were assigned to each of several steps in the evaluation process.

As a result of this review it is concluded that the hydrological balance and lake circulation and mixing modeling frameworks are sufficiently developed to address
certain water resource planning problems of the Great Lakes. The status of two other modeling frameworks, a) ice and temperature and b) ecological models is poor; and considerable expenditure and research effort are required to bring these models to the point of useful application in water resource planning. The remaining seven modeling frameworks fall in a marginal status where some key variables or phenomena may be lacking but a sufficient base has been laid for some preliminary applications of the models to planning questions. It is also concluded from the analysis that there is a pressing need for model synthesis, because many efforts in the past have been fragmented and directed to rather narrowly conceived aspects of planning problems.

A demonstration modeling framework was constructed to illustrate this process of synthesis and to demonstrate the feasibility of the application of existing modeling technology to real planning problems in the Great Lakes. The demonstration model framework includes models of:

1. Long term trends on the Great Lakes scale of conservative water quality variables such as total dissolved solids and chlorides.
2. Regional models of Western Lake Erie for chlorides and bacteria.
3. Eutrophication model of Western Lake Erie.
4. Food chain model of Western Lake Erie.

The primary emphasis in the demonstration model effort is placed on the eutrophication model. This model is structured so as to maximize its ability to respond to several planning alternatives. The model includes effects of both biological and chemical reactions with the primary variable being phytoplankton biomass. Forty-nine simultaneous nonlinear time dependent equations are solved numerically in order to compute the phytoplankton, zooplankton, and nutrient distributions to be expected from various planning alternatives.

Model verification using data from cruises in 1967-1970 and some earlier data is considered satisfactory for evaluating effects of planning activities.

A variety of applications of the demonstration model framework to Type II planning questions were carried out. It is concluded on the basis of the results of these applications that the models provide new and important insights into the consequences of proposed control actions, insights that would ordinarily not be obvious without the application of quantitative modeling and system analysis techniques.

In evaluating and ranking the problem categories to provide a basis for recommendations in any further Phase II study, four criteria were used: l) existing modeling efforts, 2) ranking of the modeling framework, 3) data availability, and 4) the degree to which the problem can be considered a Type II planning problem.

Of the seven problem areas, it is concluded that the ice category and a portion of the public health category (near shore pathogen problems) are generally not Type II planning problems. It is also concluded that Great Lakes problems associated with a) lake levels and b) erosion and sediment are being analyzed and are adequately modeled in various ways for present needs. The ranking of the four remaining Type II planning problems which was subjectively established in lieu of being objectively determined is:

1. Eutrophication
2. Water Quality
3. Public Health (regional and lake wide scale)
4. Concentrations of toxic or harmful substances

It is concluded that it is feasible to construct a Limnological Systems Analysis for these categories using the existing available data, although the degree of detail and specificity of the Limnological Systems Analysis would vary with the problem category.

A range of alternate Limnological Systems Analysis programs were evaluated in order to explore varying levels of effort and cost for a Phase II study.

1. Level l: This alternate is estimated to cost $\$ 0.7$ million with a two year completion time and represents the lowest level at which a meaningful Limnological Systems Analysis can be carried out.
2. Level 2: This level is estimated at a $\$ 2$ million cost with a three year completion time and represents a favorable balance between problem contexts that can be approached rapidly, given the present modeling status, and those problem categories which have high priority but for which modeling frameworks must be significantly advanced.
3. Level 3: The cost of this level is estimated at $\$ 3.9$ million with a three year completion time and represents a more intensive effort than Level 2. Level 3 funding is felt to be the maximum amount that can be prudently spent for a Phase II study of the use of a Limnological Systems Analysis for the Great Lakes.

The overall conclusions of this Phase I study can be summarized as follows:

1. It is feasible to construct a Limnological System Analysis for certain Type II water resource problems in the Great Lakes.
2. Mathematical modeling and system analysis techniques can provide important preliminary quantitative estimates of the effects of certain proposed water resource control actions.
3. Sufficient data presently exist for the immediate implementation of a Limnological Systems Analysis although this does not preclude the need for further extensive field efforts on the Lakes.
4. Feasible funding ranges for a Phase II Limnological Systems Analysis are from $\$ 0.7$ million to $\$ 3.9$ million.

## SECTION II

## RECOMMENDATIONS

## SECTION II

RECOMMENDATIONS

```
It is recommended that a Phase II Limnological Systems Analysis study be funded at the \(\$ 2.0 \mathrm{million}\) level with a three-year completion time.
Within this level, it is recommended that:
```

1. Existing subsystem models, parameter values, and inputs be gathered into interactive modeling frameworks.
2. Generalized computer programs be developed and modifications be made to existing models to accomodate recently evolved numerical and software techniques.
3. Applications be made of existing systems technology to those problem categories for which a reasonable degree of success for the application is assured.

The following specific problem contexts are recommended for inclusion in the Phase II study:

1. Water Quality Problems
a) Dissolved oxygen
b) Chemical interactions
2. Public health
3. Eutrophication - biomass problems
4. Food chain toxicant problems

It is recommended that the Phase II study be directed toward three spatial scales:

1. Comprehensive Great Lakes - All lakes interconnected scale
2. Lake wide scale

- Lakes Erie and Ontario

3. Regional scale

- Duluth, Minnesota area, Southern Lake Michigan, Green Bay, Saginaw Bay, Lake St. Clair.


## SECTION III

## INTRODUCTION

## Purpose and Orientation

The purpose of this report is to present an assessment of the technical feasibility and the economic practicality of applying a Limnological Systems Analysis (LSA) to water resource problems of the Great Lakes. Specific attention is directed to an evaluation of the state of the art of modeling as it applies to these interrelated problems. The overall purpose of the study is to indicate the degree of understanding of limnological phenomena as affected by both nature and man's activities. Equally important, if not more so, its purpose is to evaluate the degree to which these processes can be expressed in a valid mathematical. form within a system analysis framework. Such a framework comprises two essential elements - the mathematical forms or models and the predictive techniques.

The greater our ability to express these processes in a mathematical form within a system analysis framework, the better the basis for selection among the alternate plans control and management of the system. Systems analysis is thus one important tool available to the administrator in his decision making role as environmental planner and manager.

Systems analysis has been applied in various ways to both natural and technological systems. Natural systems comprise those phenomena whose structures have been determined without man's influence whereas technological systems have been directly designed by man to meet various objectives. Although our fundamental knowledge of both systems is approximately of the same order, the application of modeling to technological systems, such as communications, transportation, energy, and industrial production has progressed to a further degree than has the application to natural systems, such as biological and chemical cycles in natural waters and hydrological and meteorological phenomena. The reason lies in the fact that many systems in the former category were understood and, in some cases, were created in terms of mathematical models which preceded their development. The second component of
systems analysis - the predictive methodologies incorporated in the general field of operations research - has been applied to technological systems to a greater extent. The majority of these techniques which are presently available have been developed within the framework of technological rather than natural systems, many of which originated, or at least significantly advanced, during and since the second World War. One of the general directions to be taken in the future is the transformation and application of these techniques to natural systems.

This report, which is primarily concerned with the modeling element, summarizes the models which describe natural limnological systems, both those significantly modified by man and those substantially unaffected by man, and indicates the utility of models in seeking and assessing alternate solutions to the problems arising from multiple uses of the water resource. In the case of the Great Lakes, as in many other natural settings, each use with its associated effects potentially or actually influences another use. It is one of the purposes of this report to demonstrate the application of modeling and to delineate and evaluate these interactive effects.

Although emphasis in this report is placed on the state of mathematical modeling, its predictive capability is not overlooked. Without the former, the latter is impossible, and without the latter, the former is useless within the context of this project. The models reviewed are those which have been specifically applied to Great Lakes problems, those which have been developed for other areas but do not have application in the region, and lastly, those which can be constructed within the time frame of the planning activity. These models are to be evaluated not only from the viewpoint of their internal validity, but also from their utility in planning and predicting.

## Methodology

Figure 1 presents the overall methodology followed in the Limnological Systems Analysis of the Great Lakes. Two


parallel paths are followed. One path analyzes aspects of the Limnological Systems Analysis concerned with planning and administrative functions simultaneously. The second path explores the scientific and engineering features of the Limnological Systems Analysis.

The first step in the planning and administrative analysis is the identification of Great Lakes water resource uses and associated problems, both present and anticipated. This identification is followed by a definition and grouping of water resource variables identified with the resource problems. These two steps are discussed in Section IV.

The extent and availability of data is an important evaluation that provides input information for several steps in the methodological framework. This step in presented in Section $V$.

The scientific, and engineering analysis begins in Section VI with a description of the basic principles of modeling and a methodology for evaluating model status. The array of models is scanned and a grouping of the models using problem categories is prepared as part of this step. Eleven modeling frameworks result from the analysis.

Section VII reviews the state of the art of the eleven modeling frameworks. The problem context, theory, extent of verification, and application are explored in detail. The first indication of the degree of feasibility of an Limnological Systems Analysis are given in this step.

The evaluation and synthesis of the eleven modeling frameworks are given in Section VIII. An analysis is also provided of the computational feasibility of interactive synthesized modeling structures.

The demonstration model which is used as an illustration of many of the steps in the overall methodology is presented in Section IX. The demonstration modeling framework consisting of several integrated submodels also provides input to the question of feasibility of an Limnological Systems Analysis.

Section $X$ begins with an analysis of the problem categories examining each of the categories in the light of several
priority criteria. The interaction of a subjective assessment of problem priority, available data, and modeling status, in essence, sets a level of possibility. The problem categories for Phase II are then discussed and a series of four alternative programs with funding is presented. Section $X$ closes with a recommended Phase II program chosen and shaped from the alternative programs.

Before beginning the detailed review of each of these steps, it is well to provide a brief description of the Great Lakes System so that the report is placed in its proper geographical and limnological setting.

## Description of Limnological System

Extending from the heartland of North America, the Great Lakes St. Lawrence River system constitutes one of the continent's most magnificient natural resources. Great population centers have developed on these shores and an economically diverse society has evolved through the effective management of the raw materials that the basin provides. The water system represented by the lakes provides the people of the United States and Canada with economic recreation and aesthetic water uses of unmeasurable value.

In geological terms the lakes are young. Their present forms were created during the Pleistocene era by glaciers that moved across the North American continent. Glacial advances and retreats over millions of years caused numerous morphological changes which about 2,500 years ago resulted in the Great Lakes system that we know today.

The Great Lakes system is composed of five major drainage basins covering an area of 295,800 square miles; 173,470 square miles of the region is in the United States with the remaining 122,330 square miles being located within the Province of Ontario, Canada. Figure 2 shows the location of the basins, and pertinent characteristics are given in Table 1. Nearly one-third of the basin constituting 94,680 square miles is lake surface. The system forms a natural waterway 2,300 miles long extending from the head waters of
FIGURE 2
great lakes basin drainage boundaries


| Description | Lake Superior | Lake Michigan | Iake Huron | Lake St. Clair | Lake Erie | Lake Ontario | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Low Water Datum (LWD) Elevation in feet IGLD (1955) | 600.0 | 576.8 | 576.8 | 571.7 | 568.6 | 242.8 |  |
| Dimensions in miles: |  |  |  |  |  |  |  |
| Length | 350 | 307 | 206 | 26 | 241 | 193 |  |
| Breadth | 160 | 118 | 183 | 24 | 57 | 53 |  |
| Shoreline including islands | 2,980 | 1,660 | 3,180 | 169 | 856 | 726 | 9,571 |
| Areas in square miles: 1 |  |  |  |  |  |  |  |
| Drainage basin in U.S. | 37,500 | 67,900 | 25,300 | 2,370 | 23,600 | 16,800 | 173,470 |
| Drainage basin in Canada | 43,500 | 0 | 49,500 | 4,150 | 9,880 | 15,300 | 122,330 |
| Total drainage basin (land \& water | 81,000 | 67,900 | 74,800 | 6,520 | 33,500 | 32,100 | 295,800 |
| Water Surface in U.S. | 20,600 | 22,300 | 9,100 | 162 | 4,980 | 3,460 | 60,602 |
| Water Surface in Canada | 11,100 | 0 | 13,900 | 268 | 4,930 | 3,880 | 34,078 |
| Total Water Surface | 31,700 | 22,300 | 23,000 | 430 | 9,910 | 7,340 | 94,680 |
| Volume of water in cubic miles: ${ }^{1}$ | 2,935 | 1,180 | 849 | 1 | 116 | 393 | 5,474 |
| Depths of water in feet: ${ }^{1}$ |  |  |  |  |  |  |  |
| Average over lake | 489 | 279 | 195 | 10 | 62 | 283 |  |
| Maximum observed | 1,333 | 923 | 750 | 212 | 210 | 802 |  |
| Outlet river or channel | St. Mary's | Mackinac | St. Clair | Detroit | Niagara | St.Lawren |  |
| Length in miles | 70 | - | 27 | 32 | 37 | 502 |  |
| Average flow in cfs (1860-1969) | 74,500 | 52,000 | 187,000 | 130,000 | 202,000 | 239,000 |  |

${ }^{1}$ Lake level at Low Water Datum Elevation. LWD is a reference elevation for nautical charts and projects. ${ }^{2}$ Maximum natural depth.


GENERAL GREAT LAKES INFORMATION

| Lake Superior | Lake Michigan | $\begin{aligned} & \text { Lake } \\ & \text { Huron } \end{aligned}$ |
| :---: | :---: | :---: |
| 600.38 | $578.68{ }^{3}$ | $578.68{ }^{3}$ |
| 602.06 | 581.94 | 581.94 |
| 598.23 | 575.35 | 575.35 |
| 1.1 | 1.1 | 1.1 |
| 1.9 | 2.2 | 2.2 |
| 0.4 | 0.1 | 0.1 |
| 29.4 | 31.2 | 31.2 |
| 30 | 30 | 31 |
| 1.00 | 0.86 | 1.05 |
| 191 | 99.1 | 22.6 |

I I I

$$
2
$$

| $\begin{array}{l}\text { Lake } \\ \text { Superior }\end{array}$ |
| ---: |
|  |
| 600.38 |
| 602.06 |
| 598.23 |
| 1.1 |
| 1.9 |
| 0.4 |


Average on basin (land \& water) Average on lake surface
Runoff (cfs/miles squared)
Runoff (cfs/miles squared)
Detention time (years)

${ }^{3}$ The straits of

Lakes Superior and Michigan through Lakes Huron, St. Clair, Erie, and Ontario into the St. Lawrence River and finally into the Atlantic Ocean. The economic value of this waterway has been enhanced by the construction of deep-water navigation channels and canal systems between the lakes.

Lake Superior is the largest and deepest of the lakes having a surface area of 31,700 square miles and a maximum depth of 1,333 feet. Because of its size and relatively small inflow, it also has the largest displacement time of 191 years. By contrast Lake Erie, which is the shallowest of the lakes, has the smallest volume and an associated displacement time of 2.6 years. Lake Ontario has the smallest surface area of the major lakes.

The tributary streams of the Great Lakes basin are generally short with relatively small drainage basins. They range in size from drainage areas of a few square miles to the size of the Maumee River and Grand River basins which are 6,600 and 5,600 square miles, respectively. The Great Lakes basin is also characterized by thousands of small upland lakes.

On the basis of data taken at land based stations, the average annual precipitation over the basins is estimated to be about 31.5 inches, while the average annual runoff of the rivers within the region varies from 9 to 38 inches.

In 1960 there were $31,780,000$ people living in the Great Lakes region. Over 80 percent of this population was living in the United States. Economic development in the basin has proceeded from the highly agricultural economy of the late 19 th Century to the present degree of development characterized by a high degree of urbanization and industrialization. In 1963, manufacturing exceeded 40 billion dollars, almost one-fourth of the nation's total. Annual investments in industries which are dependent on the Great Lakes water resource are in excess of 1.6 billion dollars. Present United States commercial shipping on the lakes is in excess of 200 million net tons annually.

The Great Lakes constitute a major source of municipal, industrial, and agricultural water supply for the basin's population. The 18 million people who rely on water supplied by the lakes utilize 4 billion gallons per day while present industrial supply is double that figure.

Forests and woodlands concentrated primarily in northern Minnesota, Michigan, Wisconsin, and New York constitute 48 percent of the land in the region; and 39 percent of the remaining land is covered by cropland and pastures located primarily in Wisconsin, southern Michigan, northern Indiana, and Ohio. The remaining 13 percent of the land area is non-agricultural and includes the urban centers, commercial, transportation, and industrial developments as well as farmsteads, idle land, and park and recreation areas.

## SECTION IV

## WATER RESOURCE PROBLEMS AND VARIABLES

## SECTION IV

WATER RESOURCE PROBLEMS AND VARIABLES

Water Resource Uses and Problems

Public policy objectives and their associated standards are fundamental to the definition of water resource problems. Actual or potential failure to meet a standard or objective identifies a water resource problem. Water resource problems therefore are generally associated with a comparison of the level or magnitude of a variable in the water environment and the desirable or required level, as specified by a standard or public policy objective. In addition, standard and policy objectives can be identified on the basis of interference with a desirable water use or environmental status. Therefore, as shown in Figure 3, water resource problem definitions contain two major parts: the first deals with variables in the water body and implies a comparison with a standard or norm; the second identifies water use or environmental status. This latter component of problem definition generally indicates the time and space scale in which the problem must be viewed. In addition to the time and space scale, the significance of the problem or the problem priority is also important.

In the first interim report of this study [1], a variety of Great Lakes water resource uses and problems are identified and referenced in detail. Specific localities experiencing various water resource problems are listed in Reference 1 , Appendix A. A brief description of the major water uses and problems is given here.

Water Supply

This water use encompasses water supplies for domestic, municipal, industrial, and agricultural purposes. The population of the Great Lakes basin is expected to grow to 60 million over the next fifty years from a present level of over 30 million.


The entire population is not served by water supply drawn from the lakes; but most of the large population centers do use the lake as a water supply source. The total United States and Canadian withdrawals from Lakes Erie and Ontario are estimated at 685 MGD (United States) and 419 MGD (United States), respectively [2].

The major consideration associated with water supply in the Great Lakes is the quality of raw water obtained from the lake. Public health considerations, such as contamination of raw water by bacteria, virus, and toxic or harmful substances are of primary concern. The majority of water intakes within the Great Lakes are presently located to yield relatively high quality waters. As population and economic growth continue around the Great Lakes Basin, it will be necessary to insure that the influence of wastewaters discharged from treatment plants or urban and other runoff do not contaminate water intakes with increased bacterial and viral concentrations. There have been no reported outbreaks of viral disease related to water obtained from well-operated water treatment plants in the Great Lakes Basin. Thus it may be inferred that water borne viruses have been controlled in public water supplies. There is, however, little or no direct evidence or data on viral concentrations within the Great Lakes or on the number of viruses present in water supplies after treatment.

A second broad area of concern in the water supply is the quality of finished water and the cost of water treatment plant operation. Specific problems have been experienced with Great Lakes water supplies in terms of taste, odor, and color problems and with clogging of intake screens, reduced filter runs and increased chemical costs [3,4]. Municipal supplies in Milwaukee, Chicago, Cleveland, and Toledo have also been affected. These problems have been associated with cladophora growths and phytoplankton blooms, and periodically have been ascribed to the residual effects of chemicals discharged in inđustrial and municipal wastes. These water supply problems can result in increased water treatment and supply costs together with reductions in finished water quality. The available supply of treated water may be temporarily reduced if water treatment plant capacity is significantly curtailed and adequate additional
facilities are not available. Many of the taste, odor, color, and clogging problems are encountered in the summer period when water supply demands approach a maximum. Thus the maximum demand periods occur when problems tend to increase operating costs and reduce the effectiveness of installed treatment plant capacities.

A potential long range water supply problem is associated with a possible buildup of total dissolved solids (TDS), chlorides, hardness, and other dissolved chemicals in the lakes. These water supply problems are not significant at the present time, but future population and economic growth could accelerate the buildup of these materials. Beeton [5,6] and O'Connor and Mueller [7] have explored some of these problems in detail. Figure 4 shows the chloride concentrations in Lake Michigan and the components that contribute to the total concentration as estimated by Mueller and O'Connor. Present TDS in Lakes Erie and Ontario are in the order of $180-200 \mathrm{mg} / 1$ which presently meets water quality criteria. Increased population and industrial growth will result in increased mass discharges of TDS to the system. These increases coupled with increased water consumption may result in future levels of TDS which will violate acceptable criteria. This problem is explored in the Demonstration Model discussed in Section IX of this report.

Recreation and the Aquatic Ecosystem

This category of water usage is considered in both an active and passive sense. Active recreational water use includes water contact sports such as swimming, boating, scuba diving, and water skiing as well as recreational boating and fishing. It is estimated that there are nearly one million recreational boats registered in the United States region of the Great Lakes basin with some 65 more boats moored in the Great Lakes proper. Included also is the broad area of aesthetic appeal of the water for picnicking, comtemplative relaxation, and scenic beauty. In addition, this water usage category has been broadened to include passive aesthetic appeal thus reflecting the conservationist viewpoint that a legitimate natural state of the environment should be maintained with a balanced relationship existing at all trophic levels.



Potential public health problems have been identified on the Great Lakes specifically with respect to bacterial pollution of beaches and associated permanent or temporary beach closings. Appendix A of Reference [l] lists forty-six beaches on the Great Lakes which have been reported closed because of bacterial pollution.

Degradation of local waters in the Great Lakes from an aesthetic viewpoint has been reported at twenty-one locations. There are a number of phenomena which can result in loss of aesthetic appeal. Rooted aquatic plants, specifically cladophora, have been washed up onto beaches and picnic areas causing unsightly conditions and local odor problems. Massive alewife dieoffs have resulted in accumulation of these fish on beaches with potential health and odor problems. Other potential problems which could interfere with direct usage and aesthetic appeal are large inshore photoplankton blooms and accidential spills of oils, floatables, and other chemicals.

Buildup of toxic and harmful substances has also been reported in the Great Lakes. Potentially harmful materials can build up in concentrations in the water column, in the benthos, and through the various trophic levels of the food web. These accumulations can result in rendering fish unfit to eat and may cause undesirable alterations and changes in the structure of the food web. Mercury and DDT have been identified with specific problems in this type of water and terrestial environment [2]. This phenomenon can infringe directly on active water usage and on the passive aesthetic concept of legitimate state of the environment.

The problem of eutrophication in the Great Lakes is perhaps the most significant. It is manifested in part by increased biological productivity especially at the phytoplankton level and undesirable changes in species composition at one or more trophic levels in the water column or the benthos. Increased productivity in the Great Lakes has usually been associated with the major nutrients (phosphorus, nitrogen, and carbon) discharged in wastes. Figure 5 shows the general location of excessive phytoplankton growth in the Great Lakes.

FIGURE 5
H $\perp$ MO\&S NOLYN $\forall 7 d O \perp$ AHd to NOI $\perp \forall O O 7$

The low dissolved oxygen levels in the hypolimnion of the central basin of Lake Erie are apparently related to the eutrohpication process. Figure 6 displays those areas experiencing dissolved oxygen problems. The shore local areas, such as Milwaukee Harbor are generally associated with discharge of organic wastes.

Changes in the composition of the benthos have been reported in various areas of the Great Lakes. These changes can be considered to represent a perturbation of existing aquatic balances and have been attributed to wastewaters entering the lake. The alterations in bottom fauna have been associated with low dissolved oxygen levels, chemical compounds of wastes, and physical settling of both organic and inorganic solids. The impact of changes in the benthos is not fully understood with regard to its effect on the overall aquatic balance. The premise has been offered that the observed changes in benthos permeate the food web, with influences ranging from destruction of fish habitat to changes in growth rate of phytoplankton and zooplankton.

The influence of the lake levels on recreational water usage can be considered in terms of possible changes in accessibility and aesthetic appeal of beaches, swimming, and picnicking areas. In addition, boat launching sites and fishing areas can also be sensitive to the levels of the Great Lakes. Lake levels can have an impact on the extent of the littoral zone and wildlife habitat availability in shallow and near shore reaches. Erosion and incoming sediment, which are related in part to the water levels in the Great Lakes, can reduce benthos population, increase turbidity, and cover fish spawning bends.

Shoreline and Harbors

The use of private, public, and commercial waterfront property is considered in this category. Recreational use of the shoreline is particularly significant in this regard. Use of shoreline property can be directly affected by variations in


Location of dissolved oxygen problems
lake water levels through erosion, reduction of beach width, exposure of aesthetically unappealing areas, interference with boat launching facilities, approach channels, and fishing areas.

Lake water levels can have a significant impact on shoreline and harbor usage. This impact results from the several components which influence the average and instantaneous lake levels. These factors are:
a. Man-made changes in lake levels resulting from regulation of the flow basin or flow diversions into or out of the basin. These changes may be direct, such as the influence of regulation at the Sault Ste. Marie control works on Lake Superior or indirect such as the influence of Lake Superior regulation on the unregulated Lakes Michigan, Huron, and Erie.
b. Annual variations in precipitation and outflow capacity which can result in seasonal, monthly, and long term fluctuation in lake levels.
c. Fluctuations in lake levels caused by seiches, storm surges, and wind waves.

Lake level fluctuations associated with the second and third components have been cited as being responsible for the severe damage caused to shore property during the high lake level period of 1951-1952. At that time innundation and accelerated shoreline erosion resulted. During the low lake levels of 1964 some shore installations, such as marinas, became less convenient to use and there were instances when they became unusable during very low water periods. However, certain recreational areas (where the sand beach is normally narrow) had the advantages of wider beaches during the 1964 low water period.

Specific locations [8,9] have been reported to have shoreline erosion problems or local flooding by lake waters. Figure 7 shows those areas experiencing erosion problems or silting of harbors and channels.

Fishing

The species composition of the Great Lakes fishery has varied significantly over several decades [10], although the average annual total weight of fish caught has been stable, fluctuating between 60 and 80 million pounds per year from 1920.

It has been suggested that intensive exploitation and environmental factors are responsible for the changes in the species composition of the catch. In addition, the invasion of the sea lamprey and the successful establishment of the alewife in the upper Great Lakes are also responsible for the changes. The sea lamprey apparently selectively preys on the lake trout and the burbot, both deep water predators, depleting these species. This depletion coupled with intensive harvesting of chubs allows the establishment and explosive growth of alewife populations in Lakes Michigan and Huron. This postulated sequence of events points out the impact of predation and exploitation on the fishery resource.

Contamination of the fishery resource by potentially toxic and harmful materials, such as mercury, DDT, or deldrin has been a problem on the Great Lakes. Mercury concentrations above the maximum permissible level have been found in fish caught from Lake St. Clair, western Lake Erie, and the Detroit and St. Clair Rivers. As an illustration of the difficulty that has been encountered with the build-up of DDT, 28,000 pounds of coho salmon were ordered seized in 1969, because the DDT concentration limit had been exceeded. Therefore, a significant problem exists in the contamination of fish by potentially toxic and harmful substances. This contamination can occur through the food chain or through direct ingestion and concentration by the fish themselves.


9NILרIS ONV NOISOYヨ ヨyOHS

The near shore area is of extreme importance in the maintenance of adequate fishery resources. . Discharge of waters from power plants which have been used for cooling can significantly influence local areas of the littoral zone. Thermal shocks and the possible physical destruction of fish fry and eggs at the water intake structure are possible consequences. Temperature changes in the zone surrounding the discharge may either damage or enhance the food available for fish depending on the design of the discharge facility.

The influence of the lake level changes on the fishery resource occurs in the littoral zone or shallow fish spawning and growing areas.

Navigation

Navigation on the Great Lakes is primarily influenced by low lake levels and ice formation [11]. During periods of extremely low lake levels, the capacity of the Great Lakes commercial fleet can be substantially reduced as a result of decreased available channel water depths. With regard to recreational boating the effectiveness of launching facilities and marinas can be impaired or made unusable by low water levels.

Low lake levels of 1964 caused adverse effects on commercial navigation and restricted to some extent the areas where recreational craft could be operated. During the 1964 navigation season when the levels of Lakes Michigan and Huron were about one foot below datum and the available channel depths correspondingly lessened, the cargo carrying capacity of the Great Lakes fleet was reported to have been materially reduced [11].

New United States commercial navigation traffic on the Great Lakes has been estimated at 222 million tons in 1968 [11]. Commercial navigation is seasonal on the Great Lakes. Ice formation in the connecting waterways, harbors, and in portions of the open lake system impede navigation. Figure 8 indicates problem areas associated with this phenomenon.

FIGURE 8
LOCATION OF AREAS EFFECTED BY ICE JAMS

Considerable attention is currently directed towards exploration of methods of extending the navigation season, predicting time of ice formation, and the time for the lake navigation system opening in the spring. A 90-day forecast ability for navigation seems to be required to satisfy overseas shipping interest on the Great Lakes while a 30 to 60-day forecast is apparently satisfactory to interlake shippers.

Navigational use of the Great Lakes can impact two other water resource problem areas. The first of these is concerned with spills of harmful or toxic materials due to shipping accidents in transit or during loading activities. The second major potential area of concern is disposal of dredge spoils. Extensive studies of this latter problem have resulted in the development of a number of pilot programs to evaluate alternative methods of dredge spoil disposal on the Great Lakes.

Power Generation

Power generation includes hydroelectric power generation, nuclear power generation, and fossil fuel power generation. It also encompasses pump storage projects and the disposal of heat from the generation of power by fossil or nuclear fuels.

In 1965, 25 million kilowatts of installed generation capacity was in the basin. In 1970, approximately 4,070 megawatts of hydroelectric generation capacity was available with approximately 2,100 megawatts of pumped-storage capacity either installed or under construction. It has been estimated that by the year 2020,467 million kilowatts of installed generation capacity will be required to insure the self-sufficiency of the power region [12]. The major increases in power generation are expected to be associated with nuclear facilities by the year 2000.

Hydroelectric generation on the Great Lakes is influenced significantly by fluctuations in lake levels and the quantity of available water. Low lake levels and reduced outflow have
resulted in substantial reductions in the power which can be generated by hydroelectric facilities on the Great Lakes. For example, the Niagara River flow available for power production in 1964 was approximately two-thirds of the long term average amounts available. Conversely, high lake levels and increased available outflow can result in increased electrical generation by hydroelectric facilities within the basin.

Power generation and cooling water disposal can contribute to Great Lakes water resource problems. Damage to the local aquatic balance may be associated with excessive heat disposal in the immediate vicinity of power plant discharges. Power plant locations and sitings may reduce aesthetic appeal of the surrounding area. Fish spawning and habitat areas as well as wildlife propagation areas may be impaired or destroyed in the vicinity of pump intakes and discharges. Alternatively, proper management and planning may enable excess heat from power generation to be employed to obtain desirable increased lake productivity and recreational and fishing opportunities. There is a possibility of radiological contamination from nuclear installations. Finally, the proposed solution to some of the thermal pollution problems requires increased consumption of water through cooling towers at power installations. This consumptive use of water will influence lake levels and outflow and will tend to increase the build up of dissolved materials in the Great Lakes.

Given the preceding brief review of water uses, there are a variety of ways to stratify the various water use problem contexts. The definitions of problems discussed at some length in Reference [1] were grouped by categories composed of similar classes of associated variables and/or standards. Seven problem categories resulted, each of which has various associated water uses. The seven problem categories are:

1. Water Level and Flow Rates of Great Lakes
2. Erosion-Sediment
3. Ice
4. Toxic Substances
5. Organic and Inorganic Chemicals
6. Eutrophication
7. Public Health

Table 2 expands on these categories and shows the associated affected water use of each category. Note that the first three categories generally deal with physical phenomena: flow, temperature, sediment, and precipitation. The last four categories are related to the general quality of the Great Lakes system, its water plants and animals.

## Variables and Planning Activities

The problem categories shown in Table 2 involve numerous water resource variables. Some of these variables are requisites for several of the problem categories. An example is the water circulation patterns of the Great Lakes which are obviously important constituents in many of the problem categories, depending on the time and space scale under investigation.

Variables are part of the water resource problem definitions which measure the status of the environment. In this sense, the level of a variable can be considered as output from modeling or other analysis efforts which a planning institution compares to an environmental standard or norm. In addition, some variables can be directly influenced by planning alternatives which change the mass (weight) or the time and space distribution of inputs or withdrawals from the system. In this latter sense, particular variables can be considered as inputs to a modeling or other analysis effort. The most obvious example of variables which may have external controls is the class of variables associated with wastewater inputs. Applications of various levels of waste removed will result in differing levels of an input variable such as metals.

Finally, there are input variables which are not of direct interest to the administrator but are required in the modeling or analysis effort. These are inputs over which there is generally no possible environmental control, such as the lake circulation or wind field.

The water resource variables associated with each of the seven problem categories are listed and then classified into twenty kindred variable groupings.

TABLE 2
WATER RESOURCE PROBLEMS

## Problem Category

Kindred Variable Groupings/ Standards

1. Mean monthly water level and flow rates of the Great Lakes
2. Erosion-sediment
3. Ice
4. Concentrations of toxic or harmful substances per unit weight of mass of biomass in the benthos and in the water column

## Water Use

A. Available channel depths for navigation
B. Available flows for hydroelectric power generation and water diversions
C. Accessibility and useability of marinas, beaches, and lakeside parks
D. Changes in the extent and character of fish and wildlife habitat
A. Lakeshore erosion with reductions in property values and utility
B. Flooding of lakeshore areas
C. Channel dredging requirements for commercial and recreational craft
D. Changes in the extent and character of fish and wildife
A. Opening and closing of the navigation season on the Great Lakes
A. Accumulation of toxic or harmful materials in the food chain which result in changes in the aquatic balance or destruction of a portion of the chain or web

TABLE 2
(continued)

## WATER RESOURCE PROBLEMS

## Problem Category

Kindred Variable Groupings/ Standards
A. Heavy Metals
B. Persistent organics
C. Radionuclides
D. Other substances
5. Concentrations of organic and inorganic chemicals which exceed present and projected water quality standards
A. Dissolved oxygen concentration
B. pH
C. Substances influencing visual appeal
D. Nutrients
E. Other Substances
6. Eutrophication. Concentration level of biomass, species distribution, and diversity in each of the trophic levels
A. Biomass
B. Species Distribution
C. Diversity Indicies
E. Increased costs for operation of water supply facilities

## Water Use

B. Accumulation of materials in higher life forms which render them unsafe or undesirable for human use
C. Unsafe or undesirable water as a source of municipal, agricultural, or industrial water supply
A. Changes in water quality of fish and wildlife habitat
B. Reductions in the aesthetic appeal of the water
C. Changes in the aquatic balance of the biological system (eutrophication)
D. Destruction of portions of the food web
A. Changes in the aquatic balance of the biological system
B. Changes in fish and wildife habitat
C. Destruction of a portion of the food web
D. Reduction in aesthetic appeal of the water
E. Increased cost for operation of water supply facilities
F. Management of commercial and recreation fishery

TABLE 2
(continued)
WATER RESOURCE PROBLEMS

Problem Category
Kindred Variable Groupings/ Standards

Water Use
G. Management of fish and wildlife habitat
7. Public Health: Bacterial and virus concentrations in the water body
A. Water unsafe or undesirable for use as a domestic, agricultural, or industrial water supply
B. Water unsafe or undesirable for use in bathing and contact water sports
C. Contamination of fish and wildlife habitat

Table 3 presents a detailed listing of the twenty variable groupings used in this study and specific examples of variables in each grouping. This table also contains an indication of the types of function variables within a variable grouping generally served in terms of model input or model output.

The specific water resource variable and the variable grouping provide the formal linkage between the planning and administrative activities and the technical modeling efforts.

A part of the planning function consists in formulating alternative management strategies (structural and non-structural) for obtaining desired objectives. These alternatives will influence the status and quality of the environment of the Great Lakes by changes in the inputs and withdrawals from the system. Communication between the planning function and the lake based models requires that variables which are affected by the proposed planning alternative be identified, potential water resource problems specified, and the level of the variable in the inputs or withdrawals associated with the planning alternative be quantified.

Table 4 indicates the variable groupings potentially affected by major planning activity in the Great Lakes Basin. These are the model input variables to which the lake based models respond.

Table 5 presents a matrix linking planning and management activities in the Great Lakes to the output from the modeling effort. Thus the planner considers altered levels of the model input variables indicated in Table 4 and analyzes the influence of this change by comparison of the model output variables from Table 5 with a goal or standard.

TABLE 3
KINDRED VARIABIE GROUPING

| Primary Variable |
| :---: |
| Function |


| model |  |
| :--- | :--- |
| output | model |
| input |  |

1. Meteorological
a) Precipitation (over lake) - X
b) Evaporation (over lake)
c) Wind speed and direction
d) Atmospheric pressure gradients
e) Solar radiation
f) Air temperature and humidity
2. Geomorphological
a) Water depth
b) Water surface area
c) Shore slope
d) Shoreline soil type
3. Hydrodynamic
a) Water currents (circulation patterns)
b) Dispersion coefficients
c) Wave heights
d) Wave energy at shoreline
4. Flow

X -
a) River and water import flows
b) Connecting channel flows
c) Consumptive water use (flow rates)
5. Lake Level

X
a) Mean monthly lake level
b) Mean any period (i.e., weekly, seasonal, et.al.) lake level
c) Water surface tilt
d) Wave run-up
6. Sediment-Erosion

X
a) Erosion rate (ft/yrs tons/yr)
b) Accumulation rate and location of deposited sediment
c) Turbidity of water-light penetration
d) Sediment concentration (suspended solids

TABLE 3
(continued)
KINDRED VARIABLE GROUPING
7. Ice
a) Time of ice formation and breakup
b) Percent ice cover
c) Ice thickness
8. Thermal

X
a) Water temperature and temperature profiles
9. Heavy Metals
a) Arsenic
b) Chromium
c) Copper
d) Lead
e) Zinc
f) Mercury
g) Cadmium
h) Selenium
10. Persistent or harmful organic contaminants
$\frac{\text { Primary Variable }}{\text { Function }}$

| model |  |
| :--- | :--- |
| output | model |
| input |  | X

$\qquad$
n

TABLE 3
(continued)
KINDRED VARIABLE GROUPING
12. Dissolved oxygen levels
a) Dissolved oxygen
b) $\mathrm{BOD}-\mathrm{TOC}-\mathrm{COD}$
c) Nitrogenous oxygen demand
d) Bottom oxygen demand
e) Oxygen production rate by photosynthesis
f) Oxygen utilization rate by phytoplankton and rooted plants
13. pH

X
a) pH
b) Carbonates
c) Bicarbonates
d) Acidity
e) Alkalinity
f) Chemicals or other buffer systems
14. Nutrients (free and combined)

\[

\]

TABLE 3
(continued)
KINDRED VARIABLE GROUPING

## 16. Species

| Primary Variable <br> Function |  |
| :--- | :---: |
| model | model |
| output | input |

a) Individual species populations

1) Phytoplankton
2) Zooplankton
3) Periphyton
4) Bottom organisms (vertibrate and invertibrate)
5) Fish
6) Wildlife
b) Allowable fish catch, per year by species
17. Diversity X
a) Diversity index (taxonomic and other)
1) Phytoplankton
2) Zooplankton
3) Bottom organisms (vertibrate and invertebrate)
4) Fish
5) Wildiife
b) Niche
6) Breadth (total or trophic level)
7) Carrying capacity (total or trophic level)
18. Bacteria and Virus X X
a) Indicator organisms
1) Total coliform
2) Fecal coliform
b) Pathogenic bacteria
c) Other bacteria
d) Virus

TABLE 3
(continued)
KINDRED VARIABLE GROUPING
19. Substances influencing visual appeal
Primary Variable

$\frac{\text { Function }}{\text { model }}$| model |
| :--- |
| output | input X

a) Floating solids
b) Settleable solids
c) Debris
d) Oils and greases
20. Other substances X X
a) Barium
b) Boron
c) Fluorides
d) Iron
e) Cyanide
f) Chlorides
g) Sulfates
h) Total dissolved solids
i) Salinity
j) Magnesium
k) Manganese

1) Color units
m) Taste test units
n) Odor test units
o) Hardness
p) Phenols


PLANNING AND MANAGMENT FUNCTIONS RELATED TO MODEL OUTPUT VARIABLE GROUPINGS

NOTE: *Cargo Dependent

1. Meteorological
2. Geomorphological
3. Hydrodynamic
4. Flow
5. Lake Level
6. Erosion
7. Ice
8. Thermal
9. Heavy Metals
10. Persistent or Harmful
organics
11. Radionuclides
12. Dissolved Oxygen
13. pH
14. Nutrients
15. Biomass
16. Species
17. Diversity Index
18. Bacteria and virus
19. Visual appeal (variables)
20. Other Substances


## REFERENCES

${ }^{[1]}$ Limnological Systems Analysis for Great Lakes - Phase I, Preliminary Model Design, Progress Report, Prepared by Hydroscience, Inc. , Westwood, New Jersey for Great Lakes Basin Commission, Ann Arbor, Michigan, Contract No. DACW-35-71-C-0030, p 105 and Appendix, (August 1971).
${ }^{[2]}$ Pollution of Lake Erie, Lake Ontario, and the International Section of the Saint Lawrence River, International Lake Erie Water Pollution Board and the International Lake Ontario - Saint Lawrence River Water Pollution Board, Volume 1 (1969).
[3] Lake Michigan Water Quality Investigations - Biology, United States Department of the Interior, Federal Water Pollution Control Administration, Great Lakes Region, Chicago, Illinois, (January 1968).
[4] Lake Erie Report - A Plan for Water Pollution Control, United States DeparEment of the Interior, Federal Water Pollution Control Administration, Great Lakes Region, Chicago, Illinois, (August 1968).
[5] Beeton, A.M., "Environmental Changes in Lake Erie," Transaction of the American Fish Society, 90 (2):153-159 (1961).
[6] Beeton, A.M., "Eutrophication of the Saint Lawrence River Great Lakes," Limnological Oceanography, 10:240-254 (1965).
[7] O'Connor, D.J., Mueller, J.A., "A Water Quality Model of Chlorides in Great Lakes," Journal ASCE, Sanitary Engineering Division, Volume 96, No. SA4 (August 1970).
[8] "Limnology of Lakes and Embayments," Great Lakes Basin Framework Study, Appendix 4, Draft No. 1 (November 1970).

## REFERENCES

(continued)
[9] Great Lakes Water Use Map, Prepared by CCIW, Resources Research Centre, Policy Research and Coordination Branch, Department of Fisheries and Forestry, No. JN 359.
[10] "Fisheries," Great Lakes Basin Framework Study, Appendix 4, Draft No. 1 (April $14,1 \overline{971) .}$
[11] "Navigation," Great Lakes Basin Framework Study, Appendix 9, Draft No. 1, Volumes 1 and 2 (December 1970).
[12] Lake Michigan Water Quality Investigations - Physical and Chemical Quality Conditions, United States Department of Interior, Federal Water Pollution Control Administration; Great Lakes Region, Chicago, Illinois (January 1968).

# SECTION V <br> EXISTING DATA 

## Introduction

Many significant elements in the development and application of models for water resource management require the use of observed data. Analysis and review of such observations provide a chronology of the history and development of water resource problems. These data also provide a basis to develop a scientific understanding of the phenomena involved. One of the most important functions, particularly within the scope of this study, is to provide the necessary information for verification of the various models required in the analyses of water resource problems. Finally, historical data, when evaluated in conjunction with data from ongoing collection and monitoring programs, can provide a measure of the effectiveness of management efforts.

Several classes of data are required for an understanding of the water resource and its proper management. These may be grouped under the following broad classifications: physical, chemical, biological, and special categories.

## Physical Data

Data in this category relate to the geomorphology, meteorology, hydrology, and hydrodynamics of the systems, and have been accumulated for well over a century. The structure, bathymetry and pertinent dimensions are well known. Maps showing the soundings in the lakes and the connecting waterways are available from the United States Lake Survey Center, NOAA.

Lake levels have been monitored as early as 1819 in Lake Erie (Table 6). Additional gages were added to the monitoring system over the years so that now a complete U.S.-Canadian network circumscribes all the lakes (Figure 9). All gages have continuous recorders and data are available on an hourly,

TABLE 6

## SUMMARY OF WATER LEVEL GAGING STATIONS IN THE GREAT LAKES BASIN**

| Lake Basin | Number of Gages |  | Earliest Year of Record |  |
| :---: | :---: | :---: | :---: | :---: |
|  | U.S. | Canada | U.S. | Canada |
| Superior | 6 | 4 | 1860 | 1907 |
| Michigan | 8 | - | 1859 | - |
| Huron | 6 | 6 | 1874 | 1906 |
| Erie | 8 | 5 | 1819 | 1860 |
| Ontario | 4 | 5 | 1837 | 1861 |
| Connecting River | Number of Gages |  | Earliest Year of Record |  |
|  | U.S. Canada |  | U.S. | Canada |
| St. Mary's | 2 | 4 | 1867 | 1908 |
| St. Clair | 7 | 2 | 1919 | 1927 |
| Detroit | 4 | 4 | 1897 | 1925 |
| Niagara | 8 | 5 | 1930 | 1919 |
| St. Lawrence* | 2 | 4 | 1916 | 1919 |

* 15 additional gages operated in connection with the st. Lawrence River Power Project by Ontario and New York (earliest year of record 1954).
**"Levels and Flows," Great Lakes Basin Framework Study, Appendix 11, Draft No. 2 (January 1971).

monthly, or annual basis from the United States Lake Survey Center and the Canadian Department of the Environment, Water Levels Division. All lake levels reported by the above agencies are referred to the International Great Lakes Datum (1955), as agreed upon by Canada and the United States. A comprehensive historic record of the United States gaging stations is available in the United States Lake Survey Center publication, Great Lakes Water Levels, 1960-1970.

Ever since Harrington released drift bottles in the Great Lakes prior to the turn of the century, work has proceeded on establishing representative net circulation patterns in the lakes. Most of the data have been obtained for the surface current by means of floating objects such as drift cards, drift bottles, and drogues (Table 7). In recent investigations, however, current meters have been used for general inplace currents and their temporal variations. Thus far, comprehensive surveys utilizing current meters have been conducted on Lakes Michigan and Erie by the F.W.P.C.A. In these surveys, data were obtained at several depths and a number of meters were monitored under both summer and winter conditions. A similar large scale survey of Lake Ontario is scheduled to be conducted in the summer of 1972 under joint United States-Canadian sponsorship.

Data on the surface water entering the Great Lakes are available from the United States Geological Survey, Department of the Interior in the form of Water Supply Papers and from the Canadian Inland Waters Branch, Department of Environment as Surface Water Data, Ontario. These flow measurements are tabulated by water years on a daily basis. All regularly operated gages are rated as to probable accuracy of measurement for periods of both high and low flows. The period of gaging records varies; the earliest information available is from the United States streams in 1884 and Canadian streams in 1906. The coverage of gaged streams was approximately 50 percent of the drainage area in the $1930^{\prime}$ s and has increased to 64 percent at present. Estimates for the individual lakes are:

Lake Drainage Area Gaged

Superior
Michigan
Huron
Erie Ontario

53 percent
71 percent
66 percent
67 percent
63 percent

Entire Lake

$$
\begin{aligned}
& \text { Saginaw Bay } \\
& \text { Douolas Point }
\end{aligned}
$$


Douglas Point
Daie du Dore

[^0]$\frac{\text { Year }(\mathrm{s})}{1953}$
$1931-32$
$1954-55$
1955
$1962-63$
1963
1954
1956
$1962-63$
1964
1928
$1948-49$
$1964-65$
$1964-65$
Five Harbors
Western Basin
 Eastern and Central Basin
Entire Lake
əIOYSIeən
$$
\frac{\text { Extent }}{\text { Eastern Portion }}
$$
Entire Iake
 Entire Lake 5 stations

| Remarks |
| :--- |
| 4470 released, $8 \%$ recovered |
| 745 released, $70 \%$ recovered |
| 3000 released, $60 \%$ recovered |
| 1297 released, max. $26 \%$ rec. |
| 44 stations, several depths, |
| summer and winter |

$$
\begin{aligned}
& 830 \text { released, } 93 \% \text { recovered } \\
& 1 \text { station several depths } \\
& 93 \text { released, } 54 \text { recovered } \\
& 14 \text { stations, several depths, } \\
& \text { summer and some winter } \\
& 34 \text { stations, several depths, } \\
& \text { summer and some winter }
\end{aligned}
$$

## IJC CCIW

3

The areas where streamflow is gaged are shown in Figure 10. In preparing this graphical summary, two publications were especially useful. The first is Catalog of Information on Water Data - Index to Surface Water Section, together with station location maps, prepared by the Office of Water Data Coordination (OWDC); United States Geological Survey. This publication lists thousands of stations which collect water data and gives the location, period or record, type of data storage, drainage area, frequency of measurement, types of data collected, and agency reporting the data. Similar information on Canadian streams is available in Surface Water Data Reference Index, Ontario which also includes station location maps. The Canadian stations are listed in downstream order and the list of United States stations can be obtained in similar form upon request to the Office of Water Data Coordination.

In the Great Lakes region, meteorological data are available from the United States National Weather Service (NWS) and the Canadian Meteorological Branch, Department of Transport. The data are collected primarily from shore-based stations although a few stations on islands and on light ships are maintained. First order stations provide hourly weather observations such as sky conditions, ceiling, pressure, air temperature, humidity, wind direction, and speed. Second order NWS stations are primarily operated by the Federal Aviation Administration (FAA) and maintain hourly records although many do not operate twenty-four hours a day. These first and second order stations provide coverage for the entire Great Lakes Basin, as seen in Figure 11. The above system is supplemented in both countries by a large number of cooperative observers who provide daily observations of air temperature and precipitation.

Additional data, such as rate of rainfall, soil and water temperature, upper air and wind condition, sunshine and solar radiation, and pan evaporation are available. The locations of current United States and Canadian stations obtaining solar radiation, pan evaporation, and upper air/wind data are shown in Figures 12 and 13.

The availability of published meteorological data is documented in Selective Guide to Climatic Data Sources prepared by the staff of the National Weather Records center, Ashville, North Carolina. Useful summaries of the daily


FIGURE II
LOCATION OF FIRST \& SECOND ORDER
METEOROLOGICAL DATA STATIONS


LOCATION OF SOLAR RADIATION \& PAN EVAPORATION DATA STATIONS


NWS data for a particular station are available in the form of the monthly publication, Local Climatological Data, from the NWS Records Center, Ashville, North Carolina. A national monthly summary, Climatological Data, which includes daily averages of upper air/wind data as well as solar radiation data is also available. Comparable Canadian data are available in the Monthly Record - Meteorological Observations in Canada, Monthly Radiation Summary and Monthly Bulletin Canadian Upper Air Data from the Queen's Printer, Ottawa.

Chemical Data

In evaluating the availability of past and present water quality data for the lakes, attention was focused on those sources whose data were collected over large areas of the lake or whose records extended over long periods of time. These sources are listed chronologically by lake in Table 8 together with an indication of the spatial and temporal extent of the surveys and a list of variables measured. The data that are available cover most chemical and biological parameters of interest although they are for varying times and locations. Full synoptic cruises on which significant variables were all collected are rare. Often the cruises are scheduled for specific purposes, such as trace metals determinations. The present International Field Year effort on Lake Ontario is designed to gather all pertinent information during a given year. The coverage therefore spans lake-wide surveys conducted during different times of the year to a specific one hundred year record at a water intake in Lake Michigan. Ranked according to the number of major surveys, Lake Erie is first with twenty-eight entries, Lake Michigan next with thirteen entries, followed by Lake Huron with twelve, Lake Superior with ten, and Lake Ontario with seven.

As may be seen from the table, United States and Canadian federal agencies are the prime collectors of data, supplemented by state agencies and educational institutions. At the present time, the Canada Centre for Inland Waters (CCIW) and the Lake Survey of the National Oceanic and Atmospheric Administration

| TimePeriod | Sampling Periodicity | Geographical Extent | TABLE 8 <br> LAKES WATER SAMPLING DATA SUMMARY* |  | Type of Data Storage | Collecting/ Reporting Agency |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | No. of Sampling Stations | Variables Measured |  |  |
|  |  |  |  | LAKE SUPERIOR |  |  |
| $\begin{aligned} & 08 / 13 / 52- \\ & 08 / 27 / 52 \end{aligned}$ | Grab | Entire Lake | 35 | temp, pH , spec. cond., DO, tot.alk., all for several depths | P | USBCF |
| $\begin{aligned} & 05 / 03 / 53- \\ & 10 / 25 / 53 \end{aligned}$ | Irregular | Entire Lake | 105 | temp, $\mathrm{Ca}, \mathrm{Na}, \mathrm{SiO}_{2}$, tot.alk.,tot.P., spec.cond., $\mathrm{pH}, \mathrm{DO}, \mathrm{Mg}, \mathrm{N}$ (dis). | P | USBCF |
| $\begin{aligned} & 04 / 08 / 61- \\ & 12 / 11 / 61 \end{aligned}$ | Irregular | East \& NE Portion | $\begin{array}{lr} \text { Max. } & 32 \\ \operatorname{Min} . & 4 \end{array}$ | temp., conduct. (limited secchi, color and pH ) | P, C | GLI |
| $\begin{aligned} & 07 / 18 / 64- \\ & 12 / 08 / 64 \end{aligned}$ | Irregular | Entire Lake | 86 | temp, cond,turb, $\mathrm{pH}, \mathrm{alk}$ (phenol and total), DO | $\mathrm{P}, \mathrm{C}$ | GLI |
| 08/00/68 | Grab | Entire Lake | 86 | secchi, color, temp, turb, spec. cond., TDS, $\mathrm{PH}, \mathrm{T}$ Alk, $\mathrm{DO}, \mathrm{T} \mathrm{PO}_{4}, \mathrm{SO}_{4} \mathrm{~F}, \mathrm{Cl}, \mathrm{SiC}_{2}$, hard, $\mathrm{Ca}, \mathrm{Mg}, \mathrm{K}, \mathrm{Na}$, Chloro a, tot.coli., fec.coli. | $C_{r} \mathrm{P}$ | CCIW |
| $\begin{aligned} & 05 / 00 / 68- \\ & 11 / 00 / 68 \end{aligned}$ | Monthly | Eastern Portion | 72 | temp,trans, pH,Eh,T Alk,TC Alk,C,Cl, spec. cond., DO, Tot.coli., sedi. chem. macro fauna,solar rad. | C | NOAA |
| $\begin{aligned} & 07 / 02 / 69- \\ & 07 / 09 / 69 \end{aligned}$ | Grab | Southern Shore | 22 | secchi disk, carb.fix., $\mathrm{SO}_{4}, \mathrm{NO}_{3}-\mathrm{N}, \mathrm{NH}_{3}-$ N ,Ortho $\mathrm{PO}_{4}$,all at surf and bottom |  | GLRD |

[^1]





USBCF USCBF

GLRI USBCF $\begin{array}{ll}P & G L R I \\ P, C & \text { GLIRBP (HEW) }\end{array}$


| $\begin{gathered} \text { Time } \\ \text { Period } \end{gathered}$ | Sampling Periodicity | GREA <br> Geographical Extent | ```TABLE 8 (continued) LAKES WATER SAMPLING DATA SUMMARY*``` |  | Type of Data Storage | Collecting/ Reporting Agency |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | No. of Sampling Stations | Variables Measured |  |  |
|  |  |  |  | LAKE MICHIGAN |  |  |
| $\begin{aligned} & 04 / 24 / 64- \\ & 11 / 07 / 64 \end{aligned}$ | Approx. Monthly | Entire Lake | 15 | organic N | P | GLRD |
| $\begin{aligned} & 01 / 00 / 68- \\ & 12 / 00 / 68 \end{aligned}$ | Monthly | SW Part,III. Water Intakes | 12 | tot.coli.fec.coli., turb,odor, pH, tot. $\mathrm{PO}_{4}, \mathrm{Cl}, \mathrm{SO}_{4}$, radioac. , phytop., macroinvert. | P | ISWB |
| $\begin{aligned} & 04 / 07 / 69- \\ & 11 / 14 / 69 \end{aligned}$ | Biweekly | SW Part,Ill. <br> Beaches | 48 | tot.coli., fec.coli., turb, temp, pH, $\mathrm{NH}_{3}-\mathrm{N}, \mathrm{MBAS}$, tot. $\mathrm{PO}_{4}$, plank ton | P | ISWB |
| $\begin{aligned} & 07 / 22 / 69- \\ & 08 / 23 / 69 \end{aligned}$ | Grab | South and <br> North Parts | 48 | secchi, $\mathrm{C}-f i x a t i o n, \mathrm{SiO}_{2}, \mathrm{NO}_{3}-\mathrm{N}, \mathrm{NH}_{3}-\mathrm{N}$ ortho $\mathrm{PO}_{4}-\mathrm{P}$,(all at surf and bottom) | P | GLRD |
| $\begin{aligned} & 08 / 23 / 69- \\ & 06 / 11 / 70 \end{aligned}$ | Grab (3) | Entire Lake | 50 | radioactivity in water sedi.,benthos, zooplankton, phytoplankton, fish,trace elements analysis | P | GLRD |
| $\begin{aligned} & 05 / 00 / 70- \\ & 11 / 00 / 70 \end{aligned}$ | Bi-monthly | North Portion | 62 | temp,trans, $\mathrm{pH}, \mathrm{Eh}, \mathrm{T}$ Alk.,TC, alk.,Cl, spec.cond., DO,tot.coli.sediment chemistry | C | NOAA |

*See Legend at end of Table.
＊See Legend at end of Table． TABLE 8
（continued） GREAT LAKES WATER SAMPLING DATA SUMMARY＊
 TABLE 8
（continued）


## IAKE HURON

tot．Coli．，Cl，phenols，chloro．dem．，
DO，BOD, turb．TS，TVS $\mathrm{DO}, \mathrm{BOD}_{5}$, turb，TS，TVS
temp， $\mathrm{Ca}, \mathrm{Mg}, \mathrm{Si}$, cond（several depths）
temp， $\mathrm{Na}, \mathrm{K}, \mathrm{Ca}, \mathrm{Cl}, \mathrm{SO}_{4}, \mathrm{SiO} 2_{2}$, spec．cond．，
$\mathrm{DO}, \mathrm{pH}$
temp， $\mathrm{Na}, \mathrm{Ca}, \mathrm{SO}_{5}$ ，conduct， $\mathrm{K}, \mathrm{Mg}, \mathrm{P}$, Tot．
transp，color，temp，DO，pH，alk．，cond．， phenol，tot．coli．，solar radiation
transp，color，temp，DO，pH，alk．，cond．， phenol，tot．coli．，solar radiation temp，secchi，cond．，turb．pH，alk．，DO； temp，secchi，cond．，turb，pH，alk．，DO； above at 2 or more depths


## -- GR

－－－－－－－－－－－－
Southern Tip
Entire Lake
South－Central
Portion
Saginaw Bay
Entire Lake

> Georgian Bay
Entire Lake
Georgian Bay P GLI P GLI IJC
ODLF
USBCF USBCF $\begin{array}{lll}H & H & H \\ ⿹ 丁 口 O & \text { H }\end{array}$
GLI
P
or
0 a
4




| $\begin{gathered} \text { Time } \\ \text { Period } \end{gathered}$ | ```Sampling Periodicity``` | Geographical Extent | TABLE 8(continued)LAKES WATER SAMPLING DATA SUMMARY* |  | Type of Data Storage | ```Collecting/ Reporting Agency``` |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | No. of Sampling Stations | Variables Measured |  |  |
|  |  |  |  | LAKE ERIE |  |  |
| $\begin{aligned} & 01 / 1910- \\ & 02 / 1957 \end{aligned}$ | Daily | Water intake at Lorain, Ohio | 1 | temp, Ts, $\mathrm{NO}_{3} \mathrm{~F}, \mathrm{Cl}, \mathrm{SO}_{4}, \mathrm{HCO}_{3}, \mathrm{Na}+\mathrm{K}, \mathrm{Mg}, \mathrm{Ca}$, $\mathrm{Fe}, \mathrm{SiO}_{2}, \mathrm{Alk}$, monthly av.are reported for each year | P | GLRI |
| $\begin{aligned} & 01 / 1920- \\ & 12 / 1956 \end{aligned}$ | Daily | Water intake at Erie, Pa. | 1 | temp, TS, $\mathrm{NO}_{3}, \mathrm{Cl}, \mathrm{SO}_{4}, \mathrm{HCO}_{3}, \mathrm{Na}+\mathrm{K}, \mathrm{Mg}, \mathrm{Ca}$, $\mathrm{Fe}, \mathrm{SiO}_{2}, \mathrm{Alk}$, monthly av . are reported for each year | P | GLRI |
| $\begin{aligned} & 6 / 15 / 28- \\ & 9 / 15 / 28 \end{aligned}$ | Monthly | Eastern Portion | 23 | temp,alb. $\mathrm{N}, \mathrm{NH}_{3}, \mathrm{NO}_{3}, \mathrm{~B}$ coli. | P | USBCF |
| $\begin{aligned} & 06 / 07 / 29- \\ & 09 / 19 / 29 \end{aligned}$ | Monthly | Eastern and Central Basin | 62 | DO, $\mathrm{CO}_{2}, \mathrm{alk}, \mathrm{pH}, \mathrm{Cl}$, Turb., temp, transp, microplankton,macroplankton | P | USBCF |
| $\begin{aligned} & 04 / 29- \\ & 10 / 29 \\ & 04 / 30- \\ & 10 / 30 \end{aligned}$ | Irregular | Western Portion |  | temp, $\mathrm{Cl}, \mathrm{NH}_{3}, \mathrm{alk}, \mathrm{NH}_{3}, \mathrm{NO}_{2}, \mathrm{NO}_{3}, \mathrm{DO}, \mathrm{CO}_{2}$, Alk.,pH,phytoplankton, zooplankton bottom organisms | P | USBCF |
| 1946-1948 | Irregular | Western Portion | 142 | tot. coli., $\mathrm{Cl}, \mathrm{phenols}, \mathrm{NH}_{3}-\mathrm{N}$, chloro. dem., temp, DO, $\mathrm{BOD}_{5}$, turb, alk.TS,TVS | P | IJC |

[^2]TABLE 8 (continued)
GREAT LAKES WATER SAMPLING DATA SUMMARY*

| Time Period | $\begin{aligned} & \text { Sampling } \\ & \text { Periodicity } \\ & \hline \end{aligned}$ | Geographical Extent | LAKES WATER SAMPLING DATA SUMMARY* |  | Type of Data Storage | ```Collecting/ Reporting Agency``` |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | No. of Sampling Stations | Variables Measured |  |  |
|  |  |  |  | LAKE ERIE |  |  |
| 1947-1953 | Irregular | Central <br> Portion | 142 | temp, $\mathrm{DO}, \mathrm{CO}_{2}, \mathrm{alk} ., \mathrm{pH}$, secchi,all at 4 or more depths | P | GLRI |
| $\begin{aligned} & 16 / 00 / 50- \\ & 09 / 00 / 51 \end{aligned}$ | Irregular | Nearshore, Ohio |  | temp, tot. coli., DO, color, pH, spec. cond., $\mathrm{SCO}_{2}, \mathrm{Fe}, \mathrm{Cu}, \mathrm{Cr}, \mathrm{Ca}, \mathrm{Mg}, \mathrm{Na}+\mathrm{K}, \mathrm{HCO}_{3}, \mathrm{SO}_{4}, \mathrm{Cl}$, $\mathrm{F}, \mathrm{NO}_{3}, \mathrm{TDS}$, hard, bottom fauna | P | ODNR |
| $\begin{aligned} & 06 / 28 / 55- \\ & 09 / 15 / 55 \end{aligned}$ | Irregular | Entire Lake | 128 | temp (several depths) | P | ODLF |
| $\begin{aligned} & 09 / 04 / 59- \\ & 09 / 05 / 59 \end{aligned}$ | Grab | Western and Central Basin | 50 | temp, DO (generally at 3 depths),secchi | P | USBCF' |
| $\begin{aligned} & 06 / 20 / 60- \\ & 11 / 15 / 60 \end{aligned}$ | Monthly | Entire Lake | 60 | temp, DO, cond., (surface and bottom) | P | ODLF |
| $\begin{aligned} & 08 / 30 / 60- \\ & 08 / 31 / 60 \end{aligned}$ | Grab | Entire Lake | 168 | temp, DO, alk,pH,spec.cond., (all for several depths),secchi,some hourly data | P | USBCF |


| Collecting/ |
| :---: |
| Reporting |
| Agency |

GLRI
ODNR ODLF
USBCF ODLF USBCF

[^3]*See legend at end of Table.

| Time Period | Sampling Periodicity | Geographical Extent | T LAKES W <br> No. of Sampling Stations | TABLE 8 <br> (continued) <br> TER SAMPLING DATA SUMMARY* <br> Variables Measured | Type of Data storage | Collecting/ Reporting Agency |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | LAKE ERIE |  |  |
| $\begin{aligned} & 06 / 00 / 66- \\ & 09 / 00 / 66 \end{aligned}$ | Irregular | Michigan Shoreline | 40 | total coli. (Michigan Beaches) | P | MWRC |
| 08/00/66 |  |  | 105 | secchi, temp, turb, spec. cond, ,pH, alk., BOD, $\mathrm{DO}, \mathrm{PO}_{4}, \mathrm{NO}_{2}, \mathrm{Cl}$,hard, phenols, tot. coli. | C | CCIW |
| $\begin{aligned} & 06 / 00 / 67- \\ & 10 / 00 / 67 \end{aligned}$ | Bi-weekly | Entire Lake | $\begin{array}{lr} \text { Max. } & 192 \\ \text { Min. } & 77 \end{array}$ | secchi, color, temp, turb, spec. cond., pH, alk., $\mathrm{BOD}, \mathrm{DO}, \mathrm{SO}_{4}, \mathrm{Cl}, \mathrm{SiO}_{2}$,hard, $\mathrm{Cd}, \mathrm{Ca}$, $\mathrm{Cr}, \mathrm{Co}, \mathrm{Cu}, \mathrm{Fe}, \mathrm{Pb}, \mathrm{Li}, \mathrm{Mg}, \mathrm{Mn}, \mathrm{Ni}, \mathrm{K}, \mathrm{Na}, \mathrm{Sr}$, Zn,tot.coli.,fec.coli.,TCS,ortho $\mathrm{PO}_{4}$ | C, P | CCIW |
| $\begin{aligned} & 04 / 00 / 67- \\ & 11 / 00 / 67 \end{aligned}$ | Grab | Entire Lake | 63 | temp,solar rad.,trans,pH,Eh,alk.,Cl, spec. cond, co,tot.coli.sedi.chemi. | C | NOAA |
| $\begin{aligned} & 04 / 00 / 67- \\ & 08 / 00 / 67 \end{aligned}$ |  | Western Basin |  | temp,spec.cond., Cl,phenol,DO,Tot P, sol $\mathrm{P}, \mathrm{NO}_{3}, \mathrm{NH}_{3}$,org -N , tot.coli., $\mathrm{SS}_{2} \mathrm{SO}_{4}$ | C | FWPCA |
| $\begin{aligned} & 05 / 00 / 67- \\ & 01 / 00 / 68 \end{aligned}$ |  | Mid-Lake |  | secchi, temp, alk., spec. cond., $D O, B O D_{5}$ $\mathrm{COD}, \mathrm{pH}, \mathrm{Eh}, \mathrm{TS}, \mathrm{TDS}, \mathrm{Cl}, \mathrm{NH}_{3}, \mathrm{NO}_{3}$, org -N , Sol P,Tot $\mathrm{P}, \mathrm{SiO}_{2}$, turb, chlor a,seston, phytopl.sedi.chemi., macro-inverte. | $C, P$ | FWPCA |

*See legend at end of Table.

 まO $\cdot \mathrm{ON}$

Grab
Entire Lake
TABLE (continued)
GREAT LAKES WATER SAMPLING DATA SUMMARY*
Collecting/
Reporting

Agency $\begin{array}{lllll} & & 0 & 3 & 3 \\ 0 & H & \text { Hy } & \text { 号 } & 0 \\ 0 & 0 & 0 & 0\end{array}$ | Type of |
| :---: |
| Data |
| Storage |

| $a$ | $a_{1}$ | $a_{1}$ | $a_{1}$ | $a_{0}$ | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| - |  |  | - | 0 |  |

LAKE ONTARIO $\frac{\text { Variables Measured }}{\text { LA }}$
temp, $\mathrm{pH}, \mathrm{Eh}, \mathrm{alk}, \mathrm{Ca}, \mathrm{Mg}, \mathrm{Na}, \mathrm{K},(\mathrm{all}$ gen-
temp, $\mathrm{pH}, \mathrm{Lh}, \mathrm{alk}, \mathrm{Ca}, \mathrm{Mg}, \mathrm{Na}, \mathrm{K},(\mathrm{lall}$ gen-

secchi,temp,cond.,turb, pH,T alk.,DO,
temp,DO,spec.cond.Ph,TC alk.,T alk., Na,k,Ca, $\mathrm{SiO}_{2}$ (many sta. sampled at
several depths) phyto., ben.macrofauna secchi, temp,turb, spec.cond. , pH,T.alk., $\mathrm{BOD}, \mathrm{DO}$, Ortho $\mathrm{PO}_{4}, \mathrm{NO}_{3}, \mathrm{NO}_{2}, \mathrm{Cl}$, hard, phe-
nol, tot. coli. secchi, color, temp,turb,spec. cond.,TDS $\mathrm{pH}, \mathrm{T}$ alk., TC alk., $\mathrm{BOD}, \mathrm{DO}, \mathrm{PO}_{4}, \mathrm{NH}_{3}, \mathrm{NO}_{3}$, $\mathrm{Cr}, \mathrm{Co}, \mathrm{Cu}, \mathrm{Fe}, \mathrm{Pb}, \mathrm{Li}, \mathrm{Mg}, \mathrm{Mn}, \mathrm{Ni}, \mathrm{K}, \mathrm{Na}, \mathrm{Sr}, \mathrm{Zn}$, phenols,chlor a,tot.coli.,fec.coli.

106

$$
47
$$

$$
z 9
$$

 -səм әжет-рти


Irregular
Irregular
Entire Lake
xeqnbəxxI

$06 / 00 / 67-$
$11 / 00 / 67$
*See Legend at end of Table.
$\square \square ? \square ?$
TABLE 8
(continued)

| TABLE 8(continued)GREAT LAKES WATER SAMPLING DATA SUMMARY |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Time Period | Sampling Periodicity | Geographical Extent | No. of Sampling Stations | Variables Measured | Type of Data Storage | Collecting/ Reporting Agency |
|  |  |  |  | LAKE ONTARIO |  |  |
| 05/00/6811/00/68 | Monthly | Entire Lake | $\begin{array}{ll} \operatorname{Max} . & 91 \\ \operatorname{Min} . & 50 \end{array}$ | secchi, color,temp,turb, spec. cond., TDS, $\mathrm{pH}, \mathrm{T}$ alk., DO, Tot. $\mathrm{PO}_{4}, \mathrm{NH}_{3}, \mathrm{NO}_{3}$, $\mathrm{SO}_{4}, \mathrm{~F}, \mathrm{Cl}, \mathrm{SiO}_{2}, \mathrm{hard}, \mathrm{Ca}, \mathrm{Mg}, \mathrm{K}, \mathrm{Na}$, chlor a,tot.coli., fec.coli. | $C, \mathrm{P}$ | CCIW |
| $\begin{aligned} & 04 / 00 / 69- \\ & 12 / 00 / 69 \end{aligned}$ | Monthly | Entire Lake |  | secchi, color, temp, turb, spec. cond. , $\mathrm{DO}, \mathrm{PO}_{4}, \mathrm{NH}_{3}, \mathrm{NO}_{3}, \mathrm{SiO}_{2}, \mathrm{Cd}, \mathrm{CO}, \mathrm{Cu}, \mathrm{Fe}, \mathrm{Pb}$, $\mathrm{Mn}, \mathrm{Mo}, \mathrm{N}, \mathrm{Sr}, \mathrm{V}, \mathrm{Zn}, \mathrm{ch} 10 \mathrm{r}$ a,bact. counts | C | CCIW |

*See Legend at end of Table.
GREAT LAKES WATER SAMPLING DATA SUMMARY*
TABLE 8
(continued)

Time Period: First and last date of sampling.
Sampling Periodicity: Approximate time between sampling.
Geographical Extent: General area coverage of sampling
No. of Sampling Stations: Representative number of sampling stations for time period.

Variables measured: Physical, chemical, biological sediment, bacteriological

Type of Data Storage: $\mathrm{P}=$ published, $\mathrm{C}=$ computerized
Collecting/Reporting Agency:

| USBCF | U.S. Bureau of Commercial Fisheries |
| :--- | :--- |
| GLI | Great Lakes Institute, University of Toronto |
| CCIW | Canada Centre for Inland Waters |
| NOAA | National Oceanic and Atmospheric Administration |
|  | Lake Survey |
| GLRD | Great Lakes Research Division, University of |
|  | Michigan (GLRI prior to 1960) |
| CDW\&S | Chicago Department of Water and Sewers |
| GLIRBP | Great Lakes M Illinois River Basin Project, U.S, |
|  | Department of Health, Education, and Welfare |
| ISWB | Illinois Sanitary Water Board |
| IJC | International Joint Commission |
| MWRC | Michigan Water Resources Commission |
| ODNR | Ohio Department of Natural Resources |
| UWO | University of Western Ontario |
| FWPCA | Federal Water Pollution Control. Administration |
| GLFC | Great Lakes Fishery Commission |
| ODLF | Ontario Department of Lands and Forests |

(NOAA) are the primary organizations having historical continuing lakewide data collection programs. Other significant sources of recent water quality data include state agencies (Ohio Department of Natural Resources and Illinois Sanitary Water Board) and the educational institutions (University of Michigan - Great Lakes Research Division and Sea Grant Program, University of Wisconsin, University of Toronto - Great Lakes Institute, University of Western Ontario). A wealth of historical water quality data as well as general lake environmental information is located in the Van Oosten Library of the Bureau of Sport Fisheries and Wildife, Great Lakes Fishery Laboratory, Ann Arbor, Michigan. Although not probed in depth, it is felt that much more data of a localized nature is available, such as from cities having water intakes in the lakes and from power plants on the lakes.

Prior to the mid-sixties, data were generally available in the form of data reports or cruise summaries. These proved extremely useful to this project in determining the scope of the surveys. At present, much of the data are stored in computerized data banks. Based on the experience accumulated to date, it is difficult to obtain summaries in meaningful or concise forms from the computerized data banks servicing the Great Lakes. The specific data can be readily obtained with time averages and values of the parameters measured as well as some statistical analysis. However, the appropriate qualifications and methods of analyses are difficult to retrieve. Summaries, currently being developed by NOAA personnel and made available to this project, should be most helpful to future investigators.

To understand the water quality data, the quantity of waste inputs of all chemical parameters are required. The current literature contains few summaries of waste inputs to the lake. Examples of published data are shown in Table 9 where it is seen that the data are sparse and only available for downstream lakes. To supplement this information, a survey of the state and provincial agencies responsible for water pollution control was conducted as part of this project. The results of this survey (Table lo) indicate that a considerable body of information concerning waste inputs from tributary streams, direct municipal STP discharges, and direct industrial discharges is available from the state/ provincial agencies.
TABLE 9
SUMMARY OF PUBLISHED WASTE INPUTS TO THE GREAT LAKES


[^4]
*See Legend at end of Table.

*See Legend at end of rable.

TABLE 10
continued)
SUMMARY .OF AVAILABLE STATE AND PROVINCIAL WASTE INPUT DATA*
27 sem
Parameters Monitored
Sampling
Frequency Yr. of Rec.

| Source |
| :--- |
| New York State |
| Department of |
| Environmental |
| Conservation |

*See Legend at end of Table. (continued)


| $\qquad$ | Waste |  | Parameters Monitored | Sampling |  | Source |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Type | \% Known |  | Frequency | $r$. of Rec. |  |
| Pa. | Trib. |  |  | irregular |  |  |
|  | Mun. | 100 | $\mathrm{pH}, \mathrm{BOD}, \mathrm{PO}_{4}, \mathrm{SS}$, settleable solids, $\mathrm{NH}_{3}-\mathrm{N}$ | 3 to 6 mo . | varies | Pennsylvania Department of Environmental |
|  | Ind. | 100 | pH, acidity,alk., $\mathrm{PO}_{4}, \mathrm{BOD}$, (others such as heavy metals,DS) | 3 to 6 mo | varies | Resources |
| Wisconsin | Trib. | 90 | Alk, tot.coli, $\mathrm{BOD}_{5}, \mathrm{Cl}, \mathrm{color}$, hardness, TS , | monthly | 11 | Wisconsin <br> Department of <br> Natural <br> Resources |
|  |  |  | TVS, SS, VSS, DO, temp, \& radioactivity |  |  |  |
|  |  |  | Org $-\mathrm{N}, \mathrm{NH}_{3}-\mathrm{N}, \mathrm{NO}_{3}-\mathrm{N}$; tot\&sol P | quarterly | 11 |  |
|  |  |  | heavy metals | quarterly | 3 |  |
|  |  |  |  |  |  |  |
|  | Mun. | 75 | BOD $5, \mathrm{TS}, \mathrm{SS} ; \mathrm{heavy}$ metals, Org-N, $\mathrm{NH}_{3}-\mathrm{N}, \mathrm{NO}_{3}-\mathrm{N}$, tot\&sol $P$ | 5 year irreg. | $\begin{array}{r} 22 \\ 3 \end{array}$ |  |
|  | Ind. |  | $\mathrm{BOD}_{5}$, SS | 5 year | 22 |  |
|  |  |  | heavy metals | irreg. |  |  |
|  |  |  | $\mathrm{pH}, \mathrm{BOD}_{5}$,solids (pulp \& paper mills) | yearly | 42 |  |

TABLE 10

LEGEND
Province/State: Origin of waste input

Waste - Type: Trib. = tributary stream input $\quad$| Mun. = direct municipal discharge to a lake |
| :--- |
| Ind. $=$ direct industrial discharge to a lake |

Waste: \% Known Estimate of percent of flow (or waste input) known to "source". Blank columns indicate that estimates were not available.
Parmeters Monitored: List of all parameters monitored during years or record. "Data on file" indicates that some form of data is available from the "source".
Sampling Frequency Approximate periodicity of observations and Years of Record: and number of years of record
Source: State or Provincial Office which complete survey forms requesting information for this table.

Although generally not in published form, the data could be obtained by visiting the particular agency - as project personnel were invited to do by several agencies.

Specific data on the water quality of United States streams tributary to the Great Lakes are available from the U.S.G.S. publication Quality of Surface Waters in the United States. Physical and chemical concentrations are tabulated and, in several cases, suspended sediment concentrations and discharges (tons/day) are presented. Several state agencies issue periodic reports on tributary water quality. Examples include the Periodic Report of the Water Quality Surveillance Network 1965-1967 Water Years by the New York State Department of Environmental Conservation and the Report of the Water Pollution Control in the Michigan portion of the Lake Michigan Basin and its Tributaries prepared by the Michigan Water Resources Commission and the Michigan Department of Health in 1968. A listing of the locations of water quality sampling stations in the United States is found in the Catalogue of Information on Water Data, Index to Water Quality Section. published by the USGS Office of Water Data Coordination.

Physical, chemical, and bacteriological data öf Canadian streams tributary to the Great Lakes are available in the Ontario Water Resources Commission annual publication, Water Quality Data for the Ontario Lakes and Streams. In general. concentration data are available and, in a limited number of cases, yearly average discharges (kilotons/year) are given. Daily suspended sediment concentration and discharges (tons/ day) of several Canadian streams tributary to Lakes Erie and Ontario are given in the Sediment Data for Canadian Rivers, published by the Water Survey of Canada.

As has been indicated above, there is much data on the quality of streams tributary to the Great Lakes, but, in most cases, these data have not been correlated with flows. Consequently, the mass discharge rates are generally not readily available; but the proper order of magnitude of these can be estimated. For major U.S. streams tributary to the Great Lakes, the mass rates are available from the EPA Storet computer system for some parameters.

## Biological Data

Bacteriological surveys of the lakes have generally been part of more comprehensive limnological surveys in recent years. All the lakes have been sampled for total and fecal coliform bacteria, primarily in the shore areas. Early surveys by the U.S.P.H.S. International Joint Commission and the United States Bureau of Commercial Fisheries (U.S.B.C.F.) are helpful. Present conditions can be adequately assessed from the ongoing surveys of the water quality by the CCIW.

Phytoplankton measurements in the form of chlorophyll or species counts are available for all the lakes for present conditions and historical surveys are also available for western Lake Erie. Table 9 shows stations where such measurements are made. Zooplankton data are less readily available, although specific locations are well documented. Benthic animal surveys have been conducted for isolated locations.

Commercial fish production records are maintained by the U.S.B.C.F., and the Ontario Department of Lakes and Forests. Effort and catch per unit effort are tabulated. The Great Lakes Fishery Commission has compiled an historical record of catch by species and by lake for the period 1867-1960 for both United States and Canadian catches. Data on year classes for various species are recorded. An extensive collection of fish scales (used to determine age of fish) as well as a large amount of historical data on the Great Lakes fishes, fish research publications, and general basin environmental information is located in the Great Lakes Fishery Laboratory, Bureau of Sport Fisheries and Wildlife. Data on predator-prey relationships, assimilation of contaminants, toxicity thresholds for various chemicals, effects of temperature, and dissolved oxygen are also available.

Special Data

Other data for a variety of areas are available for use on the Great Lakes. Sediment bearing characteristics of tributary streams are available in the Water Supply Papers
of the United States Geological Survey and in the Water Survey of Canada publication, Sediment Data for Canadian Rivers. Chemical analyses of the bottom sediments have been performed for all the lakes. An extensive study of radioactivity in the water, sediment, benthos, plankton, and fish of Lake Michigan is available, together with estimates of the radiological wastes entering the lake. The United States Public Health Service has also accumulated data on the radioactivity of tributary streams as well as for some near shore sampling locations. Waste input data such as oils and heat have been measured.

## Summary

From the large amount of information uncovered during the investigation, it is apparent that sufficient data presently exist for preliminary model development for many of the water resource problems of the Great Lakes. The resources expended on data collection programs on the Great Lakes are proportionately greater than the effort devoted to analysis of these data. Preliminary modeling will therefore tend to use the available data and increase the impact of this information on the decision making process.

The available data have been collected by numerous agencies and investigators. It should be anticipated, therefore, that difficulties will be encountered in the use of these data for any proposed Phase II program. Data gaps will undoubtedly exist and difficulties in interpretation of the information and translation of the data will occur. In spite of anticipated problems and the shortcomings of the existing data base, it is concluded that the available information is sufficient to support "first cut" modeling effort. This will result in analysis of the existing data. In addition, the knowledge and understanding gained from modeling can be employed to guide ongoing data collection programs by providing indications of the spatial and temporal scales on which data should be collected.

Based on the analysis of available data, a need exists for continuation and possible expansion of ongoing data collection programs, particularly with respect to broad scale programs such as the International Field Year on the Great Lakes. Other data collection programs carried out in recent years by CCIW, NOAA, EPA, and other organizations should be coordinated with standardized sampling locations, measurements, procedures, and data reporting formats.

A number of questions have been raised during this study with regard to the need or desirability for central data storage facilities for the Great Lakes. It is possible that an assessment of the total needs within the basin will provide sufficient justification for development of central data storage facilities. It is concluded that the needs for data in the proposed Phase II study can be adequately met from the existing scattered data base. Consolidation of the existing data base in a central storage facility can not be justified solely by the benefits to be obtained for the proposed Phase II study.

## SECTION IV

## WA TER RESOURCE PROBLEMS AND VARIABLES

## SECTION VI

PROBLEM ORIENTED MODELS

## Introduction

In the field of water resources modeling, there are a few basic principles from which all modeling efforts are evolved or developed. These principles are discussed below.

Conservation Principle

This principle allows the establishment of the basic balances of mass and energy within a specific volume of a natural water system by accounting for the inputs and outflows of the particular constituent and its various sources and sinks. The net rate of change of these factors results in either an accumulation or decrease of the substance within the volume or in no change, i.e., an equilibrium condition exists if the inflows, outflows, and sources and sinks are in balance. It is pertinent to note that the principle may be applied to living (biological and biochemical) systems as well as inanimate (physical and chemical) systems. With respect to mass balances, the principle applies to the water budget itself and to constituents contained in the water body. The energy balances, on the other hand, are applied primarily to thermal regimes and temperature conditions throughout the lakes. Thus, this principle provides the basis for the framework of the hydrological and temperature models as well as the eutrophication, dissolved oxygen, and bacterial models. Furthermore, it plays an important role in the sediment, chemical, and fisheries models.

Momentum Principle

This scientific law (Newton's second Law of Motion) allows the description of the various hydrodynamic factors of concern in
natural water systems. It is specifically applied to an evaluation of the velocity regimes and wave patterns due to the various forcing functions found in nature such as winds and storms. It is also fundamental to the description of transport of dissolved and suspended materials as well as the erosion of shorelines. Its primary application is in the evaluation of velocity and dispersion fields of various spatial and temporal scales throughout the system. It is fundamentally the principle which underlies the hydrodynamic modeling.

Thermodynamic Principles

These principles (the first and second law of thermodynamics) can be used to provide a general description of the chemical state of the system with respect to the equilibrium condition of its constituents. They specify the necessary relationships between the Gibbs Free Energy and the chemical composition of a system. They also permit the application of thermodynamics to the calculation of chemical and thermal regimes in natural water systems and are the essential basis for the chemical models. One of the primary applications is the computation of chemical or thermal equilibrium levels for actual or assumed conditions of practical concern.

Ecological Principles

These principles refer to the more qualitative, but equally fundamental, aspects of both terrestial and aquatic resources which have been discovered to be important in the understanding of these biological systems, such as principles which provide the framework for the understanding of food chains and webs of the life-death processes of the biological organisms in the lake system. As such, they encompass or utilize in its application one or many of the previous fundamentals. This analytical framework of the ecological model which has been constructed using these principles permits definition of the transport and accumulation of critical substances through the system and its elements.

These principles may be and frequently are expressed in mathematical forms which are succinct, precise, and very general. These expressions usually take the form of differential equations which describe the rates of change of the phenomena in time and/or space and relate these changes to natural fundamental concepts. Because these relationships are so fundamental, they are also very general. Thus the continuity equation applies equally to a mass balance as well as an energy balance and to any natural water body, regardless of its specific characteristics. Furthermore, as specific phenomena are addressed within the Great Lakes settings, the mathematical structure may include one or more of these principles, e.g., the hydrodynamic equations of motion encompass both the continuity as well as the momentum principles. The physical setting is usually incorporated in the boundary conditions of these equations. Thus the fundamental principles take on a greater specificity as they are applied in greater detail to the analysis of certain phenomena within the natural environment of any water system. These equations are then further specified as they are applied in a lake system. The final set of working equations, addressing a particular problem within the Great Lakes setting, is the mathematical model of the system.

## Mathematical Models

The mathematical model of a system is developed to answer a given problem in a natural system. As described above, the development of the model utilizes one or more of the principles and their associated equations. This step results in the formation of a specific set of equations which may be algebraic, differential, either ordinary or partial, or integral. Further, they may be either deterministic or probabilistic, or contain elements of each depending on the state of knowledge of the causality chains. The equations contain constants, coefficients, or functions, which characterize the components of the system in a quantitative fashion. If the principles upon which the model is based are fundamental, and the constants or coefficients are well established, the model will accurately describe the real world. If, on the other
hand, assumptions are made either about the specific applicability of the principles or the coefficients, the less certain is the ability of the model to reproduce observed phenomena. In the vast majority of applications of systems analysis, including limnological systems, there is a broad spectrum of models ranging from those based on scientific principles to those qualitative relations which are primarily empirical in nature. This condition is characteristic of the field of water resources.

## Administrative Problem Definition

The basic requirement of the Limnological Systems Analysis is that it be directed to and responsive to planning needs of the Basin. This condition implies that water use and problems associated with or resulting from the uses can be sufficiently described by administrators and planners, so that an accurate technical and scientific translation of the problem may be made with respect to the variables of significance. This problem definition is the primary step in the application of water resource models and systems analysis techniques to the planning process. The process must contain three basic elements:

1. Specification of the water use to which the problem is related.
2. Specification of the variables of concern in sufficient detail to distinguish between possible overlaying classes of variables and effects.
3. Specification of the apparent extent of the water use interference (spatial scale) and the period of time over which the interference occurs (temporal scale).

This procedure, described in Section IV, is followed in defining the problems and the related variables of the Great Lakes. For each of the problems and their variables presented in Table 3 of Section IV, one or more models are required
for their analysis. A review was made of the many models that would be required for these problems. Out of this review, it was determined that eleven modeling frameworks could be grouped together. These eleven modeling frameworks are required to address the problems of the Great Lakes system:

```
    1. Hydrological balance
    2. Ice and lake wide temperature
    3. Thermal
    4. Lake circulation and mixing
    5. Erosion-sediment
    6. Chemical
    7. Eutrophication
    8. Dissolved oyxgen
    9. Pathogens and Indicator bacteria
10. Fishery
11. Ecological
```


## Mode1 Development

Given the problem definition by the administrator and having specified the variable or variables associated with the problems, the basic steps in the required model structure and evaluation may be initiated.

The first step involves a more detailed technical assessment of the dependent (endogenous) and output variable following the rather qualitative assessment made in the administrative definition of the previous step. Part of this step involves the specification of the appropriate time and space scale for the model which may be, and usually is, different from the time-space scale implied by the problem.

The next step involves the selection of the necessary inputs, forcing functions, variables, and parameters - the endogenous variables - which relate to the problem now more specifically defined. The key element in executing this step is relevance, i.e., only those variables that are important to the problem context are introduced to the model structure.

The third step is to provide a basis or a norm for determining the status of model structures available to deal with the specified problem. The output from the previous step is used to formulate suggested model structures along three broad lines:

1. Definitive equations with specified functional relationships and interactions.
2. Equations with only general functional relationships and interactions.
3. Qualitative descriptions of a model structure.

The basis for this classification lies essentially in the degree to which scientific knowledge, data, and model specifications and applications are available.

Evaluation Process and Model Status

Figure 14 shows the sequence of steps to be used in evaluating the status of model structures available to solve specified problems. It may be noted that the major considerations in determining model status are:

1. Basic understanding and knowledge
2. Data availability
3. Degree of model verification
4. Degree of model application

In order to provide a basis for ranking each of the modeling frameworks, numerical weights are associated with each step as shown in Figure 14.


FIGURE 14

Determination of Model Availability (Weight, 0-30)

This step requires an analysis of work that has been done in structuring models for the given problem. A bibliographic computer search has been implemented to aid in this step. A considerable number of references dealing with the Great Lakes have been put into the retrieval system. A search, using key words, can then be made of the reference file to determine what relevant progress has been made in structuring applicable models for the given problem context.

Determination of Data Availability (Weight, 0-40)

Major data sources and data banks are evaluated in this step to determine: a) where data are collected, b) time of year, c) frequency of sampling, and d) variables that are analyzed. Compilations are analyzed by lake and region to determine degree of overall coverage and to highlight data gaps, if any. The overall data structure can then be interrogated at this step to determine the available data specific to the time-space scales and problem variables defined previously.

Determination of Degree of Model Verification (Weight, 0-20)

This step is one of the more important evaluations in determining the status of a model structure. Consideration is given to whether the model has been verified at all and, if so, the number of independent verifications that have been conducted.

In all cases, an evaluation is made of the degree of success of the verification which governs, to some extent, the confidence that can be placed in the use of the model for planning purposes. Some models are only crudely verified in the sense that output conforms generally to what one expects. Other models attempt through a series of independent data analysis to verify the model by hindsight. This type of verification increases the degree of confidence in model use for predictions.

# Determination of Seriousness of Lack of Verification (Weight, 0-10) 

If a model has not been verified, this step evaluates the consequences of the lack of such verification. For some model structures the lack of detailed verification analyses may not be critical. This may be so if the problem context has been dealt with successfully in other geographical areas or other natural water systems.

On the other hand, if a model consists of a series of hypothesized equations with little follow-through, and the problem context is new, then the lack of verification could seriously affect the utility of the model. An example would be the formulation of equations of complex ecological systems which have not been tested against real data. This step, then, evaluates the impact of the lack of verification or the usefulness of the modeling structure for application and prediction.

Determination of Degree of Model Application (Weight, 0-10)

In addition to model verification an important consideration in evaluating model status is the extent of the application of the model in planning or predictive situations. This step evaluates the existing models in order to determine the degree, if any, of application of the model. Items such as the success of the analytical tools in predicting future courses of events or the degree to 'which they have proved useful in the planning process are evaluated.

Determination of Reasons for Lack of Models (Weight, 0-10)

As shown in Figure 14 this step is initiated when it has been determined that no models are available for the given problem specifications. If models do not exist, this step examines
the reasons for the deficiency. For some problem contexts, basic scientific knowledge may be lacking as to the mechanisms and pathways that are operative in a particular phenomenon. Hypotheses may be available, but for a variety of reasons models have not been structured for planning or predictive purposes. This step evaluates each problem for a determination of the reasons for a lack of modeling structure.

Determination of Degree of Effort Required to Obtain Knowledge (Weight, 0-10)

If the determination has been made that basic scientific knowledge is lacking for a modeling structure, then the effort that is required to obtain that understanding must be estimated. The estimation includes such considerations as the necessary scientific research work in the laboratory, research work in the field, and analysis of results. Each incurs a cost and requires time for accomplishment. This step estimates the cost and time required to obtain necessary basic knowledge for each problem context.

Determination of Data Availability (Weight, 0-20)

This step is similar to the Determination of Data Availability step in the parallel path except with weight, 0-20.

Determination of Degree of Difficulty in Formulation and Verification (Weight, 0-10)

If some understanding is available but a model structure has not yet been constructed, a determination must be made of the degree of difficulty in formulating a model structure and its subsequent verification. Estimates are made of data needs for verification and the degree of complexity of required model development.

Determination of Software and Computer Program Availability (Weight, 0-10)

In conjunction with formulation and verification, analyses are made of the availability of computer programs and hardware to deal with given model structures. Substantial efforts in terms of program development may be required to implement a given model.

The procedure, then, for evaluation of model status is to apply the preceding ten steps to each of the eleven modeling frameworks, determine the numerical weight of each framework, and rank the results.

Three development stages are considered:


A summary of the available models is given in the next Section followed by a summary of the ranking of model status in Section VIII.

## SECTION VII

## AVAILABLE MODELS - STATE OF THE ART

SECTION VII
AVAILABLE MODELS - STATE OF THE ART

Hydrological Balance Models

Problems and Scope

The hydrological or water balance of the Great Lakes is concerned with the overall supply, storage, and withdrawal of water from the Great Lakes system as a whole and also with the interrelations between the various mechanisms which influence the inflows and other inputs, storage, and outflows and other losses of Great Lakes water.

The primary variables of concern in the hydrological balance of the Great Lakes are the levels of the lakes and the flows between the lakes. The water resource problems are related to lake levels and connecting channel flows and include: availability of channel depths for navigation and dredging requirements; availability of flows for hydroelectric power generation and water diversions; accessibility and usability of marinas, beaches, and lake side parks; changes in the extent and character of fish and wildlife habitat; lakeshore erosion with reductions in property values and usefulness; and flooding of lakeshore areas.

The planning and management functions most directly impacted by the hydrological balance are concerned with navigation, power, and shoreline protection. A number of modeling efforts are currently being carried out in the Great Lakes which address the hydrological balance and several aspects of the associated water resource problems and planning needs.

Modeling Frameworks

A water budget for each lake is the framework employed in the hydrological balance models. In equation form, the water
budget can be expressed as follows:

$$
\begin{equation*}
\Delta S=P+R+U-E+I-O \pm D \tag{I}
\end{equation*}
$$

where:

| $\Delta \mathrm{S}$ | $=$Change in the volume of water <br> stored in lake |
| ---: | :--- |
| P | $=$ precipitation on lake surface |
| R | $=$runoff from the lake drainage <br> basin |
| $\mathrm{U}=$ | ground water contribution |
| E | $=$evaporation from the lake |
| surface |  |
| O | $=$ inflow from upstream lake |
| D | $=$outflow from lake through its |
| natural outlet |  |

Precipitation and Runoff. The independent variables, precipitation, P , and evaporation, E , in Equation (1) are related to overlake meteorological conditions. A number of studies $[1,2,3,4,5,6]$ have indicated that overlake meteorological conditions can differ from those observed at shorebased stations. The differences between shorebased and overlake precipitation have been estimated [6,7] on an average annual basis to range between 6 percent and 11 percent. This is equivalent to a constant flow rate of about 3,000 cfs or 1.9 inches in net lake level. The seasonal variation in the difference between onshore and overlake precipitation is even more marked. It has been estimated that average overlake precipitation in Lake Michigan may be 14 percent lower in the summer and 4 percent higher in the winter than
onshore values. Equivalent differences have been reported [5,6,7] in evaporation rates and other meteorological conditions.

The dependent variable, runoff, $R$, in Equation (1) is related to onshore meteorological phenomena as well as the specific character and conditions of the tributary drainage area. Gaging records for tributary streams of the Great Lakes covered approximately 50 percent of the drainage area in the 1930's and has increased to 64 percent at present. Estimates for the individual lakes are given below [7]:

Lake
Superior
Michigan
Huron
Erie
Ontario

## Drainage Area Gaged

## 53 percent

71 percent
66 percent
67 percent
63 percent

One potential difficulty in estimating the runoff, $R$, is that extrapolations to the total drainage area are required. In general, the ungaged areas are regions near the lakes whose meteorology is influenced by the proximity of the lake. Further, the characteristics of the near-lake portions of the drainage basin can be significantly different from upstream conditions because of the geological history of the area and because of man's activities such as the creation of urban and agricultural areas. When significant changes in runoff are not expected to be associated with alternative plans under study, the historical information may be used to obtain estimates of the runoff. If, however, future conditions are projected to differ significantly, it may be necessary to consider a shorebased hydrological balance model as an adjunct to the lake hydrological model.

A number of models have been proposed in the past for the computation of runoff from precipitation for small time intervals and for small tributary areas. This includes a large number of linear models, many based on the concept of the unit hydrograph. A number of methods have also been
developed which permit the computation of the unit hydrograph from records of rainfall and runoff; and computer programs for routine application are available from many sources, such as the Hydrologic Center of the Corps of Engineers.

The unit hydrograph method is still used widely, although it is generally acknowledged that the relationship between precipitation and runoff is not linear. A promising extension of the unit hydrograph approach is the use of a functional series of progressively higher order integrals, thus attempting to account for the nonlinearity of the process.

A method for the computation of the linear and nonlinear response functions has been proposed recently by Amorocho and Brandstetter [8]. It eliminates the rainfall excess computation and baseflow separation required by the unit hydrograph method. Both methods, however, assume highly lumped conditions with respect to the areal distribution of rainfall events and watershed characteristics.

More detailed approaches use extensive routine of flows starting with arbitrarily small subunits of a watershed and computing the various processes affecting runoff separately. Thus, equations are developed in varying mathematical detail for the computation of the effects on runoff of infiltration, surface retention, evaporation, and snow melt, etc. The best known and comprehensive model of this type is the Stanford Watershed model [9] which has found extensive practical applications. Its detail in both mathematical formulation and data requirements make it uneconomical, however, for large regional applications such as for the Great Lakes.

A compromise model could be considered which would provide sufficiently accurate computations of runoff, while requiring only a reasonable amount of data and computational effort for the areal lumping needed for large scale regional analyses. A regression model taking into account antecedent conditions may be satisfactory for the computation of monthly runoff values. For more detailed time and space distributions, the Stanford Watershed model or a simplified version using extensive areal lumping may be needed.

Lake Level Model. The lake level models that have been proposed to date are structured to compute average monthly horizontal lake levels using a water balance equation of the type:

$$
\begin{equation*}
\Delta S=N B S+I-O \tag{2}
\end{equation*}
$$

The variables $P, R, E$, and $U$ in Equation (1) have been grouped together into a term called the Net Basin Supply (NBS) as shown in Equation (3):

$$
\begin{equation*}
\text { NBS }=P+R+U-E \tag{3}
\end{equation*}
$$

Various efforts are in progress on the Great Lakes to compute the components of the historic and real time NBS considering both hydrologic and meteorologic processes. For example, Witherspoon [10] has developed a regional hydrologic response model for Lake Ontario considering evaporation, soil moisture, and snow melt to compute runoff to the lake from the surrounding drainage basin.

An attempt is being made by Meredith and Jones [11] to compute the components of the net basin supply (NBS) for each lake. Jones is computing lake precipitation and evaporation, and Meredith is computing land runoff. Ground water interactions with the lakes are neglected. Monthly records of NBS for the years 1946-1965 are being used for this analysis. Runoff, precipitation, and evaporation are determined from the existing records; then the computed NBS is compared with the NBS values published by the Lake Survey. Precipitation over land is determined from the existing United States and Canadian precipitation gages using weighting factors and considerable extrapolation to obtain overlake precipitation. Isohyets of monthly precipitation for the recorded years are drawn for each month.

Evaporation from each lake is computed using the Richards Irbe procedure [12] which is a modified method based on the Lake Hefner studies. This method requires knowledge of
water temperatures of the lakes. The Canadian Meteorologic Service has been conducting infrared flights once a month over all lakes except Lake Michigan, since 1950. The flights are conducted in criss-cross patterns, so that a good sampling of lake surface water temperatures is obtained for the time of the flight. An average lake temperature is computed for the entire flight. This average is used by Jones to compute lake evaporation. Little is known, however, about the temperature change during the rest of the month, and this knowledge would be required for more accurate evaporation computations. There is a need to have corresponding infrared flights over Lake Michigan. Some flights have been conducted for special purposes over limited areas, for example, last year, a flight was made over Green Bay near the University of Michigan. Actual lake water temperature measurements are spotty and do not cover many years. Some records are also available from lake steamers. Some regressions have been attempted between lake water temperatures and land air temperatures and wind, which includes their current and previous monthly values. However, these attempts are limited by the lack of data. Meredith plots isopleths for each month of the years used for the analysis to determine monthly runoff to each lake. The number of stations used for each month varies depending on the records available for each month.

Preliminary computations for two years for Lake superior and Lake Ontario show good agreement of the computed and actual NBS for Lake Ontario, but large errors exist for Lake Superior. This seems to be the result of having many fewer precipitation stations around Lake Superior and also a higher water-to-land ratio requiring more extrapolation of land precipitation records to cover the lake precipitation for Lake Superior. To date these efforts are greatly limited by the lack of data on over-the-lake precipitation and lake temperature distributions. Difficulties are also caused by the high variation in data density between the lower lakes and Lake Superior.

The results of the hydrological balance are sensitive to the measurements of connecting channel flows as represented by variables, inflow, I, and outflow, O. By way of illustration, the average flow in the Detroit River is on the order of 200,000 cfs. Therefore, every one percent error in flow measurement represents an error of $\pm 2,000 \mathrm{cfs}$.

Planning functions and management activities can influence the individual variables grouped in the net basin supply (NBS) term by alterations of meteorological conditions or the runoff characteristics of the drainage area. The remaining terms, input, $I$, outflow, $O$ and diversion, $D, i n$ Equation (1) are those variables which are most likely to be changed by various planning functions and management activities.

State of the Art

There are two lake level models currently available for the Great Lakes. Both models employ the basic water balance Equation (1) for Lakes Michigan, Huron, and Erie and use existing operating rules for Lakes Superior and Ontario. Each model contains semi-empirical equations which are developed by using regression analysis for calculations of connecting channel flows. The basic difference in the two models is found in the manner in which the net basin supply, Equation (3), is considered. The Corps of Engineers model [13,14,15] employs NBS without attempting to subdivide the individual components. In the model developed by Quinn [16] evaluation of the individual components which make up net basin supply, NBS, as indicated in Equation (3), is attempted.

The model equation currently used by the United States Corps of Engineers is:

$$
\begin{equation*}
\Delta S=N B S+I-O \pm D \tag{4}
\end{equation*}
$$

where:

$\Delta S=$| change in volume of water stored |
| :--- |
| in lake |

NBS $=$ net basin supply
$I=$ inflow from upstream lake

| $0 \quad=$ | outflow from lake through <br> its natural outlet |
| ---: | :--- |
| $D \quad=$ | artificial diversions into <br> or out of the lake |

The net basin supply in Equation (4) can be calculated from historical records of lake levels, flow in the connecting channels, and diversions. Thus, knowledge of the magnitude of its separate components is not required for the application of the model involving historical conditions.

This model has several versions, each used for a specific purpose. All versions use the same routing algorithm based on change of storage equals inflow minus outflow. Basically, two main versions exist:
(1) the forecasting model used to forecast lake levels in real time six months ahead
(2) the regulation model used to test various lake level regulation schemes and which employs either of two sources for input to NBS:
a. the historic net basin supply values
b. simulated (Markov chain) values of net basin supply [13].

Each of these two versions ( 2 a and 2 b ) can run with different regulation schemes for the flow in the connecting channels between the lakes and proposed operating rules, including possible regulation of the flows of the $S t$. Clair and Detroit Rivers.

The forecasting model uses only the existing rules for Lake Superior and Lake Ontario regulation. The regulation model uses either historic or simulated values of NBS. The simulated values are based on the statistical properties of
the historic values using a simple linear single lag autoregressive Markov chain model.

For forecasting purposes, the NBS is estimated by two steps. For the end of the first month forecast, regression equations are used relating the NBS of that month with precipitation and air temperature forecasts for that month and recorded precipitation and air temperature of the preceding month. A separate equation is developed for each lake from historical records. For the remaining five months of the six month forecast the statistical properties of the NBS are used based on historical records and adjusted for recent trends.

All computations are based on present day conditions; that is, historic records are adjusted to represent present day conditions. Constant values are used and no seasonal or other adjustments are made (for the major diversions (in or out of the lakes). A total of 680 years is simulated which is ten times the historical record on which it was based. No attempt is made to estimate any possible future changes in the hydrologic and hydraulic regime of the lakes.

All computations result in monthly averages for flows and end-of-month values for water levels in the lakes. However, smaller time steps are used to route the flows through the connecting channels (l0 equal time steps for a constant $30-$ day month for all months of the year on upper lakes, 10 equal time steps for each quarter of a constant 30-day month on lower lakes).

The channel routing is accomplished using a two stage discharge relationship derived from using regression analysis from historical records. The following equations are employed:

St. Clair River:

$$
Q=.173515(.5 \mathrm{HB}+\mathrm{GP}-540.84)^{2}(\mathrm{HB}-\mathrm{GP})^{1 / 2}
$$

Detroit River:

$$
Q=.17729(\mathrm{GP}-547.95)^{2}(\mathrm{GP}-\mathrm{CL})^{1 / 2}
$$

where:

$$
\begin{aligned}
& \mathrm{HB}= \begin{array}{l}
\text { Lake Michigan-Huron level at } \\
\\
\text { Harbor Beach }
\end{array} \\
& \mathrm{GP}=\begin{array}{l}
\text { Lake St. Clair level at Cross } \\
\\
\text { Point Yacht Club }
\end{array} \\
& \mathrm{CL}=\begin{array}{l}
\text { Lake Erie level at Cleveland, } \\
\\
\text { in feet, referenced to the } \\
\\
\text { IGLD (International Great Lakes } \\
\\
\text { Datum). }
\end{array} .
\end{aligned}
$$

Only two of the coefficients for each equation have been determined by regression in each equation; the others, including the exponents, have been held fixed. These equations are based on the record for the 1962-1968 hydraulic regimes of these rivers. For the other outflows (Lakes Superior and Ontario), the existing rule curves are incorporated into the programs.

During the winter, flow retardation values are incorporated in the computations of the flows in the connecting channels, based on conditions in each channel for the first month of the forecasting. For this purpose, the St. Clair-Detroit Rivers are broken into shorter subreaches to determine the restricting reach. Average retardation values are used for each month for the regulation studies based on historical records.

There is also some flow retardation in the summer due to weed growth in Lake st. Clair, but the model neglects this at present. However, the computations of the lake level models are sufficiently accurate for the computation of average monthly flows in the connecting channels and their effects on the lake levels.

The Lake level model as used for planning employs the historical data for calculation of NBS. Thus, the model is not verified in the sense used in this report. In view
of the use of the Corps model in near term planning, however, this lack of verification does not appear to be a significant impediment.

For the real time prediction purposes, the accuracy of the Corps model deteriorates rapidly with successive months [13]. The deterioration appears to be primarily a result of present difficulties in accurately predicting climatological conditions. Thus, a deterministic model using predicted precipitation and air temperatures is used only for the first month's prediction, while statistical properties of past records are considered for predictions during the following months.

Using the models described above, the Corps of Engineers has developed new rules for the regulation of Great Lakes levels. These include regulating the presently unregulated St. Clair and Detroit Rivers, which would increase economic benefits compared to the existing rules. These results are developed considering monthly historical records spanning 68 years and adjusted for present development. The new rules are devised to consider the hydraulic interrelationships of the lakes, while the existing rules are based on considering each lake separately. Using the Markov chain model, the new rules have been tested against monthly simulated values spanning 680 years.

A systematic mathematical optimization technique has been developed by Su and Deininger [17] to determine the optimum regulation of Great Lakes levels considering economic factors. A periodic Markov chain decision model computes monthly lake releases from each lake which maximizes long-range economic benefits rather than determining fixed rule curves.

It is significant to note that with respect to planning activities related to lake based variables (lakes, levels, and flows) on the Great Lakes, the existing Corps of Engineers model appears adequate for the present needs and in fact represents one of the few available illustrations of successful model application to Great Lakes problem analysis and planning needs.

The major shortcoming, with respect to lake based variables, is the loss of accuracy in real time prediction of lake level. This is an example of the situation wherein a model meets planning needs but does not meet operating and real time management and prediction needs. This latter shortcoming is associated with the inability to predict meteorological phenomena.

Modifications of the approach which would tend in the direction of improving accuracy from the modeling standpoint would depend on improved treatment of NBS, retardation, and hydraulic gradients in channel portions of the system. The model proposed by Quinn [16] is a step in this direction. The model calculates the values of the individual component variables of NBS rather than using the aggregate variable. It, therefore, can readily include any improvements in ability to independently measure or calculate any of these components. Both ice and weed retardation in the connecting channels have been evaluated. In addition, a model for the Detroit and St. Clair Rivers which routes unsteady flows through these connecting channels has been developed. However, the hydrodynamic routing model is not a part of the basic hydrologic balance model.

This calculation procedure should also facilitate evaluation of any changes which would influence the individual component variables of the NBS. As an example, weather modification and/or predicted long-term climatological changes could be considered in this computational framework. In addition, changes in runoff characteristics of the drainage area could also be considered directly. This could be a significant feature if large increases in urban development are to be investigated or if proposed waste disposal practices include diversion of waters from the lake for on land disposal or basic export.

An error or sensitivity analysis of this lake level model has also been presented and it should be consulted with respect to allocation of funds for improved scientific measurements between runoff, evaporation, precipitation, and connecting channel flows.

Evaluation of Model Status

Model Availability. A number of hydrological balance models are available for the Great Lakes. These models are presently operational and are being actively employed for evaluation of Water Resource planning alternative by agencies involved in management activites on the lakes.

Data Availability. The amount of information and data available for use in modeling of the hydrological balance of the Great Lakes is in excess of that available for any other aspect of the limnological system. There are several specific areas where increases in measurement accuracy and/or collection of more information would be of value scientifically. These are:
(1) Improved accuracy in measurement of connecting channel flows.
(2) Increased gaging of the drainage area adjacent to the lakes.
(3) Improved information on overlake meteorological processes such as winds, precipitation, and evaporation.

The hydrological balance models are not significantly constrained by lack of data in meeting present planning needs on the Great Lakes.

Model Verification. Hydrological balance models for the Great Lakes have generally employed the observed data in the basic modeling effort. Independent verification of the models has not been carried out. This lack of verification is not a significant impediment in the application of existing models to meet present planning needs.

Model Application in Planning. As indicated above, the hydrological balance models are presently being employed to examine planning alternatives on the Great Lakes. They
represent one of the few examples of successful application of models to planning problems and they are regarded as adequate for present needs.
${ }^{[1]}$ Bruce, J.P., and Rodgers, G.K., Water Balance of The Great Lakes Systems, Great Lakes Basin, Research Pub. No. 71, $\overline{\text { AAAS }, ~} \mathrm{pp} 40-69$ (1962).
[2] Lemire, F., Winds on the Great Lakes, Canada Dept of Transport Met. Branch, $\overline{\mathrm{CI} R-3560-T E C}-\overline{380}$ (1961).
[3]
Richards, T.L., and Loewen, P., Preliminary Investigations of Solar Radiation over the Great Lakes as Compared to Adjacent Land Areas, University of Michigan, Great Lakes Res. Division, Pub. No. 13, pp 278-282 (1965).
[4] Richards, T.L., Bragert, H., and McIntyre, D.R., "The Influence of Atmospheric Stability and Over-Water Fetch on Winds over the Great Lakes," Monthly Weather Rev., 95:5, pp 448-453 (1966).
[5] Kresege, R.E., Blust, F.A., and Roper, G.E., "A Comparison of Shore and Lake Precipitation Observations for Northern Lake Michigan," Lake Erosion, Precipitation, Hydrometry. Soil Moisture, I.A.S. H. Pub. No. 65, pp 311-323 (1963).
${ }^{[6]}$ Changnon, S.A., "Precipitation Contrast Between the Chicago Urban Area and Offshore Stations in Southern Lake Michigan," Bull. Amer. Met. Soc., 42:1, pp l-l0 (1961).
[7]"Limnology of Lakes and Embayments," Great Lakes Basin Framework Study, Appendix 4, Draft No. I (November 1970).
[8] Amorocho, J., and Brandstetter, A., "Determination of Nonlinear Functional Response Functions in Rainfall-Runoff Processes," Water Resources Research, Volume 7, No. 5, pp 1087-1117 (1971).
${ }^{[9]}$ Crawford, N.H., and Linsley, R.K., Digital Simulation in Hydrology: Stanford Watershed Model IV, Technical Report, No. 39, Stanford University (July 1966).

REFERENCES
(continued)
[10] Witherspoon, D.F., A Hydrological Model of the Lake Ontario Local Drainage Basin, Technical Bulletin No. $3 \overline{1}$, Inland Waters Branch Dept. of Energy, Mines, Resources, pp 1-13 (1970).
[11] Meredith, D.D., and Jones, D.L. , Work in Progress, January, 1972 - private communications to Battelle Pacific Northwest Laboratories.
[12]
Richards, T.L., and Irbe, J.S., "Estimates of Monthly Evaporation Losses From the Great Lakes, 1950-1968, Based on Mass Transfer Techniques," Proc. 12th Conf. Great Lakes Research, Int. Assoc. for Great Lakes Research, pp 469487 (1969) .
[13]
DeCooke, B.D., and Megarian, E., "Forecasting the Levels of the Great Lakes," Water Resource Research, Volume 3, No. 2, pp 397-403 (September 1967).
[14]
DeCooke, B.D., "Regulations of Great Lakes Levels and Flows," XI, Pan American Consultation on Cartography (1970).
[15] Megerian, E., and Pentland, R.L., "Simulation of Great Lakes Basin Water Supplies," Water Resource Research, Volume 4 , No. 1, pp 11-97 (March 1968).
[16] Quinn, F.H., "Quantative Dynamic Mathematical Models for Great Lakes Research," Ph.D. dissertation, University of Michigan (1971).
${ }^{[17]}$ Su, Shiau, Y., and Deininger, R.A., Optimal Operating Policies For the Great Lakes System, The University of Michigan, Ann Arbor, Michigan, UIRICH-ENUSA-71-3; pp 1189 (June 1971).

Ice and Lake Wide Temperature Models

Problems and Scope

The basic water resource problem associated with temperature balances and ice on the Great Lakes is the opening and closing of the navigation season on the Great Lakes. Overall heat balance calculations in the lakes are possible, but carrying these to the point of prediction for the opening and closing of the navigation season appears beyond the present state of the art. It should be noted that most of the problems in this regard are, under most circumstances, beyond the scope of Type II planning. For example, the ice problem associated with the navigation season opening and closing is primarily a reflection of an operational need for real time predictions.

Modeling Frameworks and State of the Art

Lake Wide Heat Budget Model. The heat budget calculation is an energy balance which considers the sources and sinks of heat energy such as heat transfer at the airmater interface, the heat transfer at the bottom of the water body, and heat generated by biochemical reactions if they are shown to be significant. The terms generally considered for heat transfer at the airwater interface are shortwave solar radiation, long wave atmospheric radiation, reflected solar and atmospheric radiation, longwave back radiation, evaporation, and conduction. Many studies have been conducted to evaluate these factors from easily measured meteorologic parameters, if direct measurements are not possible or not available. The best known investigations were conducted at Lake Hefner, Lake Mead, and the Salton Sea; and the relationships developed during these studies have been widely used.

There are a number of processes acting across the air-water interface to change the temperature of the water. Short and longwave radiation, evaporation, condensation, and sensible heat conduction act to produce absolute heat changes, whereas convective and turbulent mixing generate a redistribution of the thermal structure. By computing the contribution of each
of these terms to obtain a net heat gain or loss, it is possible to estimate the changes to the thermal structure. The following air-water interface processes are jointly referred to as the heat budget and can be represented by the equation:

$$
\begin{equation*}
Q=Q_{s}+Q_{c}-Q_{b}-Q_{e}-Q_{r}-Q_{h} \tag{1}
\end{equation*}
$$

where:
$Q=$ net gain or loss of heat
$Q_{s}=$ insolation
$Q_{c}=$ heat gain by condensation
$Q_{b}=$ effective back radiation
$Q_{e}=$ heat loss owing to evaporation
$Q_{r}=$ reflected radiation
$Q_{h}=$ heat conduction across interface

Precipitation may cause local temperature changes. There are also heat changes that result from heat flow through the lake bottom, the dissipation of wind and tidal energy, and heat bound or released by chemical processes; but these changes are insignificant for short term predictions.

Heat budget formulations usually require a knowledge of both air and water temperature, particularly for the evaluation of evaporation and its effect on heat transfer.

Because of the lack of information on the eddy conductivity, it is general practice to relate sensible heat transfer to evaporation. Assuming that evaporation and conduction of specific heat energy are similar processes, Bowen [1] derived a ratio for the two processes:

$$
\begin{equation*}
\frac{Q_{h}}{Q_{e}}=R=K \frac{\left(T_{w}-T_{a}\right)}{\left(e_{w}-e_{a}\right)} \cdot \frac{P_{a}}{1,000} \tag{2}
\end{equation*}
$$

where:
$\mathrm{R}=$ Bowen ratio
$\mathrm{P}_{\mathrm{a}}=$ atmospheric pressure
$\mathrm{K}=$ constant
and $T_{w}, T_{a}, e_{W}$, and $e_{a}$ are water temperature, air temperature, vapor ${ }^{\text {press }}$ sure ${ }^{\text {w }}$ at water surface, and partial pressure of water vapor in air, respectively.

The handling of the radiative terms has received extensive analysis. Battelle [2] has found that it is hazardous to the accuracy of predictions to use computed values of input solar radiation for making verification simulations. The use of statistically treated point measurement data more accurately reflects the input energy than any mathematical treatment proposed to date. This is essentially the same conclusion reached by Anderson, et.al.[3]. This variable is closely related to the atmospheric opacity as affected by air pollution in metropolitan areas. Consequently, for predictive work, close attention needs to be placed on accuracy in the forecasting of radiative inputs.

The longwave radiation exchange terms are only important in the wintertime. Errors in computation of back radiation are introduced by two problems. Usually, summer conditions are used for heat budget studies, but the Bowen ratio is apparently considerably different in the deep of winter when boundary conditions modify the evaporation/conduction ratios widely. Estimates of radiation exchange are also modified in winter. Additional research is needed in order to obtain more evaluations of thermal transfer in the winter months.

Formulae for computing evaporation vary from simple expressions relating evaporation to the wind and vapor pressure difference alone to complex considerations involving aerodynamic surfaces, occurrence of spray, and vertical profile of wind and vapor pressure. The Lake Hefner studies described by Marciano and Harbeck [4] include comparison of evaporation as computed with the equations developed by a number of authors (Sverdrup, Millar, Norris, Sutton) to measurements of evaporation based on water budget calculations. The complicated equations of Sverdrup and Sutton give satisfactory comparisons, but a simple empirical equation was devised that also gives good agreement, although the only inputs are wind speed and the difference in water vapor pressures at two levels. A similar equation developed by Rohwer [5] and slightly modified by Laevastu [6] gives comparable values and is considered more realistic for ocean evaporation. Rohwer [5] presented a comprehensive investigation of evaporation involving evaporation measurements under both laboratory and natural conditions. Observations were made with various types of equipment and procedures and included measurements in still air, in natural air flow, air under various controlled wind speeds, over a heated surface, from an ice surface, and at different altitudes. From data gathered over a six year period, Rohwer concluded that one formula has general application. Laevastu modified this formula slightly to compensate for the wind profile over the sea. This formula for calculating evaporation is:

$$
\begin{equation*}
Q_{e}=2.46(0.26+0.04 W)\left(e_{w}-e_{a}\right) \tag{3}
\end{equation*}
$$

where:

$e_{w}=$| saturated vapor pressure at sea |
| :--- |
| surface temp., mb |

$e_{a}=$ dry bulb vapor pressure, mb
$W=$ wind speed, knots

For inland lakes, the Bowen ratio $R$ is 0.61 under normal atmospheric conditions, but it can vary between 0.58 and 0.66. Although originally conceived for molecular diffusion pressures, Bowen's ratio has been shown to apply to nonlaminar flow also. Some authors have suggested theoretical modifications to Bowen's ratio, but observations have generally supported his concept. Anderson [3] found from the exhaustive Lake Hefner observations that the Bowen ratio is generally valid. Tabata [7] and Gaul and Elder [8] reviewed methods of computing sensible heat transfer and concluded that Bowen's ratio is satisfactory.

Neglecting the pressure term, which has only a small effect, and substiting for $Q_{e}$ in the above equation, the transfer of sensible heat can be found from:

$$
\begin{equation*}
Q_{h}=0.83(0.26+0.04 W)\left(T_{w}-T_{a}\right) \tag{4}
\end{equation*}
$$

This equation is applicable for surface cooling where colder air overlies warmer water. The wind is important in that the warmed air is rapidly removed by atmospheric convection. The reverse situation, where warm air lies over colder water, produces surface heating of the water. Here the stabilizing effect of the air's being cooled from below reduces the transfer of heat. To reduce the rate of surface heating, the above equation is modified in accordance with Laevastu's [6] proposed reduction of the constants so that:

$$
\begin{equation*}
Q_{h}=0.036 W\left(T_{w}-T_{a}\right) \tag{5}
\end{equation*}
$$

Roughness of the lake surface has been extensively studied and has some relation to the changes in effective transfer coefficient, however, the extension of studies to swiftly flowing rivers produces relatively dramatic deviations from the classic concept. In these cases, sensible transfer of from three to five times that predicted by the Bowen ratio occurs. Jaske [9] has attempted to compute the sensible heat exchange coefficient variation as a function of stream surface velocity, but the work is incomplete and requires additional research.

Ice Models. Ice transport and the estimation of forces and the hydraulic modifications related to ice accumulation are difficult to model. A number of excellent techniques are available for the estimation of ice forces and rates of accumulation in flowing rivers, such as the Detroit River, Lake St. Clair reach, and St. Clair River. Aerial surveys of ice are also conducted, and the results are published monthly by the Lake Survey.

Correlations are principally based on the use of Kivisild's theorem, which states [10] that the accumulation of ice is related to critical values of the Froude number ranging from 0.6 to 0.8 . In 1955, Michel tested the Kivisild criterion in a number of cases and found it applicable. In this method, the relation of the equilibrium of the upstream edge of the cover fed by ice flows can be estimated from the following:

$$
\begin{equation*}
F_{r}=\frac{V}{g y}=\sqrt{2 \frac{\rho-\rho_{1}}{\rho}(1-\xi) \frac{t}{y}\left(1-\frac{t}{y}\right)} \tag{6}
\end{equation*}
$$

where:

| $\mathrm{F}_{\mathrm{r}}$ | $=$Froude number of the flow in <br> front of upstream edge |
| ---: | :--- |
| $\mathrm{V}=$ | velocity in feet per second <br> in front of upstream edge |
| Y | $=$depth of water in feet, in front <br> of upstream edge |
| $\rho_{r} \rho_{1}=$ | specific masses of water and <br> solid ice |
| $\xi$ | $=$ porosity of the accumulation |
| $t$ | $=$thickness of the upstream edge, <br> in feet, at equilibrium |

Porosity plays a major role in the progression of ice covers and is also a very difficult quantity to estimate. An ice cover may consist of frazzle flocks of very high porosity, of solid flows of low porosity, or a combination of both [ll].

The situation with respect to the initial movement of pack ice under wind or current movement is less well developed. Ice booming is successfully carried out in bays with small reaches, and in some instances, such as the booming of hydro facilities on the St. Lawrence, some success has been noted. The reduction of this practice to numerical analysis is very difficult and, at best, essentially empirical in that the information needed for the solution of complex theoretical expressions is not available.

The Russian literature has much useful information on the modeling of ice movement. A paper by Panfilov [12] contains the following relationships which appear to be useful in defining a first approach to the deterministic modeling of ice forces involved in pack ice movement. These forces can be divided into active and reactive forces. The active forces promote ice movement in the direction of the wind or the current flow and include: (a) the frictional forces $\mathrm{F}_{\mathrm{w}}$ of water on ice acting on the lower surface of the pack, or alternatively $F_{m}$, the forces due to wind on the ice surface, and (b) the component $F_{\text {in }}$ of the ice weight in the direction of net movement. The $\bar{p}$ Pessure exerted on the upstream rim of the ice field is grouped with the active forces, but is ambiguous in some cases because of internal bridging. Reactive forces oppose the movement and comprise the shore resistance $\mathrm{F}_{\mathrm{S}}$ and the reaction $\mathrm{F}_{\mathrm{h}}$ of the structure under pressure.

At any moment, the ice condition is in dynamic equilibrium with the basic forces. As unbalance occurs because of temperature rises, flow changes, ice expansion, or other active factors, modifications of the dynamic balance occur rapidly. As a result, deformation of the edge of the ice pack and reductions in the thickness and strength of the ice occur. The resistance offered by the shore and shore based structures decreases and the forces associated with active pressures increase. This occurs until the active
forces exceed the total reaction of the structures and the shore, and the ice sheet begins to move.

The condition for limiting equilibrium of the ice sheet can be mathematically stated as follows:

$$
\begin{equation*}
F_{w}+F_{m}+F_{i c}-F_{S}-F_{h}=0 \tag{7}
\end{equation*}
$$

In order to use this approach, a number of simplifying assumptions are necessary: principally that relatively straight edges are presented by the pack and that the thicknesses are uniform enough to be represented by a mean thickness.

The resistance exerted by the structure against ice movement is related to the configuration and resiliance of the shore line structure; to the nature of the contact between shore, structure and ice; and to the thickness and strength of the ice itself. As these factors are indefinite and depend on rapidly varying local conditions under some critical stages, a rigorous quantitative evaluation of shore reaction force is very difficult [13].

It is also well known that because of the nature of ice itself, sheet movements are accompanied by ice pile-ups on the shore. These are particularly heavy at constricted entrances to bays and harbors. The pile-ups occur as a result of the disruption of the continuity of the pack at points of contact with the shore structure and involve internal forces of bending, shearing, and crushing. Consequently, the total shore-resistance force can be expressed as:

$$
\begin{equation*}
F_{s}=k_{s} R_{b} h L \tag{8}
\end{equation*}
$$

where. $k_{s}=k_{b}+k_{s r} R_{s r} / R_{b}+k_{c} R_{c} / R_{b}$. The coefficients $k$ represent each type of deformation, and $R_{b}, R_{s r}$ and $R_{c}$ are the ultimate strengths of the ice, respectively, in bending, shearing, and crushing.

Similarly, the other forces can be evaluated in terms of coupling coefficients, momentum transport to the ice from wind, and the differential movement of the pack under applied forces. The numerical summation of the forces balance equation is solved to permit a first approximation of the net forces involved.

Other detailed modeling problems might be encountered in fine detail as a number of shoreline factors are taken into consideration. These are the impacts of ice flows on structures. However, most of these transport problems are more directly related to the engineering design of individual protective works and probably need not be considered for Type II planning. If such consideration is necessary, for example, in the determination of the extra cost of protection for year round harbor operation on a large and widely distributed scale, the basic formulations for estimation are presently available.

Research on ice mechanics indicates that as the rate of loading on ice is increased, the strength of ice increases rapidly until a maximum static compressive strength of approximately 400 psi is reached at a load rate of 1,000 psi/minute. Beyond this value, the strength decreases and a value of 160 is about average for first approximation of collapse stress. When ice collides with a vertical obstruction, the leading edge will be progressively crushed. A state of progressive failure is maintained. If the structure is massive, a maximum force equivalent to the brittle strength of ice at the highest rate of loading is applicable.

Ice impacts are accompanied by dynamic oscillations which are functionally related to the thickness and condition of the ice. Measured oscillations with periods on the order of 3 to 10 seconds can be expected with appropriate resonant coupling to the response spectra of the structure itself.

However, the determination of mass ice movement from wind forces or the estimation of ice breakup by deterministic means remains in a relatively unknown state. A number of investigations have shown promise of the creation of open
passage for navigation or structures using bubbler systems; however, the closing of these relatively vulnerable passages can occur promptly with massive destructive forces. The booming of large ice masses has been attempted by the various agencies involved in the St. Lawrence Seaway, but to date no deterministic results have been reported. It appears safe to assume that in the absence of an approach, no method can be recommended without extensive additional investigation. Such an investigation is now underway by the United States Army Corps of Engineers.

Evaluation of Model Status

Model Availability. The models for prediction of the opening and closing of the navigation season require prediction of thirty to ninety-day meteorological conditions and the formation of ice in open lake areas, connecting channels, and harbors. In addition, lake wide temperature modeling would also be required. The former prediction requirements are beyond the scope of present technology. Lake wide temperature modeling, given meteorological data, is within available technological capabilities.

Data Availability. Data on historical meteorological conditions are adequate for shore stations around the Great Lakes. Over lake meteorological data are inadequate. Information allowing conversions of shore meteorological data to overlake data requires substantial additional investigations. Data on ice cover closing and opening dates and lake temperatures appear adequate for a first-cut analysis effort.

Model Verification. In view of the need to predict meteorological conditions and ice and temperature conditions on a lake-wide and local (harbor, connecting channel) scale, the problem formulation and verification would be difficult.

Software and Computer Availability. Present generation computers appear adequate to support a first cut modeling effort.

## REFERENCES

[1] Bowen, I.S., "The Ratio of Heat Losses by Conduction and Evaporation from any Water Surface," Phys. Rev., Sev. 2, Vol. 27, pp 779-787 (1926).
[2] Jaske, R.T., and Spurgeon, J.L., "Thermal Digital Simulation of Waste Heat Discharge," Water Research, Pergamon Press, Volume 2, S-777-802 (1968).
[3] Anderson, E.R., "Energy Budget Studies," Water-Loss Investigations: Volume 1, Lake Hefner Studies, U.S. Navy Electronics Lab. Tech. Report 327, pp 71-119 (1952).
[4]Marciano, J.J., Jr., and Herbeck, G., "Mass Transfer Studies," Water Loss Investigations Volume l, Lake Hefner Studies, U.S. Navy Electronics Lab. Tech. Report 327, pp 46-70 (1952).
[5] Rohwer, C., "Evaporation From Free Water Surfaces," Tech. Bull. U.S. Dept. Agric., p 271 (1931).
[6] Laevastu, T., "Factors Affecting the Temperature of the Surface Layers of the Sea," Commentat. Physico-Math., Volume 26, pp 1-136 (1960).
[7] Tabata, S., and Giovando. L.F., Prediction of Transient Temperature Structure in the Surface Layers of the Ocean, Fishery Res. Bd. of Canada, Manuscript Report (ocean), NO. 132 (1962).
${ }^{[8]}$ Gaul, R.D., and Elder, R.B., A Critical Summary of Sea Surface Heat Exchange Equations, IMR 0-40-62, U.S. Naval Oceanographic Office (1959).
[9] Jaske, R.T., An Evaluation of the Use of Selective Discharges from Lake Roosevelt to Cool the Columbia River, Battell $\overline{\text { Northwest }}$ Report, BNWI-208, pp 1-87 (1966).

REFERENCES (continued)
[10] Michel, B., and Deslaurieries, "Kivisild's Criteria for Ice Movements and Accumulation in River Described in Review Papers," Proc. of a Conf. held at Laval University, Quebec (November 1966).
[11] Gold, L.W., "Elastic and Strength Properties of FreshWater Ice," National Research Council of Canada, Conf. on Ice Pressure Against Structures, Quebec (1966).
[12]Panfilov, D.F., "Ice-Flow Conditions Upstream of Hydrostations," Tranlated from Gidrotechnickeskow Stoitel Studies, No. 5, pp 33-36 (1971).
[13] Kivisild, H.R., Ice Impact on Marine Structures, (1969).

## Thermal Models

Problems and Scope

The scope of problems considered in this grouping deals with the fate of waste heat discharged to the Great Lakes and the influence of the natural heat sources on the thermal and transport regimes of the Great Lakes. The spatial scale of these phenomena span the range from the local thermal distribution, which is affected by a particular discharge with a particular diffuser configuration, to harbor and region-wide thermal patterns, to the lake-wide thermal distributions characterized by phenomena such as the thermal bar and the thermocline. Although the detailed temperature distribution and dilution characteristics of a specific diffuser are not directly Type II planning questions and are therefore not pertinent to this discussion, the effect of a series of discharges on lake temperature is of concern.

Thermal inputs to the Great Lakes are primarily of natural origin and the effect of these inputs is clearly seen in the development of the vertical temperature distribution which prevails during the summer months in the Great Lakes and is characterized by the thermocline. The importance of the thermocline and the thermal bar is in their effect on the lake circulation and mixing and the effect of the differing temperatures on biological and chemical phenomena. Thus the interest in the large spatial scale thermal phenomena is centered on these effects; and if data is available which specifies the temperature behavior of the lakes at this spatial scale, it would be equivalent to having a model which calculates the temperature distributions.

This is not the case, however, for the problem associated with thermal effluents. The primary sources of man-made thermal inputs are fossil fuel and nuclear power plants. The quantity of waste heat to be disposed of under projected industrial development around the shores of the Great Lakes is substantial. The problems associated with this disposal are of concern; and the first requirement for an assessment of these problems is a method of calculating the expected temperature distribution, i.e., a model.

Modeling Framework and State of the Art

The state of the art of modeling these phenomena can be conveniently addressed in terms of the spatial scale implicated. On the lake-wide scale, seasonal development of the temperature distribution and associated thermocline is the major manifestation of the natural inputs and outputs of heat energy.

As incident solar radiation and other direct inputs of heat energy exceed the heat energy being lost, lake surface layers begin to warm relative to the deeper layers. Above $4^{\circ} \mathrm{C}$, the density of water decreases as its temperature increases so that the warmer surface layers are buoyant relative to the deeper layers. This buoyant force competes with the vertical turbulent mixing; and if the mixing is sufficiently weak, the buoyant forces prevail and a temperature stratification develops in the vertical direction. This stratification is characterized, in most cases, by a depth at which there is a sharp temperature change that separates the warmer surface waters from the colder deep water layers. This region of rapid temperature change is the thermocline. Examples of this rapid temperature variation for Central Lake Erie are given in the Project Hypo report [1].

Models which address this phenomenon directly have been developed over a period of years and their behavior is reasonably well understood. The classical formulation considers only the variation of temperature in the vertical direction and the governing equation is conservation of energy averaged horizontally. That is:

$$
\begin{equation*}
\frac{\partial T}{\partial t}+\frac{1}{A} \frac{\partial}{\partial z}(Q T)-\frac{1}{A} \frac{\partial}{\partial z}\left(E A \frac{\partial T}{\partial z}\right)=S_{i}-S_{O} \tag{1}
\end{equation*}
$$

where $T(z, t)$ is the horizontal average temperature, $A(z)$ is the cross sectional area at depth $z, Q(z, t)$ is the vertical flow rate, $E(z, t)$ is the vertical dispersion coefficient.
$S_{i}$ and $S$ are the inputs and outputs, or sources and sinks, respectively, of thermal energy.

Orlob [2], Dake [3], Dake and Harleman [4], Huber and Harleman [5], and Sundaram [6] discuss the basic equation of heat conservation for a lake. The solution of the above equation requires specification of the velocity field, the dispersion coefficient, and the distribution of heat sources. The references quoted above bring out the following points:
a. The temperature structure in lakes is likely to be very homogeneous horizontally. This is a consequence of the tendency for the temperature pattern to approach a stable state relatively rapidly with respect to seasonal changes. Under these conditions, the horizontal variations can be neglected. Horizontal inputs of heat may be accounted for in the source term. At certain times during the year a horizontal stratification called the thermal bar is set up in several of the Great Lakes. This phenomenon does not persist for more than a few weeks and the associated current structure is of negligible magnitude. An analytical model which reproduces some of the features of the thermal bar is given by Brooks [7].
b. The vertical diffusion coefficient, E, is in general a function of the inputs' of energy by wind stress on the water surface and of the local stability of the temperature distribution $[8,9]$. Near the water surface, the vertical diffusion coefficient, $E$, is large because of wind action, and a well mixed surface layer is found in most lakes. The thermocline is a consequence of the gravitational stability damping out wind induced turbulence. However, during periods of heat loss from the water surface, unstable temperature distributions may generate mixing and result in thermocline erosion or overturning of the lake as in the late fall.
c. The heat source, $S_{i}$, is usually composed of the following components:
a) solar radiation absorbed at the water surface
b) solar radiation absorbed within the water body
c) lateral inputs of heated or colored water, either natural or man-made (usually power plant discharges)

In summary, thermocline formation is at a reasonably well developed stage with a variety of analytical and numerical models available as summarized in Table ll. To date, most applications have been made to small lakes. The models depend on observed data to establish the vertical dispersion coefficient, although some progress has been made to relate the vertical dispersion coefficient to wind and lake physical characteristics. Typically the horizontal variations are neglected, and only the temperature distribution as a function of depth and time is the object of the analysis. The seasonal time scale is usually considered, although a diurnal model has been investigated [10]. The seasonal models have been verified in a number of cases for reservoirs and small lakes, but no detailed Great Lakes data has been analyzed.

The thermal bar phenomenon, which is peculiar to large lakes, has been observed to occur and last for a few weeks [ll]. During this time it has an effect on the horizontal mass transport near the shoreline. It is essentially a vertical thermocline which then merges with the normal thermocline. An analytical model has been proposed [7,12,13] which exhibits the features of the thermal bar; and realistic models of near shore transport which are concerned with weekly variations during the period of the thermal bar should consider this phenomenon. No Great Lakes applications have yet been attempted.

## TABLE 11

## CHARACTERISTICS OF THERMAL MODELS

## Investigators

|  | Orlob, |
| :---: | :---: | :---: |
| Harleman,et.al. | et.al. Sundaram,et.al. |
| $[3,4,5,15,16,17]$ | $[2,21] \quad[6,18,19,20]$ |


| Analytical |  |  |  |
| :--- | :--- | :--- | :--- |
| Numerical | x | x |  |
| Wind Stress |  |  | x |
| Variable E | x | x |  |
| Mixed Surface Layer | x | x | x |
| Arbitrary Geometry | x | x |  |
| Arbitrary Heating | x | x | x |
| Absorbed Radiation | x | x | x |
| Lateral Inflows | x | x |  |
| Power Plant Dynamics |  |  |  |

Thermal discharges entering the lakes constitute an input of heat, which may substantially affect the thermal regime of the near shore. Models for temperature distributions in lakes have been proposed, and in some cases, verified [14].

The development of thermal models can be either straightforward or very difficult depending on whether the thermal discharge being considered substantially modifies the velocity and dispersion characteristics of the water body. The local velocities and dispersion are surely modified, but whether a substantial modification occurs at larger spatial scales is the key issue. If not, and this is the conventional assumption, models can be and have been constructed based on conservation of energy. If so, only complex numerical models which are currently being constructed offer hope for a solution. As discussed in the hydrodynamics modeling review, models of this sort have been constructed for oceanic circulation and are currently being contemplated for Great Lakes applications during the International Field Year on the Great Lakes.

Evaluation of Model Status

Model Availability. Models are available for seasonal thermal phenomenon although they have been applied primarily to small lakes and reservoirs. The vertical distribution of temperature has been the primary concern since in this case the buoyant forces are a significant factor. The physics of this phenomenon is well understood although details of the turbulent structures are still a matter of investigation. Models for the distribution of waste heat have been proposed and applied, although not to a Great Lakes situation directly. Comprehensive models which include the equations of fluid motion as well as conservation of heat energy are in the process of development.

Data Availability. Temperature data for detailed modeling of thermocline development in the Great Lakes is available for each of the lakes to some degree. Extensive data for

Lake Erie has been collected during Project Hypo and is being collected during the International Field Year on the Great Lakes for Lake Ontario. Temperature distributions which result from individual waste heat inputs are also available as part of surveillance monitoring which is currently being conducted.

Model Verification. Vertical thermocline models have been verified in smaller lakes and reservoirs, but no Great Jakes applications are yet available. Horizontal temperature distribution models have also been verified in non-Great Lakes applications.

Model Application in Planning. Great Lakes applications are lacking; however, applications elsewhere have been made in the design of detailed diffuser devices and in the assessment of their effect on the receiving water [22].

## REFERENCES

${ }^{[1]}$ Burns, N.M., Ross, C., Project Hypo, CCIW (February 1972).
[2] Orlob, G.T., Development of a Mathematical Model for Prediction of Temperatures in Deep Reservoir, $\overline{\text { Water }}$ Resource Engineers, Walnut Creek, California (1966).
[3] Dake, J.M.K., and Harleman, D.R.F., An Analytical and Experimental Investigation of Thermal Stratification in Lakes and Ponds, M.I.T., Department of Civil Engineering, Ralph M. Parsons Laboratory for Water Resources and Hydrodynamics, Technical Report, No. 99 (September 1966).
[4]
Dake, J.M.K., and Harleman, D.R.F., "Thermal Stratification in Lakes - Analytical and Laboratory Studies, " Jour. Water Resources Research, Vol. 5, No. 2 (April 1969).
[5] H
Huber, W.C., and Harleman, D.R.F., Laboratory and Analytical Studies of the Thermal Stratification of Reservoirs, $\overline{\text { M.I.T., Department of Civil Engineering, Ralph M. Parsons }}$ Laboratory for Water Resources and Hydrodynamics, Technical Report No. 112 (October 1970).
[6] Sundaram, T.R., et.al., An Investigation of the Physical Effects of Thermal Discharges into Cayuga Lake, Cornell Aeronautical Laboratory, CA: NO. VT-2616-0-2 (November 1969).
[7] Brooks, I., and Lick, W., "Lake Currents Associated with the Thermal Bar," Proc. 14th Conference Great Lakes Research (1971).
[8] Li, Ting Y., "Formation of Thermocline in Great Lakes," Proc. 13th Conference Great Lakes Research (1970).
[9] Li, Ting Y., "Maintenance of Thermocline in a stratified Lake," Proc. 14th Conference for Great Lakes Research, University of Toronto, Toronto, Canada (1971).

## REFERENCES

(continued)
[10] Foster, Theodore, D., "A Convective Model for the Diurnal Cycle in the Upper Ocean," J. Geophysical Research (76) 3 (January 1971).
[11] Rodgers, G.K., "The Thermal Bar in the Laurentian Great Lakes," Proc. 8th Conference Great Lakes Research (1965).
[12] Elliot, G., Elliot, J., "Small-Scale Model of the Thermal Bar, Proc. 12 th Conference Great Lakes Research, Great Lakes Research Division (1969).
[13] Elliot, G., Elliot, J., "Laboratory Studies on the Thermal Bar," Proc. 13th Conference Great Lakes Research, Great Lakes Research Division, Part 1 (1970).
[14] Csanady, G.r "Waste Heat Disposal in the Great Lakes," Proc. 13 th Conference Great Lakes Research, Great Lakes Research Division, Part 1 (1970).
[15] Markofsky, M., and Harleman, D.R.F., A Predictive Model for Thermal Stratification and Water Quality in Reservoirs," M.I. T., Department of Civil Engineering, Ralph M. Parsons Laboratory for Water Resource and Hydrodynamics, Technical Report No. 134 (January 1971).
${ }^{[16]}$ Ryan, P.J., and Harleman, D.R.F., Prediction of the Annual Cycle of Temperature Changes in a Stratified Lake $\overline{\text { or Reservoir, M.I.T., Department of Civil Engineering, }}$ Ralph M. Parsons Laboratory for Water Resources and Hydrodynamics, Technical Report, No. 137 (April 1971).
[17] Huber, W.C., Harleman, D.R.F., and Ryan, P.J., "Temperature Prediction in Stratified Reservoirs," J. Hydraulics Division, Am. Soc. Civil Engineers, 98 (HYA) p 645 (April 1972) .

REFFRENCES
(continued)
[18] Sundaram, T.R., and Rehm, R.G., "Formation and Maintenance of Thermoclines in Stratified Lakes Including the Effects of Power Plant Thermal Discharges," AIAA, 8th Aerospace Sciences Meeting, N.Y. (January 19-21, 1970).
[19] Sundaram, T.R., and Rehm, R.G., "The Effects of Thermal Discharges on the Stratification Cycle of Lakes," AIAA 9th Aerospace Sciences Meeting (January 1971).
[20] Sundaram, T.R., Rehm, R.G., Rudinger, G., and Meritt, G.E., A Study of Some Problems on the Physical Aspects of Thermal Pollution, Cornell Aeronautical Laboratory, $\overline{\mathrm{CA}} \overline{\mathrm{NO}} \overline{\mathrm{VT}}-\overline{2790-\mathrm{A}-1}$ (June 1970).
[21] Orlab, G.T., Development of a Mathematical Model for Prediction of Temperatures in Deep Reservoirs, Water Resources Engrs., Walnut creek, California (I966).
[22] Harleman, D.R.F., Stolzenback, K.D., and Jirka, G., A Study of Submerged Multi-port Diffusers with Applications to the Shoreham Nuclear Power Station, M.I.T. , Depart ment of Civil Engineering, Technical Report (June 1971).

Lake Circulation and Mixing Models

Problems and Scope

This modeling section presents a portion of the overall hydrodynamic system and includes the general circulation of the Lakes (both steady and time variable), transient motions leading to variable lake levels, and the random and smaller scale motions resulting in lake mixing or dispersion. The general hydrological system including water balances, lake level forecasting, and climatological effects are considered in a preceding subsection. Lake circulation is assumed to incorporate the mean, i.e., the relatively large-scale (lake-wide), approximately organized motions that generally vary seasonally. Mixing and dispersion are considered as the results of random movements of smaller spatial-temporal scales.

The importance of understanding and predicting water movements in the Great Lakes is expressed in two fundamental ways. First, water movements and lake level fluctuations directly impact such problem contexts as flooding, shore line erosion, and harbor and channel improvements. Second, the general circulation and changes in water level are important input information for many other aspects of the limnological systems, such as water quality, eutrophication, and general ecological models. These interactions occur principally through circulation, mass transport of water, and lake mixing and dispersion processes. The overall role and the importance of the hydrodynamic modeling framework are schematically depicted in Figure 15.

There are a variety of time and space scales associated with the general interactive scheme shown in Figure 15. The direct impact of the hydrodynamic modeling framework on water resource problems is concerned with short term transient phenomena such as wind setup and wave action during storms. Such transient forcing functions also play important roles in the general lake circulation. Seasonal variability in circulation coupled with variable density effects is an important time scale in the interaction of circulation models and other limnological

ROLE OF HYDRODYNAMIC MODELING OUTPUT
systems. Finally, quasi-steady-state circulation and mixing patterns, usually thought of as occurring within a seasonal time frame, can also be significant inputs to subsequent limnological models.

Space scales of interest extend from general lake-wide circulation scales to scales characteristic of the nearshore area to harbor circulation or regional water movements. As with other modeling frameworks, the problem context often dictates the time-space scale of importance. However, as discussed further below, the time-space grid of some hydrodynamic modeling efforts may be at a considerably finer scale than that required or even possible for the other limnological modeling. Such fine grids are often occasioned by computational necessity rather than by the nature of the problem under consideration.

In contrast to some other modeling contexts discussed in this section, the subject of water movement and Great Lakes circulation has been the subject of a considerable amount of scientific literature over the years. The subsections which follow are intended as an overview of the state of the art followed by an evaluation of the status of circulation modeling.

Modeling Framework

There are two primary forcing functions leading to water movement in the Great Lakes: (a) wind stress on the water surface and (b) density differences resulting from heat transfer through the water surface. The motions resulting from these inputs are further modified, depending on the time-space scale, by the rotation of the earth, bottom topography, shoreline configuration, and river inflows.

Field observation and analytical and numerical modeling studies have permitted a general description of the lake circulation. The wind as the primary forcing function sets up large-scale mass movements in the Great Lakes in all seasons. The character of these motions varies strongly with the density distribution in the lake, so that there are pronounced seasonal differences in flow regimes. Early in
the spring, the lakes are typically homogeneous and their motions are dominated by seiches as modified appropriately by friction. During the spring warm-up period, a warm ring of water surrounds a cold core with a primary circulation in geostrophic equilibrium (a balance between Coriolis and pressure forces), a secondary flow with sinking near the $4^{\circ} \mathrm{C}$ isotherm, and seiches and internal oscillations of the spring thermocline superimposed on this pattern. In the summer, the baroclinic motions (resulting from intersecting isobaric and density surfaces) give rise to strong coastal jets accompanied by upwelling or downwelling near shore. There are also a number of near-inertial oscillations (which dominate current velocities at midake), while seiche movements are present during other seasons. The irregular topography of the lakes and fluid and boundary friction complicate the picture even further: topography controlled gyres are presumably present under certain conditions, although there is no permanent steady-state current pattern (of appreciable amplitude) present. A complete and wholly satisfactory synthesis of the lake movements does not yet exist. However, within the framework of the general fluid flow equations, various portions of the problem have been explored in detail. All, however, begin from the generalized equations of motion.

One form of the equations for the general hydrodynamic modeling framework, which is sufficient for a starting point in this discussion, is:

$$
\begin{gather*}
\frac{\partial u}{\partial t}+u \frac{\partial u}{\partial x}+v \frac{\partial u}{\partial y}+w \frac{\partial u}{\partial z}=-\frac{1}{\rho} \frac{\partial p}{\partial x}+f v+F_{x}  \tag{1}\\
\frac{\partial v}{\partial t}+u \frac{\partial v}{\partial x}+v \frac{\partial v}{\partial y}+w \frac{\partial u}{\partial z}=-\frac{1}{\rho} \frac{\partial p}{\partial y}-f u+F_{y}  \tag{2}\\
\frac{\partial w}{\partial t}+u \frac{\partial w}{\partial x}+v \frac{\partial w}{\partial y}+w \frac{\partial w}{\partial z}=-\frac{1}{\rho} \frac{\partial p}{\partial z}+g+F_{z}  \tag{3}\\
\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{4}
\end{gather*}
$$

$$
\begin{gather*}
\frac{\partial T}{\partial t}+u \frac{\partial T}{\partial x}+v \frac{\partial T}{\partial y}+w \frac{\partial T}{\partial z}=\frac{\partial}{\partial x}\left(K_{x} \frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_{y} \frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_{z} \frac{\partial T}{\partial z}\right)+Q_{T}  \tag{5}\\
\rho=f(T, p) \tag{6}
\end{gather*}
$$

where $x, y$, and $z$ are the spatial coordinates ( $z$ positive downward), $t$ is time, $u, v$, and $w$ are the $x, y$, and $z$ components of the velocity, $p$ is the pressure, $\rho$ is the fluid density $T$, is water temperature, $g$ is acceleration of gravity, f is the Coriolis parameter, $\mathrm{K}_{\mathrm{x}}, \mathrm{K}_{\mathrm{y}}$, and $\mathrm{K}_{\mathrm{z}}$, are components of the heat dispersion coefficients, and $F_{X}$, and $F_{Y}$, and $\mathrm{F}_{\mathrm{z}}$ represent other forces in $\mathrm{x}, \mathrm{y}$, and z directions and incorporate eddy viscosity and wind stress components. Equations (1), (2), and (3) express the conservation of momentum, Equations (4) and (5) are the continuity (mass balance) equations for the fluid and temperature, respectively, and Equation (6) is an equation of state relating the density, temperature, and pressure fields.

Even though Equations (1) through (6) do not include all possible effects and associated mathematical terms, they are still difficult to deal with numerically and impossible to handle analytically. The framework is, therefore, often reduced considerably in complexity by a series of assumptions depending on the problem context.

Further, the frictional terms are considered at several levels of complexity ranging from simple linear friction to more complicated non-linear forms or by assuming that friction is proportional to the horizontal Laplacian of the velocity. Density effects are either ignored (homogeneous water body) or incorporated indirectly. Some large models attempt to compute the density field simultaneously with the velocity field. The handling of boundary conditions at the lake surface, the bottom sides, and islands also bears heavily on the final form of the complete equations and available methods of solution. It is clear, then, that while one can write the equations that theoretically represent the flow field under any conditions, the actual
implementation of the equations is not direct, and various simplifying assumptions must often be made. In the historical development of Great Lake circulation models, therefore, the first efforts began with simplified forms of the Equations (1) through (6) which could be studied analytically. The analytical studies continue to increase understanding of lake circulation and have also provided information as to which terms in the governing equations need to be retained and how the solutions may best be obtained numerically.

Although Equations (1) through (6), in theory, permit the description of all fluid movements in time and space, and hence should produce output that directly reflects dispersion effects, this is generally not possible in practice. Therefore the handling of mixing and dispersion of water has generally been through externally supplied sets of dispersion coefficients, as in Equation (5), rather than through attempts to compute such effects internally. Some recent models, however, do carry out such internal computation.

State of the Art

As indicated above, one can proceed in many directions from the basic Equations (1) through (6). In the Great Lakes the progression has generally been from the analytical formulation and solution to the numerical simulation, the latter usually incorporating more non-linear and topographic effects. In order to summarize the work that has been done to date, it is convenient to group the efforts as follows:
A) Steady-state
B) Transient

1) Homogeneous
2) Stratified
3) Homogeneous
4) Stratified

Within these four categories, the work can be further divided into three sub-categories:
a) theoretical (analytical) studies, usually based on a linearized theory and simple lake models
b) numerical computer modeling
c) laboratory model simulation of entire lake basins or of isolated phenomena

In addition, as often mentioned throughout this report, it is important to review the field observations available for each category to show to what degree evidence exists to support conclusions resulting from the above modeling efforts.

Steady-state Homogeneous Models. Extensive literature exists, especially in the oceanographic field, on steady-state circulation models. Much of the earlier literature did not concern itself with lateral boundary problems. The models proceeded from the simplest geostrophic case to more complex numerical models. Thus, if in Equations (1) through (6), assumptions of a homogeneous density, non-accelerated, frictionless, linearized situation are made, the resulting equations are simply:

$$
\begin{align*}
& 0=-\frac{1}{\rho} \frac{\partial p}{\partial x}+f v \\
& 0=-\frac{1}{\rho} \frac{\partial p}{\partial y}-f u  \tag{7}\\
& 0=-\frac{1}{\rho} \frac{\partial p}{\partial z}+g
\end{align*}
$$

The horizontal motion resulting from these equations is the simple geostrophic flow. By incorporation of the hydrostatic equation, a very crude estimate of the fluid velocity can be made if the density (temperature-pressure) field is known. This approach has been used by Ayers $[1,2,3]$ to deduce orders of magnitude of velocity and transport. It should be stressed, of course, that the geostrophic estimates of transport are only
grossly approximate. For example, Ayers [l] estimated a transport of $473,000 \mathrm{ft}^{3}$ per second across a section of Lake furon compared to an outflow of $216,000 \mathrm{ft}^{3}$ per second of the St. Clajr River. This is about the order of verification that can be expected, although, as discussed below, other attempts to correlate geostrophic currents to measured current in certain lake areas have been somewhat more successful. Noble [4] has provided some evidence for geostrophic circulation in Lake Michigan. It is generally recognized, however, that models based solely on the geostrophic approximation can be subject to large errors. For any really serious modeling effort on lake circulation, therefore, one must incorporate other effects into the modeling framework which more realistically reflect the prototype situations.

A more reasonable steady-state model incorporates wind stress at the surface and places a bottom in the lake. Friction and density effects are ignored. The equations then become [5]:

$$
\begin{align*}
& g \frac{\partial}{\partial x}=A_{z} \frac{\partial^{2} u}{\partial z^{2}}+f v \\
& g \frac{\partial \eta}{\partial y}=A_{z} \frac{\partial^{2} v}{\partial z^{2}}-f u \tag{8}
\end{align*}
$$

where the free surface is $z=\eta(x, y)$, the bottom is at $z=$ -h ( $x, y$ ) and $A_{z}$ is the coefficient of vertical eddy viscosity. The boundary conditions are:

$$
\begin{align*}
& A_{x}\left(\frac{\partial u}{\partial t}\right)=\tau_{x} \text { at } z=0 \\
& A_{x}\left(\frac{\partial v}{\partial t}\right)=\tau_{y} \text { at } z=0  \tag{9}\\
& u=0 ; v=0 \text { at } z=-h
\end{align*}
$$

where $\tau_{x}$ and $\tau_{y}$ are the components of the wind stress acting on the lake surface. Analytical solutions exist for this set of equations and permit evaluation of the behavior of the
fluid field and surface under different wind stress conditions. Some of the approaches used to obtain analytical solutions are quite ingenious. For example, Welander [5] in dealing with Equation (9) temporarily assumes $\partial \eta / \partial n$ as known, obtains solutions for $u$ and $v$ in terms of $\partial n / \partial n$, integrates the solution vertically, and inserts the result in the mass continuity equation to obtain a single partial differential equation of elliptic type. The water level equation can then be solved numerically for any closed basin using the boundary condition that the normal flow vanishes at the coast.

Other analytical models of steady rotating homogeneous fluid flow in closed basins have been studied by Birchfield [6,7], and Janowitz [8]. In these models, Ekman layers (vertical velocity gradients due to wind stress or bottom friction) are formed below the surface and at the bottom. The transport from these layers delivers fluid to the side-wall boundary layers which acquire rather different characteristics when the walls have a slight slope. Thus, nearshore conditions require further attention, but it is probably true that away from the shores the Ekman drift is confined to relatively shallow layers at the lake surface and bottom.

When more complex topography is incorporated, a numerical model must be used. Such numerical models of steady motion in a homogeneous lake have been described by Murthy and Rao [9] and Simons [12] among others. Murthy and Rao use a steady-state homogeneous linear model that is vertically integrated over depth but incorporates a smoothed bathymetry. The equations of motion in integrated form are:

$$
\begin{align*}
-f V & =-g H \frac{\partial \eta}{\partial x}+\frac{1}{\rho}\left(\tau_{s x}-\tau_{b x}\right)  \tag{10}\\
f U & =-g H \frac{\partial \eta}{\partial y}+\frac{1}{\rho}\left(\tau_{s y}-\tau_{b y}\right)
\end{align*}
$$

where:

$$
V=\int_{-H}^{\eta} v d z ; \quad U=\int_{-H}^{\eta} u d z
$$

and $H=H(x, y)$. $H$ is the depth of water, ${ }^{\tau}$ sx and ${ }^{\tau}$ sy are $x$ and $y$ components of surface wind stress and $\tau_{b x}$ and $\tau_{b y}$ are the x and y components of bottom frictional stress. The surface stresses are used as the forcing functions while the bottom stresses are assumed to be linearly proportional to volume transport. Murtiny and Rao applied this numerical model to Lakes Erie, Huron, Michigan, and Superior. Grid sizes varied from about 5 km across Lake Erie to 15 km in Lake Fiuron.

Gedney and Lick [10,11] have constructed a steady-state numerical model of Lake Erie assuming homogeneous conditions. The approach is similar to that of Welander's [5] which is discussed above. The vertical eddy viscosity is independent of depth. Since Lake Erie is stratified during the summer, the results are considered to apply only during fall, spring, and non-ice winter conditions. Island geometry is incorporated. A single equation for the stream function is obtained from the vertically integrated continuity equation and equation of motion. With the stream function calculated numerically, velocity components and water levels can be computed. In the island region, a grid size of 0.8 km (about 2800 grid points) was used while outside the region, a 3.2 km (about 2250 grid points) grid was employed.

The Gedney-Lick study also represents one of the few attempts to directly compare the velocity output from the model to observed current information. Some current data on Lake Erie were available from the Environmental Protection Agency during spring and fall of 1964. Figures 16 and 17 show some of these comparisons which, in general, agree qualitatively and approximately quantitatively. In addition, for this Feasibility Study, net velocity field in the Western Basin of Lake Erie was computed from the Gedney-Lick output and used in a chloride demonstration model. The results of this computation, utilizing chlorides as a tracer which acts as a verification of the flow field, are quite good. The analyses and results are discussed in detail in the Demonstration Model. The results from these models show large topography-controlled gyres, which are presumably present in the lakes during months of nearly homogeneous density distributions.

Horizontal velocities at a constant $9.9 \mathrm{M}(32.8 \mathrm{ft})$ from surface. Wind direction, W50S; wind magnitude, 10.1 meters per second( 22.7 mph ); friction depth, 27.4 meters $(90.0 \mathrm{ft})$; rivers: Detroit, Niagara.


[^5]COMPARISON OF COMPUTED AND OBSERVED CURRENT MEASUREMENTS

Other numerical models of steady homogeneous circulation in lakes have been constructed by Cheng and Tung [13] and Cheng [14] using a finite element method of solution. This computational approach has a distinct advantage over more traditional methods of finite differencing in the handling of boundary conditions. Thus, complicated lateral boundaries or islands in the lake are readily incorporated. Numerical results for transport in Lake Erie are qualitatively similar to other work although no detailed verification is attempted.

Hydraulic models have also been constructed, and they provide a means for estimating circulation in homogeneous water. Rumer [15] has constructed models of Lake Erie and recently of Lake Ontario. The Lake Erie model has a horizontal length scale of l:200,000 and a vertical scale of l:500. These models again show the effects of topography similar to those exhibited by the numerical models and also the effects of river inflow and outflow.

Field observations following periods of relatively steady winds under homogeneous conditions indeed indicate gyres of the kind predicted by numerical and physical laboratory models, although the evidence is not quite conclusive. The presence of opposite Ekman drifts in the surface and bottom layers of Lake Erie has been shown by drift cards and bottorn drifter observations. Rumer's hydraulic model shows that the inflow from the Detroit and Niagara Rivers tends to follow the south shores of Lakes Erie and Ontario, respectively [16]. Verification of the numerical models has been generally supported by observed surface current directions from drift cards and bottles. Murthy and Rao [9] report good qualitative agreement of their numerical circulation pattern with observed drifts in Lakes Erie, Michigan, and Superior. For Lake Huron, the authors consider the general circulation pattern agreement remarkable. It should, of course, be added that most observed motions in the Great Lakes are unsteady and the lakes during important times of the year, from a limnological point of view, are not homogeneous.

Steady-State Stratified Models. Analytical studies of steady state circulation in stratified lakes have been published by Csanady [17,18] and Huang [19].

For example, Csanady [18] developed an analytical model based on geostrophic flow along the shore in the warmer regions during spring, which also conserves potential vorticity. By dividing the nearshore region into two density regimes, the basic features of observed phenomena are indicated by this frictionless steady linear model. The analytical models, therefore, predict baroclinic motions with a velocity maximum close to shore (coastal jets).

During the early part of the season, only the nearshore regions contain warm water; the inclined spring thermocline separates this part from the rest of the lake. Baroclinic motions of appreciable amplitude are predicted to be confined to these nearshore regions by the theory. Later in the season the thermocline is continuous across the lake except that it may tip up or down near the shores where the coastal jets are located. The water transport arriving by Ekman drift in the shore zone is removed horizontally by the coastal jets. The observed facts of a spring thermal regime in Lake Ontario consisting of a thermocline surface of wedge or lens shape in the shore zone and a summer regime characterized by tilting of the across-lake thermocline are generally explained by some of the above analytical models. Additional discussion of the nearshore circulation is given below.

A numerical model of flow in stratified bodies of water has been constructed by Bennett [20]: a model lake consisting of an infinitely long rotating channel in which all variables are assumed independent of the long axis. A constant slope region and a flat interior region are used. The results are similar to those obtained by the analytical models: an inclined thermocline accompanied by a geostrophic current near shore.

Laboratory studies by Gliiot and Elliot [21,22] have shown a friction-dominated circulation to be set up by surface heating in a shore zone of a physical model of constant slope. This is associated with a thermal structure similar to that observed during the spring warming period (the thermal bar). The circulation which results is essentially perpendicular to the shore and involves sinking of water in the neighborhood of the spring thermocline and upwelling at mid-lake and near the shores.

From the point of view of verification analyses, many of the features of the modeling effort have been observed in the Great Lakes. For example, Rodgers [23,24] has studied and observed in great detail the motion of the thermal bar during the spring. Figure 18 shows some of the field results. The steady-state stratified models generally provide similar results to those observed. A direct comparison between observed currents and currents estimated by a simple geostrophic calculation was made by Smith and Ragotzkie [25] off the Keeweenaw Pensinsula in Lake Superior. Computed current velocities reached 15 cm per second but were generally less than $10 \mathrm{~cm} / \mathrm{sec}$. The normal component of the measured currents was often $30 \mathrm{~cm} / \mathrm{sec}$ in some parts of the cross-section; and significant speeds extended down to 60 m , the assumed level of "no motion" used in the geostrophic calculations. While this work indicates the non-verification of the simple geostrophic current, in this case it does provide direct observation of the coastal jets predicted by the above models.

A similar analysis of the coastal current of the south shore of Lake Ontario has been made by Scott, et.al. [26]. For this case, comparison of computed (geostrophic) and observed current indicated good agreement. Average measured transport was 2.35 ( $\mathrm{km}^{3}$ /day) compared to an average computed transport of 2.26 ( $\mathrm{km}^{3} /$ day) .

It appears that the circulation observed by Elliot and Elliot [21] is also present in the real lakes, in the manner of a secondary flow. Parallel to the thermocline surface there are geostrophic motions of considerable intensity, as may be expected from theory, although the details are by no means easy to interpret. Ragotzkie and Bratnick [27] have demonstrated the prolonged existence of slow upwelling during the summer in the middle of Lake Superior (which remains in a "spring regime" essentially all summer). A detailed study of flow in the coastal zone near Oshawa, Ontario has also turned up a number of coastal jets [28].

While many of the observed features of lake circulation agree with theoretical predictions of steady-state baroclinic models, it should be noted that none of the above observed motions are in fact steady.


FIGURE 18
PROGRESS OF THE THERMAL BAR FROM WINTER TO FULL SUMMER STRATIFICATION

Time Variable Homogeneous Models. Transient motions of a homogeneous lake (seiches) have been the subject of a large number of analytical studies. One outstanding investigation is that of Platzman [29] who studied wind tides of Lake Erie. In basins the size of the Great Lakes, the effects of rotation are noticeable in that high water rotates around the basin counter-clockwise. Apart from this distortion, seiches are mass-movements of water from one end of the lake to the other. One should add that the same seiches occur also when the lake is stratified, only then they are accompanied by other motions. There are an infinite number of modes which seiches may take (depending on how they are excited, i.e., what distribution of wind stress causes them) but only the few lowest frequencies are of practical importance. The periods of these are of the order of 10 hours in most of the lakes [30].

Platzman's work provides a great degree of analytical insight to the behavior of transient motions and wind set-up in Lake Erie. His model is essentially the classical Ekman wind driven circulation model and incorporates vertical eddy viscosity, Coriolis forces, pressure gradients, surface wind stress, and bottom friction. A numerical differencing scheme is used to compute model results for Lake Erie conditions. Platzman also carries out an extensive verification analysis of his model as discussed below.

Other numerical and analytical studies of unsteady motions in homogeneous closed basins have also been carried out with reference to unsteady motions resulting from unsteady wind stress. For example, Paskavsky [31] has developed a numerical model of Lake Ontario under a uniform density regime. His model incorporates the non-linear field acceleration terms such as:

$$
\frac{{ }^{2} \cdot u}{v} \frac{u}{x}+v \frac{\partial u}{\partial y}
$$

bottom and lateral friction, and topographic and Coriolis effects.

Simulations are conducted under transient wind fields starting as a cyclonic disturbance passing across the lake in an east










- K7т̦тиәр snoauəbourou




- [0ع] [TəMypoy Kq suostueduon әwos smoys ZT



-spโəبモ










- -sodund



-sțsKiture sțu7 uṭ pəzuəsəxd






Oliserved (heavy curves) and computed (light curses) Puffaleminus-Toledo set-up. Dotted portions of observed set-up curves FIGURE 19

TABLE 12
COMPUTED AND OBSERVED PERIODS OF THE FIRST FIVE MODES OF LONGITUDINAL FREE OSCILLATION OF LAKE MICHIGAN
(Adapted from Rockwell [30])

| Mode | Computed (hours) | Observed (hours) | Error \% |
| :---: | :---: | :---: | :---: |
| 1 | 8.83 | 9.0 | - 1.9 |
| 2 | 4.87 | 5.2 | - 6.3 |
| 3 | 3.53 | 3.7 | - 4.6 |
| 4 | 2.85 | 3.1 | - 8.1 |
| 5 | 2.39 | 2.5 | - 4.4 |

a flow structure practically indistinguishable from the coastal jets of the steady-state models, except that they have a wave-like amplitude distribution along the basin perimeter. One should note again that these baroclinic motions associated with intersecting density and pressure surface occur side by side with barotrophic seiches (where density and pressure surfaces remain parallel) producing a quite complex total response in a stratified lake. One important practical conclusion is that a stratified lake extracts much more energy from the wind than a homogeneous one.

Large numerical models of unsteady motion in stratified waters have been constructed by oceanographers. For example, West [37] has developed a two-layered prognostic model of the circulation in the Gulf of Mexico. Each layer includes horizontal and vertical momentum exchange, Coriolis effect, and non-linear advection of momentum. Bottom topography is included but wind effects are excluded, the primary function being the inflow through the Yucatan Strait. Surface and internal gravity waves are filtered out by several approximations. The model is spun up to a steady-state which is then used as an initial condition. A one year prediction is then made of the baroclinic and barotrophic modes of circulation in the Gulf using a seasonally varying input flow. The spin-up process takes about 3 hours of CDC-6500 CPU time while the 360 -day prediction takes 10 hours which "explains why a parametric study was not pursued..." [37]. The spatial grid used in this study was 20 km on a model size of 1420 x 800 km or a total of about 2800 grid points. The time step is approximately 1.5 hours, the circulation predicted by the baroclinic model has the same basic features as the observed data in the Gulf of Mexico. The barotrophic mode of the circulation could not be verified because of a lack of data. Similar models have been constructed of the North Pacific Ocean [38], the Indian Ocean (which includes monsoon effects), and the North Atlantic [39].

A rigid three-dimensional finite difference grid was used by Friedrich [39] who developed a 14-layer non-steady model of ocean circulation, including temperature and salinity
effects, based on the work by Bryan [40], and tested in the North Atlantic Ocean. A period of 80 years was simulated using a $5^{\circ}$ grid, followed by 70 years using a $3^{\circ}$ grid. A time step of 2.22 hours is used for the $3^{\circ}$ grid computations. A grid of $26 \times 37$ horizontal cells and 14 vertical layers (a total of 13,468 cells) requires nine seconds of computer time per time step on the UNIVAC l108. Consequently, more than 10 hours of computer time are required to simulate one year of real time. This points out the present limitations of three-dimensional models, in general. These limitations make it extremely expensive to test such models and to perform sensitivity analyses to evaluate the importance of various parameters.

For the Great Lakes area, Simons [41] has formulated a three dimensional solution of the hydrodynamic equations, based on vertically integrated equations for each layer. The model is being tested at present in Lake Ontario, initially using a 5 km horizontal grid. A time setup of one minute is used. It is interesting to note that Simons concludes from his initial work with a homogeneous model that the effects of non-linear acceleration terms do not seem to be significant enough to justify their inclusion in the model.

From a verification viewpoint, detailed comparisons between model output and observation has generally not been possible. As indicated several times above, however, numerous features generated by the models such as coastal jets, tilting of thermoclines, and approximate order of magnitude of current speeds and directions are observed in the prototype.

There is extensive evidence for the existence of Poincare waves during summer condition in the Great Lakes. Mortimer [42,43] has reviewed this evidence and interpreted it in terms of the theoretical concepts. Away from the shores, currents are produced mostly by Poincare waves during the summer, while close to shore more persistent velocities occur. The temperature structure of Lake Michigan, in particular, shows the presence of both Poincare and Kelvin waves quite clearly. Similar conclusions follow from the Lake Ontario observations of Csanady [28]. The latter study also shows the presence of near-inertial oscillations during the spring period with the thermal bar present.

Near-Shore Circulation. The primary reason for singling out the nearshore zone for special attention is, of course, the concentration of human activity with the associated waste residuals. However, there is also a scientific reason as discussed above. The shore zone is a singular region on several counts: the Ekman drift arriving from midlake has to accommodate itself to the presence of the shores, while the depth of water reduces gradually to zero. Under summer and spring conditions, it has already been indicated that a peculiar thermocline structure develops in the shore zone (see Figure 18) which may be accompanied by coastal jets. Thus one may legitimately describe the coastal zone (of some 10 km width) as a boundary layer of greater than usual complexity. The relatively small width of this layer also means that inertial forces cannot be neglected in it (linear theory is not applicable except as a crude approximation). All the forces: wind stress, Coriolis force, pressure gradient, friction, and inertial forces play an important role. An additional complication is caused by surface waves arriving from midlake, usually at an oblique angle against the shore. Their momentum parallel to the shore causes longshore currents in a beach-zone (of at most a few hundred meters width).

It is possible to focus on phenomena in a shore zone, ignoring the lake-wide circulation, except insofar as it causes an inflow of mass (Ekman drift) or of momentum into the shore zone. A well known theory of this type is that of edge-waves along a sloping shore [44]. In a shore-zone model of constant slope (usually of order $10^{-2}$ to $10^{-3}$ ), wave-like modes of motion are found to exist. These are trapped at the shores in the sense that they decay exponentially with distance from shore.

The baroclinic Kelvin wave, which was discussed above in the lake wide context, is also such a trapped wave in a basin with vertical boundaries. In a more realistic shore-zone model containing a sloping beach, many baroclinic Kelvin type waves are possible [34]. These all have long wave-lengths (comparable to basin dimensions) and their frequency is quite low, a small fraction of the inertial frequency.

Thus, given the otherwise rapid variation of flow structures, their velocities are nearly in geostrophic equilibrium. Kelvin waves in the Great Lakes are not possible if the water is homogeneous because the basins are too small or, putting it another way, the propagation speed of surface seiches are too fast.

Edge waves and Kelvin waves are both transient motions even if the latter appears steady. Strictly steady baroclinic currents (coastal jets) have also been found in theoretical studies of shore zones [34]. These are similar to the coastal jets in basins with vertical shores, but are displaced shoreward. Their driving mechanism is the influx of water from midlake.

All the above theoretical conclusions were derived from linearized theory. However, the Rossby number based on the width-scale of shorebound currents is often in the order of unity, which means that inertial forces are not negligible. The physical mechanism involved is quite simple: when water drifts from midlake into a strong coastal current, some considerable convective accelerations are exerted.

Bottom friction must play an important role, at least in the shallower parts of the coastal zone. These effects have already been referred to in connection with some lakewide theoretical models involving friction (a frictional boundary layer). Elliot's [21] laboratory model also shows a Erictionally controlled (thermally induced) circulation. There has been, however, no theoretical (analytical) attempt to describe the structure of coastal currents subject to friction in a sloping shore zone.

The theory of longshore currents in a beach zone, generated by incident wave-trains, has been treated recently by LonguetHiggins [45] and others. The component of the momentum transport by the waves parallel to the shore causes a radiation stress in the same direction, which maintains a longshore current against the force of friction. The current is concentrated on the inshore side of the breaker line and its velocity is proportional to wave orbital velocities (but is about one order of magnitude less). Under conditions typical of the wave climatology of the Great Lakes, namely, during
major storms when wave heights are of the order of 5 meters, the longshore currents are only rarely comparable to other lake currents ( $10 \mathrm{~cm} / \mathrm{sec}$ or more). Also, these currents are confined to a smaller inner portion (order 100 m width) of the coastal zone. The mass transport they cause is nevertheless very important from the point of view of sediment movement and presumably also for the initial transport of pollutants from small sources offshore or of any pools of effluent trapped along the shore.

Most experimental studies of shore zone currents are based on continuous records of one or a few moored current meters. Statistical analysis of long records obtained in this manner provides some valid data on current climatology, but gives very little information on nearshore circulation patterns. A good recent study of this kind is that of Birchfield and Davidson [46] which shows the considerable differences between nearshore and offshore currents. Some more detailed studies have been reported by $5 m i t h$ and Ragotzkie [25] and Csanady [28] from Lake Superior and Lake Ontario, respectively. The Lake Ontario data in particular shows clearly the existence of a distinct shore zone of approximately 7 km width, in which the water movements are more nearly persistent (current like) than wave-like. Their complex structure could be interpreted as the combined result of wind stress, Coriolis, and non-linear accelerations as well as pressure and friction forces. One important practical conclusion is that records of one, or even a few, moored current meters would be quite inadequate to elucidate the details of the complex shore current structure and could lead to some deceptive conclusions.

Summarizing this discussion briefly, theory suggests that there is a special nature in nearshore circulations where a coastal boundary layer of rather complex characteristics develops. Inflow of mass and momentum into this boundary zone forces the circulation there, while several free modes of motion (edge waves, Kelvin waves) are also possible. In a narrow zone radiation, stress of surface waves sets up longshore currents over beaches. Experimental data confirm the complexity and special nature of shore zone circulations and demonstrate the inadequacy of point measurements of currents.

Mixing and Dispersion. The above discussion has centered primarily around the regular, somewhat deterministic, structure of water movements in the Great Lakes. For certain applications, however, notably, attempts to describe the paths and distribution of pollutants, attention must also be focused on the random smaller scale oscillations in fluid flow. These oscillations give rise to a mixing and dispersion of material discharged into the lakes. In addition to temporal changes in flow, spatial gradients also contribute to the dispersive phenomena. As indicated in the previous subsection, currents in the Great Lakes tend to be concentrated in narrow bands around the boundary so that there are strong velocity gradients at these locations. Within the Ekman layer at the surface or at the bottom, velocity varies both in magnitude and direction.

Another kind of mass transport by (nearly) steady motions is that caused by secondary flow of the type associated with the spring thermocline. As indicated above, a confluence (sinking) occurs near the $4^{\circ}$ isotherm. The cells of secondary circulation inshore and offshore of this confluence effectively produce large scale mixing. Semi-permanent confluences have been observed in the Great Lakes in other places and other times; all are, presumably, evidence of secondary circulation of various sorts which produce mixing.

In wave-like motions, particle paths are almost closed curves. There is a second-order effect giving rise to a net mass transport velocity (Stokes Velocity). This residual velocity in surface waves has been studied in detail, but not in Poincare waves, edge waves, or other long waves common in the Great Lakes. Such wave-like motions dominate the flow at least during the summer and outside the shore zone where there is neither theoretical nor experimental information on mass transport velocities.

The importance of wave-like motions in causing dispersion is demonstrated by a recent study of Ahrnsbrak and Ragotzkie [47]. A simple diffusion-advection model is used to compute the effective eddy diffusivities (or dispersion coefficients) in Green Bay. Values ranges from $0.25 \cdot 10^{5} \mathrm{~cm}^{2} / \mathrm{sec}$ at the southernmost end of the Bay to about $3 \cdot 10^{6} \mathrm{~cm}^{2} / \mathrm{sec}$ at a distance of 30 km . Northward, diffusivities decreased to about $0.7 \cdot 10^{6} \mathrm{~cm}^{2} / \mathrm{sec}$.

The authors continue their analyses by using the output from a numerical model of Green Bay circulation to independently check the observed dispersion coefficients. The model used takes into account wind stress, free oscillation in the Bay, and a forcing function at the mouth of the Bay which is due to the seiching of Lake Michigan. The eddy diffusivity is assigned equal to the product of the root mean square velocity residuals (due to seiche activity) and a time scale (taken as one-half the free period of the Bay). Agreement between diffusivities predicted from the model and the observed results is quite good. The results, therefore, show that seiche movements in Green Bay are responsible for the observed decay of Fox River water concentration along the axis of the bay. Seiche movements at this location are of relatively large amplitude, and there is, of course, a considerable variation of velocity between the bottom of the bay and the surface. It appears that this velocity gradient, coupled with vertical diffusion and the temporal variation of seiche velocities, produces the relatively high longitudinal diffusion.

The direct effects of turbulence in causing dispersion have been studied experimentally and related to the classical theory of turbulent diffusion in connection with dye plume and patch observations [49,50]. The problem is very similar to oceanic diffusion in that the apparent diffusivity increases as the size of the cloud grows, due to the increasing range of flow nonuniformities encountered. For example, Murthy [50] found that the $4 / 3$ law resulting from the similarity theory of turbulence is suggested by dye diffusion studies in Lake Ontario. His results indicate that the horizontal dispersion coefficient ( $K$ ) is given by $K=\left(6 \times 10^{-3}\right) L^{4 / 3}$ where $K$ is in $\mathrm{cm}^{2} / \mathrm{sec}$ and L in cm . A summary of some horizontal dispersion coefficients obtained by different workers in the Great Lakes is given in Table 13.

Vertical mixing in the Great Lakes has been found to be generally quite small (2-3 orders of magnitude less than horizontal dispersion, i.e., approximately $1 \cdot 10^{2} \mathrm{~cm}^{2}$ per second) except when there is strong momentum flux into the water and/or heat flux into the air at the free surface. Strong mechanical and/or thermal turbulence develops in the latter cases and mixes the surface layer down to a sharp thermocline. Mixing across the thermocline also occurs (as

TABLE 13

## SUMMARY OF SOME HORIZONTAL DISPERSION COEFFICIENTS IN THE GREAT LAKES

| Area | Method | Approximate Range ( $\mathrm{cm}^{2} / \mathrm{sec}$ )* | Reference |
| :---: | :---: | :---: | :---: |
| Green Bay | Tracer and Circulation Model | 100-200•104 | 47 |
| Lake Michigan, Lake Erie | Drogues | $3-5.5 \cdot 10^{4}$ $3-4 \cdot 10^{4}$ | 48 |
| Lake Erie, Lake Huron | Dye | $.05-.03 \cdot 10^{4}$ | 49 |
| Lake Ontario | Dye | 0.1 ${ }^{10} 10^{4}(100 \mathrm{~m}$ scale) | 50 |
| Little Traverse Bay | Dye | $.02 \cdot 10^{4}$ | 51 |
| Lake Erie | Current Meters | $.02-4.0 \cdot 10^{4}$ | 52 |
| $* 10^{4} \mathrm{~cm}^{2} / \mathrm{sec}=$ | 3 square mil | s/day |  |

demonstrated by the gradual sinking of the thermocline during the season, mainly in September/October), but the exact mechanism of this is not well understood. Some preliminary diffusion experiments involving dye release below the thermocline [53] have shown no measurable upward diffusion.

Evaluation of Model Status

Model Availability. It is clear from the preceding discussion that a great variety of models exists to describe circulation and mixing in the Great Lakes. The models extend from steady state representations of the general circulation to numerical analyses of time variable motions and include specific models of near shore boundary phenomena under variable density regimes. Models of water movement and dispersion in the Great lakes are in more plentiful supply than any other component of the limnological systems and there is little lack of fundamental knowledge.

Data Availability. Relatively speaking, more is known about circulation in the Great Lakes than many of the other components in the limnological systems. This is the result of measurements of all types (drift bottles cards, bottom drifters, current speed and and direction, temperature) made over a period of many years, some work extending over 80 years. From the many direct and indirect measurements (primarily of surface currents), a broad picture of the general circulation pattern during different seasons has emerged. It is true, then, that for most long-term planning purposes, the general lake circulation has been observed and is continuing to be observed at a level sufficient for many limnological problem contexts.

There are data gaps, however, primarily in more local transient situations. For example, reliable current measurements in the Lakes have yet to be made on any large scale of the coastal jet effect and the nearshore ( $0-10 \mathrm{~km}$ ) circulation during thermal bar conditions. Also, although some comprehensive current measurement programs (e.g., on Lake Michigan) have been carried out from fixed buoy stations, the data are not readily available. Measured current data
of this type would be of significant value in verification studies associated with simulation models. The present International Field Year on the Great Lakes effort will provide valuable data for verification of hydrodynamic models.

Model Verification. In general, the models of circulation and mixing have been compared to observed data, summaries of data, or general qualitative descriptions. The comparisons have indicated a reasonable degree of model veracity, sufficient for many planning purposes. In some cases, the verification analysis has been extensive (i.e., Platzman's work), while in other cases only general features are compared. For some modeling situations, its quite difficult to obtain information that would verify model output due to the physical difficulties of measuring current regimes in large bodies of water. Also, some modeling structures analyze only certain features of water movement (e.g., motions due to transient storms). A meaningful comparison between model output and observed data would require an extensive and intensive sampling program coupled with data analysis to provide observed movement that could be compared to model output. In general, however, verification of hydrodynamic model output has been good to excellent and many of the models can presently provide a strong basis for limnological planning.

Model Application in Planning. For planning purposes, models of lake water movements have been applied primarily in lake level forecasting during storm surges. There apparently has been little direct incorporation of circulation modeling in models of chemical and/or eutrophication problem contexts. The demonstration model discussed in this report shows an example of the necessity of a verified circulation model for planning problems associated with water quality and eutrophication. It should be recognized, however, that fluid flow calculation in almost all cases can be decoupled from modeling activities of limnological problems. Thus, although hydrodynamic model output is often not included explicity in the water problem contexts, it invariably has been incorporated as sets of exogenous input generated by other analyses.

Finally, it should be noted that additional significant advances in hydrodynamic modeling are in the offing and rely on the complete use of large computers for successful implementation. These advances will most likely be of sufficient extent to answer many of the remaining loose ends of circulation modeling, certainly with respect to planning activities of the limnological systems. For example, within a few years, it is entirely conceivable that a realistic, synthesized model of lake circulation will be available and will incorporate such features as: a) time variable wind stress, b) realistic lake geometry including a nearshore zone, probably on denser network of grid points, and c) time variable density effects.

In conclusion, the present modeling structures for lake circulation and mixing are well developed for planning purposes. Furthermore, the historical interest in lake water movements, coupled with a firm physical basis for understanding the phenomena, has generated a momentum among researchers and modelers in the Great Lakes Basin. Such momentum is probably sufficient to develop as complete a modeling structure as would be desirable or necessary for planning purposes for the next several years.

## REFERENCES

[1] Ayers, J.C., "A Dynamic Height Method for the Determination of Currents in Deep Lakes," Limnology and Oceanography, Vol. No. 3 (July 1956).
[2] Ayers, J.C. et.al., Currents and Water Masses of Lake Huron, Great Lakes Research Division, University of Michigan, Pub. No. l (February 1956).
[3] Ayers, J.C., et.al., Currents and Water Masses of Lake Michigan, Great Lakes Research Divison, University of Michigan, Pub. No. 3, p 169 (1958).
[4] Noble, V.E., "Evidences of Geostrophically Defined Circulation in Lake Michigan," Proc. loth Conf., Great Lakes Research, pp 289-298 (1967).
${ }^{[5]}$ Welander, P., "Wind Action on a Shallow Sea: Some Generalization of Ekman's Theory," Tellus, Vol. IX, pp 4552 (1957).
[6] Birchfield. G. E., "Wind-Driven Currents in a Long Rotating Channel," Tellus, Vol. 19, pp 243-249 (1967).
[7] Birchfield, G. E., Wind-Driven Currents in a Large Lake or a Small Ocean, Report 70-6, Series in Applied Mathematics, Northwestern University (1971).
[8] Janowitz, G.s., "The Coastal Layers of a Lake When Horizontal and Vertical Ekman Numbers are of Different Orders of Magnitude," Proc. 14th Conf. IAGLR (1971).
[9] Murty, T.S., and Rao, D.R., "Wind Generated Circulation in Lake Erie, Huron, Michigan, and Ontario," Proc. l3th Conf. IAGLR, pp 927-941 (1970).

## REFERENCES

(continued)
[10] Gedney, R., and Lick, W., "Numerical Calculations of the Steady-State Wind Drive Currents in Lake Erie," Proc. 13th Conf. IAGLR, pp 829-838 (1970).
${ }^{[11]}$ Gedney, R., and Lick, W., "Numerical Calculations of the Wind-Driven Currents in Lake Erie and Comparison with Measurements," NASA Tech. Memo. X-67804, p 19 and figures (April 1971).
[12] Simons, T.J.. "Development of Numerical Models of Lake Ontario," Proc. 14th Conf. IAGLR (1971).
[13] Cheng, R.T., and Tung, C., "Wind-Driven Lake Circulation by the Finite Element Method," Proc. 13th Conf. IAGLR, pp 891-903 (1970).
[14] Cheng, R.T., "Numerical Investigation of Lake Circulation Around Islands by the Finite Element Method," To appear in Int. J. for Numerical Methods in Engineering, P 16 and figures.
[[15] Rumer, R.R., and Robson, L., "Circulation Studies in a Rotating Model of Lake Erie," Proc. 1lth Conf. IAGLR, pp 487-495 (1968).
[16]
Howel, J., Kiser, K., and Rumer, R., "Circulation Patterns and a Predictive Model for Pollutants Distribution in Lake Erie," Proc. l3th Conf. IAGLR, pp 434-443 (1970).
[17] Csanady, G.T., "Wind-Driven Summer Circulation in the Great Lakes," J. Geophys. Res., Vol. 73, pp 2579-2589 (1968).
[18] Csanady, G.T., "On the Equilibrium Shape of the Thermocline in a Shore Zone," J. Phys. Oceanog., Vol. 1, p 92-104 (1971).

REFERENCES
(continued)
[19] Huang, J.C.K., "The Thermal Current in Lake Michigan," J. Phys. Oceanog., Vol. l, pp 105-122 (1971).
[20] Bennett, J.R., "A Numerical Simulation of the Thermal Bar," Proc. 14th Conf. IAGIR (1971).
[21] Elliot, G.H., and Elliot, J.A., "A Small Scale Model of the Thermal Bay," Proc. 12th Conf. IAGLR, pp 553-557 (1969).
[22] Elliot, G.H., "A Mathematical Study of the Thermal Bar," Proc. 14th Conf. IAGLR (1971).
[23] Rodgers, G.K., "The Thermal Bar in Lake Ontario, Spring, 1965 and Winter, 1965-1966," Proc. 9th Conf. IAGLR, University of Michigan, pp 369-374 (1966).
[24] Rodgers, G.K., and Sato, G.K., "Factors Affecting the Progress of the Thermal Bar of Spring in Lake Ontario," Proc. I3th Conf. IAGLR, pp 942-950 (1970).
${ }^{[25]}$ Smith, N.P. and Ragotzkie, R.A., "A Comparison of Computed and Measured Currents in Lake Superior," Proc. 13th Conf. IAGLR, pp 967-977 (1970).
[26] Scott, H.T., Jekel, P., and Fenlon, M. "Transport in the Coastal Current Near the South Shore of Lake Ontario in Early Summer," Proc. 14th Conf. IAGLR (1971).
[27] Ragotskie, R.A., and Bratnick, M., "Inferred Temperature Patterns on Lake Superior and Inferred Vertical Motions," Proc. 8th Conf. IAGLR University of Michigan, pp 349-357 (1965).

REFERENCES
(continued)
${ }^{[28]}$ Csanady, G.T., Coastal Current Regimes in Lake Ontario, to be published.
[29]Platzman, G.W., "The Dynamical Prediction of Wind Tides on Lake Erie," Meteorological Monograph, Volume 26, p 44 (1963).
[30] Rockwell, D.C., "Theoretical Free Oscillations of the Great Lakes," Proc. 9th Conf. IAGLR, University of Michigan, pp 352-368(1966).
[31] Paskavsky, D.F., "Winter Circulation in Lake Ontario," Proc. 14th Conf. IAGLR, p 20 and figures (1971).
[32] Csanady, G.T., "Large-Scale Motion in the Great Lakes," J. Geophys. Res., pp 4151-4162 (1967).
[33] Csanady, G.T., "Motions in a Model Great Lake Due To a Suddenly Improved Wind," J. Geophys. Res., 72, pp 64356447 (1968).
[34] Csanady, G.T., "Baroclinic Boundary Currents and LongEdge Waves in Basins With Sloping Shores," J. Phys. Oceanog., Vol. I, p 92-104.
[35] Birchfield, G.E., "Response of a Circulation Model Great Lakes to a Suddenly Improved Wind Stress," J. Geophys. Res. 74, pp 5547-5554 (1969).
[36] Johnson, M.A., and Mortimer, C.H., Theory of Internal Waves in Large Basins with Particular Application to Lake Michigan, Spec. Report, Centre for Great Lakes Studies, University of Wisconsin, Milwaukee (1971).
[37] West, R.T., "A Baroclinic Prognostic Numerical Model of the Circulation in the Gulf of Mexicor" ph.D. Dissertation, Texas A\&M University, p 66 and ix (May 1970).
[38] O'Brien, J.J., "A Two Dimensional Model of the Wind-Driven North Pacific," Investigation Pesquera, Vol. 35 (1), Barcelona, pp 331-349 (February 1971).
[39] Friedrich, H.J., Preliminary Results from a Numerical Multilayer Model for the Circulation in the North Atlantic, $\overline{U D C}, 551.46 \overline{5.45}$ pp 145-164 (1969).
[40] Bryan, K., "A Numerical Method for the Study of Ocean Circulation," Jour. of Computational Phys., Vol. 4, pp 347367 (1969).
[41] Simons, T.J., "Development of Numerical Models of Lake Ontario," Proc. 14th Conf. IAGLR (1971).
[42] Mortimer, C.H., "Frontiers of Physical Limnology with Particular Reference to Long Waves in Rotating Basins," Proc. 6th Conf., IAGLR, University of Michigan, pp 9-42 (1963)
[43] Mortimer, C.H., Large-Scale Oscillating Motions and Seasonal Temperature Changes in Lake Michigan and Lake Ontario, Special Report No. $1 \overline{2}$, Center for Great Lake Studies, University of Wisconsin, Milwaukee (1971).
[44]Reid, R.O., "Effects of Coriolis Force on Edge-Waves," J. Marine Res., Vol. 16, pp 109-144 (1958).
[45] Longuet-Higgins, M.S., "Longshore Currents Generated By Obliquidy Incident Sea Waves," J. Geophys. Res., Vol. 75, pp 6778-6801 (1967).

## REFERENCES

(continued)

## [46]

Birchfield, G.E., and Davidson, D.R., "A Case Study On Coastal Currents in Lake Michigan," Proc. 10th Conf. IAGLR, pp 264-273 (1967).
[47] Ahrnsbrak, W.F., and Ragotzkie, R.A., "Mixing Processes in Green Bay," Proc. 13th Conf. IAGLR, pp 880-890 (1970).
[48] Okubo, A., and Farlow, J.S., "Analysis of Some Great Lake Drogue Studies," Proc. IOth Conf. IAGLR, pp 299-307 (1967).
[49] Csanady, G.T., "Turbulence and Diffusion in the Great Lakes," Proc. 7th Conf. IAGLR, University of Michigan, pp 326-3 39 (19 $\overline{64}$ ).
[50] Murthy, C.R., "An Experimental Study of Horizontal Diffusion in Lake Ontario," Proc. 13th Conf. IAGLR, pp 477489 (1970).
[51] Noble, V.E., Measurements of Horizontal Diffusion in the Great Lakes," Proc. 4th Conf. IAGLR, University of Michigan, pp 85-95 (1961).
[52] Palmer, M.D., and Izatt, J.B., "Lakeshore Two Dimensional Dispersion," Proc. 13th Conf. IAGLR, pp 495-507 (1970).
[53] Murthy, C.R., and Csanady, G.T., "Outfall Simulation Experiment in Lake Ontario," Water Research, in press (1971).

> Erosion and Sediment Models

Problems and Scope

The water resource problems associated with erosion and sediment-related phenomena on the Great Lakes include: lakeshore erosion with reductions in property values and utility, flooding of lakeshore areas, channel dredging requirements for commercial and recreational craft, and changes in the extent and character of fish and wildife habitat. In addition, there are problems associated with channel dredging requirements and the impact of the dredged sediment on habitat, which, while significant, are generally associated with local conditions and are therefore not considered in Type II planning activities.

The major lake-wide problems are associated with shoreline erosion and local flooding. The Corps of Engineers is in the process of studying the economic impact of lake level regulation on flooding and erosion. This study utilizes a model for ultimate water level calculations which is described below.

Modeling Frameworks and State of the Art

Erosion. Prediction of wave heights is a critical factor in the evaluation of erosion and the design of facilities and protective works. A great deal of research, centered in Holland, has been performed in the field. The size, shape, and frequency spectrum of waves is a function of the induced wind turbulence, the duration of exposure, the length of the exposure path or fetch, and the shape of beach as it determines reflective energy additions. The term "setup" is used to identify the incremental wave height related to tilting of the lake surface from extended exposure to low atmospheric pressure associated with storms. Lake Erie, which is shallower than the other Great Lakes, is more responsive to storm surges, and has been extensively studied.

The classical calculation for storm surges on the Great Lakes has been described by Platzman [1,2,3]. These dynamic models are used by a number of agencies for safety analysis, and are accepted by the United States Atomic Energy Commission for analysis of storm surges for nuclear plant safety analysis. Keulegan [4] has also made extensive contributions to the analysis of drag forces involved in determining the coefficient of wind stress and sea roughness over Lake Erie. The United States Weather Bureau currently uses a regression method based on the work of Harris and Angelo [5] and described by Richardson and Pore [6]. The computer simulation using the statistical approach is preferred by the Weather Bureau for operational reasons which are principally related to inputs during stormm conditions.

The state of the art in the analysis and prediction of beach erosion, littoral drift, dune formation, and the development of engineered structures which influence and control these effects has been presented by the Corps of Engineers [7]. This compilation has been obtained from the worldwide experience of the Corps in the development and management of shorelines. It represents the basis for the modeling of beach conditions with respect to water and wave motion. Most of the established principles have been embraced by a majority of investigators and, where exceptions exist, the literature has ample contributions from the Dutch and other European sources which permit sophisticated modeling. A number of studies $[8,9,10,11]$ are available in the Great Lakes which contain data that can or have been used for initial model development and verification of shoreline processes.

A number of investigators, including the Beach Erosion Board of the Corps of Engineers, have confirm.l the use of Hunts' empirical relation for application to a natural beach [12]:

$$
\begin{equation*}
\mathrm{R}=2.3 \mathrm{~T} \overline{\mathrm{H}} \tan \alpha \tag{I}
\end{equation*}
$$

where:

$$
\mathrm{R}=\text { maximum wave run-up }
$$

$\mathrm{T}=$ wave period
$\mathrm{H}=$ peak to valley wave height
$\alpha=$ beach slope

The work of Battjes [13] has extended and confirmed the application of Hunts' rule to various wave forms ranging from steep sided to swell waves. In his summary of the work, distributions of run-up of breaking waves are derived by assigning to individual waves in an irregular wave train a run-up value according to Hunts' formula. Explicit expressions for the run-up are obtained for waves of which the squares of heights and periods have a bivariate Rayleigh distribution. The extremes of this distribution are limiting cases for a young sea and a fully developed sea.

The actual transport of materials in an eroding shore requires additional steps beyond the physical description of wave form, run-up, and energy input information. Methods for the estimation of beach transport have been made by a large number of investigators as well as by the Corps of Engineers who summarized the techniques [7]. A criticism by some foreign investigators of the methods used to date is that they are largely empirical and take into account neither longshore currents nor the material size and slope of the beach.

Bijker [14] has published the results of intensive investigations to determine if longshore current and bed roughness are related to the longshore transport. His work represents the state of the art in the description of longshore transport.

Despite the large amount of dynamic and statistical modeling of the details of beach and shore erosion, ultimate reliance on the statistics of shoreline regression is still required. No general approach has been found having satisfactory predictive capability for the many varying conditions involved in shoreline erosion on the Great Lakes. In general, where regular beach conditions prevail, excellent
dynamic prediction of both physical run-up and transport are possible. When irregular land formations and overhanging cliffsides are involved, there are no known techniques to predict undercutting and toppling of such features other than long term statistics. In the latter case, observations of Great Lakes shoreline contours have been under extensive investigation for a long period of time, and the inventory of beach regression information appears adequate to handle most problems during the near term.

Flooding. The term "ultimate water level" has been adopted in the Great Lakes to designate an extreme water level associated with a storm on the lakes. The ultimate water level calculation considers three phenomena. The undisturbed water level which is a result of the hydrological balance for the lake makes up one of the components of the ultimate water level. This undisturbed water level is either a measured value or it may be calculated employing one of the lake level models described previously. Winds associated with storms on the lakes cause the water surface of the lake to tilt in the direction of the wind, lowering the water level along the up-wind shore and raising the water level on the down-wind shore. The third component of the ultimate water level is the maximum vertical height of waves which reach the shore and runup the beach.

The Corps of Engineers [15] has calculated ultimate water levels for thirty-six significant reaches of the Great Lakes. The ultimate water levels for a reach allows for average conditions and actual levels may vary locally. Data from fifteen water level gaging stations and sixteen weather stations are used to determine storm water levels for corresponding wind speeds and directions. The maximum instantaneous lake level observations each month are adjusted to constant conditions of water diversion and flow. The lake surface tilt is then determined from analysis of the observed information.

Wave run-up is calculated employing Equation (2):

$$
\begin{equation*}
\mathrm{R}=2.3 \mathrm{MTH}^{-0.5} \tag{2}
\end{equation*}
$$

where:
$\mathrm{R}=$ wave run-up
$\mathrm{M}=$ beach slope
$\mathrm{H}=$ wave height

The wave periods and wave heights are obtained employing hourly wind data and equivalent fetch lengths in conjunction with available deep water wave curves [7]. Average wind speed and direction are computed from hourly wind data at each weather station for periods of one to twenty-four hours duration before the time each storm water level is recorded. Land station wind speeds are increased by a factor of 1.2 to adjust the wind for overlake conditions. The Corps of Engineers' calculations for wave height thus obtained are compared to the maximum height of a wave that can be sustained at the storm water level and water depth. The lesser of the wave heights is used in Equation (2) to calculate wave run-up.

The Corps of Engineers' calculation procedure (or model) discussed above is used for a comparison of alternative lake regulation systems and planning activities. There are a number of limitations associated with the approach: short period surges from squalls are not included in the analysis; and in general, local phenomena, the amplitudes of which are dependent upon local beach and shoreline configurations, are not included in the model.

The Corps of Engineers' model for calculation of ultimate water level has been used in the comparison of lake regulation plans. This model employs observed data on the Great Lakes in conjunction with empirical curves developed for marine conditions. There has been no independent verification of the model reported to date. The Corps' model deals with broad scale regional conditions. The lack of verification has been recognized in application of this model; however, the model is employed primarily to develop comparisons between alternative regulation schemes. The
absolute output from the model in terms of ultimate water levels should, as indicated by the Corps, be used with extreme caution. This is because of the lack of verification and the fact that the model does not include the effect of short period surges and local phenomena.

In spite of the limitations that are associated with the existing ultimate water level model, it represents one of the few applications of models and computational procedures to planning and problem evaluation on the Great Lakes. The lake level model and the ultimate water level model are used together in this planning process, and they are both adequate to meet present planning needs in the Great Lakes.

Evaluation of Model Status

Model Availability. The shoreline erosion process has not been modeled in detail on the Great Lakes. However, the Corps of Engineers has developed a very broad scale shoreline erosion model which enables examination of the economic impact of alternative lake level regulation plans on lake shore erosion.

Data Availability. Some data are available on shore line erosion and could provide a beginning data base for a more detailed analysis of this problem. Significant additional data collection would be required for an adequate formulation and verification of detailed erosion models.

Degree of Verification. The existing model is not verified from the standpoint of direct physical measures of erosion. However, the model does use available physical data and economic information. Because of the broad scale nature of the existing modeling effort, lack of verification does not appear to be a significant impediment.

Planning Application. The existing modeling effort appears to adequately meet present broad scale Type II planning needs.
[1] Platzman, G.W., "A Numerical Computation of the Surge of 26 June 1954 on Lake Michigan," J. Geophys. Res., Vol. 6, pp 407-438 (1958).
[2]
Platzman, G.W., "A Procedure for Operational Prediction of Wind Set-up on Lake Erie," Report No. ll, University of Chicago, ESSA Contract E-91-67.
[3] Platzman, G.W., "The Dynamic Production of Wind Tides on Lake Erie," Meteorological Monographs, Volume 4, No. 26, p 44 (1963).
[4]
Keulegan, G.H., "Hydrodynamic Effect of Gales on Lake Erie," Journal of Res., U.S. Nat. Bureau of Standards, Volume $\overline{50, \text { No. }} \overline{2}$, $\overline{\mathrm{pp}} 99-109$ (1953).
[5]
Harris, D.I., and Angelo, A., "A Regression Model for Storm Surge Productions," Monthly Weather Review, Volume 91, Nos. 10 to 12 (1963).
[6] Richardson, W.S., and Pore, N.A., "A Lake Erie Storm Surge Forecasting Technique," ESSA, Tech. Memo. WBTM, TDL 24 (1964).
${ }^{[7]}$ U.S. Army Corps of Engineers, Technical Report No. 4, Beach Erosion Board (1966).
${ }^{[8]}$ Fox, W.T., and Davis, R.A., "Profile of a Storm Wind, Waves and Erosion on the Southeastern Shore of Lake Michigan," Proc. 13th Conf. IAGLR, pp 233-241 (1970).
[9] Bajournas, L., and Duane, D.B., "Shifting Offshore Bars and Harbor Shoaling," Journal of Geophysical Res., Volume 72, No. 24 (1967).

## REFERENCES

(continued)
[10]
Berg, D., and Duane, D.B., "Effects of Particle Size Distribution on Stability of Artifically Filled Beaches, Presque Isle, Peninsula," Proc. 11th Conf. IAGLR, p 161 (1968).
[11] Coakley, J., "Natural and Artifical Sediment Tracer Experiments in Lake Ontario," Proc. 13th Conf. IAGLR, p 181 (1970).
[12] U.S. Army Corps of Engineers, "Water Levels on the Great Lakes - Report on Lake Regulation," Appendix C, pp 6-12 (1965).
[13] Battjes, J.A., "Run up Distributions of Waves Breaking on Slopes," Jour. Waterways, Harbors, and Coastal Engineering Div., ASCE (February 1971).
[14] Bijker, E.W., "Littoral Drift as Function of Waves and Currents," Proc. llth Conf. Coastal Eng., Volume 1, Part 1 and 2, Chapter $\overline{26}$, Volume II, Part 3 and 4, ASCE pp 415-432 (1968).
[15] U.S. Army Corps of Engineers, "Levels and Flows," Great Lakes Basin Framework Study, Appendix H, Draft 2, Volume 1 and 2 (January 1971).

## Chemical Models

Problems and Scope

Chemical models which are of practical use for planning needs are concerned with the reactions that may occur among the various aqueous chemical species in natural waters. Problems which are specifically chemical in nature include the effects of direct discharges of substances to the lakes, such as strong acids and bases, dissolved solids, heavy metals, and other toxic ions and gases. In addition, discharges of dredge spoils, mine tailings, and other solids can appreciably affect the lake chemistry through dissolution reactions. Interaction between the chemical and biological regimes is the second major class of chemical effects which impact planning alternatives. Examples of biological consequences are the phytoplankton populations and their nutrient supply, the bacteria populations and their oxidation reduction reactions, and the effects of toxic chemical species on these forms as well as on other flora and fauna such as fish and wildife.

Two general classes of chemical models have been developed. The first considers only the steady-state or equilibrium configuration of a chemical system and is based entirely on chemical thermodynamic principles. Given the total concentration of the components being considered (e.g., cations such as calcium and magnesium, and anions such as carbonate and sulfate) and the temperature, it is possible to calculate the steady-state concentration of the various aqueous chemical complexes and condensed species (gases and solids) which are included in the model. Calculations of this sort have been pursued for the major ion chemistry at the Great Lakes spatial scale with quite encouraging results. An example of the application of equilibrium chemical models to planning problems is a recent calculation on a lake wide scale of the probable effect of NTA discharges on Lake Ontario water chemistry, specifically, its effect caused by forming heavy metal complexes. In addition, the development of large scale computer models with the capacity to include tens of components and hundreds of possible complexes and solids is now in progress.

The second class of chemical models which have been formulated are concerned with the kinetic behavior of chemical reactions, i.e., the transient approach to steady-state. Unlike equilibrium models, there is no general workable theory for the aqueous chemical reactions which are of interest. The available models are specific for certain reactions such as the rate of hydration of carbon dioxide. The rates can also be affected by the presence of certain other components, such as the rate of oxidation of $\mathrm{Fe}(\mathrm{II})$ in the presence of silica. These models can be included in problem contexts if their importance is suspected. For example, oxidation-reduction reactions, which tend to occur at slow rates, would be included in chemical models associated with hypolimnetic dissolved oxygen calculations. However, if a class of reactions are known to attain equilibrium rapidly with respect to the time scale of the problem being considered, their detailed kinetic behavior is of little concern and the well developed equilibrium models can be used.

Thus chemical models are required for the analysis of problems that are explicitly chemical and also as submodels for inclusion in larger modeling frameworks.

Modeling Framework

The principle which underlies all chemical equilibrium models proposed to date is derived from two fundamental thermodynamic principles: the first and second laws of thermodynamics [1]. These laws state, respectively, that for a closed isolated system, energy is conserved; and that the equilibrium composition of the system is reached at maximum entropy. An equivalent statement of the second law is that all permissible processes result in an increase in the entropy of the system. For systems of variable composition (e.g., chemical systems) in which the reactions occur at constant temperature and pressure, the relevant state function which corresponds to the entropy in an isolated system is the Gibbs free energy $G=\Sigma \mu_{i} n_{i}$. At constant temperature and pressure, the criterion for equilibrium is that the Gibbs free energy is minimum at the equilibrium configuration of the system.

This condition and the equations of mass balance which insure that all the elements present are conserved are the bases for all chemical equilibrium models which have been proposed.

Gibbs Free Energy and Chemical Potential. To make the Gibbs Free Energy specific, it is necessary to obtain an expression for the chemical potential $\mu_{j}$. Under certain idealized assumptions, the chemical potential for the $i^{\text {th }}$ dissolved species is given by the equation:

$$
\begin{equation*}
\mu_{i}=\mu_{i}^{o}+R T \ln [i] \tag{1}
\end{equation*}
$$

where $R$ is the universal gas constant, $T$ is the temperature in ${ }^{\circ} \mathrm{K}$, [i] is the activity of th omponent $i$, and $\mu \mathrm{i}$ is the standard free energy of the $i^{\text {th }}$ component. For dilute solutions the activity is approximately equal to the mole fraction or the concentration of the dissolved species so that the Gibbs free energy can be given explicitly in terms of the mole fraction concentration, $n_{i}$, and the standard free energy values, $\mu_{j}^{\circ}$, i.e.:

$$
\begin{equation*}
G=\sum_{i} n_{i}\left[\mu_{i}^{o}+\operatorname{RT} \ln \left(n_{i}\right)\right] \tag{2}
\end{equation*}
$$

The mass balance equations have the general form:

$$
\begin{equation*}
\sum_{i} a_{i e} n_{i}=b_{e} ; \quad e=1, \ldots, E \tag{3}
\end{equation*}
$$

where $a_{i}$ is the number of moles of element $e$ contained in component $i$ and $b$ is the total mole fraction concentration of element $e$. Thêre are as many of these equations as their are elements in the system being considered. One additional set of equations which guarantees uniqueness and constrains the solution to be physically meaningful is:

$$
\begin{equation*}
n_{i} \geqslant 0 ; i=1, \ldots, N \tag{4}
\end{equation*}
$$

which states that all concentrations at equilibrium must be non-negative. Thus the equilibrium concentrations, $n_{i}$, are uniquely given by the conditions that $G$, given by Equation (2) be minimum subject to the conditions of Equations (3) and (4). And in order to calculate the equilibrium composition of a chemical system all that is required is the standard Gibbs free energies, $\mu$, , which, for most well understood chemical reactions, are available.

Mass Action Equations. Although the above approach is completely general, it is not the conventional formulation for the equations of chemical equilibrium. However the conventional equations, commonly referred to as the equations of Mass Action, are directly derivable from the above conditions. As an example consider the single reaction given by the chemical equation:

$$
\begin{equation*}
\underset{i}{\sum} \nu_{i} A_{i} \not{ }_{j}^{\sum} \nu_{j} A_{j} \tag{5}
\end{equation*}
$$

where $\nu$ is the stoichiometric coefficient and $A_{i}$ is the molecular formula of the $i^{\text {th }}$ chemical component. ${ }^{i}$ The condition for equilibrium is given by a minimum value of the Gibbs Free energy G. It can be shown that this results in the condition:

$$
\begin{equation*}
\sum_{i} \nu_{i} \mu_{i}=\sum_{j} \nu_{j} \mu_{j} \tag{6}
\end{equation*}
$$

An example of such an equation which corresponds to the carbon dioxide-bicarbonate equilibrium reaction is:

$$
\begin{equation*}
[\mathrm{H}]\left[\mathrm{HCO}_{3}\right] /\left[\mathrm{H}_{2} \mathrm{CO}_{3}\right]=\mathrm{K} \tag{7}
\end{equation*}
$$

where [ H$],\left[\mathrm{HCO}_{3}\right]$, and $\left[\mathrm{H}_{2} \mathrm{CO}_{3}\right]$ are respectively the molar concentrations of the reactants $\mathrm{H}^{+}$, $\mathrm{HCO}_{3}{ }^{-}$, and $\mathrm{H}_{2} \mathrm{CO}_{3}$. This result can be generalized to include many reactions, all occurring simultaneously. The resulting equilibrium equations are all of the form of Equation (6). In addition to these
equilibrium equations the elemental mass balance equations (Equation (3)) and the positivity conditions (Equation (4)) must also be satisfied.

Condensed Species - Dissolution and Precipitation. In the preceding section the equations presented apply to dissolved ions and complexes in the aqueous phase. However under certain conditions it is possible for condensed species (e.g., minerals) to form or, conversely, for these minerals to dissolve. In order to include this possibility within the framework presented, the chemifal potential for a condensed species is needed. For the $c$ ch condensed species the chemical potential is given by:

$$
\begin{equation*}
\mu_{c}=\mu_{C}^{\circ} \tag{8}
\end{equation*}
$$

This equation states that the chemical potential for the $c{ }^{\text {th }}$ species is independent of the concentration of the species as long as it is present in abundance. This is also the assumption made concerning the water in which the reactions occur. The condensed species are then included in the formula for the Gibbs free energy and in the elemental mass balances, and the formulation is as previously given.

Condensed species may also be included in the mass action equation formulation. For this case the mass action equation is given as an inequality, i.e.:

$$
\begin{equation*}
{ }_{i}{ }^{n}{ }_{i}{ }^{\nu}{ }_{i c} \leqslant K_{c} \tag{9}
\end{equation*}
$$

where, as before, $v$ ic are the stoichiometric coefficients of the condensed speciés and $K_{C}$ is the solubility product. The procedure for incorporating ${ }^{\text {chis }}$ equation into the calculation is somewhat different from the one used when the Gibbs free energy equation is employed. If the condensed species is known to exist, then Equation (9) is treated as an equality and the elemental mass associated with the mineral is incorporated into the mass conservation laws. On the other
hand, if there is a possiblity that the condensed species represented by Equation (9) may form, the calculation is done assuming it is not present and the final concentrations are compared to Equation (9). If this inequality is satisfied then the mineral does not precipitate; if the inequality is violated then precipitation will occur and the inequality becomes an equality.

Computational Methods. The computational methods which are currently available for solving chemical equilibrium problems are based on the two sets of formulations possible for the chemical equilibrium problem. The equations based on the Gibbs free energy are:

$$
\begin{align*}
G= & \sum_{i} n_{i}\left[\mu_{i}^{o}+R T \ln n_{i}\right]+\sum_{c} \mu_{C}^{o} n_{c}  \tag{10a}\\
& \sum a_{i e} n_{i}+\sum a_{c e} n_{c}=b_{e}  \tag{10b}\\
& n_{i} \geqslant 0 ; n_{c} \geqslant 0 \tag{10c}
\end{align*}
$$

where the problem is to minimize the Gibbs free energy given by Equation (10a) subject to the elemental mass balance conditions, Equation (10b), and the positivity conditions, Equation ( 10 c ). This approach to chemical equilibrium calculations was initiated by White, et.al. [2], who formulated the problem as a non-linear convex programming problem. It is interesting to note that if no dissolved species are present, and only condensed species are being considered, then the problem is a classical linear programming problem. With the addition of the dissolved species, however, the problem becomes a non-linear programming problem, specifically a convex programming problem, for which algorithms have been and are being developed. Perhaps the most readily available implementation of these algorithms is described in the series of memoranda issued by the Rand Corporation [10].

Methods of computation based on the mass action equation have a longer history. In fact, the first proposed numerical methods dealt with the equations in this form. The problem, stated mathematically, is to solve the following equations:

$$
\begin{gather*}
\sum_{i} v_{i j} \ln n_{i}=k_{j}  \tag{lla}\\
\sum_{i} v_{i c} \ln n_{i} \leqslant k_{j}  \tag{llb}\\
\sum \sum_{i} a_{e i} n_{i}+\sum_{c} a_{e c} n_{c}=b_{c}  \tag{llc}\\
n_{i}>0, n_{c} \geqslant 0 \tag{lld}
\end{gather*}
$$

These simultaneous non-linear equations are difficult to solve analytically except for simple situations in which the problem reduces, essentially, to finding the roots of an algebraic equation. The inclusion of condensed species represented by Equation (1lb) adds an additional difficulty to the formulation since this equation is an inequality. However, methods are being developed which can satisfactorily cope with this added complexity.

An excellent summary of the currently available chemical equilibrium computation with the emphasis primarily on gaseous systems is available [3], and the literature relating to the application of these techniques to aqueous systems is currently growing. Thus the computational aspects of large scale chemical equilibrium models are well in hand, although they do require rather extensive numerical calculations that can only be implemented on a computing machine.

Restrictions. There are some inherent difficulties with equilibrium thermodynamic models as they are classically formulated when directly applying these models to natural water systems such as the Great Lakes. As expressed by stumm and Morgan [4] they include the facts that:
A. Natural systems are continuous flowing systems. That is, they exchange mass with their surroundings and, therefore, are not necessarily at thermodynamic equilibrium, although they may be at a temporal steady-state.
B. Pertinent chemical equilibria may have been ignored or important solid or aqueous species left out of the formulation.
C. The thermodynamic data upon which calculation is based may be incorrect. Temperature and activity corrections may be necessary.
D. There may be inadequate chemical characterization of the species involved. For example, dissolved versus suspended components may not be properly measured.
E. Although certain reactions are thermodynamically possible, they may in fact occur at very slow rates so that if an equilibrium is calculated based only on thermodynamic considerations, the resulting equilibrium configuration may represent a configuration which will occur only in the distant future and may not be representative of the time scale within which the problem is being formulated. An example of such a slow reaction is the dissolution or precipitation of quartz.

State of the Art

Mineral Dissolution Models. The use of thermodynamic equilibrium models which are based on the assumption that lake waters are in thermodynamic equilibrium with certain minerals has been pursued and applied to the Great Lakes in a series of important papers by Kramer [5,6]. The models all share a common basis in addition to their being all equilibrium models. A group of minerals are assumed to exist and to be in chemical equilibrium with the overlying lake waters. The partial pressure carbon dioxide, temperature, and perhaps ionic strength (which is directly correlated to total dissolved solids) are specified. Then, depending upon the minerals chosen, the resulting equilibrium concentration of the output variables are calculated. The number and type of minerals included within the model determine the chemical ions which are included in the calculation.

Perhaps the most straightforward of these models is the calcite model applied by Kramer [5] to the Great Lakes. The assumptions are that the lake waters are saturated with respect to carbon dioxide and calcite. In addition, the TDS concentration is used to calculate the ionic strength of the medium. The results of the calculations are the pH of the overlying water, the calcium concentration, and the alkalinity. For $5^{\circ} \mathrm{C}$ and atmospheric saturation of carbon dioxide, the results are $\mathrm{pH}=8.38, \mathrm{Ca}=33 \mathrm{ppm}$, and alkalinity $\left(\mathrm{CaCO}_{3}\right)=81 \mathrm{ppm}$. These results compare reasonably well to lake wide average pH , calcium, and alkalinity concentrations in the carbonate Great Lakes (Michigan, Erie, and Ontario).

The more complex mineral dissolution models proposed by Kramer and his students all follow the pattern of the calcite model. The major structural features of these models are outlined in Table 14. The minerals considered, the assumptions made, the input constants, the input variables, and the outputs are listed. In general, the variables considered cover the major cations and anions in the lake water, as well as the pH , phosphate, and fluoride concentrations. The input constants are usually the carbon dioxide concentration, the temperature, the chloride + sulphate concentrations, and the total phosphate concentration.


A notable feature of these models is that the calculations which result are compared to observed measurement in the Great Lakes as shown in Figure 20. (See also Figures 5, 6, and 7 , reference [6]; Table 3, reference [5], Figure 5, reference [7]). Although the models are not verified in every detail and they include measured observations as part of their input constants, they do represent the major features of the aquatic chemistry of the variables considered. In addition, since the models are based on well understood theoretical foundations, they can be incorporated into larger modeling frameworks. In particular they can be joined to biological models. The obvious importance of the carbon dioxide concentration in establishing the equilibria that are observed and the possibility of either phosphate dissolution or precipitation due to the apatite minerals provide an important biological aspect of these models.

Calculations of similar types have been explored and applied to oceanic settings by a variety of investigators, among them Sillen, Garrels and Thompson, Kramer, and others [4] in an attempt to construct model oceans.

Aqueous Chemical Equilibria Models. A second class of chemical equilibrium models which have been presented by the various authors, and for which an application to a Great Lakes problem has been made, is a calculation presented by Childs [8] to assess the effect of discharging NTA, a proposed substitute for phosphate in detergents, on Great Lakes water chemistry. These models consider only the aqueous ions and complexes and generally do not consider the possibility of mineral precipitation and dissolution. The inputs to such a model are the total concentrations of the various metal ions and ligands which make up the chemical species being considered in the model. For the calculation presented by Childs the metals and ligands considered are presented below:

| Metals (M) |  |  |
| :--- | :--- | :--- |
| $\mathrm{Ca}_{2}{ }^{+}$ | $\mathrm{Ca}_{2}{ }^{+}$ | $\mathrm{Cu}_{2}{ }^{+}$ |
| $\mathrm{CO}_{2}{ }^{+}$ | $\mathrm{Fe}_{2}{ }^{+}$ | $\mathrm{Fe}_{3}{ }^{+}$ |
| $\mathrm{Pb}_{2}{ }^{+}$ | $\mathrm{Ni}_{2}{ }^{+}$ | $\mathrm{Zn}_{2}{ }^{+}$ |
| $\mathrm{Mn}_{2}{ }^{+}$ | $\mathrm{Ba}_{2}{ }^{+}$ | $\mathrm{Mg}_{2}{ }^{+}$ |
| $\mathrm{Na}^{+}$ | $\mathrm{Sr}_{2}{ }^{+}$ | $\mathrm{Hg}_{2}{ }^{+}$ |

Ligands (L)
$\mathrm{NTA}_{3}{ }^{-}$
$\mathrm{PO}^{3^{3}}$
$\mathrm{CO}^{\mathrm{m}^{3}}{ }^{-}$
$\mathrm{SO}^{4}{ }^{-}$
$\mathrm{Cl}^{4^{3}}$

LE GEND
--- DATA BOUNDARY

- CALCULATED SOLUBILITY FOR INDICATED CO
CONCENTRATION
(a) CALCITE

(b) DOLOMIT

(c) OH-APATITE


FIGURE 20
COMPARISON OF OBSERVED VS CALCULATED SOLUBILITY PRODUCTS (a) CALCITE (b) DOLOMITE (c)OH-APATITE

Three general classes of reactions are considered: ligand protonations which have the form:

$$
\begin{equation*}
\alpha(L)+\beta(H)=\left(L_{\alpha} H_{\beta}\right) \tag{12}
\end{equation*}
$$

where $L$ is the ligand in question and $H$ represents the proton and $\alpha$ and $\beta$ are the stoichiometric coefficients; metal hydrolysis reactions which have the form:

$$
\begin{equation*}
\alpha(\mathrm{M})+\beta(\mathrm{OH})=\left(\mathrm{M}_{\alpha} \mathrm{OH}_{\beta}\right) \tag{13}
\end{equation*}
$$

and metal ligand complexations which are either of the form:

$$
\begin{equation*}
\alpha(M)+\beta(L)+\gamma(H)=\left(M_{\alpha} L_{\beta} H_{\gamma}\right) \tag{14a}
\end{equation*}
$$

or of the form:

$$
\begin{equation*}
\alpha(\mathrm{M})=\beta(\mathrm{L})+\gamma(\mathrm{OH})=\left(\mathrm{M}_{\alpha} \mathrm{L}_{\beta} \mathrm{OH}_{\gamma}\right) \tag{14b}
\end{equation*}
$$

Childs considers 8 protonation reactions, 20 hydrolysis reactions, and 71 complexation reactions for a total of 99 chemical reactions. The equilibrium constants for these reactions are obtained mainly from the published literature. The calculations are performed using a modification of the algorithm proposed by Perrin [9]. Two kinds of calculations are presented by Childs. The first involves the speciation to be expected in normal lake water containing no NTA. The results indicate that, for no NTA addition, over a range from pH 6 to pH 9 , the metals included in the model appear primarily as free ions with the exception of copper (II), iron (III), lead (II), and iron (II), which appear as carbonates or hydroxides. Only a small amount of phosphate and carbonate and no chloride ion is complexed. The second calculation involves the addition of NTA to normal lake water.

Childs calculates that for a 1 micromole/liter concentration of NTA and pH 8 , 50 percent of the copper ion is complexed and 20 percent of the lead ion is complexed with the NTA ion. This is a marked change from the speciation in normal lake water where the copper is primarily copper carbonate and the lead is a free ion, lead hydroxide, and lead carbonate. In addition, all of the added NTA is complexed with one or another of the metals and no free NTA concentration is calculated. This change in the heavy metal speciation may have important biological effects. Another possible avenue that such a calculation points to is the possibility that NTA in aqueous systems may cause liberation of heavy metals which are bound in the sediments. If, as is indicated in the calculations, the complexing ability of NTA is sufficiently active, then it is possible that heavy metals, which exist as stable minerals or are bound in the organic sediment, would be resoluabilized and would enter the aqueous phase as NTA complexes.

Large Aqueous - Solids Equilibrium Calculations. There has occurred recently the development of efficient computational algorithms for large chemical equilibrium calculations for both solid and aqueous species. The older and more limited program has been under development by the Rand Corporation for some time [10]. Although it does not appear to have been applied to problems in a natural waters setting as yet, the capabilities of the program are adequate for at least initial applications. A recent described chemical equilibrium program by Morel and Morgan [11] has the largest capacity of any program available to date. Their published example calculations include 20 metals, 31 ligands, 730 complexes, and 64 possible solids, almost an order of magnitude larger than those preceding it. Thus the computational aspects of equilibrium chemical models are well in hand.

Nonequilibrium Chemical Models. A number of the restrictions discussed above are directly related to the principle assumptions underlying chemical equilibria models, namely that of thermodynamic equilibrium. For many chemical reactions this is an excellent assumption. However for some important reactions, especially those mediated by bacterial action, the rate at which the reaction occurs can be important depending on the time scale of the problem being considered. Unfortunately there is no general,
workable theory from which the reaction kinetcs for anything but the simplest reaction can be calculated. For reactions of interest in natural water systems, it is necessary to use empirical reaction kinetics formulations based on laboratory data. For example the oxidation of $F e(I I)$ has been found to follow the kinetic law [12]:

$$
\begin{equation*}
\frac{\mathrm{d}\left[\mathrm{Fe}^{2+}\right]}{\mathrm{dt}}=-\mathrm{k}\left[\mathrm{Fe}^{2+}\right]\left[\mathrm{O}_{2}\right][\mathrm{OH}-]^{2} \tag{15}
\end{equation*}
$$

Other oxidation-reduction reactions have been studied and some kinetic results are available [4]. In particular, nitrification reactions can be adequately described in some cases by simple linear kinetic models [13].

Processes at the Air-lake and Lake-bottom Interfaces. The
chemical processes and reactions that occur in the Great Lakes are influenced by transport processes which exchange dissolved gases between the atmosphere and the lake water and between the lake water and the lake bottom or benthos. In addition, processes and reactions occuring in the sediments themselves can have an impact on lake water chemistry. Of the two interfaces, the air-lake interface is better understood and reasonable models are available to describe the transport phenomena [14]. Although these models have been developed and applied primarily to rivers and estuaries, the problems involved in these applications to a lake setting are understood and suitable approximations can be proposed.

This situation with respect to the sediment-water interface is less well understood. A quantity of qualitative and quantitative information is available [15] and speculations concerning mechanisms which control transport and reaction are available; however, the modeling effort is in its infancy.

Evaluation of Model Status

Model Availability. A number of chemical equilibria models are available for both the major ion chemistry and some of the heavy metals. The framework is well understood, since it is the basis of aquatic chemistry; and the computational tools are available for application to Great Lakes problem settings. For those applications which require non-equilibrium calculations, the availability depends on the particular reactions of concern. Computational difficulties can occur if the reaction rates are fast with respect to the other rates in the problems. Airwater interface transfer of gases is reasonably well understood in streams and estuaries although no applications to lakes have been uncovered. Models of sediment-water interface phenomena are not as yet available.

Data Availability. Open water data for verification of chemical equilibrium models of the major ions is in plentiful supply. Some of the earliest data available for the Great Lakes are measurements of the concentrations of the major ionic constituents. Heavy metals data are only currently becoming available due in part to the growing concern regarding their introduction and accumulation in the Great Lakes. Sediment data and experimental results are available, although they appear to be scattered and sketchy. One problem is that the relevant variables and modeling framework are not available to assess the utility of the data that are available.

Model Verification. Only the major ion-mineral dissolution models have been verified and these to order-of-magnitude in some cases and to lake wide averages in other. However, the verifications are quite encouraging. Models of heavy metal complexes and models of biologically active species such as phosphate are not verified per se, although in the latter case, they provide some basis for comparison with observed data.

Model Application. Only the simplest chemical model, that for which there is no chemical reactions considered (the chloride demonstration submodel), has been used for planning applications. No other chemical models have been used in this way.
${ }^{[1]}$ Lewis, G.N., Randall, M., Thermodynamics, 2nd Edition, Revised, Pitzer, K.S., Brewer, L., McGraw Hill, New York (1961).
${ }^{[2]}$ White, W.B., Johnson, S.M., Dantzig, G.B., "Chemical Equilibrium in Complex Mixtures," J. Chem. Phys., 28, pp 75175 (1958).
${ }^{[3]}$ Van Zeggeren, F., Storey, S.H., The Computation of Chemical Equilibria, Cambridge University Press (1970).
[4]Stumm, W., Morgan, J.J., Aquatic Chemistry, Wiley-Interscience, New York (1970).
[5] Kramer, J.R., "Theoretical Model for the Chemical Composition of Fresh Water with Application to the Great Lakes," Pub. No. 1l, Great Lakes Research Division, pl47 (1964).
[6]
Kramer, J.R., "Equilibrium Models and Composition of the Great Lakes in Equilibrium Concepts in Natural Water Systems," Advances in Chemistry, Series No. 67 Am . Chem. Soc., Washington D.C. (1967).
${ }^{[7]}$ Sutherland, J.C., Kramer, J.R., Nichols, L., Kurtz, T.D., "Mineral-Water Equilibria, Great Lakes: Silica and Phosphorus," Pub. No. 15, Great Lakes Research Division, p 439 (1966).
[8]
Childs, C.W., "Chemical Equilibrium Models for Lake Water which contains Nitrilo triacetate and for Normal Lake Waters," Proc. 14th Conf. IAGLR (1971).
[9] Perin, D.D., Sayce, J.S., "Computer Calculation of Equilibrium Concentration in Mixtures of Metal-ions and Complexing Species," Talanta, 14, p 833 (1967).

## REFERENCES

(continued)
[10] Shapley, M., Cutler, L., "Rand's Chemical Composition Program: A Manual," R-495-PR, Rand Corporation, Santa Monica, California (June. 1970).
[11] Morel, F., and Morgan, J.J., "A Numerical Method for Computing Equilibria in Aqueous Chemical Systems," Environmental Science and Technology, 6 (1) p 58 (January 1972).
[12] Stumm, W., Lee, G.F., "Oxygenation of Ferrous Iron," Ind. Engr. Chem. Volume 53, p 143 (1961).
[13] rhomann, R.V., O'Connor, D.J., DiToro, D.M., "The Effects of Nitrification on the Dissolved Oxygen in Streams and Estuaries," Technical Report, Environmental Engineering and Science Program, Manhattan College, New York (1971).
[14] O'Connor, D.J., Dobbins, W.E., "Mechanism of Reaeration in Natural Streams," Journal Sanitary Engineering Division, ASCE, Process paper 1115 (December 1956).
[15] Mortimer, C.H., "Chemical Exchanges between Sediments and Water in the Great Lakes - Speculations on Probable Regulatory Mechanisms," Limnology and Oceanography, 16(2), p 387 (March 1971).

## Eutrophication Models

Problems and Scope

Problems that are classed as eutrophication problems have in common an excessive growth of either microscopic or macroscopic species of aquatic plant life. Accompanying this excessive growth is usually a predominance of species which are in some way less desirable than those which had predominanted before eutrophication had occurred. The usual chain of events starts with an increasing quantity of inorganic nutrients being discharged to the body of water às a consequence of increasing population and industrial growth. The increased available nutrients, in conjunction with other factors, enhance the growth of aquatic plant life. The changes which make up the eutrophication problem spring from this enhanced growth.

In particular, the microscopic plant life of a lake, the phytoplankton, can reach population densities which are in themselves detrimental to water use. Such changes can occur either lake-wide or basin-wide, such as in the western basin of Lake Erie or in smaller regions, such as harbors or embayments. If conditions are suitable, macroscopic aquatic plants, primarily of the rooted variety, can proliferate along the shoreline. Excessive cladophora growth in Lake Erie and Ontario is an example. These phenomena tend to occur on a seasonal time scale. With increasing fertilization, phenomena characterized by shorter time scales such as patch blooms are also in evidence. Surface scums of phytoplankton and their resultant accumulations on shorelines as well as the formation of rooted aquatic windrows are a serious consequence of severe eutrophication.

In addition to the general increase in population densities as measured by the biomass of phytoplankton, observed changes in species predominance can also occur. Less desirable forms of phytoplankton, particularly blue-green algae, can also be a consequence of overfertilization. These forms can cause taste and odor problems in water supplies and their decomposition on shorelines gives rise to noxious odors.

In addition to directly phytoplankton-related problems, interactions occur between the increased biomass and the other water quality variables. In particular, the phytoplankton cause a dissolved oxygen variation and affect the major nutrients required for phytoplankton growth (inorganic carbon, nitrogen, and phosphorus). The utilization of dissolved carbon dioxide can have important consequences on the carbonate balance, pH , and the attendant chemical systems.

The excessive phytoplankton which are produced eventually settle to the bottom of the lakes and can seriously interfere with benthic organisms. Also, the decaying phytoplankton exert a demand on the dissolved oxygen resources and release organic forms of carbon, nitrogen, and phosphorus. The water transparency suffers at high phytoplankton biomass concentrations, and this condition hampers those predator which depend upon visual identification of their prey.

The increased phytoplankton and possible species changes can influence the overall production of zooplankton and their species distribution. This can have an effect upon the higher order carnivores such as fry or adult fish which rely either upon phytoplankton or zooplankton as their primary food sources. The competitive structure of fish predator-prey relationships can also be altered.

Accordingly, large scale changes in the population structure of the phytoplankton, which are the primary producers of organic material in lakes, can be expected to have widerange effects. Therefore, the quantitative description of phytoplankton and zooplankton population distribution is of primary importance in the successful formulation of a limnological systems analysis. Without such a quantitative description it is unlikely that successful analyses and predictions of the probable effects of remedial actions directed at water quality modifications can be successfully accomplished.

The basis of such a modeling effort is available within the scientific discipline of limnology which is concerned with the cause and effect interactions that govern the behavior
of aquatic species in lakes. An impressive amount of qualitative information has been assembled regarding the various phytoplankton species, zooplankton species, and their interactions. However, only recently has substantial quantitative information become available. In the absence of detailed quantitative descriptions of phytoplankton behavior, the problem of eutrophication has been addressed in several empirical fashions. Perhaps the first attempt to set quantitative relationships between the quantity of nutrients, which must be present in order that eutrophication problems do not occur, was attempted by Sawyer [1]. More detailed empirical relationships have been suggested since then. However, the predictive value of empirical correlation is open to question, and thus more detailed cause and effect structures have been attempted. These formulations will be discussed subsequently.

The formulations which have proven most successful to date deal with the phytoplankton and zooplankton populations characteristized solely in terms of the biomass concentrations. Although a large amount of qualitative and some quantitative information is available with regard to individual species or genera of both phytoplankton and zooplankton, this information has not as yet been utilized in the construction of model structures. Thus the problems which are related to subtle shifts in species compositions of the population cannot as yet be addressed. However, the success of biomass calculations indicate that successful models can be developed to this level of detail. It is likely that similar models can be successfully developed for rooted aquatics and perhaps benthic algal species. The rooted aquatic plants are a significant problem in Great Lake eutrophication and some projected work for development of such models has been scheduled during the International Field Year on the Great Lakes (IFYGL).

Modeling Frameworks

Empirical Relationships. As is common in the development of most engineering-scientific disciplines, the initial attempts
to understand phenomena are based on observed correlations between controllable variables and the variables which are associated with the problem of concern. An initial attempt to formulate an empirical relationship between the inorganic nitrogen and inorganic phosphorus concentrations, which were required for eutrophication problems not to occur, was formulated by Sawyer [1] based on his experiences in the Wisconsin lakes.

Although it is apparent with hindsight that this formulation is a vast oversimplification, considering all the factors which can contribute to excessive phytoplankton biomass growth, it has the advantages of being a simple description. In particular, Sawyer suggests that if, before the spring, development of phytoplankton concentrations of inorganic nitrogen and phosphorus is below $0.30 \mathrm{mg} / 1 \mathrm{nitrogen}$ and 0.015 $\mathrm{mg} / \mathrm{l}$ phosphorus, then it is probable that eutrophication will not occur during the spring, summer, and fall months.

A more sophisticated version of such correlation has been developed by Vollenweider [2]. He suggests that the primary important variables are the areal discharge rate of phosphorus and the average depth of the lake. Shallow lakes with excessive nutrient inputs tend to be eutrophic. His contention is supported by a series of observed conditions in many lakes. As with all empirical correlations, the primary difficulty with such a formulation is the uncertainty as to the underlying cause and effect relationships and the range over which prediction is possible, based on such a correlation.

Attempts to structure phytoplankton models based on more fundamental relationships have been attempted. The structures have become increasingly more complex but their construction is based on generally accepted ideas concerning the growth and death of phytoplankton and zooplankton populations and the effect of environmental parameters on these processes.

Principles of Kinetically Structured Models. The point of view adopted in the construction of the models discussed below has been frequently advocated as a basis for the development of a mathematical biology. As expressed by

Lotka [3] in his Elements of Mathematical Biology, the method of approach is as follows:

It now behooves us to establish, with respect to the problem of evolution Iin the sense of the time history of a biological system, either over short or long time spans], a viewpoint, a perspective, a method of approach, which has hitherto received its principal development and application outside the boundaries of biological science.

This perspective is that which contemplates an evolving system as an aggregation of numbered or measured components of several specified kinds, and which observes and enregisters the history of that system as a record of progressive changes taking place in the distribution, among those components, of the material of which the system is built up.

It is thus that physical chemistry views the progressive changes in a system comprising several chemical species, that is to say elements, compounds phases, etc. It describes the system by enumerating these components, by stating their character and extent (mass); and by further indicating the values of certain quantities or parameters, such as volume or pressure, temperature, etc., which, together with the masses of the components, are found experimentally to be both necessary and sufficient, for the purposes in view, to define the state of the system. With the instantaneous state of the system thus defined, physical chemistry investigates by observation and by deductive reasoning (theory) the history, the evolution of the system, and gives analytical expression to that history, by establishing relations, or equations, between the variations defining these states (after the manner set forth above), and the time.

It is commonly found that these fundamental equations assume the simplest, the most perspicuous form, when they are written relative to rates of
change of the state of the systems, rather than relative to this state itself... . In the language of the calculus, the differential equations display a certain simplicity in form, and are therefore, in the handling of the theory at least, taken as the starting point, from which the equations relating to the progressive states themselves, as functions of time, are then derived by integration.

With the outlook gained in our preceding reflections, we envisage the life-bearing systems, in the progress of evolution, as an assembly of a number of components: biological species; collections or aggregations of certain inorganic materials such as water, oxygen, carbon dioxide, nitrogen, free and in various combinations, phosphorus, sulfur, etc.

These components are placed in various relations of mutual interaction under specific conditions of area, topography, climate, etc. Under these conditions each may grow, decay, or maintain equilibrium. In general the rate of growth, $d x / d t$, of any one of these components will depend upon, will be a function of, the abundance in which it and each of the others is presented; this rate of growth will also be a function of the topography, climate, etc. If these latter features are defined in terms of a set of parameters $P_{1} P_{2} \ldots P_{j}$, we may write:

$$
\begin{gathered}
\frac{d X_{1}}{d t}=F_{1}\left(X_{1}, X_{2}, \ldots X_{n} ; P_{1}, P_{2} \ldots P_{j}\right) \\
\frac{d X_{2}}{d t}=F_{2}\left(X_{1}, X_{2}, \ldots X_{n} ; P_{1}, P_{2} \ldots P_{j}\right) \\
\vdots \\
\frac{d X_{n}}{d t}=F_{n}\left(X_{1}, X_{2}, \ldots X_{n} ; P_{1}, P_{2} \ldots P_{j}\right)
\end{gathered}
$$

Thus Lotka envisioned the formulation of a mathematical theory of biological growth and change (what he called evolution) in terms of sets of differential equations specifying the rates of change of the dependent variables $X_{i}$ as functions of the dependent variables $X_{i}$, and the environmental parameters, $P_{1}, \ldots P_{j}$.
These equations, which are most commonly associated with chemical kinetics, form the basis of the kinetic interaction among the dependent species being considered. In addition to the interactions among the species, the second principle on which phytoplankton biomass models are based is the principle of conservation of mass. This principle simply states that the mass of each species being considered must be accounted for in one way or another. For the models being considered herein, the primary mechanisms associated with this principle are the transport mechanisms which serve to advect and disperse the various components being considered. The function of the hydrodynamic and transport models is to provide the quantitative description of these transport phenomena. The primary emphasis of this discussion will, therefore, center on the kinetic interactions which appear to be the primary controling mechanisms for the development of phytoplankton biomass.

Phytoplankton Growth Kinetics. The behavior of a natural assemblage of phytoplankton is a complicated function of the species of phytoplankton present and their differing reactions to the environmental variables which affect their development. For simplified models, which consider only the biomass of the phytoplankton, the primary variables which have been investigated are temperature, solar radiation intensity, and nutrient concentrations. The complex data pertinent to this problem have been reviewed by Hutchinson [4], Strickland [5], Lund [6], and Raymont [7]. A review of pertinent Great Lakes literature is contained in the Framework Study [8]. The detailed references for the kinetic behavior as described below is available [9].

The influence of temperature on the growth rate of phytoplankton has been experimentally investigated by a number of researchers. It has been found that at optimum
conditions of light and nutrients the growth rate varies approximately linearly with respect to temperature and indicates an approximate doubling of the growth rate for an approximate doubling of the temperature over the temperature range from $10^{\circ}$ to $20^{\circ} \mathrm{C}$. Observed growth rates at $20^{\circ} \mathrm{C}$ lie in the range from 1.5 to 2.5 per day (base e).

The growth rates observed under optimal conditions are not directly applicable to the natural setting in a lake where the light intensity and nutrient concentrations are not optimal. The effect of varying light intensity on the growth rate of the population is that at low light intensities, the growth rate increases approximately linearly with respect to incident light intensity and as the intensity increases the rate reaches the maximum and then decreases as higher intensities are encountered. This effect, coupled with the decrease of light intensity as a function of depth in natural waters (as measured by the extinction coefficient), reduces growth rate of the natural population.

Similar effects have been demonstrated with respect to the concentration of the macronutrients required for phytoplankton growth (carbon, nitrogen, phosphorus, silica for diatoms). At a low concentration of the specific nutrient, the growth rate appears to be linearly proportional to the nutrient concentration available; and as the nutrient concentration is increased, the growth rate eventually reaches the value specified by the available light intensity and temperature.

These effects are graphically presented in Figure 21. Figure 2lA represents the growth rate as a function of temperature at optimal conditions. Figures 21B and 21C indicate the effect of incident light intensity as a function of depth. Thus, at a particular depth a specific light intensity occurs which results in the growth rate to be expected at that temperature. Figure 21d, a graph of the Michaelis Menton function, shows a hypothesized functional form for the behavior of the growth rate as a function of a required nutrient concentration. It has been further hypothesized [9] that these effects are multiplicative and that the resulting growth rate is a function of the product of the reduction due to nutrient limitations and non-optimal light intensities.

## PHYTOPLANKTON <br> GROWTH RATE:

## A. TEMPERATURE EFFECT

B. SOLAR RADIATION EFFECT
C. EXTINCTION EFFECT

Errect

D. NUTRIENT EFFECT


Phytoplankton Death Rate. In addition to the growth of phytoplankton, an additional series of phenonema causes a loss of phytoplankton population biomass. Endogenous respiration, the rate at which phytoplankton lose biomass because of their metabolic energy requirements, is a primary mechanism. Experimental information has shown that the respiration rate is also approximately linear with respect to temperature, again exhibiting a doubling in rate for a twofold increase in temperature. Reported respiration rates at $20^{\circ} \mathrm{C}$ are on the order of 0.1 per day (base e). Thus at optimal conditions the growth rate of a phytoplankton population is on the order of 10 to 20 times its respiration rate.

Phytoplankton are the primary producers of organic material in aquatic systems and form the base of the food chain for all aquatic animals. Thus an important contribution to the rate at which phytoplankton population biomass is removed is the rate of predation by the microscopic animals in the next level of the food chain, the zooplankton. Experiments have indicated that the rate of phytoplankton removal by zooplankton grazing is approximately proportional to the zooplankton concentration and increases as a function of temperature. Figure 22 presents a graphical illustration of these relationships.

Zooplankton Growth and Death Rates. Since the zooplankton form an essential part of the mechanism by which phytoplankton populations are influenced, it is important that the kinetics of zooplankton growth and death be formulated.

Zooplankton growth occurs because the zooplankton graze on phytoplankton and, in some cases, on smaller zooplankton forms and particulate organic material that may be present. For that portion of the zooplankton (herbivorous) which grazes primarily on phytoplankton, the rate at which the zooplankton grow is directly related to the concentration of their primary food, the phytoplankton. A graphical presentation of the relationships that have been observed is presented in Figure 22. The growth rate of zooplankton is linearly proportional to the phytoplankton population present at low phytoplankton population, but becomes independent of phytoplankton concentration at large concentrations. This effect is similar to the phytoplankton

## PHYTOPLANKTON <br> DEATH RATE:

A. RESPIRATION TEMPERATURE EFFECT

B. ZOOPLANKTON GRAZING EFFECT

ZOOPLANKTON GROWTH \& DEATHRATES

C. ASSIMILATION EFFECT

D. RESPIRATION TEMPERATURE EFFECT


FIGURE 22
PHYTOPLANKTON DEATH RATE,ZOOPLANKTON GROWTH AND DEATH RATES, INTERACTIONS
nutrient relationships given in Figure 22D. In addition, a temperature effect has been observed, which increases the grazing rate as a function of temperature.

Zooplankton death rates are functions of the population's respiration which appears to increase as a function of temperature and predation on the zooplankton by higher forms. The mechanisms which control the zooplankton population are less well understood than the phytoplankton population so that the functional forms of the mechanisms and, indeed, the proper biomass variables to be utilized for trophic levels above the herbivorous zooplankton are as yet unclear. However, the major outlines of the kinetic interactions of phytoplankton and zooplankton with respect to the environmental variables temperature, solar radiation, and the important nutrients are reasonably well understood.

State of the Art

Seasonal Phytoplankton Model. The first phytoplankton biomass model which incorporates major features that influence phytoplankton population kinetics was proposed by Riley [l0]. This model followed the prescription given by Lotka in establishing the differential equation which relates the growth of phytoplankton biomass, $P$, to the mechanism of growth (phytosynthesis), $P_{h}$; respiration, $R$; and zooplankton grazing, G. Riley proposed the equation:

$$
\begin{equation*}
\frac{d P}{d t}=\left(P_{h}-R-G\right) P \tag{I}
\end{equation*}
$$

where:

$$
\begin{align*}
P_{h} & =\frac{p I_{o}}{K_{e} Z_{e}}\left(1-e^{\left.-K_{e} Z_{e N M}\right)}\right.  \tag{2}\\
R & =r_{1} e^{r_{2} t}  \tag{3}\\
G & =g H \tag{4}
\end{align*}
$$

The inputs to the model are temperature, $T$, solar radiation, $I_{\text {; }}$; the extinction coefficient of the water body, $\mathrm{K}_{\mathrm{e}}$; depth of the euphoric zone, $z$; nutrient concentration (in this case phosphate), $N$; and zooplankton biomass concentration, $H$. The parameters which govern the biological rates in Equations (2), (3), and (4), are the growth rates, p; the death rate and its temperature coefficient $r_{1, r_{2}}$; the grazing coefficient, G; and a nutrient reduction constant, M. For a given set of inputs, the parameters are chosen either from experimental information or to fit the observed data, and the resulting phytoplankton biomass concentration is compared against observed data. Two verifications [11,12] are shown in Figure 23, Part A and B. The resulting agreement, considering the simplified nature of the model, indicates that the major environmental relationships appear to be correctly formulated.

Seasonal Zooplankton Model. Following the structure of the seasonal phytoplankton model discussed above, Riley [13] presented a seasonal zooplankton model. The mechanisms included in the kinetic structure of the model are the zooplankton assimilation and grazing of available phytoplankton, A; respiration, R; carnivore predation, C; and natural death, D. The equation which he developed is:

$$
\begin{equation*}
\frac{d H}{d t}=(A-R-C-D) H \tag{5}
\end{equation*}
$$

where:

$$
\begin{align*}
& A=g P \leqslant A_{\max }  \tag{6}\\
& \mathrm{R}=\mathrm{r}_{1}^{\prime} \mathrm{e}^{r_{2}^{\prime} T}  \tag{7}\\
& \mathrm{C}=\mathrm{CS} \tag{8}
\end{align*}
$$

The inputs to the model are the observed temperature, $T$; the observed phytoplankton concentration, $P$; and the carnivore biomass concentration, $S$. The parameters that specify the behavior of the population are the grazing coefficient, $G$;


C. GEORGES BANK


FIGURE 23
VERIFICATIONS OF RILEY'S PHYTOPLANKTON AND ZOOPLANKTON MODELS
the maximum growth rate, $A_{\text {max }}$; the respiration coefficients, ri, rl; the carnivore grazing rate, $c$; and the rate of natural death, D. The resulting model was applied to the observed zooplankton population for Georges Bank and the result is shown in Figure 23, Part C. Again, the reasonable agreement between observed populations and predicted results indicates substantial agreement.

Steady-state Vertical Distribution of Phytoplankton and Nutrients. In an important contribution to the modeling of phytoplankton population and their interactions, Riley, Stommel, and Bumpus [14] presented a phytoplankton model which includes the transport mechanisms that characterize the vertical transport in offshore oceanic waters. In addition, the phytoplankton and nutrient equations are coupled so that their solutions are interdependent. Thus the reliance on observed nutrient concentration is dropped from the modeling framework and instead a more advanced attempt to model both phytoplankton and nutrient concentration as a function of depth is attempted. The equations for phytoplankton and nutrient concentrations which were used are:

$$
\begin{align*}
& 0=\frac{\partial P}{\partial t}=\frac{\partial}{\partial z}\left(E \frac{\partial P}{\partial z}\right)-V \frac{\partial P}{\partial z}+\left(P_{h}-R_{p}-G H\right) P  \tag{9}\\
& 0=\frac{\partial N}{\partial t}=\frac{\partial}{\partial z}\left(E \frac{\partial P}{\partial z}\right)-a_{n p}\left(P_{h}-R_{p}\right)+a_{n h} R_{h} \tag{10}
\end{align*}
$$

In addition, a vertically integrated zooplankton equation is included:

$$
\begin{equation*}
0=\frac{d H}{d t}=\left(\overline{\mathrm{GP}}-\overline{R_{h}}-c \bar{S}\right) \tag{11}
\end{equation*}
$$

The kinetic structure of the phytoplankton and zooplankton equations are similar to those previously used. The nutrient equation includes the effect of nutrient uptake by phytoplankton, $-a_{n p} P_{h}$; and the nutrient excretion by plankton, $a_{n p} R_{p}$, and by zooplankton $a_{n h} R_{h}$. These equations represent the first interacting model for phytoplankton
populations to have been formulated and, in addition, they take into account the vertical transport structure. These are two important improvements over the previous modeling attempts. A number of comparisons were made between observed data and the resulting theoretical predictions. These are illustrated in Figure 24 for two months using data from Georges Bank.

Although there are differences between the observed data and the calculated distributions, it is clear that the resulting calculations indicate the approximate shape and levels of phytoplankton biomass as measured by chlorophyll and nutrient as measured by phosphate concentrations.

In an attempt to further incorporate the interactive structure of phytoplankton, zooplankton, and nutrient concentrations, a model was developed by steele [15] which utilizes a series of differential equations expressing the kinetic interaction and, in addition, utilizes a simple two layer approximation for the effect of spatial distribution in depth. The two layers were taken to represent the epilimnion and hypolimnion of a lake. The equations as given by Steel and shown below follow the structure of the previous models for phytoplankton and nutrients and incorporate a more empirical formulation for zooplankton:

$$
\begin{align*}
& \begin{array}{r}
\frac{d P}{d t}=\left(K_{1} N-K_{2}-g H-M \quad\right) P \\
\text { Growth Resp. Pred. Exchange }
\end{array}  \tag{12}\\
& \frac{\partial H}{d t}=K_{3} P-K_{4} H^{2}  \tag{13}\\
& \text { Empirical } \\
& \frac{d N}{d t}=-\left(K_{5} N \quad-\quad K_{6}\right) P+M\left(N_{B}-N\right)  \tag{14}\\
& \text { Uptake Regener- Exchange } \\
& \text { ation }
\end{align*}
$$

GEORGES BANK
MAY 1940
PHOSPHATE


FIGURE 24
VERTICAL DISTRIBUTION - VERIFICATIONS

Calculations employing these equations and numerical integration techniques suitable for computer use were performed. The predicted variation of the populations are in accord with reasonable behavior and, in particular, show the effect of varying the rate of exchange between the epilimnion and hypolimnion.

Seasonal Variations of Phytoplankton, Zooplankton, and Nutrients. The incorporation of seasonal effects as well as direct interactions of the phytoplankton, zooplankton, and a single nutrient was investigated by Davidson and Clymer [16] with further calculations presented by Cole [17]. The equations utilized are similar to those discussed above, but they explicitly include the seasonal variation of temperature and solar radiation in the phytoplankton growth rate. Although the simulation is hypothetical, it exhibits the behavior characteristic of the spring bloom.

A more detailed model of the seasonal phytoplankton distribution which includes a predator of the zooplankton in addition to the phytoplankton, zooplankton, and phosphate has been presented by Parker [18]. This model is not just hypothetical, but the computed results are compared to data from Kootenay Lake. The general patterns are approximately reproduced for the initial growth of phytoplankton and zooplankton, although the subsequent behavior is not properly reproduced, and the details of the temperature dependence of the phytoplankton growth rate are questionable.

Interactive models which consider a wider range of mechanisms than those presented above have also been developed. A hypothetical eutrophication model with interactions between dissolved and suspended organic matter, a sediment layer, and attached plants as well as phytoplankton, herbivores, fish, and dissolved nutrients has been structured by Brezonik [19]. This model is an example of the general class of linear ecological models which are discussed in a subsequent subsection (Ecological Models). The major drawback of such a model is the lack of realistic formulations for the interaction mechanisms. For example, instead of the non-linear coupling, which is characteristic of the phytoplankton-nutrient interactions, a simple linear coupling is used. This framework is further analyzed in the Demonstration Model section, and its drawbacks are detailed.

An interactive model with nonlinear kinetic coupling which also attempts to include a wider range of mechanisms and variables in the formulation has been presented by Chen [20].

Two groups of phytoplankton, differentiated by their growth rate-nutrient dependence; zooplankton; inorganic nitrogen; phosphorus; and organic detritus are included in this formulation. The transport regime considered is a one dimensional stream, and the calculations presented are hypothetical. However, further applications of this model which include verification attempts are underway.

A phytoplankton, zooplankton nutrient model has been developed by Di Toro, O'Connor, and Thomann [9] and applied to the Sacramento-San Joaquin Bay Delta estuary in California. An example of the verification achieved is shown in Figure 25. Since an extension of this effort comprises the eutrophication sub-model of the demonstration model, its description is included in a later section. The primary thrust of this effort is to derive the phytoplankton growth and death rates from available laboratory and field data, and to incorporate a realistic vertically integrated representation of the interaction of the incident solar radiation, extinction coefficient, and growth rate-light dependency of the phytoplankton.

Models which emphasize other aspects of the eutrophication phenomena have also been formulated. A detailed, though hypothetical, model of the rotifer life cycle has been presented by King and Paulik [21]. The influence of the stoichiometric composition of algae and bacteria on predicted seasonal variations has been investigated by Verhoff, et.al.[23]. Inorganic chemical effects have been incorporated within a lake eutrophication model [23], which includes the vertical transport structure, algae and bacteria, but not zooplankton. Efforts are underway to apply this model to the Lake Erie Time Study data. The influence of upwelling phenomena on phytoplankton growth in coastal waters is being investigated along the line described above [24], with nitrogen as the primary nutrient. Comparison to observed data is being used to verify the simulation.




Evaluation of Model Status

Model Availability. The previous section has reviewed the models and modeling framework available for use in assessing the eutrophication phenomena. All models to date have concentrated on aspects of the seasonal distribution of biomass, primarily phytoplankton and zooplankton, and the effects of the major nutrients, nitrogen and phosphorus. This is a promising first step because some eutrophication problems are directly related to the excess biomass produced by overfertilization. However, other problems are related more to the detailed biological changes which accompany this increased biomass: for example, species changes in the biomass composition with bluegreen algae becoming predominant. Also, the nutrient recycling accomplished in the benthos has not been considered in a convincing way. Links to the food chain above the zooplankton may be required as well as more detail in the predation effect of zooplankton grazing and the influence of particulate organic detritus. Thus, although models are available they do not address the full range of eutrophication problems, nor do they include all the known interaction mechanisms which may influence eutrophication in the Great Lakes.

Data Availability. Data surveys which include the requisite variables for the construction of first cut biomass eutrophication model are available for each of the Great Lakes. Although the data is not complete, either spatially or temporally, and there are gaps which may prove troublesome, the construction and verification of biomass model can proceed through initial verification. Detailed data is available on Lake Erie and Lake Ontario. The latter will be greatly augmented by the International Field Year on the Great Lakes effort.

Model Verification. The model verification efforts to date have been restricted to only a few applications. However, the general agreement achieved has been encouraging and indicates that the major features of the phenomena are understood. There are, of course, many questions of detailed mechanisms and pathways which are still only hypotheses. One drawback is the lack of Great Lakes verifications, although
the eutrophication submodel of the demonstration model adds weight to the applicability of eutrophication biomass models to Great Lakes settings. Thus, some model verifications are available.

Model Application in Planning. Direct planning applications of eutrophication models to the Great Lakes are lacking, and only preliminary planning results have been produced elsewhere. This is primarily due to a lack of verifications of the model in sufficient degree to warrant detailed planning investigations. However, the structure of the models is such that the models lend themselves to answering planning questions. This is also illustrated using the demonstration model.

## REFERENCES

[1]Sawyer, C.N., "Fertilization of Lakes by Agricultural and Urban Drainage," J.N.E. Water Works Association, 61(109), (1947). See also Sawyèr, C.N., "Basic Concepts of Eutrophication," J. Water Pollution Control Federation, 38(5), p 737 (May 1966).
[2] Vollenweider, R.A., "Scientific Fundamentals of the Eutrophication of Lakes and Flowing Waters," Organization for Economic Cooperation and Development Directorate for Scientific Affairs (OECD), Paris, France (1968).
[3] Lotka, A.J., Elements of Mathematical Biology, Reprint, Dover, New York (1956).
[4] Hutchinson, G.E., A Treatise on Limnology Volume II, Introduction to Lake Biology and the Limnoplankton, J. Wiley and Sons, New York (1967).
[5] Strickland, J.D.H., "Production of Organic Matter in the Primary Stages of the Marine Food Chain," Chemical Oceanography, Volume $I_{r}$ (ed.) J.P. Riley and G. Skivow, Academic Press (1965).
${ }^{[6]}$ Lund, J.W.G., "The Ecology of the Freshwater Phytoplankton," Biol. Res. 40 , pp 231-293 (1965).
[7] Raymont, J.E.G., Plankton and Productivity in the oceans, Pergamon Press (1963).
${ }^{[8]}$ Great Lakes Basin Framework Study, Draft No. 1, (April 1971).
[9] Di Toro, D.M., O'Connor, D.J., Thomann, R.V., "A Dynamic Model of the Phytoplankton Population in the SacramentoSan Joaquin Delta," in Nonequilibrium Systems in Natural Water Chemistry, Adv. in Chem. Series 106, Am. Chemical Soc., Washington, D.C., pp 131-180 (1971).

## REFERENCES

(continued)
[10]Riley, G.A., "Factors Controlling Phytoplankton Populations on Georges Bank," J. Marine Res., 6(1), pp 54-73 (1946).
[ll] Riley, G.A., "Seasonal Fluctuations of the Phytoplankton Populations in New England Coastal Waters," J. Marine Res. 6(2), pp 114-125 (1947).
[12]
Riley, G.A., and Von Arx, R., "Theoretical Analysis of Seasonal Changes in the Phytoplankton of Husan Harbor, Korea," J. Marine Res., $8(1), \mathrm{pp}$ 60-72 (1949).
[13]
Riley, G.A., "A Theoretical Analysis of the Zooplankton Population of Georges Bank," J. Marine Res., 6(2), pp 104113 (1947).
[14]
Riley, G.A., Stommel, H., Bumpus, D.F., "Quantitative Ecology of the Plankton of the Western North Atlantic," Bull. Bingham Oceanog. Coll. 12(3), pp 1-169 (1969).
[15] Steele, J.H., "Plant Production on Fladen Ground," J. Marine Res. Biol. Assoc., United Kingdom, 35, pp 1- $\overline{3} 3$ (1956).
[16]
Davidson, R.S., and Clymer, A.B., "The Desirability and Applicability of Simulating Ecosystems," Annals, N. Y. Acad. of Sci., 128(3), pp 790-794 (1966).
[17] Cole, C.R., A Look at Simulation Through a Study on Plankton Popülation $\frac{\text { Dynamics, Report BNWL }}{1} 4 \frac{1}{85}$, Battelle Northwest Laboratory, Richland, Washington (1967).
[18]
Parker, R.A., "Simulation of an Aquatic Ecosystem," Biometrics, $24(4)$, pp 803-822 (1968).
[19]
Brezonik, P.L., "Application of Mathematical Models to the Eutrophication Process," Proc. llth Conference, G.L.R.D. p 16 (1968).

REFERENCES
(continued)
[20] Chen, C.W., "Concepts and Utilities of an Ecologic Model," J. San. Engr. Div., Proc. A.S.C.E., Vol. 96, No.SA5, $\overline{\mathrm{pp}} \overline{1085}-\overline{1097}$ (October 1970).
${ }^{[21]}$ King, C.E., Paulik, G.J., "Dynamic Models and the Simulation of Ecological Systems," J. Theoret. Biol. 16, pp 251267 (1967).
[22] Verhoff, F.H., Echelberger, W.F., Tenny, M.W., Singer, P.C., Cordeiro, C.F., "Lake Water Quality Prediction Through Systems Modeling," Environmental Health Engr., University of Notre Dame, Indiana, 46556.
[23] Prober, R., Haimes, Y., Teraguchi, M., Moss, W., "An Ecosystem Model of Lake Algae Blooms," Symposium on Biol. Kinetics and Ecological Modeling, A.I.Ch.E., 69 th $\overline{\text { National Meeting, }}$ Cinncinati, Ohio (May 1971).
[24] Walsh, J.J., "Simulation Analysis of Trophic Interaction in an Upwelling Ecosystem," Proc. Summer Computer Simulation Conference, Boston, Massachusetts (1971).

## Dissolved Oxygen Models

Problems and Scope

The concentration of dissolved oxygen is one of the most significant water quality parameters in all natural water systems. Its presence is usually associated with high water quality conditions, which deteriorate as the concentration of dissolved oxygen decreases to a level of degradation when the water is devoid of the gas. It takes on particular significance in lakes, especially those undergoing eutrophication in which the concentration in the hypolimnion, at times during the annual cycle, is greatly reduced and, under certain conditions, reduced to levels that are barely measureable.

Dissolved oxygen is a controlling factor in many biological and chemical processes. It is of utmost importance to many forms of aquatic life. Fish, in all stages of their development - egg, larvae, young, and adult - depend on oxygen. It is furthermore highly related to the chemical process of oxidation and reduction, and the level of its concentration is critical in the precipitation and release of many chemicals. It could well be the controlling factor in the recycling of nutrients and minerals.

Under aerobic conditions and at proper pH levels, iron and manganese in their oxidized form are relatively insoluble and usually complexed with other compounds such as phosphate and organic substances. These complexes precipitate and are subject to settling under conditions commonly encountered in lakes. They accumulate and remain in the bed material, provided an aerobic condition is maintained in the hypolimnetic waters overlying the bed and the surface of the benthos contains dissolved oxygen. The interface between oxidizing and reducing conditions is found a short distance ( 1 cm ) below the benthal surface. If the dissolved oxygen is depleted in the hypolimnion, the interfacial layer is reduced and the iron and manganese are solubilized and phosphate is released. Furthermore, the gaseous end products of anaerobic decomposition diffuse to the bed surface and are introduced
to the overlying water. The concentration of dissolved oxygen is thus critical, not only to recycling of minerals but also to nutrients that enter the biological food chain and ultimately lead to the problem of eutrophication.

Although many physical and chemical factors come into play in the oxygen balancc of lakes as indicated by the examples above, the proccsses involving the utilization of dissolved oxygen are primarily bioloqically and biochemically oxidative in nature. They are usually the result of the bacterial and enzymatic breakdown of organic matter and the respiration of a variety of aquatic organisms, notably the phytoplankton. The most significant factors initiating or controlling these reactions are the thermal regime and circulation patterns for the specific geomorphological structure and the amount and concentration of oxidizable substances and nutrients in the system. The orqanic materials are oxidized by bacterial activity while the inorganic nutrients are predominantly assimilated by the phytoplankton. Both these metabolic processes may occur in the overlying water or at the interface with the benthos, the relative importance depending on the nature of the inputs and the structure of the lake.

The upper epiliminion, in general contrast to lower hypolimnion, is usually characterized by higher concentrations of dissolved oxygen, higher temperatures during the spring to fall period, and more intense mixing and circulation. These conditions are more conducive to greater metabolic activity of both bacteria and phytoplankton. The end-products of these processes, such as dead or dying cclls and partially or totally oxidized residues settle through the upper into the lower zone where further oxidation at a slower ratc takes place. Ultimately these substances settle to the bottom where they accumulate, and the final stages of oxidation take place provided oxygen is present and available. If oxygen is depleted by these processes, nutrients and minerals are reduced, released, and recycled as described above.

In addition to the factors which utilize oxygen, account must be taken of the mechanisms which replenish it. These are two: atmospheric reaeration and photosynthetic production. The atmosphere in contact with the lake surface is the
ultimate source of oxygen for reaeration. The rate at which it passes from the atmosphere through the air-sea interface into solution depends on the deficit created by the sinks of dissolved oxygen described above and by the condition of the water surface. The greater the surface renewal as determined primarily by winds and waves, the greater the rate of transfer. Oxygen is also derived from photosynthetic activity of rooted plants and phytoplankton as a by-product of carbon synthesis. The latter source is obviously limited in time to the daylight hours and in space by the vertical limit of light transmission in the euphotic zone. This zone occupies a more significant portion of the water depth in the near shore areas than it does in the mid-lake regions.

The transport of dissolved oxygen both horizontally and vertically is brought about by the velocity field with its fluctuations and gradients. These regimes are primarily the result of wind action on the lake surface and of density differences between different layers and zones and the lake. These in turn are created by the meteorlogical conditions and their interplay with the earth and water surfaces, particularly by differentials in pressure and temperature. Ultimately the predictability of the velocity field is tied to weather prediction techniques which yield, relatively speaking, only reasonable short-term projections. Thus, verification of transport and constituent models may be accomplished a posteriori with knowledge of the wind patterns; but long-term projection must be based on probabilistic analysis.

In describing the overall oxygen balance of a lake, account must be taken not only of the biological and chemical reactions, but also of the amount of organic material and nutritive substances which are introduced into, stored in, and flow from the system by natural phenomena and man's activities. These are the factors in the mass balance over which there exists some control for planning purposes, in contrast to chemical and physical reactions over which minimal control can be exercised. The analysis must therefore incorporate the inputs and outflows of the system as well as the various mechanisms involved in the reactions. The degree to which the thermal regimes and hydrodynamic effects must be included depend on the nature and scale of the problems, specifically on the time and space scales.

The time scale associated with a significant problem of dissolved oxygen is seasonal, from spring through fall. During this period the dissolved oxygen can be markedly depressed in hypolimnion reaching a maximum and possibly a steady state during the summer. The analysis of vertical distribution therefore may be approached practically from two bases: a steady-state analysis during the most severe period and a time variable analysis directed to the seasonal variation. This analysis may extend from spring, when the concentrations are reasonably uniform over the vertical plane, to the fall when thermal and circulation conditions again produce uniformity. The space scale for these problems may be conveniently divided into near shore and mid-lake regions initially, with subsequent modeling frameworks incorporating an overall spatial analysis.

The second significant problem area relates to the dissolved oxygen depression in shore regions receiving the discharge of polluted rivers and/or the effluents from municipal and industrial waste treatment plants. This analysis may be developed on a steady-state two dimensional horizontal scale. Consideration may be given to the transient problems arising from short term discharges, such as storm water overflow.

Lastly, the long term projections, in which elements of a simplified completely mixed system approach may be incorporated, should be considered.

Modeling Framework

The basis for construction of dissolved oxygen models is the principle of conservation of mass as expressed by the three dimensional advective-diffusion equation:

$$
\begin{equation*}
\frac{\partial c}{\partial t}+\nabla \cdot[-E \nabla c+U c]=\sum_{i} S_{i} \tag{1}
\end{equation*}
$$

where $c$ is the concentration of dissolved oxygen, $E$ is the diagonal matrix of diffusion coefficients, $U$ is the velocity vector, and $\delta S_{;}$is the sum of all the sources and sinks of dissolved oxygen. The sources, as discussed previously, include atmospheric reaeration (which appears as a boundary condition at the lake surface) and photosynthetic productions. The sinks include all the chemical and biological reactions which utilize oxygen. Reactions which occur at the lake bottom are included as a boundary condition at the lake bottom.

The general three dimensional equation is too complex and general to be solved directly in practical applications. Usually at least one spatial dimension may be suppressed by averaging (e.g., over depth for a shallow water body). Sometimes two dimensions may be suppressed by considering only depth variations and taking horizontal averages.

State of the Art

Although limited applications of the basic equations have been reported on the water quality analysis of lakes [1], it is believed that the basic understanding of the phenomena and the general experience gained in other natural bodies of water is sufficient to warrant a presentation of what the state of the art may be in the immediate future. As has been stated by a number of limnologists (e.g., [2]), the distribution of oxygen in a stratified lake has been studied more than many other aspects of limnology. Coupling this with experience from other natural systems and the advances made in the modeling of these phenomena, there is every reason to assume that sufficient progress can be made over the short term to justify its inclusion in this section of the report.

Vertical Distribution. Assuming initially that the horizontal transport is not significant in the formation of the vertical profile of temperature and dissolved oxygen, the problem may be analyzed as follows:

$$
\begin{equation*}
\frac{\partial C}{\partial t}=\frac{\partial}{\partial z}\left(E_{z} \frac{\partial C}{\partial z}\right)-K L(z, t)+P(z, t)-R(z, t) \tag{2}
\end{equation*}
$$

The time rate of change of dissolved oxygen is the result of two factors. The first term is the transport due to vertical dispersion. The second is the oxidation of dissolved organic matter with concentration $L(z, t)$, which is acting as a sink of dissolved oxygen. The coefficient, $K$, is the temperature function and varies over the season. It may also vary over depth as does the dispersion coefficient, $\mathrm{E}_{\mathrm{Z}}$. The last two terms, $P$ and $R$, represent the photosynthetic contribution and respiratory sink of dissolved oxygen which is due to the phytoplankton. Both terms are functions of temperature and, in addition, the photosynthetic source is a function of light and nutrient concentration. These parameters may be assigned from measurement on background data or calculated by using the phytoplankton-eutrophication model. The analysis may be envisioned as a two layered model with an interfacial resistance at the thermocline or as a series of vertical segments in which the coefficients and parameters may vary from element to element over the total depth and, in time, over the season or year. In any case, the boundary conditions are the oxygen transfer at the surface and oxygen utilization at the bed:

$$
\begin{align*}
E_{O} \frac{d c}{d z}=K_{L}\left[c_{S}-c_{0}\right] & \text { at } z=0  \tag{3}\\
E_{h} \frac{d c}{d z}=B & \text { at } z=h \tag{4}
\end{align*}
$$

The solution of these equations is a straightforward matter and should yield a preliminary analysis of some value. The most important step in this analysis is the development of a relationship between the concentration of organic matter, $L$, and the benthal uptake, $B$; and a relationship between the phytoplankton parameters, $P$ and $R$, and the inputs of organic matter and nutrients. The most extreme condition is a short term steady-state in mid or late summer at maximum temperature and minimum dispersion - that period of maximum stability and greatest utilization of oxygen. This steady-state is a more simplistic view, yet it may provide sufficient information,
even on a preliminary basis, to use for planning needs, again provided some correlation can be made between the inputs and the dissolved organic and benthal demands. Sufficient data should be available for the preliminary analysis but efforts will have to be made to extend the data collection, both spatially and temporally, for the next modeling step.

Horizontal Distribution of Oxygen. This problem is associated with the near shore distribution of organic and chemical pollutants in the vicinity of river outlets and waste treatment effluents. It is a common problem in all the Great Lakes and has been frequently reported. These substances cause a reduction in dissolved oxygen in the large scale plumes within which oxidation is taking place. The resulting oxygen distribution, taking into account the various sources and additional sinks, is suggested as follows:
$\frac{\partial c}{\partial t}=\frac{\partial}{\partial x}\left(E_{x} \frac{\partial c}{\partial x}\right)+\frac{\partial}{\partial y}\left(E_{y} \frac{\partial c}{\partial y}\right)-U \frac{\partial C}{\partial x}-K L(x, y)-B(x, y)+K_{a}\left(C_{S}-c\right)$

The origin of the axis is the river mouth at the shoreline or the location of the discharge diffusor. The $x$ and $y$ coordinates refer to the axis in the horizontal plane with its dispersion components, $E_{x}$ and $E$. The $x$ coordinate arbitrarily represents the maxjor adZective direction, with velocity $U$, the term, $K L$, is the sink due to oxidation of the organic matter, whose concentration is $L$ and reaction coefficient, K. This may be the output of another model which links the mass emission rates to the dissolved oxygen concentration. It would be appropriate to classify this input in accordance with its carbonaceous and nitrogenous components and thus carry two subsystems instead of one as indicated by the single term, L. This has practical planning implications since separate control may be exercised over these components in some cases, and since cost and technology factors also come into play. The benthal uptake, if present, is represented by $B$ and the atmospheric reaeration by the last term, in which $C_{\text {g }}$ is the saturation value of dissolved oxygen at the prevailing temperature, and $K_{a}$ is the oxygen transfer coefficient.

The analysis assumes vertical uniformity of oxygen over the area of concern, which may be a realistic assumption in the relatively shallow shore waters. If however, a vertical profile exists, the vertical dispersion term is introduced and replaces the surface transfer and benthal uptake terms which enter as boundary conditions, as described above.

The first three terms of Equation (5) represent the transport field which may be the output of the hydrodynamic model or may be obtained from measurement and specified in terms of the dispersion and advective coefficients as shown in the equation.

If the phytoplankton contribution is significant in the dissolved oxygen analysis, the photosynthetic and respiratory terms must be added in Equation (5). In areas of severe depression of dissolved oxygen, these terms may not be significant, but farther from the shoreline and in high quality shore areas, they may be. The individual situation would indicate the importance of these factors, in any case, it would have to be taken into account in any projections to analyze controls which would affect water quality improvements.

If these factors are integrated and averaged over the depth, and this procedure is valid in some circumstances, the dissolved oxygen equation is simply Equation (5) with the additional $P$ and $R$ terms. If, on the other hand, significant variations over the depth exist, it may be more appropriate to introduce this variation in the fundamental equation which then becomes:

$$
\begin{gather*}
\frac{\partial c}{\partial t}=\frac{\partial}{\partial x}\left(E_{X} \frac{\partial c}{\partial x}\right)+\frac{\partial}{\partial y}\left(E_{Y} \frac{\partial c}{\partial y}\right)+\frac{\partial}{\partial z}\left(E_{z} \frac{\partial c}{\partial z}\right)-\frac{\partial}{\partial x}\left(U_{X} c\right)-\frac{\partial}{\partial z}\left(U_{z} c\right)-  \tag{6}\\
K L(x, Y, z, t)-R(x, Y, z, t)+P(x, y, z, t)
\end{gather*}
$$

which is Equation (1) in component form. The oxygen exchange at the air-water interface and the uptake at the lake bottom are used as boundary conditions. The solution of the equation
is more complicated due to the three dimensional aspects of the problem and the uncertainties associated with the vertical variations of phytoplankton. It is, however, a problem of some significance and should be regarded as one of the required models for practical planning in the immediate future.

Large Scale Completely Mixed Systems. Long term analysis of dissolved oxygen conditions (more than 10-20 years) may be approached on this relatively simple basis as a preliminary step. Each lake may be segmented in 3 or 4 spatial elements and the time interval of the analysis may be taken as one-quarter to one year in length. Interactions between lakes could be examined as well as individual lakes on the segmented basis. Critical to this analysis would be the overall nutrient and organic balances and the correlations between these inputs and the commonly measured parameters in water quality. In any case, it is a recommended step for planning purposes on a long-range scale.

Evaluation of Model Status

Model Availability. Conceptual frameworks exist for dissolved oxygen balance models and the significant sources and sinks are known. However, direct applications to the Great Lakes is lacking. The importance of phytoplankton photosynthesis and respiration as well as the influence of benthic processes, both bacterially mediated and algal related, are the major sources of uncertainty. A less severe difficulty, because some information is available $[3,4,5]$, relates to the surface reaeration coefficient which must also be quantified for Great Lakes application. In addition, the aqueous reactions related to bacterial oxidation of organic carbon and ammonia must be included.

Model Verification and Data Availability. As indicated above, because little application of modeling has been performed on lake systems, model verification is likewise lacking. Assuming that approaches described above are in the appropriate direction, the major emphasis in the next phase is the application and the verification of these models within the framework of a specific location and problem. There are a
number of locations in the Great Lakes which lend themselves to the type of analysis described above. Furthermore, from the point of view of a preliminary analysis, sufficient data are presently available to justify this step. Data availability varies markedly from area to area where this problem exists, but, in general it is sufficient for present purposes. In particular, Lake Erie data are readily available as a result of Project Hypo [6]. Based on these analyses, recommendations would be forthcoming for additional data, if required. Similarly, application of dissolved oxygen modeling to planning needs has not been conducted in lake systems. As indicated above, dissolved oxygen modeling has been applied to many river and estuarine systems and this experience should provide an excellent basis for translation to the lake system. To date, however, there does not appear to have been any significant area of application to planning needs.

Model Application in Planning

In view of the above, it is felt that a reasonable basis exists for a useful modeling framework for certain relatively simple problems, and additional efforts are required for some more advanced problems in dissolved oxygen analysis. Sufficient data appear to be available for verification purposes; whatever is required for preliminary analysis could be collected without great difficulty and expense. Additional efforts should be directed to measuring the organic and nutrient inputs to the lake system. Although preliminary planning results have been produced elsewhere, direct planning applications of eutrophication models to the Great Lakes are minimal. The reason is that there is not a sufficient degree of model verification to warrant detailed planning investigations. However, the structure of the models enables the analyst to answer planning questions.

## REFERENCES

[1] Bella, D.Q., "Dissolved Oxygen Variations in Stratified Lakes," J. Sanit. Div. A.S.C.E., 96 (SA5), p 1129 (October 1970).
[2] Hutchinson, G.E.r A Treatise on Limnology, J. Wiley and Sons, New York, pp-5 $\overline{75-652}$ (1957).
[3] O"Connor D.J., Dobbins, W.E., "Mechanism of Reaeration in Natural Streams," Trans. A.S.C.E., 123, p 641 (1958).
[4] Churchill, M.A., Buckingham, R.A., and Elmore, H.I., "The Prediction of Stream Reaeration Rates," Tennessee Valley Authority, Chattanooga, (1962).
[5] Owens, M., Edwards, R., and Gibbs, J., "Some Reaeration Studies in Streams," Int. J. Air Water Pollution, Pergamon Press, Volume $8, \overline{\mathrm{pp}} 46 \overline{9}-4 \overline{96}(\overline{1964)}$.
[6]
Burns, N.M., Ross, C., "Project Hypo," Canada Centre for Inland Waters, Burlington, Canada, U.S. Environmental Protection Agency, Fairview Park, Ohio, (February 1972).

Problems and Scope

The presence of pathogenic and other indicator bacteria in waters to be used for water supply or recreational purposes is a direct reflection of sewage pollution and is of general public health concern. Although incidences of waterborne communicable diseases have decreased rapidly in recent years, continual awareness of the potential problem must be maintained, especially as water use and contact with possibly contaminated water increases.

The variables concerned in this class of problems include such pathogens as Salmonella; indicator bacteria such as the coliform, fecal coliform, and fecal streptoccoci groups; and viruses. Specific pathogens and viruses have been isolated from sewage effluents and are known to survive for varying periods in water. The isolation of such specific organisms is generally difficult and time consuming. As a result, groups of bacteria are often used as indicators of known pollution, although it should be stressed that the absence of such indicators does not assure absence of pathogens.

In the Great Lakes, the problems associated with bacterial and viral discharges are primarily confined to a relatively small space scale. Thus, the bacterial quality of open lake water is excellent in all of the Great Lakes. Concern with bacterial contamination is evident in the near shore ( $0-10 \mathrm{miles}$ ) and harbor areas and tributary rivers and streams which are most heavily used for water based recreation and municipal water supply. For example, as indicated previously in this report, forty-six beaches on the Great Lakes have been reported closed because of bacterial pollution. This is reflected in the fact that about one-third of the U.S. Lake Erie shore is affected either continuously or intermittently by bacterial contamination [1]. Figures 26 and 27 show the overall spatial scale of the problem which is generally confined to the shoreline


[^6]

Coliform densities in 10 -mile zone along west shore of Lake Michigan.


$$
\nabla-1-2 b
$$

Coliform densities in 10 -mile zone along east shore of Lake Michigan.
regions and harbor areas. (Note difference in coliform concentrations between the one mile and ten mile stations in Figure 27). Along the shoreline, however, the problem may extend for twenty to thirty miles or more indicating a general area-wide bacterial contamination due to urban and suburban development. The contrast between the west and east shores of Lake Michigan, as shown in Figure 27, illustrates the point.

Pathogens have been isolated in Great Lakes waters and are present in tributaries to the Great Lakes ([3], [4], and [5]). No specific data have been reported on the isolation of viruses in the near-shore area or tributary stream.

The time scale associated with the above space scale ranges from steady-state and seasonal scales to short term hour scales. The former time frame is related to the general level of urban development (see Figure 27). The latter time scale is associated with the transient discharges of combined and separate sewer overflows which emit high concentrations of bacteria, but only for a short duration of time during and after periods of rainfall. In the city of Milwaukee, for example, beaches on Lake Michigan are closed for variable periods of time after a storm to allow bacterial concentrations to return to the levels required for swimming. Figure 28 shows some results for Big Bay Beach in Milwaukee [6] and illustrates the transient nature of the bacterial problem.

Also, a variety of the scales ranging from daily to seasonal is of importance in bacterial levels at water supply intakes throughout the Great Lakes. Bennett [7] has investigated the daily and weekly changes of bacteria concentration in the intake of two water treatment plants on Lake Ontario near Toronto. A wide range of variability was found, depending on short term meteorological effects and longer term seasonal trends.

In summary, the time-space scales for the pathogen and indicator bacteria sub-system range from 0-10 miles in the near shore and harbor areas, and from steady-state to short term transient problems as a result of combined and separate storm sewer overflows.

## L己 ヨコロロリ」



DZ－1－



FIGURE 28
BACTERIOLOGICAL DATA AT BIG BAY BEACH, WHITEFISH BAY, WISCONSIN

Modeling Frameworks

The basis for the modeling of the bacteria systems is the dispersion-advection equation:

$$
\begin{equation*}
\frac{\partial c_{b}}{\partial t}+\nabla \cdot\left[-E \nabla c_{b}+U c_{b}\right]=-K_{b}(x, y, z, t) c_{b}+W_{b}(x, y, z, t) \tag{1}
\end{equation*}
$$

where $c_{b}$ is the concentration of bacteria, $K_{b}$ is the rate of die-off of the bacteria, and $W_{p}$ is the direct discharge of bacteria (other terms have been previously defined). The reaction kinetics in Equation (1) are usually assumed to be first order, but they are in fact generally complex functions of space and time through other exogenous variables. Specification of $K_{b}$ is then central to the application of the modeling framework.

It should also be noted that the form of Equation (1) does not result in an aftergrowth of bacteria, a phenomenon also observed, especially after chlorination. For example, Scarce et.al. [8] observed increases in bacteria of 200 - 300 percent after one day in chlorinated samples, the aftergrowth effect generally lasting about one to two days.

State of the Art

Steady-State. The one-dimensional steady-state form of Equation (l) has been applied extensively in analyses of bacterial distributions in rivers and estuaries. Applications to distributions of bacteria in lakes has been limited. O'Connor [9] obtained solutions to a two-dimensional version of Equation (1) given by:

$$
\begin{equation*}
0=E\left(\frac{\partial^{2} c}{\partial x^{2}}+\frac{\partial^{2} c}{\partial y^{2}}\right)-K_{b} c \tag{2}
\end{equation*}
$$

The justification for the use of this equation is that in the absence of a well defined steady-state current pattern and variable wind speeds and directions, the effects of the resulting variable current movements can be incorporated in the constant dispersion coefficient. An advective velocity can also be incorporated if it exists. Figure 29 shows the orientation of axes after transformation of Equation (2) to polar form and assuming $C$ is constant within $45^{\circ}$ in either side of a given radius. The solution is in terms of Bessel functions. Some early data are available for the distribution of coliform bacteria in Lake Michigan in the vicinity of the Indiana Harbor. The results of comparing the analytical solution to observed data are shown in Figure 30. The interesting point to note is that a simplified version of the complete equation does quite well in verifying the order of magnitude of the observed data.

More complex situations involving complicated geometry or circulation patterns require a finite-difference form of Equation (1). Under steady-state, then, such modeling contexts involve sets of algebraic equations. The application of such models to a finite grid in Western Lake Erie is presented in the Demonstration Model section. Applications of steady-state multi-dimensional models have not been made for pathogens or viruses.

Non-Steady-State. The transient bacterial problem is related principally to overflows from combined and separate sewers. Thus, the forcing function, $W_{B}(t)$, has a probabilistic component and depends largely ${ }^{B}$ on the random occurrences of rainfall of variable amounts. The modeling framework for the combined sewer overflow problem is therefore simple in principle but quite difficult to apply in practice. In a dispersion-advection situation the bacterial density can be readily calculated by numerical integration of the finite-difference form of Equation (1). The difficulty lies in obtaining reliable information on the transient bacterial inputs. For some planning purposes, however, it may be sufficient to construct a simulation using past records of rainfall, drainage characteristics, and typical values of bacterial concentrations in both combined and separate overflows. There are few instances available where a transient bacterial model has been verified, although the difficulty

FIGURE 29
COORDINATE SYSTEM FOR DISCHARGE OF BACTERIA INTO A LAKE

DAILY AND MONTHLY AVERAGES
WITH ADVECTIVE VELOCITY
COMPARISON OF OBSERVED AND COMPUTED VALUES (SOLID LINES)
FOR BACTERIA IN LAKE MICHIGAN VICINITY INDIANA HARBOR
THREE MONTH AVERAGES



is often not with the modeling framework as much as with the proper determination of the inputs. With the exception of present ongoing work in Milwaukee River and Harbor, a detailed modeling analsyis of transient coliform bacteria has not yet been conducted on the Great Lakes, although efforts in this direction are underway [10].

Evaluation of Model Status

Model Availability. The conceptual modeling framework for indicator groups is quite simple and consists of the dispersion-advection equation with a first order die-away of the bacteria. The modeling structure is therefore available and sufficient for most planning questions. A model of this type is presently being applied in the Great Lakes [11]. Attempts have not yet been made to structure modeling frameworks for specific pathogens or viruses, although, on the surface, there does not appear to be any strong reasons to doubt that a first order die-away model might also apply to these situations. It should also be noted that the model of Equation (1) does not include the aftergrowth phenomenon or the possible interaction of phytoplankton populations and the death rate of bacteria and pathogens. Hedrick [12] has, for example, indicated an apparent toxicity effect of algae on the population of Shigella. In the presence of a natural assemblage of lake phytoplankton, the rate of die-away is about 15/day or a one percent survival after approximately seven hours. From a planning point of view, however, incorporation of this type of interaction may not be warranted, because results using lower die-away rates will be conservative estimates and algal toxicity can be considered as a type of safety factor. In summary, then, models of indicator bacteria, and in some cases, pathogens, are readily available and have proved useful in a number of water resource planning problems, but have not yet been applied in the Great Lakes setting, although efforts are currently being made in this direction.

Data Availability. A review of the data sources for indicator bacteria shows a considerable amount of available data suitable for modeling purposes. Information is also generally available on the inputs, waste discharges, and tributary inflows; and an
increasing amount of information is being generated on the typical bacteria and pathogenic characteristics of combined and separate sewer overflows. Data are generally lacking on virus distribution, but this is a situation common to almost all water bodies, and is a subject of continuing research.

Model Verification. Where steady-state models of bacteria distribution have been tested in rivers, estuaries, and harbors; verification has generally been adequate for planning purposes. It should be recognized that a verification of bacterial data is generally considered adequate when comparisons between observed and computed data agree within an order of magnitude. The data are generally quite variable and subject to substantial variation because of sampling and measurement errors, so that refinement of the model is generally not warranted.

Attempts at verifying time variable bacterial or pathogen data resulting from combined sewer overflows have not generally been made. Leendertse and Gritton [14] have compared some computed transient bacterial profiles in Jamaica Bay (subject to considerable loading from combined sewer overflows) to average bacterial data collected in the Bay. Results were reasonably good, although the verification was not with time variable data.

In general, for most purposes, verification of bacterial models has been adequate. Although specific verifications have not been carried out for the transient case, the high die-off rates and generally dominant advective flow regime indicate a favorable prognosis for such verifications. This lack of verification, therefore, for combined sewer overflow problem setting is not considered serious. Indeed, the difficulty lies essentially in determining reliable input data (e.g., overflow loads and rainfall) rather than in the modeling structure itself.

Degree of Application to Planning. As mentioned above, existing models of indicator bacteria distribution have generally not been applied to the Great Lakes setting to answer planning questions. In fact, it is surprising that even the simple modeling framework describing bacteria in natural waters, coupled with generally good data on the
near shore area of the Lakes, has not been used to answer some of the planning problems associated with beach closings on the Great Lakes. Where such models have been applied elsewhere, the results have contributed in a meaningful way to the decision making process, especially with regard to the order of priority and effectiveness of various environmental control schemes.

## REFERENCES

[1] Lake Erie Report, A Plan for Water Pollution Control, United States Department of Interior, F.W.P.C.A., Great Lakes Reg., p 107 (August 1968).
[2] Scarce, L.E., "The Distribution of Bacterial Densities in Lake Michigan," Proc. Eighth Conference, G.L.R., University of Michigan, pp 182-196 (1965).
[3] Clemente, J., and Christensen, R.G., "Results of a Recent Salmonella Survey of Some Michigan Waters Flowing Into Lake Huron and Lake Erie," Proc. loth Conf. IAGLR, pp l-ll (1967).
[4]
Petersen, J.L., "The Occurrence of Salmonella in Streams Draining Lake Erie Basin," Proc. 10th Conf., IAGLR, pp 7987 (1967).
[5]
Dutka, B.N., Poplow, M.J., and Yurack, V., "Salmonellae Isolation from Surface Waters," Proc. Ilth Conf. IAGLR, pp 531-537 (1968).
[6]
Ernest, L.A., "Effects of Storm Water on Bathing Beaches in Separate and Combined Sewer Areas," Report presented at 38th Mtg., Central States W.P.C.A., Albert Lea, Minnesota, p 19 (June 1965).
[7] Bennett, E.A., "Investigations of Daily Variations in Chemical, Bacteriological, and Biological Parameters at Two Lake Ontario Stations Near Toronto Part II - Bacteriology," Proc. 12th Conf., IAGLR, pp 21-38 (1969).
${ }^{[8]}$ Scarce, L.E., Rubenstein, S.H., Nad Megregian, S., "Survival of Indicator Bacteria in Receiving Waters Under Various Conditions," Proc. 7th Conf., GLRD, University of Michigan, pp 130-1 $\overline{39}$ (19 $\overline{64)}$.

## REFERENCES

(continued)
[9] O'Connor D.J., "The Bacterial Distribution in a Lake in the Vicinity of a River Discharge," Proc. , Second Industrial Waste Conference, Texas Water Pollution Control Federation, p 6 (June 1962).
[10] Hydroscience, Inc., "Time Variable Analysis of the Milwaukee River and Harbor," in progress.
[11] Canale, R., University of Michigan, in progress.
[12]
Hedrick, L.R., Meyer, R., and Kossoy, M., "Survival of Salmonella, Shigella, and Coliforms in Lake Michigan Water," G.I.R.D., University of Michigan, Publication No. 9, pp 159-171 (1962).
[13] Leendertse, J.J., and Gritton, E.C., "A Water Quality Simulation Model for Well Mixed Estuaries and Coastal Seas: Volume III, Jamaica Bay Simulation," N.Y.C. Rand Institute, N.Y., N.Y., R-709-NYC, p 73 (July 1971).

## Fishery Models

Problems and Scope

The fisheries problem in the Great Lakes incorporates the following aspects: changes in species composition, a decline in value of commercial catch, a rapidly rising demand for a viable sport fishery, and conflicting claims over the impact of environmental changes and commercial effort.

The full dimensions of the problem are explored in detail in the Framework Study [1] and are not repeated here. Extensive catch and effort data exist [2] for previous years and provide specific information which documents species shifts and apparent population changes. Eleven species of fish assumed important roles prior to 1950. Three species either invaded or were introduced to the Great Lakes region, and five species have been or will be introduced in the near future. The changing species composition associated with market demands for certain species (e.g., lake whitefish) has resulted in a general decline in dollar value of the total catch. The volume of commercial fish catches has remained at about 75 million pounds annually since 1920. During 1967, about 40 percent of the total catch was from Lake Erie and 46 percent (including alewives) from Lake Michigan [3]. As commercial valuation of the catch has declined, there has been a general increase in sport fishing pressure. Fish stocking for sport is assuming increased importance as evidenced by early successes in planting of coho salmon. In 1967, sport fishermen harvested 55,000 fish in 232,000 man-days of effort, representing approximately a $\$ 3,700,000$ effort [4]. The sport fishery is therefore extensive and, although generally confined to the near-shore area, sport fishing pressure is expected to exert a continuing influence on species population.

There are several sub-systems that interact with the Great Lakes fisheries sometimes in subtle and, as yet, poorly understood ways. Figure 31 outlines the major sub-systems which are known to influence species numbers and composition. The relative degree of impact has not yet been adequately quantified, although evidence indicates that

commercial fishing, water quality, and unplanned introduction of new species play fundamental roles. The purpose of a systems analysis of the Great Lakes fishery is to develop a structure which incorporates the interactions shown in Figure 31 in a manner sufficient to aid future management schemes.

Modeling Framework and State of the Art

Population Data. One of the major difficulties in structuring a fishery model is the uncertain nature of the data on population distribution. Classically, catch statistics and fishing intensity are utilized as measures of population and predation, respectively. Often the catch and effort data are expressed in relative terms. Thus, let $\mathrm{p}=$ percent abundance above some base period (where $\mathrm{p}=100$ is the average pounds caught per fishing intensity during base period), and $x=$ percent fishing intensity during base period. Then data on catch and effort can be displayed and analyzed in relative terms. Figure 32 (from [5]) shows some data of this type for northern Green Bay. It is obvious that care must be taken in interpreting data such as are shown in Figure 32. Abundance data are only a crude measure of population dynamics and are never a substitute for actual intensive sampling of the population. In the absence of such data gathering efforts, abundance data afford at least a first approximation of the population data needed to develop a modeling structure of fish population dynamics. The lake herring (Figure 32A) show a typical predator-prey relationship, while lake whitefish (Figure 32B) appear to display some transient modes that are not readily explainable by simple predator-prey interactions.

Yield Models. The yield of a fishery has assumed an important historical role, because the early impetus for modeling fisheries came from a desire to exploit a fishery at a maximum level. Yield is considered in production terms, i.e., pounds of fish produced from a fishery. Early models looked toward describing yield as a function of various properties of the fish and fishery and are being used extensively in the West Coast salmonoid fisheries,


GREEN BAY-WALLEYE


GREEN BAY - LAKE WHITEFISH


FIGURE 32
CATCH AND EFFORT DATA FOR NORTHERN GREEN BAY

North Sea, and North Pacific fisheries, among other locations. Yield models have apparently not been extensively applied to the Great Lakes. A general review of yield models is given in [6] and [7]. The analysis generally begins by writing a growth equation for a species as [6]:

$$
\begin{equation*}
\frac{l}{p} \frac{d p}{d t}=r(p)+g(p)-M(p)-F(x)+n \tag{1}
\end{equation*}
$$

where $p=$ fish biomass; $r, g$, and $M$ are rates of recruitment, growth, and mortality, respectively; $F=$ fishing mortality, $\mathrm{x}=\mathrm{a}$ function of fishing effort, and n represents external environmental conditions. Most models to date have ignored the external environmental conditions and have considered the fishery over some average environmental regime.

Under steady state, $d p / d t=0$, and introducing yield $y=$ $F(x) p$, one obtains (ignoring environmental effects):

$$
\begin{equation*}
y=F(x) p=[r(p)+g(p)-M(p)] p \tag{2}
\end{equation*}
$$

Beverton and Holt [8] attempt to explicitly account for the individual terms in (2), while Schaefer [9] chooses a logistic functional form for the fishery growth and a linear relationship between fishing mortality and fishing effort, i.e., $F=c x$. Therefore, Schaefer's model is:

$$
\begin{equation*}
c x=k(P-p) \tag{3}
\end{equation*}
$$

where $P$ is the asympototic equilibrium population.
The Beverton-Holt model begins by examining the number of recruits, $R$, at age $t_{r}$ entering an exploitation area, but not retained by the size of the gear until age $t_{c}$. Natural mortality only operates during this period.

The yield is:

$$
\begin{equation*}
\frac{d y}{d t}=F(t) N(t) W(t) \tag{4}
\end{equation*}
$$

where $W=$ fish weight. The Beverton and Holt model is given by:

$$
\begin{equation*}
\frac{d y}{d t}=(F)(R)\left[e^{-M\left(t_{C}-t_{r}\right)} e^{-(F+M)\left(t-t_{C}\right)}\right]\left[g L \delta\left(1-e^{-K\left(t-t_{o}\right)}\right)\right] \tag{5}
\end{equation*}
$$

where $W(t)$ is given in the last term in brackets, $L=$ asymptotic length of fish, and $\mathrm{K}, \mathrm{q}$, and $\delta$ are known coefficients from empirical data.

Computations are often carried in relative terms, because estimates of the absolute number of recruits may not be at hand. One can then make estimates of $Y / R$, the yield per recruit to the fishery. A fishery manager is then in a position to estimate the magnitude and direction of the effect of a change in fishing intensity or gear. Equation (5) is integrated over a fishable life span to get the total yield of a fishery. Response surface analysis can then be carried out to determine appropriate optimal yield conditions [8,9]. Numerous applications of the basic yield models have been developed $[10,11,12]$ and utilized extensively for fishery management.

Silliman [12] describes the application of a simple yield model with variable coefficients. The model was implemented on an analog computer and applied to a variety of fish situations including the lake trout of Lake Michigan. His results are shown in Figure 33 and indicate that the simulation was good from 1929 to 1944. After that date, actual catches of lake trout were always below calculated catches, indicating some other influence was operative. The parasitic sea lamprey was assumed as one of the causes of this decline in actual catch.

A generalized computer program has also been developed for equilibrium yield calculations [13]. Walters [14] has developed a more general simulation model for yield studies.


The model uses age-specific natural mortality rates, growth rates, and relative fecundities. Any stock-recruitment relationships can be used. One interesting result from this work is the great differences in a fifty-year yield computation using the Beverton-Holt equation assuming constant recruitment and the computer model using variable coefficients. The Beverton-Holt equation predicted maximum yield for high fishing rates and low entry ages to the fishery. The computer model predicted maximum yield at low fishing rate and high age at entry.

Deterministic-Statistical Models. Recently, a species specific model has been proposed which attempts to describe species interactions in the Great Lakes [5]. The growth equation for each species is given by:

$$
\begin{equation*}
\frac{1}{p_{i}} \frac{d p_{i}}{d t}=b_{i}-a_{i i} p_{i}-c_{i} x_{i} \tag{6}
\end{equation*}
$$

where $p_{i}=$ population (abundancel of the $i^{\text {th }}$ fish species, and $x_{i} \xlongequal{\underline{1}}$ fishing intensity of $i$ species and $a_{i j}, b_{i}$, and $c_{i}$ are constants. In the form of Equation (6), ithe ítelative growth term is expressed as the sum of growth (b), death, or self-species interaction ( $a_{i j} p_{i}$ ) and predation by fishing ( $c_{i} x_{i}$ ). If other species interact with the $i$ th species, then ${ }^{1}$ the interactions are included as product interactions. Therefore for $n$ interactive species:

$$
\begin{align*}
& \frac{1}{p_{1}} \frac{d p_{1}}{d t}=b_{1}-a_{11} p_{1}-a_{12} p_{2} \ldots-a_{1} p_{n}-c_{1} x_{1} \\
& \frac{1}{p_{2}} \frac{d p_{2}}{d t}=b_{2}-a_{21 p_{1}}-a_{22} p_{2} \ldots-a_{2} p_{n}-c_{2} x_{2}  \tag{7}\\
& \vdots \\
& \frac{1}{p_{n}} \frac{d p_{n}}{d n}=b_{n}-a_{n}{ }_{1} p_{n}-a_{n} p_{2} \ldots-a_{n n} p_{n}-c_{n} x_{n}
\end{align*}
$$

The relative growth rate is estimated from abundance data by a $\alpha$ differencc approximation. Stepwisc multiple regression techniques are then used to find the constants $a_{i j}, b_{i}$, and c for a complete set of ahundance and fishing intensity data fôr each species (sce, for example, Figure 32). Time lags are introduced by regressing relative growth rates of species $i$ with population of spocies $j$ at some earlier time. Goodness-of-fit data on the outcome of the regression analysis, however, are not given.

The model was applied to data in Northern Green Bay and included analysis of eight fish species and one predator (sea lamprey). The results show that there was little effect of fishing intensity on relative growth rates which is somewhat surprising, as the authors themselves indicate. For example, casual inspection of Figurc 32 indicates an apparent correlation of lake herring with fishing intensity and a lag of about four to six years.

Attempts were not made to use the regression model obtained to generate the actual population (abundance) data that were used. It should be noted again that the regression model fits relative growth ratns and not the actual abundance data.

Stochastic Models - Leslie Matrices. Year class behavior and species interaction can be examined by stochastic models using the basic principles of Markov chains and formulating the problem in a Leslie matrix form [15]. If:

$$
\begin{gathered}
P_{i}=\text { fraction of year class } i \text { that survives from } t \text { to } t+1 \\
R_{i}=\text { fraction of year class } i \text { that is new born at } t+1
\end{gathered}
$$

Then the population at $t=1$ is:

$$
\left[\begin{array}{c}
\mathrm{N}_{0}  \tag{8}\\
\mathrm{~N}_{1} \\
\cdots \\
\vdots \\
\mathrm{~N}_{\mathrm{m}_{1}}
\end{array}\right]=\left[\begin{array}{cccc}
\mathrm{R}_{0} & \mathrm{R}_{1} & \cdots & \mathrm{R}_{\mathrm{m}} \\
\mathrm{P}_{0} & 0 & \cdots & 0 \\
0 & \mathrm{P}_{1} & \cdots & 0 \\
\vdots & & & \\
0 & \cdots & \cdots 0 & P_{m}
\end{array}\right]_{0-1}\left[\begin{array}{c}
\mathrm{N}_{0} \\
\mathrm{~N}_{1} \\
\mathrm{~N}_{2} \\
\vdots \\
\mathrm{~N}_{\mathrm{m}}
\end{array}\right]
$$

or:

$$
\begin{equation*}
(N)_{1}=[M]_{0}-1(N) \tag{9}
\end{equation*}
$$

where [M] in the form given is known as a Leslie matrix. The population at time $t$ is therefore:

$$
\begin{equation*}
(N)_{t}=\prod_{i=1}^{t}[M]_{i-1, i}^{(N)_{0}} \tag{10}
\end{equation*}
$$

or, for a constant Leslie matrix:

$$
\begin{equation*}
(N)_{t}=[M]^{t}(N)_{0} \tag{11}
\end{equation*}
$$

More generally, one can consider $(\mathbb{N})_{t}$ as a vector of biomass measures of several categories of animals (year, class, species, competitors, etc.). The maxtrix [M] $\mathrm{H}_{\mathrm{i}}$, is then viewed as the probabilities of transition from one category to another category during the interval i-l,i. [M] is therefore referred to as a transition matrix. Equation (10) is then called a non-stationary Markov chain, while Equation (11) is a stationary Markov chain.

This approach has been explored and applied in detail by Riffenburg [16] to a sardine-anchovy model. The interactions are shown in Figure 34. Nine categories were used (including biomass lost from system). An encouraging feature of this work


FIGURE 34
SARDINE ANCHOVY MODEL
is the verification analysis using data collected during 1950-1959. The results of comparisons between predictions and observations are good and permit the generation of projections with at least some degree of confidence. Also, Pella [17] has developed a Poisson process representation of searching for schooling fish combined with empirical estimates of factors such as fishing density and weather.

Simulation Models. In this category, there are a number of models which do not necessairily attempt to incorporate detailed physiological mechanisms in analytical form, but rather include phenomena in a decision-rule context. Thus the models are built up from a series of simple rules, e.g., for population i, it takes twelve days to pass through region $y$ and an average of 60 percent of the fish are caught with a standard deviation of 20 percent from a normal distribution. The difficulty with these models is the lack of generality, i.e., basic underlying theory and principles that interact with apparently differing phenomena can be masked by the simulation procedure. On the other hand, simulation models permit easy inclusion of complex interactions and spatial detail, are readily constructed, and, most important, tend to be more easily understood by decision makers. Some simulation models are given in [18], [19], and [20].

Evaluation of Model Status

Model Availability. The previous subsection has indicated the variety of models of fisheries resource that are available. The models range from single species exploitation (yield) models to more complex interactive species models including large simulation models that have been used extensively in management decisions relating to maximum fishery yields.

Thus there does not appear to be any fundamental lack of basic knowledge which would hamper the construction and application of existing models to the Great Lakes fishery. It should be recognized, however, that all of the existing models really deal with a present biological structure and are not structured in an ecological predictive sense. The
existing fishery models have little external environmental effects built into them. For example, the models do not permit the prediction of the effects of long term depletion of oxygen in the hypolimnion in the fisheries resource.

Data Availability. In terms of the type of data available for fisheries model construction, there is ample data available on the Great Lakes. As mentioned previously, good records of catch and effort exist for the major species and for major geographic areas. Of course, as with all fishery data, information on actual numbers of population is minimal. Planned work as part of the International Field Year of the Great Lakes effort on Lake Ontario will hopefully provide estimates of the population of major species in that lake.

Model Verification. Where attempted, verification of catch data with a fishery model has generally been satisfactory. This is true solely within the context of the utilization of model results. Output from fishery models is used most often to indicate direction of changes and not, necessarily, absolute magnitude of population changes.

Model Application and Planning. In terms of the Great Lakes fisheries problem, two major additions must be made to the more traditional modeling structure: a) more detailed incorporation of species interactions, possibly in predatorprey and competition modeling frameworks, and b) incorporation of external environmental effects of fish reproduction, behavior, and survival. These additions do not produce any conceptual difficulties, because much is already known about the physiological mechanisms of the major species, and effects of environmental water quality on fish are also well documented.

## REFERENCES

${ }^{[1]}$ Great Lakes Basin Framework Study, Appendix No. 8, Fish, Draft No. 1, (April 1971).
[2]"Commercial Fish Production in the Great Lakes 18671960," Great Lakes Fishery Commission, Technical Report, No. 3, Ann Arbor, Michigan, p 166 (July 1962).
[3] NacNish, C., and Lawhead, H.G., "History of the Development of Use of the Great Lakes and Present Problems," in Proc. of Great Lakes Water Resource Conference, Toronto, Canada, sponsored by EOC and ASCE; pp 3-48 (1968).
[4]
Cain, S.A., "Conflicts of Recreation and Other Uses of the Great Lakes," in Proc. of Great Lakes Resources Conference, Toronto, Canada, sponsored by EIC and ASCE, pp 123-153 (1968).
[5] Walter, G., Hooman,M., "Mathematical Models for Estimating Changes in Fish Populations with Applications to Green Bay," Conference \#52, Conference for Great Lakes Studies, University of Wisconsin, Milwaukee, Wisconsin (1971).
[6]Schaefer, M.G., and Beverton, R.J.H., "Fishery Dynamics - Their Analysis and Interpretation," The Sea, Volume II, Hill, M., (ed.), J. Wiley and Sons, New York, New York, pp 464-483 (1960).
${ }^{[7]}$ Cushing, D.H., Fisheries Biology, A Study In Population Dynamics, University of Wisconsin Press, Madison, Wisconsing p 200 (1968).
[8] Beverton, R., and Holt, S., On The Dynamics of Exploited Fish Populations, Agricultural and Fish Investigations, Section 2, p 533 (1957).

REFERENCES
(continued)
[9] Schaefer, M.B. "Some Aspects of the Dynamics of Populations Important to the Management of Commercial Marine Fisheries," Bull. Inter-American Trop. Tuna Commission, 1, pp 26-56 (1954).
[10] Paulick, G.J., and Gales, L.E., "Allometric Growth and the Beverton and Holt Yield Equations," Trans. American Fish Soc., Volume 43, No. 4, (October 1964).
[11] Ricker, W.E., Handbook of Computations for Biological Statistics of Fish Populations, Fish. Res. Bd. Canada, Bulletin No. $1 \overline{19, ~ p 300(1958) .}$

Silliman, R.P., "Analog Computer Simulation and Catch Forecasting in Commercially Fished Populations," Trans. America Fish Soc., No. 3, pp 560-569 (1969).
[13] Paulik, G.J., and Bayliff, W.H., "A Generalized Computer Program for the Ricker Model of Equilibrium Yield Per Recruitment," J. Fish. Res. Bd., Canada, 24(2), pp 249259 (1967).
[14] Walters, C.H., "A Generalized Computer Simulation Model for Fish Populations Studies," Trans. Amer. Fish. Soc., No. 3, pp 505-512 (1969).
[15] Pielou, E.C., An Introduction to Mathematical Ecology, Wiley Interscience, New York, $\overline{\text { New }}$ York, p 286 (1969).
[16] Riffenburgh, R.H., "A Stochastic Model of Interpolation Dynamics in Marine Ecology," Journal Fish. Res. Ba., Canada, Vol. 26, No. $11,2,8 \overline{43-2, ~} \overline{880}(1 \overline{969)}$.
[17] Peall, J.J., "A Stochastic Model for Purse Seining In a Two-Species Fishery," J. Theoret. Biol., Volume 22, pp 209-226 (1969).

## REFERENCES

(continued)
[18] Royce, et.al., Salmon Gear Limitations in Northern Washington Waters, University of Washington, Volume II, No. I, Cut No. 145, (February 1963).
[19] Olsen, J.C., "Steelhead Trout Growth and Production in Fern Lake, Washington Determined by Sampling and a Simulation Model," pH.D. Thesis, University of Washington, College of Fisheries, p 194 (1969).
[20] Paulik, G.J., and Greenough, J.W., Management Analysis for a Salmon Resource in Systems Analysis in Ecology, $\overline{\text { K.E.F. }}$. Watt ( $\overline{\text { Ed. }}$ ), Academic Press, New York, New York, pp 215-252 (1966).

## Ecological and Food Chain Models

Problems and Scope

Ecological models in the context of this subsection are considered to be analytical structures of broad segments of the aquatic ecosystem. Several trophic levels are generally included and the models attempt to analyze behavior and interactions of numerous ecological variables. The so-called compartment models and models of trophic level concentration of chemicals (food chain models) are included in this category. Models directly related to lake enrichment and eutrophication are treated separately in a preceding subsection.

The substantial difficulty in constructing models of the ecological system is primarily related to the lack of a basic scientific set of laws which describe biological behavior. This can be seen when one contrasts the state of the art of modeling hydrodynamic phenomena as opposed to biological phenomena. In the former case, the Navier-Stokes equations, together with well known and tested energy balance and continuity equations, comprise the foundation for predicting water movements. This is not intended to minimize the difficulty of implementing these basic equations in any specific problem setting; rather, the observation is intended to form a contrast to the ecological problem area for which such a basic starting set of equations simply does not exist, nor is there any hope in the foreseeable future for constructing such equations. Aside from continuity statements, rigorous deterministic equations for prediction of biological behavior at all trophic levels are generally not available, and indeed, such behavior is characterized as much by its stochastic properties as by its deterministic structure.

Nevertheless, ecological models have been constructed along several lines. These models include (a) descriptions of portions of the biological setting called compartment models; (b) closely related food chain models of concentration of chemicals and radioactive substances through various trophic
levels; (c) detailed models replete with complex interactions attempting to describe the details of trophic level behavior; and (d) classification models (e.g., niche analysis) coupled to sets of deterministic equations.

The basic difficulty of these analyses (aside from problems of verification and data availability, discussed more fully below) lies in the difficulty of predicting a sequence of biological events. Thus the models do not generally attempt, nor in fact are they claimed to do so by their creators, to predict, for example, the evolutionary patterns of specific species under a variety of environmental conditions. Rather, the models are largely descriptive in nature and the degree to which they can be perturbed from existing conditions is generally not known.

Modeling Framework and state of the Art

The subsections previous to this have discussed the details of the present state of the art of a series of water resource models. The nature of these models indicates the separate and often disparate lines of inquiry from which they are drawn. In order to respond to today's planning problems, the need to synthesize the apparently separate sub-models into a unified system must be satisfied. Ecological models to date have not generally responded to this need, but rather have retained a type of descriptive character of specific phenomenon.
Attainment of a goal of interactive sub-systems at the present time is not without its difficulties and dimensions of infeasibility.

In discussing the structure of complex interactive ecological models, it is useful to define an elemental component which can act as a type of building block for comparing differing levels of analysis. The notion of a "compartment" is useful for this purpose. A compartment is considered as any water resource or ecological variable, suitably located in space. Positioning in time will be governed by the problem context. The concept of a compartment arises, on the one hand, from the finite difference approximation of partial differential equations which will express mass balances. Continuous space is therefore replaced by discrete finite elements or spatial
compartments within which are located (usually uniformly distributed) the variable of interest. On the other hand, the concept of a compartment arises from quantitative ecological models where the continuum of the environment is also replaced by finite, discrete, interacting trophic levels. Physical volume in the spatial domain corresponds to mass of the ecological variable in the trophic level or state domain. Other analogies also apply; for example, residence time in spatial compartments correspond to mean ages in state compartments.

Figures 35, 36 , and 37 diagramatically depict a ten compartment model with no spatial definition, a ten compartment model with spatial definition, and a seven compartment food chain model.

The species of each compartment need not be expressed in detail; therefore, a black box concept is used. Internal mechanisms which are probably of a non-linear nature are not examined. Rather, the compartments are defined in a manner consistent with the type of problem under examination. The model building ecologist therefore carries out a type of free-body analysis before beginning construction of his model. Attention is usually directed to a portion of the ecosystem, the degree of specificity of compartments depending heavily on the aims of the investigator. Thus, one investigator describes compartments including shore birds of various maturity, their food source compartments, and their nesting compartments. Only passing attention is given to other ecological compartments, such as carnivorous fish, because the investigation context is shore birds. The ecological concepts of this type of first analysis have been reviewed adequately by Watt [1], Dale [2], Patten [3,4], and Mankin and Brooks [5]. A bibliography of modeling in ecology has also been prepared [6]. Much of the more recent work has been conducted as part of the International Biological Program.

Mathematical Structure. With the general notion $\mathrm{th}^{f}$ a compartment in mind, one can define $C_{i f}$ as the $i^{\text {th }}$ variable located in the $r$ spatial position. Interactions between variables can be considered as linkages, such linkages including, for example, physical transport of a variable from location $r$ to location s. Alternately, one can consider




FIGURE 37
A FOOD CHAIN MODEL
causal linkages which transform variable $i$ to variable j. The linkages may be complex functions of the compartments that are interacting (as, for example, nutrient limited growth kinetics in a phytoplankton compartment) or may be independent of the compartments, as in fluid flow transport between spatial compartments of the same variable.

From these notions of compartments and linkages, a variety of model structures can be considered. A simple model of six compartments consisting of three variables positioned at two spatial locations can be considered as a starting point. Diagramatically, the six compartments can be displayed as in Figure 38.

As shown, the variables $C_{1}, C_{2}$, and $C_{3}$ are each found in two spatial compartments. The double subscript in Figure 38 therefore indicates the variable and its location. Thus, $\mathrm{C}_{21}$ is the second variable located at the first position. The interaction between compartment \#l and \#4 is possible only through compartment \#2. That is, variable $C_{1}$ must be physically transported from location 1 to location 2, before it can causally interact with variable $C_{2}$. Note that at this stage, the analytical nature of the links is not specified; it may be linear or non-linear or constant or time variable. Further, the link may be empirical, probabilistic, or deterministic functions of characteristics of the compartment or contiguous compartment.

In general, one can consider a link as $K_{j}$, which represents the causal transformation of variable i もdrvariable jat location r. It is convenient to distinguish further the mass transport links as $\mathrm{F}_{\mathrm{i}}$, which represents the bulk transport of variable $i$ from loteration $r$ to location $s$. Let $K * C$ symbolically represent a causal transformation operation and $\mathrm{F} \cdot \mathrm{C}$ represent a mass transport operation. (Note that for some compartment realizations as between \#8 and \#9 in Figure 36, the transport need not be passive, but might involve active transport as in the self-mobility of fish). With this notation, any output variable can be written as:

$$
\begin{equation*}
\frac{d C_{i r}}{d t}=\sum_{s} F_{i, r s} \cdot C_{i s} \pm \sum_{j} K_{i j, r}{ }^{*} C_{j r}+g_{i r}(t) \tag{1}
\end{equation*}
$$


where $g_{i, r}(t)$ represents an input forcing function of variabler at location r. This expression really represents a discrete version of a mass balance around the $i$ th compartment, and is composed of transport over all s spatial compartments bordering on $r$ p plus the causal transformation of all $j$ variables linked to $i$, all located at the $r^{\text {th }}$ position.

For a total of $m$ variables and $n$ spatial locations, there are mn compartments which represent mn differential equations to be solved. Under steady state, there are mn algebraic equations to be solved. For example consider twenty-five resource or ecological variables that are interactive in complex ways, i.e., none of the twenty-five variables can be analyzed separately or sequentially. For some lake wide problems, a spatial grid size of 200 compartments might be reasonable. For Lake Michigan, this represents a resolution of horizontal gradients on the order of miles, with no vertical definition. Thus 5,000 possibly non-linear and time variable equations must be solved, which is not an inconsiderable number.

If one attempts to construct compartment models on the same spatial scale as the usual hydrodynamic grid size, the size of the problem increases substantially. For example, for Lake Erie the number of spatial compartments for some hydrodynamic models is about 5,000. For the same twenty-five variables, this results in a total of 125,000 equations. Except for large scale numerical weather forecasting, implementation and operation of problem sizes of this magnitude have not been accomplished. This issue of computability is further addressed in a later section.

Nevertheless, the framing of a limnological problem in the broad context of compartment analysis has significant utility in describing the nature of the problem to be investigated. Notwithstanding the computational difficulties of large numbers of equations, it is useful to attempt to construct at least the broad outlines of a meaningful ecological model, which through compartmentalized structure can respond to a variety of problems.

Linear Compartment Analysis. A special case of the general mathematical compartment structure as given by Equation (1) is to consider all compartment interactions as linear [7]. A large number of ecological and food chain models have been constructed under this assumption. It is generally made because of two basic reasons. First, it is difficult, if not impossible, to specify in non-linear detail all the complex mechanisms that may exist in a given problem context. Further, it is not necessarily clear that such a detailed specification is any better than a broad linear interactive system. Secondly, the mathematical and computational aspects of solving large systems of interactive linear equations (perhaps with time variable coefficients) are well understood.

Conversely, the arguments against the linear assumption also revolve about two notions: (a) even though it may not be possible to specify all details mechanistically, certain features of the problem context may be clearly non-linear (e.g., nutrient limitation) and to ignore such an obvious well known non-linear structure is to invite difficulty in the projection of future conditions; (b) although linear systems may represent an adequate description of past or present conditions, the degree to which such a system can be perturbated (e.g., future changes in one compartment) is not clear. This is explained mathematically below. Equation (1) can be written in a linear form as:

$$
\begin{equation*}
\frac{d C_{i r}}{d t}=\sum_{s=1}^{n} F_{i, r s} C_{i s} \pm \sum_{j=1}^{m} K_{i j, r} C_{j r}+g_{i, r}(t) \tag{2}
\end{equation*}
$$

In this time variable form, the coefficients may still be temporally variable. This may not produce difficulty with regard to the transport links, but the specification of time variable kinetic links is usually not accomplished independently of the data at hand. For many applications (e.g. [8], [9], [10], [11]), a steady state approximation is made initially to aid in determining the order of the kinetic interaction coefficients. In the case of steady state, the resulting vector equation can be written as:

$$
\begin{equation*}
[\mathrm{K}] \quad(\mathrm{C})=(\mathrm{g}) \tag{3}
\end{equation*}
$$

where [K] is an mn $x \mathrm{mn}$ matrix of interactions, (C) is an nm x 1 vector arranged in such a way that the first n elements represent the distribution of $C_{\text {ir }}(r=1 \ldots n)$, the second set of $n$ elements represents. the ${ }^{1}$ distribution of $C_{2}$ ( $r=1 . \ldots n$ ) and so on. The vector ( $g$ ) is interpreted similarly ${ }^{r}$ for the input forcing functions.

There are two interpretations that can be placed on Equation (3). First, the set of equations can be viewed as representing an equilibrium situation. In this case, it is necessary to have all elements of (C) positive for physically realistic results. That is, the problem would lose its meaning if after solution of (C) it was discovered that the phytoplankton compartment became negative. It can then be shown that all terms off the main diagonal of [K] must be negative for an all-positive vector (C). The major ecological consequence of this restriction is that direct inclusion of predation or other similar effects is not possible. Mathematically, this
 Equation (3) is ${ }^{r}$ interpreted as an absolute steady state equation, then the preceding restrictions must be met.

The alternate interpretation of Equation (3) is to consider the solution to be a deviation from some equilibrium level. The two interpretations are shown schematically in Figure 39. Reexpressing Equation (3) to delineate this interpretation gives:

$$
\begin{equation*}
[\mathrm{K}](\delta \mathrm{C})=(\delta \mathrm{g}) \tag{4}
\end{equation*}
$$

where $\delta$ indicates departures from equilibrium. Negative values in the solution vector are now allowed, because such values represent deviations from the equilibrium point. The difficulty with this interpretation is that the equilibrium solution may not generally be known, therefore the degree to which Equation (4) is meaningful is not known. This is depicted in Figure 39B. Presumably, an allowable


FIGURE 39
perturbation is one which does not result in a physically unrealizable result and mathematically describes a solution trajectory along the same solution path as would be generated with a more realistic non-Iinear model. It is not possible to know these paths and degree of physical realizability a priori. Indeed, if they were known, it would not be nécessary to carry out the linear analysis.

For food chain models, i.e., the transfer of radionuclides or pesticides through various trophic levels, a similar compartment procedure is followed [12], [13], [14], and [15]. In this case, however, the mass balance is now taken around the trophic level and fluxes are computed into and out of the given level located at a specific physical place. The emphasis in Equation (2) is then on the second term on the righthand side, namely, the transfer of material between causal compartments.

Food chain models have been extensively applied to study the movement of radioactive substances in the environment. The capacity of an organism to accumulate radioactive substances is expressed by the ratio of its radioactivity to that of the aqueous medium or preceding food link. This ratio is called the concentration factor and is defined as:

$$
\begin{equation*}
K=c / c^{\prime} \tag{5}
\end{equation*}
$$

where $c$ is the concentration in the organism and $c^{\prime}$ is the concentration in the water.

Concentration factors are generally based on the wet weight of the organism. The concentration factors are constant for a very wide range of concentration values in the water and are usually the same for both the radionuclide and the stable form for most elements. It has been found that pH , light, and $\mathrm{CO}_{2}$ can influence the concentration factor K , for Sr and Zn in plants. Concentration factors are often reported for equilibrium conditions.

Concentration factors within freshwater, terrestrial, and marine ecosystems are well known. The kinetics of concentration factor changes also seem to be well defined. Some results are from laboratory experiments, some from controlled field tests, and others are measured directly in natural waters. Table 15 after Ayres [16] lists concentration factors, phytoplankton, zooplankton, and benthos for several elements in Lake Michigan.

Gustafson [17] has used a simple model to predict future levels of tritium in the Great Lakes from nuclear power production. Tritium is an important component of the wastes discharged by such installations and is produced during the routine operation of a reactor. Apparently, the movement of tritium through food chains involves no trophic level effect or reconcentration along the food chains leading to human consumption. Therefore, a mathematical model need not consider such phenomena. As a result, it is possible to write a continuity equation based on a single homogeneous volume, $V$, that is:

$$
\begin{equation*}
v \frac{d c}{d t}=W-Q C-v C / \lambda \tag{6}
\end{equation*}
$$

where $W$ is the mass input rate, $Q$ is the flow rate, and $\lambda$ is the half-life of tritium (12.4 years). The mass inputs result from natural sources, weapons tests, and nuclear power plants. The author estimates each of these loadings for each of the Great Lakes. The equilibrium concentration of tritium for each Lake is then calculated assuming uniform mixing. In each case, the equilibrium concentration is well below the maximum permissible concentration for tritium in drinking water, which is $3,000 \mathrm{pci} / \mathrm{ml}$.

However, the uniform mixing assumption may result in an over optimistic estimate of future tritium levels. As an example, a large nuclear power plant (2,000 MW) can be expected to discharge approximately 2,000 curies/year. If such a plant were to locate near Traverse City on the west arm of Grand Traverse Bay, significantly higher tritium levels would be expected. Indeed, the calculation indicates that for the size and detention time of Grand Traverse Bay, the equilibrium value is about twenty times the value estimated for complete mixing through the Lake.

## TABLE 15

## ILLUSTRATIVE RECONCENTRATION FACTORS [16]

| Element | Phytoplankton <br> Lightly <br> Contaminated | 161 | Zooplankton <br> Lightly <br> Contaminated |
| :--- | :---: | :---: | ---: | | Benthos |
| :---: |
| Ca |

Linear models have been constructed by ecologists largely for descriptive purposes, i.e., to better understand the observed data. Transfer coefficients are usually obtained from the data directly, thereby weakening the utility of the model framework for prediction purposes. Thus, verification of the models in the sense of attempting to independently evaluate the analysis is usually not done. This is explored more fully below in Evaluation of Model Status.

Non-linear Analysis. The basic structure is given by Equation (1). Attempts are made throughout the analysis to incorporate those phenomena which are known to be non-linear. For example, in the linear model, nutrient uptake must be incorporated in the self-decay coefficient of the nutrient balance equation. In the non-linear modeling framework, nutrient uptake can be explicitly incorporated in the nutrient balance equation in a manner that more closely reflects the phenomenon.

A variety of computer codes now exist to aid the analyst in structuring his model of a selected portion of the ecosystem. For example, a Continuous System Simulator (CSS) [18] has been developed for ecologists as part of the IBP. This program incorporates features that ecologists are most likely to use and can be contrasted to more general simulation languages, such as CSMP (Continuous Systems Modeling Program, IBM) and MIMIC (CDC). One of the obvious advantages for the ecologist using the program is that the language is somewhat more familiar. For example, the following transfers between compartments are incorporated in CSS:
a) linear flow
b) product interactions
c) proportional to input
d) Michaelis-Menton
e) proportional to product of input and compartment
f) logistic

These programs tend to be limited in scope and are generally useful for exploratory model building only. Spatial detail is usually not included explicitly, and considerations of verification of the model are not discussed.

O'Neill [19] has conducted an error analysis of ecological models aimed toward determining the degree to which introduction of nonlinearities results in a more reliable model. By examining several modeling levels, O'Neill was able to test the hypothesis that a more complex model is a more accurate model. The total uncertainty accompanying a model prediction is analyzed and is observed to consist of two components (Figure 40): system uncertainty, high at the linear level; and measurement uncertainty of the parameters which tends to increase with the introduction of more complex phenomena in the non-linear models. The degree to which these errors propagate through the system and result in an uncertain prediction may not be intuitively clear. It is interesting to note that in a comparison of relative error between a simple three compartment linear model and a similar non-linear version, the linear model was calculated to be more accurate over a wide range ( $\pm 50$ percent of the equilibrium model. The non-linear model was more accurate outside this limit.

Niche Analysis. The above modeling framework deals exclusively with deterministic sets of equations. There is an obvious limit to the extent of determinism that one can build into an ecological structure. Predation and competition efforts among numerous biological species are difficult to analyze explicitly and some other means should be at hand for characterizing biological systems. The niche concept represents an attempt to analyze species interactions and community structure.

Basically, the niche represents the ecological requirements for the existence of a specie. More specifically, Levins [20] has defined the niche as a measure of fitness in environmental space. Two important measures of niche are considered; the niche breadth and the niche dimension. One can arrange relative frequencies of species in given environments
LINEAR MODELS

MODEL COMPLEXITY
in a matrix form, with elements $P$ ih, where the subscript i represents the specie and subscript $h$ represents the environment. The quantity $\mathrm{P}_{i h}$ therefore represents the proportion of species $i$ in environment $h$ of the total number of species in all environments. The niche breadth is then defined as:

$$
\begin{equation*}
\mathrm{B}_{\mathrm{i}}=1 / \sum_{\mathrm{h}} \mathrm{P}_{\mathrm{ih}}{ }^{2} \tag{7}
\end{equation*}
$$

which is a measure of the spread of specie $i$ over all environments. Abundant species therefore have broader niches, i.e., $B$ tends to be larger.

The niche dimension or diversity is computed for a fixed environment over all species. Therefore, let $f_{i n}$ be proportion of individuals of species i in envirônment $h$ of all individuals in the same environment $h$. The niche diversity is therefore:

$$
\begin{equation*}
D_{h}=-\sum_{i} f_{i h} \log f_{i h} \tag{8}
\end{equation*}
$$

The dimensionality of the niche therefore depends on the degree to which the species divide the total environment among themselves. Levins [20] also structures the competition structure between species by estimating the probability that two species will interact. These interaction coefficients are then used in growth-death equations of the species. Thus, the equations for specie i is given by:

$$
\begin{equation*}
\frac{d x_{i}}{d t}=\frac{r_{i} x_{i}}{K_{i}}\left(k_{i}-x_{i} \sum_{j} \quad d_{i j} x_{j}\right) \tag{9}
\end{equation*}
$$

where $X_{i}$ is the $i^{\text {th }}$ specie, $r$ is the growth rate, $K$ is the equilibrium population (the "carrying capacity"), and the $\alpha_{i j}$
are $[20]$ :

$$
\begin{equation*}
\alpha_{i j}=\sum_{h} p_{i h^{\prime}}{ }_{j h}\left(B_{i}\right) \tag{10}
\end{equation*}
$$

The coefficients $\alpha_{i j}$ are known as the competition coefficients. Transport terms have, of course, theen dropped from Equation (10). Under steady state, the $i^{\text {th }}$ equilibrium community is:

$$
\begin{equation*}
k_{i}=x_{i}+\sum \alpha_{i j} X_{j} \tag{11}
\end{equation*}
$$

or, in matrix form:

$$
\begin{equation*}
[A](X)=(K) \tag{12}
\end{equation*}
$$

where (X) and (K) are column vectors of species and carrying capacities, respectively, and the community matrix is:

$$
[A]=\left[\begin{array}{ccccc}
1 & \alpha_{12} & \alpha_{13} & \cdot & \cdot \\
\alpha_{21} & 1 & \alpha_{23} & \cdot & \cdot \\
\alpha_{31} & & & 1 & 1 \\
\cdot & & & &
\end{array}\right]
$$

Equation (12) can be solved for ( $K$ ) if present species numbers $\alpha_{\text {j.'s are }}$ known. The carrying capacity vector ( K ) is therefore a useful measure of the degree which a specie has filled the environment. Lane and McNaught [21] and Lane [22] have applied these analyses to zooplankton communities in Lake Michigan. The analyses permit the determination that habitat selection, rather than resource allocation, is operative in the zooplankton structure. The habitat selection is accomplished though diel vertical migration.

It should be noted that the theory of the niche is still in its infancy. As a consequence, there is still a large measure of subjectivity associated with the theory. This is particularly true for the determination of the environment groupings. No a priori objective selection of environments can yet be made, and it is obvious from the above that such selections can influence results markedly. Moreover, the theory does not yet result in a predictive framework, but rather has aided in further understanding and recasting of observed data.

Evaluation of Model Status

Model Availability. As indicated above, there are a variety of ecological models available, most are in a linearlized compartment form. Although gaps exist in the detailed knowledge of species interaction, a considerable body of knowledge exists on which to base a meaningful framework. Unfortunately, such a framework to date has relied exclusively on the observed data and very little prediction of the consequences of environmental actions has been attempted.

Data Availability. The extent of data availability specifically related to transfers between ecological compartments is not very great. Isolated regional studies have been conducted but there has not generally been any Great Lakes-wide attempt at ecological modeling. Data that exists from other sources may be applicable to the Great Lakes Limnological Systems Analysis. For example, $0^{*}$ Neil [23] has compiled a variety of transfer matrices that may be useful in linear compartment analysis.

The distribution of radioactivity within the food web in the Great Lakes has been studied only to a limited extent. Risley and Abbott [24] have described gross beta levels in plankton and sediments for Lake Erie, and Risley [25] for Lake Michigan. The Michigan Water Resources Commission has measured the distribution of radioactivity in Lake Michigan and tributary streams as well as in the plankton, periphyton,
filamentous algae, crayfish, and minnows in the immediate area of the Big Rock Nuclear Power Plant. During 1969 and 1970, Ayres [16] conducted an extensive survey of the distribution of radionuclides in Lake Michigan. The work was supported by several pówer companies and was conducted to evaluate and forecast the damages that would result from the expansion of nuclear fuel plants on Lake Michigan. Radiological analyses were performed on the water, benthos, phytoplankton, zooplankton, sediment, and fish. Concentrations of Cs-l37, $\mathrm{Zn}-65, \mathrm{~K}-40$, Ra-226, and gross beta activity were obtained.

Model Verification. As noted above, ecological models to date are almost totally descriptive in nature and have not been verified in the usual sense. That is, the transfer coefficients between compartments have not been used to verify model output against observed data. The capacity of ecological models for predicting environmental changes is, therefore, not known.

This lack of verification is generally serious. As stated, it implies a reduced utility of the models in answering planning questions. Nevertheless, for the first time, attempts are being made to provide some structure to-the complex nature of the ecological systems. At the very least, then, although ecological modeling is in its infancy, it provides some basis for elucidating interactive variables, the degree of interactive variables, and the degree of interaction and importance of the various ecological variables.

Model Application in Planning. Ecological models have generally not been applied in a limnological planning context. This is due primarily to the largely descriptive nature of the present state of the art. However, once an ecological model is constructed as outlined above it may provide a useful basis for planning purposes. This would be especially so if the ecological modeling framework were imbedded in or attached to portions of the Limnological Systems Analysis that are better understood.
${ }^{[1]}$ Watt, K.E.F., Ecology and Resource Management: A Qualitative Approach, McGraw-Hill Book Company, Inc., $\bar{N} e w$ York, New York, p 450 (1968).
[2] Dale, M.G., Systems Analysis and Ecology, Ecology Volume 51, No. $1, \mathrm{pp} \mathrm{1-16}$ (1970).
[3]Patten, B.C., "Ecological Systems Analysis and Water Quality," Proc. 3rd Annual Symp. on Water Resources Research, Ohio state, $\overline{\mathrm{p}} 37$ (1967).
[4] Patten, B.C., Editory, Systems Analysis and Simulation in Ecology, Academic Press, New York, p 607 (1971).
[5] Mankin, J.B., and Brooks, A.A., Numerical Methods for Ecosystem Analysis, ORNL-IBP 7l-1, Oak Ridge National Laboratory, Oak Ridge, Tennessee, p 99 (June 1971).
[6] O'Neill, R.B., et.al., A Preliminary Bibliography of Mathematical Modeling in Ecology, ORNL-IBP-70-3, Oak Ridge National Laboratory, Oak Ridge, Tennessee, p 97 (October 1970).
[7] Funderlic, R.E., and Heath, M.T., Linear Compartmental Analysis of Ecosystem, ORNL-IBP-71-4, Oak Ridge National Laboratory, Oak Ridge, Tennessee, p 50 (August 1971).
[8] Williams, R.B., "Computer Simulation of Energy Flow in Cedar Bog Lake, Minnesota Based on the Classical Studies of Lindeman," in Systems Analysis and Simulation in Ecology, B.C. Patten, Ed. , Volume I, Academic Press, pp 543-582.
[9] Eberhardt, L.L., "Similarity, Allometry, and Food Chains," J. Theor. Biol., 24:43-55, (1969).

## REFERENCES

 (continued)${ }^{[10]}$ Bledsow, L.H., and Van Dyne, G., "A Compartment Model Simulation of Secondary Succession," in Systems Analysis and Simulation in Ecology, B.C. Patten, Ed., Volume I, $\overline{\text { Aca }}$ demic Press $\overline{\mathrm{pp}} \overline{480-485 \text { (1971). }}$
[ll] Bloom S.G., et.al., Mathematical Simulation of Ecosystem - A Preliminary Model Applied to a Lotic Freshwater Environment, Bat $\overline{t e l l}$ Memorial Institute, Columbus Laboratory, Columbus, Ohio, p 32 (1969).
[12] Eberhardt, L.L., et.al., "Food Chain Model for DDT Kinetics in a Freshwater Marsh," Nature, Volume 230, pp 60-62 (March 1971).
[13] Harrison, et.al., "Systems Studies of DDT Transport," Science, Volume 170, (October 1970).
[14] Kaye, S.V., and Ball, S.J., "Systems Analysis of a Coupled Compartment Model for Radionuclide Transfer in a Tropical Environment," Proc. 2nd National Symposium on Radiology, Ann Arbor, Michigan pp 731-779 (1967).
[15] Riley, G.A., "Theory of Food-Chain Relations in the Sea," in The Sea, (M.W. Hill, Ed.) Volume $2 \mathrm{pp} 438-463$ (1963).
[16] Ayers, J.C., Lake Michigan Environmental Survey, Special Report No. 49, G.L.R.D., University of Michigan (1970).
[17] Gustafson, "Future Levels of Tritium in the Great Lakes from Nuclear Power Production," Proc. 13th Conf. G.L.R.D., Part 2, p 839 (1970).
${ }^{[18]}$ Sollins, P., CSS: A Computer Program for Modeling Ecological Systems, $\overline{O R N L}-\overline{1} B \bar{P}-71-5$, 0 ak Ridge $\overline{N a t i o n a l ~ L a b o r a t o r y, ~}$ Oak Ridge, Tennessee p 96 (August 1971).

REFERENCES (continued)
[19] O'Neill, R.V., Error Analysis of Ecological Models, Decid. Forst Brochure, Memo Report \#71-15, Oak Ridge National Laboratory, Oak Ridge, Tennessee, p 31 (September 1971).
[20] Levins, R., Evaluation in Changing Environments, Princeton University Press, Princeton, New Jersey, p 120 (1968).
[21] Lane, P.A., and McNaught, D.C., "A Mathematical Analysis of The Niches of Lake Michigan Zooplankton," Proc. 13th Conf. IAGLR, pp 47-57 (1970).
[22] Lane, P.A., "A Comparative study of the structure of Zooplankton Communities," pH.D. Thesis, State University of New York, Albany, New York p 202 (Unpublished).
[23] O'Neill, R.V., Examples of Ecological Transfer Matrices, ORNL-IBP-71-3, Oak Ridge National Laboratory, oak Ridge, Tennessee, p 26 (June 1971).
[24] Risley, C., Jr., Abbott, W.L., "Radioactivity in Lake Erie and its Tributaries," Proc. 9th Conf. GLRD, (1966).
[25] Risely, C., Jr., "Radioactivity in Lake Michigan and its Tributaries," Proc. 8th Conf. GLRD, (1965).

## SECTION VIII

MODEL SYNTHESIS FOR PLANNING NEEDS
VIII. MODEL SYNTHESIS FOR PLANNING NEEDS

As indicated in Section VI, the main thrust of the Limnological Systems Analysis is the application of problem oriented models to specific planning situations. The preceding review of the state of the art of available models indicates that a variety of models and submodels have been developed in various degrees. In general, the modeling frameworks tend to be concerned with specialized areas. There is a notable absence of integrated and synthesized modeling frameworks aimed at Great Lakes planning problems. The purposes of this section are to summarize the existing model status, describe the process of model synthesis, and examine the level of computational feasibility of interactive modeling frameworks.

## Summary of Evaluation of Model Status

The ranking of each of the preceding eleven modeling frameworks is summarized in Table 16 and Figure 41. It should be recalled that this ranking was performed by using the relative weights of key aspects in the evaluation of model status, as shown in Figure 14, Section VI. Further, although cost is not explicitly included in the above individual analyses, it is included implicitly in the evaluation, i.e., a relatively low score reflects, among other things, the difficulty of bringing a specific modeling framework to a point where it can be useful as a planning tool.

As indicated previously, each of the eleven modeling frameworks includes, in varying degrees, submodels of component phenomena. Thus, the eutrophication modeling framework includes a submodel of nutrient flow in the water column which in turn includes a model of the water circulation. In addition, the ranking shown in Table 16 represents a type of average ranking over all the lakes. Although geographical differences do occur in some of the modeling frameworks, the ranking is an attempt to present an overall picture of relative modeling status.
TABLE 16
RANKING OF MODELING



| Planning <br> Application <br> $(0-10)$ |
| :---: |
| 8 |
| 5 |
| 3 |
| 7 |
| 5 |
| 5 |
| 0 |
| 5 |
| 2 |
| 0 |


S
1 1

| PROBLEM AND MODEL CONTEXT |  |
| :---: | :---: |
| 1 HYDROLOGICAL |  |
| 2 thermal balance |  |
| 3 ICEaLAKE WIDE TEMP |  |
| $4 \begin{aligned} & \text { LAKE CIRCULATION } \\ & \text { AND MIXING }\end{aligned}$ |  |
| 5 SEDIMENT-EROSION |  |
| 6 CHEMICAL |  |
| 7 EUTROPHICATION |  |
| 8 DISSOLVED OXYGEN |  |
| 9 PATHOGENS a VIRUS NDICATOR BACTERIA |  |
| 10 fishery |  |
| 11 ECOLOGICAL AND |  |
|  | considerable expenditure and research effort required. no presently viable modeling framework, CONCEPTUAL FRAMEWORK MAY BE UNTESTED: SOME KEY VARIABLES OR PHENOMENA LACKING. MARGINAL VERIFICATION AND/OR APPLICATION. <br> planning applications direct. some adoitional effort required for great lakes problems. |

status
FIGURE 41

The ranking as shown in Figure 41 indicates that two modeling frameworks are in the upper level of a marginal status: hydrological balance and lake circulation and mixing. This reflects the longstanding scientific and planning interest in the problems associated with these areas. At the other end of the scale, two models are in the poor to poor-tomarginal category: ice and temperature, specifically the modeling of ice buildup and breakup phenomena, and general ecological and food chain models. The latter represents the lowest modeling development status of the eleven analyzed models and reflects the relative recent interest in constructing models related to the aquatic ecosystem. The low score is primarily a result of little available data and no verification analysis of the ecological models. The remaining seven groups fall in the intermediate category representing, as indicated, the fact that some key variables or subproblem contexts have not been adequately modeled. No model framework fell in the advanced development stage, primarily because some subproblems have not been modeled in sufficient detail for planning applications.

Model Synthesis

The essence of the process of model synthesis is to interrelate individual submodeling frameworks into a structure that is aimed at a particular planning problem. The steps followed in model synthesis in this study are shown in Figure 42.

As indicated, seven general categories of water resource problems are identified. These categories are listed in Table 2, Section IV, together with the associated water use interferences. The eleven modeling frameworks shown in Figure 42 are identified with the frameworks analyzed in Section VII and are ranked in Figure 41. For each problem category, selections were made from the individual eleven modeling areas to provide a synthesized modeling structure. The overall rank of the modeling structure is the arithmetic average of the ranks of the component models. The computational feasibility of implementing each modeling

FIGURE 42
STEPS IN MODEL SYNTHESIS
structure was then analyzed in terms of the total number of equations to be solved simultaneously. This step is discussed more fully below. The results of the model synthesis for each problem category then provide an important input to the formulation of Alternative Phase II plans as presented in Section $X$.

An example of the process of model synthesis is shown in Figure 43. The three component modeling frameworks for problem category \#4, toxic and harmful substances, are \#4 Lake Circulation and Mixing, \#6 Chemical, and \#ll Ecological and Food Chain. The average ranking of this synthesized modeling structure is 42 , placing the structure in Development Stage II.

The results of the application of the process of model synthesis to each of the seven problem categories are given in Table 17.

It should be noted that if a modeling context is in a specific developmental stage it does not necessarily imply an accompanying need for action in that area. Thus, although the ice and temperature modeling sector has a poor status, this does not imply that effort should necessarily be expended in a Phase II study to improve the status of that modeling framework. The priority of the problems must be interacted with the models and the model status to determine candidates for inclusion in further study. This interaction requires the specification of problem priorities. The subjective establishment of such priorities as part of this study is given in a subsequent section.

With respect to the state of model development, it is concluded from the above analysis that several frameworks are at a stage where certain planning problems can be approached on the Great Lakes. Other dimensions of planning problems, however, will require significant additional effort to bring the modeling to a point useful for planning. The analysis also indicates the need for model synthesis because so many efforts in the past have been fragmented and directed to more narrowly-conceived aspects of planning problems.

## TABLE 17

PROBLEM CATEGORIES AND RELATED MODELING FRAMEWORKS
$\left.\begin{array}{lll}\begin{array}{c}\text { Problem Categories } \\ \text { (See Table 2) }\end{array} & & \text { Component Modeling Frameworks }\end{array} \begin{array}{l}\text { Average } \\ \text { Rank }\end{array}\right)$

Determination of Computational Feasibility

The construction of large, interactive Limnological Systems Analysis requires not only that the component models be available, but that they can be easily interfaced. Furthermore, it is highly desirable that as many models as possible be implemented using the same modeling framework and computer programs. That this may be the case is suggested by the fact that the majority of the models are based on the principle of conservation of mass or energy (See Section VI). The exceptions are hydrodynamic equations which conserve momentum and mass and the equilibrium chemical relations which, in addition to mass conservation, also are constrained by a free energy minimum condition. The equations which are based on the conservation principle are quite similar in their mathematical expressions and their modeling structures.

This fact leads to a number of simplifications which can be exploited in the synthesis of individual models into larger frameworks and in the design of generalized computer programs for their implementation. Each conservation law is expressed as a partial differential equation in time and space variables. In the numerical solution of these equations, the spatial domain is considered to be a grid, and the equations are written in terms of finite differences. For steady-state conditions, the result is a set of simultaneous algebraic equations. If the equations are also linear, their solution is straightforward. For non-steady-state models (linear or nonlinear), the result in explicit finite difference equations specifies the manner in which the solution evolves in time. Again the solution of such sets of equations is straightforward. Thus, if computer programs (software) are designed which can solve both the steady-state linear equations and the non-steady state equations, considerable savings in development effort can be effected. Furthermore, if the software is designed to be flexible, so that it can be used to structure the appropriate equation sets for each planning model as required, additional advantage is gained. These observations lead to the recommended computer program development: the available models can all be developed using the same computer programs, the exceptions being the hydrodynamic and the equilibrium
chemical models. Although the hydrodynamic model is indeed based on the conservation principle, specialized software has already been developed by many scientists and engineers for its solution. For the chemical model, the resulting mathematical problem is equivalent to a set of non-linear algebraic equations with special properties, for which special solution techniques and software are also available. The only remaining problem is one of interfacing, which can be easily resolved in view of theoretical similarities discussed above.

In order to gauge the complexity of an integrated model, the notation of a compartment has been evolved. A compartment is defined as one dependent variable at one grid point. The dependent variable could be the magnitude of a velocity, the concentration of a substance, or the number of organisms; and the grid point is a specific location in any of the three dimensions in the lake. Thus, a series of five dependent variables at 500 horizontal locations and at two depths in the lake constitute a 5,000 compartment model. In the eutrophication model discussed in the next section, seven variables are modeled in seven spatial segments. This represents a forty-nine compartment model. Data have been obtained on the computational running times of a variety of models, principally in atmospheric and hydrospheric simulations. The number of compartments was determined in each case. Some results are shown in Figure 44 , which indicate there is a tendency for the larger machines to be more efficient in the evaluation of a compartment step.

The utility of the concept stems from the fact that over a wide range of model compartment types, the total time required to execute a computation on similar third generation computers is approximately proportional to the number of compartments in the model. Using this relationship, it is possible to estimate the execution time of a model based on the number of compartments involved, the time step used in the simulation, and the total real time of computation. Thus, for a one year simulation with a computational time step of 0.1 days, the relationship between number of compartments and central processing unit (CPU) time required for execution is that which is given in Figure 45. The upper solid line corresponds to the large third generation computers, the lower solid line to the newly developed pipeline or parallel


FIGURE 45
TIME TO COMPUTE ONE YEAR SIMULATION
$\Delta t=0.1$ DAY
(CPU TIME)
processors. The execution time for a seasonal simulation of 50,000 compartments (3-10 hours) appears to be the feasible limit. Most current large numerical meteorological and hydrodynamic models appear to have this range as their upper limit of execution time per run.

Figure 45 also contains the probable upper limit of computational power that appears to be technically feasible today, the dashed lines representing the ultimate computer [7]. For such a machine, 1,000,000 compartments and beyond become feasible.

Figure 46 is another representation of computation complexity versus execution time. For various grid sizes, the computational time required for a given number of dependent variables is indicated for two lake sizes. For this figure, it is again assumed that the simulation is for one year at a time of $t=0.1$ day. In addition, it is assumed that a five layer model is contemplated with the various horizontal spacings indicated. Based on the surface area of each of the Great Lakes, the number of compartments required for a given number of water quality variables is calculated. The number in parenthesis is the number of spatial compartments based on the horizontal spacing and the five levels assumed.

Assuming the feasible limit at ten hours of execution time, 5-10 variables would be the feasible limit for a five km spacing; 15-40 variables for a ten km spacing; and 16-150 variables for a twenty km spacing. For development purposes, it is reasonable to use an order of magnitude below the limits so that the number of variables allowed decreases accordingly. For this condition, a five km model is infeasible; a ten $k m$ model with l-4 variables and a twenty km model with 6-15 variables are the feasible development models.


* NUMBER OF SPATIAL COMPARTMENTS FOR INDICATED GRID SIZE

FIGURE 46
TIME TO COMPUTE ONE YEAR SIMULATION AT FIVE LEVELS IN DEPTH FOR VARIOUS HORIZONTAL SPACINGS

$$
\Delta t=0.1 \text { DAY }
$$

## REFERENCES

[l] O'Connor, D.J., Thomann, R.V., DiToro, D.M., "Dynamic Water Quality Forecasting and Control," Environmental Engineering and Science Program, Manhattan College, (1972).
[2] West, R.T., "A Baroclinic Prognostic Numerical Model of the Circulation in Gulf of Mexico," Ph.D. Dis., Texas A\&M University, p 66 and ix (May 1970).
[3] Kasahara, A., "Computer Experiments in the Global Circulation of the Earth's Atmosphere," Fall Joint Computer Conf., 1968, AFIPS, Vol. 33, Thompson Book Company, Washington, D.C. (1968).
[4] Friedrich, H.J., "Preliminary Results from a Numerical Multilayer Model for the Circulation in the North Atlantic," UDC, 551. 465.45, pp 145-164 (1969).
[5] Langlois, W.E., "Digital Simulation of the General Atmospheric Circulation Using a Very Dense Grid," Fall Joint Computer Conf., 1971, AFIPS Press, Montvale, New Jersey, pp 97-103 (1971).
[6] Mintz, V., "The General Circulation and Long Range Prediction," Advances in Numerical Weather Prediction 19651966 Progress Report, Travelers Research Center, Hartford, Conn.
[7] Ware, W.H.; "The Ultimate Computer," IEEE Spectrum pp 8491 (March 1972).

## SECTION IX

## DEMONSTRATION MODEL

## SECTION IX

DEMONSTRATION MODEL

## Introduction

In order to illustrate the steps of model construction, synthesis, verification, and application to planning problems, an example - a demonstration model - is presented in this section. The modeling structures are formulated to illustrate a range of time-space scales as well as a range of significant limnological planning and problem settings. The primary emphasis in the demonstration model is directed toward the high priority problem of lake eutrophication and the interaction of other water resource variables within this problem context. The spatial scales included in the demonstration models are the entire Great Lakes, considered as completely mixed systems, and Western Lake Erie with a spatial scale from 5 to 40 km . The time scales considered include decades which are associated with Great Lakes wide space scales, seasonal variations of water resource variables in Western Lake Erie, and steady-state distributions.

Figure 47 illustrates the separate submodels of the overall demonstration model. The input and output variables associated with the framework are listed in Figure 48. As indicated, five submodels are formulated and analyzed with major emphasis on the eutrophication model. The models and the associated problem structure are:

1. Total dissolved solids (TDS) and chloride model - Great Lakes scale: this submodel permits estimation of long range buildup of dissolved solids due to such factors as municipal and industrial discharges as well as the secondary effects of urban growth, such as the use of salt for deicing. High concentrations of TDS affect the quality of water for municipal and industrial water supply purposes and can have secondary effects on the chemical balances of the lakes.


FIGURE 48
PRIMARY INPUT OUTPUT VARIABLES
DEMONSTRATION
MODEL


$$
\begin{aligned}
& \text { OUTPUT VARIABLES } \\
& \text { TDS - CHLORIDES } \\
& \text { BACTERIA } \\
& \text { ZOOPLANKTON BIOMASS } \\
& \text { PHYTOPLANKTON BIOMASS } \\
& \text { NITROGEN } \\
& \text { PHOSPHORUS } \\
& \text { CADMIUM }
\end{aligned}
$$

2. Chloride model for Western Lake Erie: this submodel provides a means for determining a valid steady-state water circulation pattern for Western Lake Erie using chloride as a tracer. In addition, with output from the previous submodel, estimates can be made of the long term effects of increased TDS and solids on the municipal water supplies of the local regions in the Western Lake Erie.
3. Bacteria model: the coliform bacteria demonstration model is constructed to highlight its role in comprehensive planning. The problem addressed relates to the water quality of bathing areas. Attention is directed toward the Western Lake Erie region. The submodel uses the water circulation pattern determined from the chloride submodel.
4. Eutrophication model for Western Lake Erie: primary attention is directed toward construction of this non-linear, non-steady-state model of lake eutrophication. The motivation for this effort is the high priority assessment associated with problems of increased lake fertilization. The model draws on the chloride submodel output which verifies a water circulation pattern for Western Lake Erie. The primary input variables include the rates of discharge of nutrients from municipal, industrial, and agricultural sources. The primary output variables include phytoplankton and zooplankton biomass and nutrient concentrations. The structure illustrates the utility
of such models in the planning of large scale nutrient removal programs and assesses the interactions which exist with other water resource variables, such as lake levels and river inflow.
5. Food chain model: this submodel is constructed to illustrate the methods employed and restrictions implied in the construction of linear food chain models which relate to problems of the concentration of potentially toxic substances in aquatic ecosystems. Cadmium is selected as the toxic substances for this example. Steadystate conditions are analyzed for seven segments in Western Lake Erie. The growth characteristics of the phytoplankton and zooplankton generated by the eutrophication model are used. Cadmium is traced through the water, phytoplankton, zooplankton, fish, and lake birds sections of the aquatic ecosystem. The model illustrates the linking of non-linear and linear submodels as well as the difficulty of verifying food chain models.

The framework of the entire demonstration model illustrates the importance of model synthesis - the process of formulating an interactive model structure from a number of available submodels. In terms of compartments, a total of eighty-five interactive compartments are formulated in the eutrophication and food chain models. An additional one hundred eighty compartments are analyzed, although they are not interactive simultaneously.

In the development of the demonstration model, emphasis is placed on the three major steps of limnological systems analysis: (a) model construction based on known phenomena and laws, (b) model verification using whatever observed data is available, and (c) model application to real problem settings.

Model of Chloride and Total Dissolved Solids

An increase in the concentration of conservative substances, such as chlorides and total dissolved solids, has been observed in the Great Lakes over the past 100 years [1]. That change, both as observed in the past and projected into the future, may be analyzed on the basis of assuming the Great Lakes to be a chain of completely mixed bodies of water in which the concentration of the constituent is considered to be uniform spatially within the time scale of the analysis. The model is used to illustrate an application to both the assessment of future water quality conditions and the evaluation of present water quality standards [2,3].

Basis of Analysis

Consider a lake, the volume of which is $V$, receiving fresh water flow, $R$, from the rivers in the drainage basin and, in some cases, an inflow, I, from the upstream lake. The mass rate of waste discharged from the population and industrial sources is $W$, which may be composed of a number of individual components. It is clear that both the flows and the volume ( $R, I, V_{r}$ ) are variable in time, yet considering the time scale in question, these variations are assumed to have a minimal effect. The long term pattern of flow and volume is characterized by constant values upon which are superimposed cyclic or random variations. A mass balance is constructed taking into account the inflow and outflow and the various sources and sinks of material. The differential equation which expresses this mass balance is:

$$
\begin{equation*}
\frac{d c}{d t}+\frac{c}{t_{0}}=\frac{W(t)}{v} \tag{1}
\end{equation*}
$$

where $c(t)$ is the annual average concentration. The wastewater discharge, $W$, is a time variable quantity and the flows and volume are assumed constant in accordance with the above analysis. The parameter, $t$, is the detention time, $V / Q$, in which $Q$ is the flow out of the lake.


#### Abstract

In order to evaluate the effect of inputs to upstream lakes on water quality in downstream lakes, it is convenient to consider a series of completely mixed bodies. The output from one acts as an input to the downstream lake, which in turn feeds the next. Identifying the lakes from 1 (the first, $\mu$ pstream) to $n$ (the most downstream) the equation for the $n$ lake is:


$$
\begin{equation*}
\frac{d c_{n}}{d t}+\frac{c_{n}}{t_{o n}}=\frac{Q_{n-1}}{V_{n}} c_{n-1}+\frac{W_{n}(t)}{V_{n}} \tag{2}
\end{equation*}
$$

where the input from the upstream lake is $Q_{n-1} C_{n-1}$.

Verification

In order to apply the above equations in a quantitative fashion, it is necessary to assign values to the various parameters, $V_{n}$ and $Q_{n}$, and the inputs, $W_{n}$. These values are abstracted from references [4,5,6]. The major components of the water balance of the lakes are the inflows and outflows, runoff, and precipitation and evaporation, the magnitudes of which vary considerably from year to year. A water balance is shown in Figure 49 which indicates the average values of all components for the period 1900-1960. The volumes, flow rates, and detention times are presented in Table 18.

The magnitude of the chloride inputs are derived from municipal, industrial, and road deicing sources, which are superimposed on a background concentration due to natural sources within the hydrologic structure $[7,8,9]$. The time variable nature of the problem is due to the increase over the century of the population and the associated industrial growth. These data are taken from the above references and amplified by additional data and analysis, based on the historical increase of chlorides and total dissolved solids in the lakes.

hYDROLOGICAL BALANCE

| Lake | Volume cubic miles | TABLE 18 | Detention Time in years |
| :---: | :---: | :---: | :---: |
|  |  | PARAMETERS |  |
|  |  | Mean flow cubic feet/second |  |
| Superior | 2,940 | 71,800 | 191 |
| Michigan | 1,170 | 55,000 | 99.1 |
| Huron | 850 | 175,400 | 22.6 |
| Erie | 113 | 201,000 | 2.6 |
| Ontario | 404 | 238,000 | 7.9 |

The computed concentration in accordance with the model is shown in Figure 50 with various sets of observations for the five lakes. The contribution of each of the components is shown in Figure 51 for Lakes Erie and Ontario. An equivalence between the chloride and total dissolved solids exists and a comparable analysis of the total dissolved solids may also be performed.

Application

The calculated TDS mass discharge rates in each basin for 1970 were extrapolated from the growth projections provided by the Great Lakes Basin Commission Staff. For the Regional Development Objective (DEV), the National Economic Development Objective (NED), and the Environmental Quality Objective (ENV), the projected mass discharges of total dissolved solids are inputed to the TDS Great Lakes model. This step is an illustration of the need to convert alternative management strategies, in this case rates of population and economic growth, into discharges to the Great Lakes system. The model output is shown in Figure 52 for the period 1970 to 2000. Also shown is the IJC and U.S.P.H.S. standard for the lake. This is an example of the comparison of model output to a standard or policy objective. It is evident from this figure that either control requirements for TDS removal or a more rigorous evaluation of standards will be needed for the future. The output can also provide an indication of the time horizon required for action.

The model has also been applied to illustrate the methods of testing alternative control policies. Under the NED population projections, the effect of estimated municipal and industrial consumptive and evaporative losses proved to be minimal as did the effect of fifty percent diversion of out-of-basin municipal flows. However, the results of fifty percent removal of in-basin TDS loads into Lake Michigan and Lake Erie, which is equivalent total industrial control in these two basins, had a significant effect as demonstrated for Lake Erie in Figure 53. In this example, compliance with the existing IJC standard is obtained for approximately forty years by the control alternative.


SOURCE COMPONENTS FOR LAKES ERIE \& ONTARIO

SヨYヲา ヨヘI」<br><br>OG ヨyחפIt






Lake Erie Western Basin Chlorides and Coliform Models

A successful Limnological Systems Analysis must deal with a variety of problems and associated variables which are characterized by various time and space scales. The purpose of this demonstration model is to illustrate the techniques and principles which are available to construct models that have a relatively small spatial scale and for which the assumption of temporal steady-state can be made. An illustration is also provided of the techniques whereby hydrodynamic modeling results can be successfully used in the specification of the transport regime. The resulting transport structure can then be used to calculate the distribution of water quality variables.

Hydrodynamic Model

The hydrodynamic model of Lake Erie developed by R.T. Gedney [10] for calculation of wind driven currents in Lake Erie is used. This model is described in Section VII of this report. Model results were obtained detailing the horizontal currents produced in the Western Basin of Lake Erie for a given wind condition at a series of depths. The magnitude and direction of these predicted currents compare favorably with observed current information [11] as shown in Figure 54. This comparison was undertaken because the Gedney model was verified only for main lake circulation. The resulting agreement is quite encouraging because the data used for the comparison were not available during the hydrodynamic model construction and the precise wind conditions which correspond to the observations were not used for the model calculations.

In order to use the calculated three dimensional velocities for a two dimensional water quality model it is necessary to calculate the depth averaged velocities. The resulting depth averaged net horizontal circulation pattern is shown schematically in Figure 55. Average velocities range from less than $0.1 \mathrm{ft} / \mathrm{sec}$ to $0.5 \mathrm{ft} / \mathrm{sec}$ near the mouth of the Detroit River.


FIGURE 54
(
FIGURE 55

The basic principle utilized in the mathematical modeling of water quality parameters is that of conservation mass, that is, the mass in a system must be accounted for by the various sources and sinks. Such a calculation is effectively accomplished by subdividing the study area into a series of interactive volumes which are chosen on the basis of geometric characteristics. The segmentation adopted for the Western Basin is shown in Figure 56: a series of eighty-eight twodimensional segments.

Segmentations of this sort are based on a knowledge of the location of the significant gradients of the water quality variables of concern. Thus the region near the Detroit and Maumee Rivers is divided into segments smaller than the regions further removed from the main sources of the variables of interest. This procedure results in a model which has greater spatial resolution in the areas of interest for a fixed total number of segments. Because the computation time for solution of a steady-state model increases proportionally to the total number of segments to a power of between two and three, there is an effective upper limit to the size of such models. Although the computational limit of present day computers is well beyond the number of segments employed for this demonstration model, the principle is well illustrated by this example. A similar reduction in segment size is employed in the hydrodynamic model [10], for which the Western Basin grid size is one-half that of the remainder of the lake.

Having defined the physical characteristics of the system, it is possible to formulate a series of mass balance equations for each finite segment and solve for resulting steady-state equations for the concentration of the water quality variable of concern. The steady-state mass balance equation for segment $j$, for a conservative substance, is:

$$
\begin{equation*}
0=\sum_{k}\left[-Q_{k j}\left(\alpha_{k j} c_{k}+\beta_{k j} c_{j}\right)+E_{k j}^{\prime}\left(c_{k}-c_{j}\right)\right]+W_{j} \tag{3}
\end{equation*}
$$


TWO DIMENSIONAL - 88 COMPARTMENT MODEL OF WESTERN LAKE ERIE
where:

$c_{j}=$| concentration of the water quality |
| :--- |
| variable in segment $j$ |


$Q_{k j}=$| net advective flow from segment $k$ |
| :--- |
| to segment $j$ |


$\alpha_{k j}=$| weighting factor for the finite |
| :--- |
| difference approximation used |

$\beta_{k j}=1-\alpha_{k j}$

$E_{k j}^{\prime}=$| bulk dispersion coefficient between |
| :--- |


$W_{j}=$| mass input rate to segment $j$ |
| :--- |

In order to establish a value for the dispersive mass transport coefficients, $E_{k j}^{\prime}$, and to further verify the net circulation pattern employed, it is desirable to compare the concentration calculated for a conservative quantity with observation of that variable.

## Chloride Concentration Verification

The Western Basin of Lake Erie is characterized by sharp gradients of chloride concentrations due to the lateral stratification of the chloride concentration of the Detroit River. The relatively low concentrations in the central portion by contrast to the higher concentrations at the edges produces a central core of relatively low chloride concentration which persists in the central portion of the basin. The magnitude of these differences and the resulting shape of the plume are the basis for establishing the magnitude of the dispersion coefficient and assessing the validity of the calculated hydrodynamic circulation.

The circulation pattern utilized for the chloride comparison is that which results for a wind velocity of $11.8 \mathrm{mi} / \mathrm{hr}$
directed at $W 67^{\circ} \mathrm{S}$ as shown in Figure 55 , the choice being dictated by the availability of the hydrodynamic model output. This appears to be a reasonable choice for the prevailing wind direction for the summer period considered. The influence of changing wind directions on the net circulation was not investigated. The chloride concentrations for the incoming Detroit River are obtained from the IJC sampling stations [12] and FWPCA survey data [13]. The total inflowing chloride mass is $1.8 \times 10^{7} \mathrm{lb} /$ day for a Detroit River flow of 193,000 cfs. Of secondary importance is the Portage River contribution of $4.5 \times 10^{4} \mathrm{lb} /$ day at a flow of 400 cfs., which is included for completeness since the hydrodynamic model flow calculation includes the Portage. The horizontal, depth averaged, dispersion coefficient utilized for the calculations is $1.0 \mathrm{mi}^{2} /$ day, a value which appears reasonable for the Western Basin.

The comparison between observed data and the calculated profile is shown in Figure 57. The observed data are for the period June-July 1967 [13]. The resulting agreement is quite satisfactory; the calculated profile has the appropriate shape and magnitude with the central core of lower concentrations clearly delineated.

The successful verification of the observed chloride profile indicates that the major features of the transport structure of the Western Basin are known in a quantitative fashion, and this information can now be used to analyze other water quality variables, in this case, the coliform distribution.

Coliform Model Development and Verification

The modeling of the distribution of coliform bacteria and the effects to be expected from proposed remedial actions are an example of an analysis which is quite complex in principle. However, for preliminary planning purposes, an order of magnitude analysis is relatively straightforward. The complexity arises from the requirement that the growth and mortality kinetics of the coliform group of bacteria must be established in order to characterize their behavior in natural waters and these kinetics are likely to be

complex. However, for the purposes of order of magnitude calculations which are the common first step in preliminary plans, these complications are not significant because it has been found that an adequate description of their kinetics is available using a first order reaction with a rate coefficient that is dependent on temperature and the salinity of the receiving water (See Section VII).

Employing first order kinetics the conservation of mass equation which describes the concentration of coliform bacteria, $c_{j}$, in the $j^{\text {th }}$ segment is:

$$
\begin{equation*}
0=\sum_{k}\left[-Q_{k j}\left(\alpha_{k j} c_{k}+\beta_{k j} c_{j}\right)+E_{k j}^{\prime}\left(c_{k}-c_{j}\right)\right]-V_{j} K_{j} c_{j}+W_{j} \tag{4}
\end{equation*}
$$

where $K_{j}$ is the reaction rate for coliform bacteria in the $j^{\text {th }}$ segment? This equation, therefore, is identical to Equation (3) with the exception of the reaction kinetic term which indicates that the substance being considered is non-conservative and follows a first order reaction. The assumption of temporal steady-state has also been made as in the case of the chloride verification.

The reaction rate coefficient for coliform bacteria in the Western Basin is not known; thus it is necessary to use actual observations to establish its value. Figure 58 presents a comparison between the June-July 1967 FWPCA survey [13] mentioned previously and other sources [14] and the model output, using a reaction rate of l.0/day, a common value for fresh waters. The quantity of coliforms discharged into the Western Basin is established from observed data in the Detroit, Maumee, Raisin, and Portage Rivers as reported by the IJC. The resulting comparison is judged to be acceptable if the variability of coliform count data which is due to measurement uncertainties is kept in mind.

COMPARISON OF MODEL RESULTS AND OBSERVED DATA

## Planning Application

As a simple example of the type of application that can be made using this model, a calculation is presented in Figure 59 for a hypothetical planning alternative which establishes a treatment procedure that removes 99.9 percent of the coliform bacteria entering the Western Basin from the Maumee River. The resulting effect is quite localized: the region around the Maumee is noticeably improved with projected concentrations between 10 and 100 MPN/ 100 ml ; yet the majority of the basin is unaffected. The coliform which remain, however, are due not only to the residual Maumee River source but also to the effect of the other sources. Thus with such a model it is possible to quantify not only the extent to which narrowly conceived treatment alternatives can be expected to improve conditions throughout the basin, but also the extent to which there are interactions among the various sources of coliform bacteria in producing observed coliform distributions.

Lake Eutrophication Model

The eutrophication model is a small scale test of the operational feasibility of Limnological Systems Analysis. Its construction was undertaken to illustrate the principles and techniques employed and to demonstrate the utility of such an analysis, once it is available. The water resource problem addressed is eutrophication. The choice is dictated primarily by the complexity and interactive nature of the biological, physical, and chemical mechanisms which underlie the phenomena, and by the conflicting testimony of experts in various scientific disciplines - which appears to be a reflection of this complexity - as to the causes and possible cures of eutrophication in the Great Lakes. In addition, a eutrophication model is a necessary component of a successful Limnological Systems Analysis. Finally, the model as it now stands can be used in a preliminary investigation of the possible consequences of certain planning alternatives.

Model Construction - Spatial and Temporal Scales

As a compromise between a lakewide model and a small model on the scale of a harbor, a regional model is chosen. Data availability and the extent to which eutrophication has progressed suggests the choice of the Western Basin of Lake Erie as the location for the eutrophication submodel construction and verification.

The seasonal variation of biomass is the relevant time scale. This is generally the period for which eutrophication models have been developed previously and it is the time scale which characterizes a significant portion of the eutrophication problem. The spatial scale of the model is in the order of the characteristic lengths of the Western Basin, and the approximate length of the spatial segments is chosen to be approximately 40 km . The rationale for this choice and its justification are discussed subsequently.

Model Structure - Variables and Equations

The eutrophication model is structured so as to maximize its ability to respond to planning alternatives. The major variable groupings - physical, chemical, and biological that are incorporated in the model are listed in Table 19. The exogenous variables are supplied externally and the endogenous variables are computed internally. The planning alternatives that can be addressed are determined by the exogenous variables and the effects that can be elucidated are determined by the endogenous variables.

The basis of the model is a series of conservation of mass equations which relate the endogeneous variables to each other and also interrelate the exogenous and endogenous variables. The model is an extension of previously proposed phytoplankton biomass models (see Section VII) and it includes the effects of biological phenomena (predator-prey relations), chemical reactions (nitrification), and the other attendant interactions which provide the nutrients necessary for phytoplankton growth.

```
ー - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - 
                    TABLE 19
EUTROPHICATION SUB-MODEL VARIABLES
Exogenous
    Physical Variables
    Temperature
    Lake Level
    Solar Radiation
    Photoperiod
        Detroit River Inflow
        Water Clarity
    Chemical Variables
    Detroit River Chemical Quality
    Maumee River Chemical Quality
    Biological Variables
    Detroit River phytoplankton and zooplankton biomass
    Maumee River phytoplankton and zooplankton biomass
Endogenous
    Physical Variables
    Extinction coefficient (euphotic zone depth)
    Chemical Variable
    Organic Nitrogen Organic Phosphorus
    Ammonia Nitrogen
    Orthophosphate
    Biological Variables
    Phytoplankton biomass (chlorophy11)
    Zooplankton biomass (carbon)
```

A typical conservation of mass equation used in the model for concentration $c_{i j}$ of substance $i(i=1, \ldots 7$ ) in segment $j$ has the general form:

$$
\begin{equation*}
v_{j} \frac{d c_{i j}}{d t}=\sum_{k} Q_{k j} c_{i k}+\sum_{k} E_{k j}^{\prime}\left(c_{i k}-c_{i j}\right)+\sum_{k} S_{i j k} \tag{5}
\end{equation*}
$$

$V_{j}$ is the segment volume, $S_{i j k}$ is the $k^{\text {th }}$ source ( + ) or sink $(-)$ of substance $i$ in segment $j ; E_{k j}^{\prime}$ is the bulk rate of transport of $c_{i k}$ into and $c_{i j}$ out of segment $j$ for all segments $k$ adjacent to segment $j$, and $Q_{k j}$ is the net advective flow rate between segments $k$ and $j .{ }^{k}$ Numerical integration of these equations gives the seasonal distribution of the endogenous variables in each of the seven spatial segments of the model. Thus a total of forty-nine compartments are considered.

Model Construction - Transport Regimes

A basic requirement for a modeling effort based on conservation of mass is an adequate representation of the mass transport mechanisms in the Western Basin. The representation chosen for the demonstration models is based on the advectiondispersion formulation of mass transport. Advective mass transport is accomplished by the average unidirectional net motions of the water. Dispersive mass transport is accomplished by the mixing motions of the water body such as the smaller scale circulations. The mass conservation equation which results from this formulation is a partial differential equation in the time and space variables as discussed in Section VII. In order to implement the solution of such an equation on a computer it is convenient to express the equation in terms of finite differences. For the spatial variables this corresponds to dividing the water body into a series of segments or cells which are chosen so that the assumption of spatial homogeneity within each segment is reasonable. Seven segments are chosen as a compromise between the requirements of homogeneous concentrations and computational complexity.

With the number of spatial segments chosen it remains to choose their locations and size. This is dictated primarily by the requirement that the shallower coastal regions, which are the productive regions for phytoplankton, be delineated from the deeper central portions.

For the spatial scale and segmentation chosen, the advective flows are established primarily by the Detroit River inflow and its passage through the Western basin into the central basin. The assignment of these flows for the demonstration model is made primarily on the basis of observed and computed flow patterns as discussed in the detailed Western Basin steady-state model. Figure 60 presents such a circulation pattern [12] with the prevailing directions of flow also indicated.

In addition to the advective flow, it is necessary to assess the magnitude of the mixing flows between adjacent segments. This is accomplished in an indirect way by the use of a conservative tracer, in this case, chloride concentration. The procedure is to establish the mixing or exchange flows in such a way that the observed tracer concentration distribution is matched by that calculated by the model. A result of such a comparison is shown in Figure 61. This comparison is judged to be adequate within the spatial and temporal scale adopted for the demonstration model. Figure 62 specifies both the exchange and advective flows which are used for this comparison.

Model Construction - The Kinetic Structure of the Endogenous Variables

The endogenous variables which are considered in the eutrophication demonstration model are listed in Table 19. In order to construct a model which includes these variables it is necessary to specify quantitatively the variable interactions among themselves and with the exogenous variables. This is accomplished below beginning with the biological variables.


FIGURE 61
CHLORIDE CONCENTRATIONS IN WESTERN LAKE ERIE
COMPARISON OF MODEL RESULTS AND OBSERVED DATA
(20)

The Phytoplankton System. The basis of the phytoplankton equation is the principle of conservation of phytoplankton biomass which relates the rate of change of biomass, measured in this demonstration model as chlorophyll concentration, to the rates of growth and death of the population, and to the transport structure. Thus if $P_{j}$ is the chlorophylla ${ }_{a}$ concentration in segment $j$, then:

$$
\begin{equation*}
V_{j} \frac{d P_{j}}{d t}=\sum_{k} Q_{k j} P_{k}+\sum_{k} E_{k j}^{\prime}\left(P_{k}-P_{j}\right)+\left(G_{P j}-D_{P j}\right) P_{j} V_{j} \tag{6}
\end{equation*}
$$

where $E_{k j}^{\prime}$ and $Q_{k j}$ are the transport coefficients discussed previously. $G_{P j}$ is the phytoplankton growth rate and $D_{P j}$ is the phytoplankton death rate in the $j^{\text {th }}$ segment.

The formulation of the phytoplankton growth rate for a depth-averaged model is based on the following reasoning [15]: At optimal conditions of light availability and nutrient concentration the growth rate of a population is dependent on temperature only, and for moderate temperature ranges, it is directly proportional. The effect of non-optimal light intensity is to reduce the growth rate. If $I_{s}$ is the optimal or saturating light intensity, then it has been proposed [16] that the reduction in growth rate due to an intensity $I$ is given by:

$$
\begin{equation*}
F(I)=\frac{I}{I_{S}} \exp \left[-\frac{I}{I_{S}}+1\right] \tag{7}
\end{equation*}
$$

In the natural environment the light available at any depth $I(z)$, varies inversely with depth according to the equation:

$$
\begin{equation*}
I(z)=I_{0} e^{-K} e^{z} \tag{8}
\end{equation*}
$$

where $z$ is the depth (positive downward), I is the surface light intensity, and $K_{e}$ is the extinction coefficient. In
addition, the surface light intensity varies throughout the day. For the time scale of this model, however, it is adequately represented as a constant $I_{\text {, }}$, the mean daily incident solar radiation, which is incldent on the basin for fraction of a day, the photoperiod.

For a model which is depth averaged and with a time scale on the order of a week, it is appropriate to use a depth averaged, time averaged growth rate reduction factor, r, due to non-optimal light. The result, using Equation (7) and (8), is [15]:

$$
\begin{equation*}
r=\frac{e f}{K_{e} H}\left[e^{-\alpha_{1}}-e^{-\alpha_{o}}\right] \tag{9}
\end{equation*}
$$

where:

$$
\begin{align*}
& \alpha_{1}=\frac{I_{a}}{I_{s}}=e^{-K_{e} H}  \tag{10}\\
& \alpha_{0}=\frac{I_{a}}{I_{s}} \tag{11}
\end{align*}
$$

and:

$$
\begin{aligned}
& \mathrm{H}=\text { depth of the segment } \\
& \mathrm{f}=\text { photoperiod } \\
& \mathrm{K}_{\mathrm{e}}=\text { extinction coefficient } \\
& \mathrm{I}_{\mathrm{s}}=\text { optimal light intensity } \\
& \mathrm{I}_{\mathrm{a}}=\text { mean daily light intensity } \\
& \mathrm{e}=2.718 \ldots
\end{aligned}
$$

The effect of non-optimal nutrient concentrations is to further reduce the growth rate. The form of the reduction
factor chosen is the same as that adopted by Monod for bacterial growth, namely, the Michaelis-Menton expression: $N /\left(K_{\mathrm{mN}}+\mathrm{N}\right)$, where $N$ is the nutrient concentration and $K_{\mathrm{mN}}$ is the Michaelis or half-saturation constant for that nutrient [17]. Based on the available data and theoretical formulations the two nutrients considered are total inorganic nitrogen: $C_{N}\left(\mathrm{NH}_{3}+\mathrm{NO}_{3}\right.$, assuming $\mathrm{NO}_{2}$ concentrations are negligible) and orthophosphorus: $C_{p}\left(\mathrm{PO}_{4}\right)$. Based on an analysis of a set of laboratory experiments [18], and for lack of a better assumption, it is assumed that the growth rate reduction due to low nutrient concentrations is expressable as a product of two Michaelis-Menton expressions: $\mathrm{C}_{\mathrm{N}} \mathrm{C}_{\mathrm{p}} /\left(\mathrm{K}_{\mathrm{mN}}+\right.$ $\left.c_{N}\right)\left(K_{m p}+c_{p}\right)$ ) where $K_{m N}$ and $K_{m p}$ are the half-saturation constants for total inorganic nitrogen and orthophosphorus respectively. The growth rate expression is then assumed to be the product of these reduction factors:

$$
\begin{equation*}
G_{p}=K_{1}(T) r \frac{c_{N}}{K_{m N}+c_{N}} \frac{c_{p}}{K_{m p}+c_{p}} \tag{12}
\end{equation*}
$$

where $K_{1}(T)$ is the temperature dependent saturated growth rate.

The formulation of the phytoplankton death rate follows a previous analysis [15]. It is assumed that the phytoplankton biomass is reduced by its endogenous respiration, which is assumed to be proportional to the temperature, and by the grazing of the zooplankton population, which is assumed to be proportional to the zooplankton biomass concentration $Z$. Thus the death rate expression is given by:

$$
\begin{equation*}
D_{P}=K_{2}(T)+C_{g}(T) Z \tag{13}
\end{equation*}
$$

where:

$$
\begin{aligned}
& \mathrm{K}_{2}(\mathrm{~T}) \quad=\quad \text { temperature dependent endo- } \\
& \text { genous respiration rate } \\
& \text { constants }
\end{aligned}
$$



Thus equations (12) and (13) specify the growth and death rates of the phytoplankton and, therefore, also specify the behavior of the phytoplankton population's interaction with temperature, light, extinction coefficient, depth, nutrient concentrations, and zooplankton predation.

The Zooplankton System. The conservation of zooplankton biomass equation is analogous in form to that of the phytoplankton (Equation (6)):
$V_{j} \frac{d Z_{j}}{d t}=\sum_{k} Q_{k j} Z_{k}+\sum_{k} E_{k j}^{\prime}\left(Z_{k}-Z_{j}\right)+\left(G_{Z i}-D_{Z j}\right) Z_{j} V_{j}$
where $G_{Z j}$ and $D_{Z j}$ are the growth and death rates of the zooplankton population whose biomass concentration, $Z$, is expressed as its equivalent organic carbon concentration. Assuming that there is sufficient phytoplankton biomass to provide the food source for the zooplankton which affect the phytoplankton (i.e., the herbivorous zooplankton which are the zooplankton of concern) then their growth rate is directly related to their grazing of the phytoplankton which can be formulated as [15]:

$$
\begin{equation*}
G_{z}=a_{1} a_{z P} \frac{K_{m P}}{P+K_{m P}} c_{g}(T) P \tag{15}
\end{equation*}
$$

where:

$$
\begin{aligned}
& \mathrm{a}_{1}= \begin{array}{l}
\text { the conversion efficiency of } \\
\text { zooplankton }
\end{array} \\
& \mathrm{K}_{\mathrm{mP}}=\quad \begin{array}{l}
\text { the half-saturation constant for } \\
\text { the phytoplankton biomass grazed }
\end{array} \\
& \mathrm{a}_{\mathrm{ZP}}=\begin{array}{l}
\text { a conversion factor, in this case } \\
\text { the carbon/chlorophyll ratio of } \\
\text { the phytoplankton population }
\end{array}
\end{aligned}
$$

The formulation of the zooplankton death rate presents somewhat of a problem, because in addition to their endogenous respiration rate the zooplankton are being preyed upon by the upper levels of the food chain. In order to simplify the model framework it is necessary to introduce this effect empirically as an additional death rate constant. Thus the zooplankton death rate is expressed as:

$$
\begin{equation*}
D_{Z}=K_{3}(T)+K_{4} \tag{16}
\end{equation*}
$$

where:

$\mathrm{K}_{3}(\mathrm{~T}) \quad=\quad$| the temperature dependent |
| :--- |
| endogenous respiration rate |

$\mathrm{K}_{4} \quad=\quad$ empirical mortality constant

The Nitrogen System. The major components of the nitrogen system included in this demonstration model are non-living organic nitrogen, $c_{3} ;$ ammonia nitrogen, $c_{4}$; and nitrate nitrogen, C5. In natural waters there is a step-wise transformation, mediated by bacteria, of the organic nitrogen to ammonia nitrogen which itself is subsequently transformed to nitrite and then to nitrate nitrogen. The first of these steps can be an important source of inorganic nitrogen for phytoplankton growth, which is the reason for its inclusion, whereas the second step, referred to as nitrification, can have important consequences in the dissolved oxygen balance of lakes (Section VII). The kinetics of these transformations are assumed to be first order reactions with temperature dependent rate coefficients.

Two sources of detrital organic nitrogen are considered: (1) the organic nitrogen produced by phytoplankton and zooplankton endogenous respiration (the assumption being that only organic forms of nitrogen result from this process) and (2) the organic nitrogen equivalent of the grazed but not
metabolized phytoplankton excreted by the zooplankton. The nitrogen that results from these processes is not completely recycled into the nonliving organic nitrogen system because, as is shown in the verification section, the data indicate a substantial loss of total nitrogen from the Western Basin. It is hypothesized that this loss is due to settling of the particulate fraction of the total nitrogen. In order to incorporate this effect into a depth averaged formulation, only a fraction, $\beta$, of the nonliving organic nitrogen source due to phytoplankton and zooplankton processes is recycled; the remainder is assumed to be removed, presumably by settling. The other sink of organic nitrogen included in the formulation is the transformation of organic nitrogen to ammonia nitrogen and, as discussed above, this is assumed to be described by first order kinetics.

The primary kinetic source of inorganic nitrogen is via the organic nitrogen transformation into ammonia nitrogen. It is assumed that there are no direct kinetic pathways from organic nitrogen to nitrate nitrogen. The primary sink of the inorganic nitrogen forms is the phytoplankton uptake. In order to conform with the suspicion that ammonia nitrogen is preferentially used by phytoplankton, a preference coefficient is introduced: $\alpha=C_{4} /\left(K_{m N}+C_{4}\right)$ which specifies that the form of inorganic nitrogen $u$ 登lized by growing phytoplankton is ammonia ( $\mathrm{C}_{4}$ ) until its concentration reaches the vicinity of the inorganic nitrogen half saturation constant, at which point the nitrogen source shifts to nitrate ( $\mathrm{C}_{5}$ ). The algebraic forms used for these kinetic interactions are shown in Table 20 , and the conservation equations are versions of Equation (6) and (14) with Table 20 indicating which sources and sinks are included in each equation.

The Phosphorus System. The formulation of the phosphorus conservation of mass equations is somewhat simpler than the nitrogen equations because only two forms of phosphorus are considered: nonliving organic phosphorus, $c_{6}$; and orthophosphorus, C7. The mechanisms which are included parallel those for the nitrogen systems with the exception of nitrification, for which it appears there is no phosphorus counterpart. The sources and sinks which result are shown in Table 21 and the conservation equation follows Equations (6) and (14) in form.




#### Abstract

The complete kinetic interactions of the endogenous variables are shown in Figure 63. The cyclical structure of the pathways is apparent: the primary production which converts inorganic nutrients to the phytoplankton; the secondary production of zooplankton accomplished by their grazing on phytoplankton; the mortality and excretion pathways which release organic material in detrital and soluble form; the deposition pathway which accounts for whatever settling of the particulate fraction of the organic material occurs; and the regeneration pathways which convert organic forms into inorganic forms that are then available for the primary production pathway.


## Data Sources

The limnological data base for the eutrophication demonstration model has been derived primarily from the survey data collected by two groups:

1. Canadian Centre for Inland Waters
(CCIW)
2. Environmental Protection Agency (EPA)

The data encompasses the bulk of the data used in the model verification. The spatial distribution of the monitoring stations is shown in Figure 64. In general, CCIW sampling locations were visited eight times each year and the EPA stations were visited four times during 1967-1968. Additional observed data were reviewed from numerous sources including those listed in Table 22.

Observed data were retrieved from the available sources and each data set was then analyzed for its compatability with other comparable data sets and its applicability to the modeling effort. The resulting data base was then subdivided into three data types:


KINETIC PATHWAYS OF THE ENDOGENOUS VARIABLES

FIGURE 64 (IN WUTER QUALITY MONITORING LOCATIONS IN WESTERN LAKE ERIE

TABLE 22
DATA SOURCES

## Agency

1. National Ocean and Atmospheric Administration, Lake Survey Center
2. United States Geological Surveys
3. Great Lakes Research Division, University of Michigan
4. Michigan Water Resource Commission
5. International Joint Commission
6. Great Lakes Study Center University of Buffalo
7. Frans Theodore Stone Limnology Laboratory, Ohio state University
8. U.S. Bureau of Commercial Fisheries
9. Great Lakes Institute, University of Toronto

Data
temp. transparency, $\mathrm{pH}, \mathrm{Eh}$, alkalinity, $\mathrm{Cl}, \mathrm{spec}$. cond., tox.col., sediment chem., $N$ series, t.phos., ortho. p.
tributary flows
limnological data
phosphorus data - Detroit R.
lake level information, coliform data
transport data
limnological data
temp, $\mathrm{NH}_{3}, ~ a l k ., ~ \mathrm{~N}, \mathrm{NO}_{2}$, phytoplankton, zooplankton
limnological data, solar radiation data

1. Open water Iimnological data
2. Tributary stream influent data
3. Physical, meteorological, and hydrological data

Canadian Centre for Inland Waters Cruise Data

Since 1967, CCIW has been involved in data collection activities on Lake Erie. Each year a series of cruises is conducted for the purpose of monitoring a broad base of limnological variables. These cruises normally take place on a monthly schedule, beginning immediately following the ice breakup and concluding some time in November. On each cruise, 60 to 80 limnological sampling stations are visited of which approximately 10 percent are in the Western Basin of Lake Erie. At each station, samples are collected at the surface and at three meter intervals to the bottom. These samples are then analyzed for their chemical, physical, and biological properties according to the schedule presented in the Data Availability Section of this report. The CCIW data base for Lake Erie was made available for the purpose of this analysis. Data that had been collected in the Western Basin were then separated for a detailed analysis of its spatial and temporal variations.

The Federal Water Pollution Control Administration Cleveland and Detroit Program Office Limnological Data

A large body of limnological and tributary data is available through the Environmental Protection Agency STORET system. The availability of these data is summarized in the data section of this report. In 1967-1968 the Detroit Office of the EPA conducted four extensive surveys of Lake Erie's Western Basin. During the same period the Cleveland Program Office collected samples on four midlake cruises which began at Toledo Harbor and terminated at Buffalo, New York. The
overall spatial coverage in the basin totaled 38 sampling stations each of which was monitored at surface, mid-depth, and bottom. These data as well as data collected by the EPA during 1965-1966 were retrieved from STORET.

Tributary information is available through STORET for most rivers discharging to Lake Erie. The two that are of primary importance in the modeling effort are the Detroit River and the Maumee River. Both rivers are sampled on a regular basis.

Since 1913, the International Joint Commission has maintained a water quality surveillance network at the mouth of the Detroit River. Monitoring stations are located at 500 feet intervals across the width of the river. Each of the fifteen stations is visited monthly. A complete record of the IJC network data collected since 1965 is included in the STORET sys tem.

In addition, STORET is also a repository of data collected at Water Pollution Surveillance Systems network stations by the U.S. Department of Health, Education, and Welfare. Two of these stations, one on the Detroit River below Trenton, Michigan, and the other at the mouth of the Maumee, provided pertinent biological data inputs for the modeling effort.

## The Exogenous Variables

The exogenous variables which are supplied to the demonstration model are of two types: (1) the variables which characterize conditions within the Western Basin, in this case, water temperature, solar radiation intensity and photoperiod, lake level, and light extinction coefficient; and (2) the variables which characterize the conditions at the boundary of the model which in this instance are required for the seven biological and chemical endogenous variables being considered.

The variation of water temperature from April through October 1970, the period chosen for the verification, is shown in Figure 65. The data shown are from the 1970 CCIW cruises. For each model segment the variation indicated is that used in the subsequent calculations.


[^7]The photoperiod and solar radiation variation used for the period of concern in shown in Figure 66. The data indicated are for 1961 [19]. It is assumed that the data are representative of 1970 conditions. The seasonal variation of both temperature and solar radiation are major contributions to the seasonal variations of the phytoplankton growth rate during the period of non-nutrient limited growth.

The variation of extinction coefficient as a function of time also contributes substantial variations to the phytoplankton growth rate. The variations observed in secchi depth are shown in Figure 67. Secchi depth is converted to extinction coefficient using the relationship [20]:

$$
\begin{equation*}
\mathrm{K}_{\mathrm{e}}=1.9 / \mathrm{sec} \text { chi depth } \tag{17}
\end{equation*}
$$

which corresponds to the assumption that the secchi depth is the 15 percent light penetration depth. The variations assumed for two segments of the model are shown in Figure 68.

The boundary conditions employed for the modeling calculations are obtained from two sources. The open water boundaries at segments 4, 5, and 6 are obtained directly from the nearest available CCIW sampling stations outside the Western Basin. The Detroit River inflow concentrations are obtained from the available IJC data, an example of which is shown in Figure 69.

It is significant to note the marked lateral variation of concentrations across the Detroit River, with the central section concentrations being a factor of at least two lower than the shoreline region. This lateral variation is a major reason for the two shoreline regions and the central region entering separate segments as shown in boundary condition Figures 70 through 75. The Maumee River inflow quality is set using the observed variations obtained from STORET. The locations and identification of the sampling stations are shown in Figure 64. The temporal variations of the boundary condition used for the 1970 verification calculation are shown in Figures 70 through 75 with the exception of the


FIGURE 66
VARIATION OF PHOTOPERIOD AND SOLAR RADIATION



FIGURE 68
VARIATION OF EXTINCTION COEFFICIENT IN TWO COMPARTMENTS OF THE WESTERN LAKE ERIE MODEL


FIGURE 69
LATERAL DISTRIBUTION OF NUTRIENTS AT THE MOUTH OF THE DETROIT RIVER

FIGURE 70
CHLOROPHYLL CONCENTRATIONS OF TRIBUTARY STREAMS
CHLOROPHYLL BOUNDARY CONDITIONS

ORGANIC PHOSPHORUS CONCENTRATIONS OF TRIBUTARY STREAMS
ORGANIC PHOSPHORUS BOUNDARY CONCENTRATIONS



zooplankton boundaries which were set at constant values for both the Detroit River ( 0.02 mg Carbon/l) and the western boundary ( 0.4 mg Carbon/l) as estimated from the available data.

The Maumee River inflow contributions are treated as direct mass inputs because of the seasonal variation of the river flow. The values used are listed in Table 23.

In order to establish the Maumee River phytoplankton boundary condition it is necessary to have a relationship between the two measurements of phytoplankton biomass available: chlorophyll concentration and total phytoplankton cell counts. Since both these measurements are available in the Western Basin for the EPA cruises, a linear regression relating the two measurements, constrained to have zero intercept, was performed with a resulting correlation coefficient of 0.82. The regression equation is:

Total Chlorophyll $(\mu \mathrm{g} / \mathrm{l})=\frac{\text { no. of cells } / \mathrm{milliliter}}{70}$
which corresponds to the line indicated on Figure 76, a plot of the total chlorophyll versus the counts data for the EPA Western Lake Erie survelliance data, 1967-1968. The ratio of chlorophyll ${ }_{a}$ to total chlorophyll is assumed to be 0.75 .

## Verification Procedure

With the boundary conditions and exogenous variables established as in the previous sections, the effect of external variations of the variables of concern on the Western Basin are specified. It remains to establish the parameters which specify the internal kinetics of the seven dependent variables. The structure of the kinetics have been presented in Tables 20 and 21; the constants and their temperature dependence are required in order to complete the model. This is done by initially using whatever laboratory experimental data are available to set the probable range of the constants [15]. Then detailed comparisons are



FIGURE 76
RELATIONSHIP BETWEEN TOTAL ALGAL CELL COUNTS AND
TOTAL CHLOROPHYLL MEASUREMENTS IN WESTERN LAKE ERIE
made between the observed data in the Western Basin and the computations of the model in order to fine tune the values. The result of this exercise, if successful, is a set of parameter values which is compatible with the observed behavior of the phytoplankton, zooplankton, and nutrients. The kinetic parameters which result from the verification procedure are listed in Table 24. Comparisons with available experimental information and other modeling studies [21,22] indicate that for those parameters which have been investigated, the values tabulated are in the range of reported values and temperature dependencies. However, these kinetic parameters are not necessarily the unique set which gives the best verification. A computational procedure to find this set, though desirable, is beyond present capabilities for a model of this complexity and the quantity of data available.

In order to strengthen the model verification it is advisable to compare the model to a situation which was not considered in the initial verification. For this demonstration model such a comparison has been made using a composite set of data from the years 1928-1930.

Verification Results

Phytoplankton Chlorophyll. The comparison of the model calculations and the 1970 CCIW survey, data for chlorophyll are shown in Figure 77. The magnitudes and shapes of the a calculated curves are in reasonable agreement with the observations, although some systematic deviations are present. In order to appreciate the importance of the kinetic transformations, a calculation has been made assuming that chlorophyll is a conservative substance. The result for segment 7 tolgether with the verification calculation and the available CCIW data for 1967 and 1970 is shown in Figure 78. An equilibrium concentration of less than $10 \mu \mathrm{~g} \mathrm{chla/l}$ results, as compared to peak concentrations of $30 \mu \mathrm{~g}$ ch1 $\mathrm{a} / 1$ for the verification, which indicates the importance of the kinetic interactions. That is, chlorophyll would behave as a conservative variable if the net growth rate ( $G_{P}-D_{p}$ )

TABLE 24

## KINETIC PARAMETERS

## Phytoplankton

Saturated Growth Rate

$$
\begin{aligned}
& K_{1}(T)=0.1+0.06 \mathrm{~T}(/ \text { day }) \\
& I_{S}=350 .\left(I_{y} / \text { day }\right) \\
& K_{\mathrm{mN}}=25 .(\mu \mathrm{g} \mathrm{~N} / 1)
\end{aligned}
$$

Optimum Light Intensity $I_{s}=350$. (Iy/day)
Inorganic Nitrogen Michaelis Constant

Orthophosphorus
Michaelis Constant
$K_{m p}=10 .(\mu \mathrm{g} P / 1)$
Endogenous Respiration Rate
$\mathrm{K}_{2}(\mathrm{~T})=0.004 \mathrm{~T}(/$ day $)$
Grazing Rate

$$
C_{g}(T)=0.012+0.021 \mathrm{~T}(1 / \mathrm{mg} \mathrm{C-day})
$$

Zooplankton
Assimilation Efficiency $a_{I}=0.65(\mathrm{mg} \mathrm{C} / \mathrm{mg} \mathrm{C})$
Carbon - Chlorophyll Ratio

$$
\mathrm{a}_{\mathrm{ZP}}=50 .\left(\mathrm{mg} \mathrm{c} / \mathrm{mg} \operatorname{ch} l_{\mathrm{a}}\right)
$$

Chlorophyll Michaelis Constant

$$
\mathrm{K}_{\mathrm{mP}}=60 \quad\left(\mu \mathrm{~g} \mathrm{Ch} \mathrm{C}_{\mathrm{a}} / \mathrm{I}\right)
$$

Endogenous Respiration Rate

$$
K_{3}(T)=0.0007(T-5)^{2}(/ \text { day })
$$

Empirical Mortality Constant

$$
\mathrm{K}_{4}=0.015 \quad(/ \text { day })
$$

## TABLE <br> 24

## KINETIC PARAMETERS <br> (continued)

Nitrogen
Organic Nitrogen- Ammonia Rate$K_{34}(T)=0.002 \mathrm{~T}(/$ day $)$Nitrification Rate$K_{45}(T)=0.002+0.0025 \mathrm{~T}(/$ day $)$Nitrogen - ChlorophyllRatio$a_{\mathrm{NP}}=7\left(\mathrm{mg} \mathrm{N} / \mathrm{mg} \mathrm{Ch} 1_{a}\right)$Nitrogen - ZooplanktonCarbon Ratio$a_{N Z}=0.14(\mathrm{mg} \mathrm{N} / \mathrm{mg} \mathrm{C})$
Fraction Recycled $\beta=0.3(\mathrm{mg} \mathrm{N} / \mathrm{mg} \mathrm{N})$
Phosphorus
Organic Phosphorus-Orthophosphorus Rate
$K_{67}(T)=0.02 T(/$ day $)$Phosphorus - ChlorophyllRatio$a_{p P}=1 \cdot\left(\mathrm{mg} \mathrm{p} / \mathrm{mg} \mathrm{Chl}{ }_{\mathrm{a}}\right)$
Phosphorus - Zooplankton Carbon Ratio $a_{p z}=0.02(\mathrm{mg} \mathrm{p} / \mathrm{mg} \mathrm{C})$Fraction Recycled$\beta=0.3(\mathrm{mg} \mathrm{p} / \mathrm{mg} \mathrm{p})$

FIGURE 77
CHLOROPHYL VERIFICATION
COMPARISON OF MODEL RESULTS AND OBSERVED DATA


CHLOROPHYLL RELATIONSHIPS IN COMPARTMENT NUMBER 7 COMPARISON OF MODEL RESULTS AND OBSERVED DATA
were everywhere zero. Under such a situation less than $10 \mu \mathrm{~g}$ chl /l is predicted whereas actual observations are in the range from 10 to $30 \mu \mathrm{~g} \mathrm{chl}{ }_{\mathrm{a}} / \mathrm{l}$.

Zooplankton. The zooplankton biomass calculated for 1970 is shown in Figure 79. Unfortunately the CCIW zooplankton data for this period are not available at this time, therefore a direct comparison is not possible. However, an estimate of the population biomass can be made using historical data [23]. Zooplankton population counts reported by various workers are shown on Figure 80. The more recent data indicate a population of between 500 and 1000 individuals/l. If an organic carbon content of $1.5 \mu \mathrm{~g} \mathrm{C} /$ individual is used as an average between adult and juvenile forms, the observations exceed the peak zooplankton carbon concentrations calculated. Also it appears that the shapes of the calculated zooplankton biomass concentrations are somewhat different from that observed. This suggests that another food source such as detrital organic material is an important component of the zooplankton nutrient source.

Nitrogen. The verification results for the three forms of nitrogen considered are shown in Figures 81 to 83. The organic nitrogen data is a filtered measurement and therefore comparable to the soluble fraction of the nonliving organic nitrogen. Because the particulate fraction of the organic nitrogen is not directly available from the model calculations, this comparison is not precise. However, within the limited amount of available data the results are encouraging.

The ammonia and nitrate nitrogen comparisons are direct since the computed variables and the measurements correspond. The agreement between the calculations and the data is quite good, with the major features of the data being reproduced.

The significant kinetic features of the nitrogen system which are included in this model are phytoplankton uptake, organic-inorganic conversion, nitrification, and settling. The first two effects can be separated from the last since only settling removes nitrogen from the water column whereas the others are transformations for which total nitrogen is conserved. Thus for a situation where no settling occurs,

FIGURE 79
THE ZOOPLANKTON SYSTEM
MODEL RESULTS


UNITS: mg Nitrogen/L

## 18 з 3 กリー

NOIIVOI
DATA

FIGURE 82
AMMONIA NITROGEN VERIFICATION
COMPARISON OF MODEL RESULTS AND OBSERVED DATA

FIGURE 83
NITRATE NITROGEN VERIFICATION
COMPARISON OF MODEL RESULTS AND OBSERVED DATA
total nitrogen, defined as the sum of organic, ammonia, and nitrate nitrogen, is a conservative variable. If settling is not allowed in the model, the result for total phosphorus as well as total nitrogen is as shown in Figure 84. The data are the appropriate sums from all 1970 Western Basin CCIW cruises; the theoretical curves are the volume average total nitrogen and total phosphorus concentrations computed for no settling and based on the 1970 boundary conditions. The discrepancy indicates a removal mechanism and the likely candidate is settling. With settling included, the results are as indicated on Figure 84.

Phosphorus. The verification results for the two forms of phosphorus considered are shown in Figures 85 and 86. Total phosphorus data and the computed total phosphorus, the sum of the nonliving phosphorus ( $\mathrm{c}_{6}$ ), orthophosphorus ( $\mathrm{C}_{7}$ ), and the phosphorus equivalents of the phytoplankton $\left(a_{p P}{ }^{P}\right)$ and zooplankton $\left(a_{p z} Z^{Z}\right)$, are compared in Figure 86. The relative lack of change in total phosphorus is a result of its being a conservative variable, with the exception of the settling of the nonliving fraction. The behavior of the orthophosphorus is directly a result of phytoplankton uptake. The apparent discrepancy at the low phosphorus concentration ( $<20 \mu \mathrm{~g} \mathrm{PO} 4-\mathrm{P} / \mathrm{l}$ ) is probably due to the difficulty of accurately measuring orthophosphorus at these low concentration levels.

Hindcast and Verification

A hindcast to the year 1930 was employed to determine the effectiveness of the phytoplankton model in predicting the environmental effects of a set of conditions that are completely different from those employed in the verification task. The year 1930 was chosen for two reasons: first, there is a significant base of observed data collected by Wright, et.al., [24] of the U.S. Department of Interior, Fish, and Wildife Service during the period 1928-1930, and second, conditions that existed in 1930 are far enough removed from the 1970 limnological conditions in the basin that a significantly different set of events could be modeled. Data collected


FIGURE 84
TOTAL NITROGEN AND PHOSPHORUS IN WESTERN LAKE ERIE COMPARISON OF MODEL RESULTS AND OBSERVED DATA


$\backsim$




FIGURE 86
VERIFICATION OF THE TOTAL PHOSPHORUS SYSTEM
COMPARISON OF MODEL RESULTS AND OBSERVED DATA
in 1930 consists of algal cell counts, crustacean zooplankton counts, albuminoid nitrogen, free ammonia, nitrate and nitrite. No phosphorus measurements were made. A bi-weekly sampling program was conducted at 16 stations located in the Western Basin during the summers of 1928 through 1930 .

These data were analyzed in the same manner as the verification data base. Temporal plots of all variables were prepared for each model segment. Tributary influent information for phytoplankton, zooplankton, and nitrogen are available for the Maumee River. These data were used in estimating total mass discharges to the system. In addition, comparisons were made with expected per capita waste inputs and land drainage inputs. A favorable comparison results.

Because phosphorus discharge information was not available for the survey period, phosphorus inputs were established on the following basis: 1) phosphorus detergent use was insignificant during the survey period, 2) an approximation of the phosphorus mass input rate can be achieved by multiplying the total nitrogen mass input rate by an appropriate physiological phosphorus to nitrogen ratio. This ratio, assumed to be 1 mg Phosphorus $/ 7 \mathrm{mg}$ Nitrogen, was applied to the total inorganic nitrogen mass inputs to estimate the phosphorus mass input.

The absence of observed data with regard to sunlight and the light extinction properties of the water column necessitates using the values for the 1970 verification. All reaction rates and transport phenomena employed in the 1970 verification are employed in the hindcast. The results of the 1928-1930 hindcast are presented in Figures 87 through 93. The total phytoplankton counts data are converted to equivalent chlorophyll concentration using Equation (18). A reasonable agreement with observed data results for all systems. However, there is some discrepancy in the crustacean zooplankton concentrations along the western shore of the basin. The data indicate higher concentrations than are calculated. This could be due to the existence of higher zooplankton concentration at the mouths of the two tributaries, the Maumee and the Rasin Rivers, where these samples were collected, as well as an inadequate modeling of the zooplankton food sources as discussed previously.


FIGURE 88
ZOOPLANKTON HINDCAST TO 1930
COMPARISON OF MODEL RESULTS AND OBSERVED DATA


[^8]
FIGURE 90
AMMONIA NITROGEN HINDCAST TO 1930
COMPARISON OF MODEL RESULTS AND OBSERVED DATA
FIGURE 91
NTTATE NITROGEN HINDCAST TO I930
COMPARISON OF MODEL RESULTS AND OBSERVED DATA


FIGURE 93
INORGANIC PHOSPHORUS HINDCAST TO 1930
MODEL RESULTS

Hindcasting is a valuable tool in systems analysis because it provides a means of checking the creditability of the model. A model that is able to reproduce an historic set of conditions that are far removed from the verification conditions has greater reliability and can be used with more confidence to predict water quality conditions in a future situation. The results of the 1928 -1930 hindcast provide an encouraging test of the Western Basin eutrophication model.

Applications to Planning

The primary purpose for constructing a Limnological Systems Analysis is to have available a method for assessing the effects of planning alternatives. The demonstration model is a small scale example of such a planning tool and it is the purpose of this section to demonstrate some of the types of planning questions for which the eutrophication model can provide guidance. However, these applications are not intended to represent absolute projections of future conditions; they are presented for illustrative purposes only.

The increase in the eutrophication of the Western Basin over the past fifty years is well documented, as shown in Figure 94 [25]. To form a basis for developing plans to control this water quality problem it is necessary to estimate the effect of the projected increases in human population and their effect on eutrophication. Three population projections are available for the region: The Regional Development Objective (DEV) for relatively rapid growth, the National Development Objective (NED) which projects relatively moderate growth, and the Environmental Quality Objectives (ENV) for a relatively slower growth during the planning period [26]. These population projections are shown in Table 25 . The exogenous variables of the demonstration are adjusted to reflect these projections as follows: The nutrient inputs to the Western Basin are computed based on the projected increase of urban runoff and municipal and industrial contribution in


FIGURE 94
HISTORICAL TRENDS IN WESTERN BASIN EUTROPHICATION

TABLE 25

## POPULATION PROJECTIONS FOR LAKE ERIE BASIN [26] (in millions)

| Growth <br> Projection | $\underline{1980}$ |  | 1990 |  | 2000 |  | $\underline{2010}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

NOTE: Western Basin Population is 57 percent of basin total.
${ }^{1}$ Environmental
${ }^{2}$ National Economic Development growth
${ }^{3}$ Developmental
accordance with the population increase on a per capita basis. Agricultural runoff is assumed to be constant for the period. Thus it is assumed that no nutrient removal programs are instituted during the projection period. All other exogenous variables are held at 1970 values. The results of these projections are shown in Figure 95. The chlorophyll concentration shown is the summer average for model segment 7 , adjacent to the Maumee River. The increases are, of course, more pronounced for the rapid growth projected by the Regional Development Objectives than for the more modest growth envisioned by the Environmental Quality Objectives. Such projected increases, and indeed current phytoplankton population levels, are cause for concern so that it is necessary to investigate possible control measures. The currently favored control policy is aimed at removal of the phosphorus entering the basin. Projected conditions for both an 80 percent removal policy and a 95 percent removal policy in addition to a total ban on detergent phosphorus are shown in Figure 96. The moderate National Development Objective population projections are used for these calculations. If it is assumed that the 1930 level of population is the desired standard, then until 1990 the standard will be achieved and by 2010 the standard will be exceeded by both control policies. It is important to note that the level of removal assumed for the more stringent control policy may not be presently technologically feasible.

An alternate policy that appears to be feasible using presently available technology is to remove 80 percent of the phosphorus and 50 percent of the nitrogen being directly discharged to the basin. The projected result is shown in Figure 97. It appears that for such a policy it is possible to attain the assumed standard through 2010.

Thus a direct use for a eutrophication model is made in assessing the efficacy of control policies specifically designed to alleviate eutrophication. In addition, other planning alternatives can be investigated, such as the effect of lake level changes on eutrophication (a shallower body of water is more productive than a deeper one, all else being the same) or an agricultural land use policy which results in a 50 percent decrease in the phosphorus content of the agricultural runoff. The projected results



FIGURE 96
PHYTOPLANKTON CONCENTRATIONS V. TIME FOR PHOSPHORUS REMOVAL POLICIES
(NED-Population Growth Used)


FIGURE 97
PHYTOPLANKTON CONCENTRATIONS Vs. TIME
FOR NITROGEN AND PHOSPHORUS REMOVAL POLICY (NED-Population Growth Used)
are shown in Table 26 which also summarizes the calculations previously described. These two planning interactions have no more than a 10 percent effect on the projected phytoplankton populations for 1970 conditions. These variations are within the probable error of the projections so that the precise magnitude of the effect is in doubt, although it is likely to be small.

The types of planning interactions that can be investigated are limited only by the exogenous variables incorporated in the model. The effects that can be projected are limited by the endogenous variables and the realism and verification of the model. Thus on a relative basis, the projected phytoplankton changes are more reliable than the projected zooplankton population changes because the data available for verification for the latter are weaker. Also it should be reemphasized that all the projections made above are in the nature of a demonstration of the utility of a eutrophication model and are not projections of future events.

A Food Chain Model of Cadmium in Western Lake Erie

Introduction

The build-up of certain substances, such as heavy metals in the ecological food chain, has been the subject of considerable study in recent years. Ecologists have attempted to analyze the flow of such material into various sectors of the ecosystem. Planners and environmental managers have attempted to control the release of such substances, often with little guidance on the expected environmental response to various levels of control actions. A mathematical model of the transfer of material in the food chain would provide a means for generating some information to guide the environmental manager on the consequences of differing policies.

The purposes of the model, therefore, are to:

$$
\text { TABLE } 26
$$

$$
\begin{aligned}
& \text { ILLUSTRATIVE APPLICATION OF THE PHASE I } \\
& \text { LIMNOLOGICAL SYSTEMS ANALYSIS DEMONSTRATION MODEH }
\end{aligned}
$$

$$
\text { Pop. }{ }^{3} \quad \text { Pop. }{ }^{3}
$$

$$
2^{\prime} \text { Lake }{ }^{3}
$$

$$
\begin{gathered}
\text { Phos. } \\
\text { Removal } \\
50 \% \mathrm{Agr.} \\
\hline
\end{gathered}
$$

$$
\begin{gathered}
- \\
\Delta 1 \text { to }_{\mu g / 1^{5}} 2
\end{gathered}
$$

$$
\begin{gathered}
\text { Phos. } \\
\text { Removal } \\
\begin{array}{c}
95 \%+\text { Deterg. } \\
- \\
- \\
10-15 \mu \mathrm{~g} / 1^{2} \\
15-20 \mu \mathrm{~g} / 1^{2}
\end{array}
\end{gathered}
$$

$$
\begin{aligned}
& \text { NOTES: } \\
& { }^{1} \text { These levels are for the moderate growth population levels. } \\
& { }^{2} \text { The same algae levels can be obtained with an } 80 \% \text { phosphate removal policy plus } \\
& \text { Nitrogen removal and } 2010-50 \% \text { Nitrogen removal. } 25 \% \\
& { }^{3} \text { Values are micrograms/liter of chlorophyll for Western Lake Erie in Section VII of the } \\
& \text { Demonstration Model (near the Maumee River) } \\
& { }^{4} \text { The information presented should be considered as an illustration of the type of results } \\
& \text { obtainable from application of eutrophication model to analysis of planning problems } \\
& \text { rather than a projection of future conditions. } \\
& { }^{5} \text { Change in chlorophyll levels from } 1970 \text { conditions. }
\end{aligned}
$$

> 1. Examine the structure of the build-up of potentially toxic substances in the food chain.
> 2. Determine what data would be required for a verification of the model.
> 3. Determine the utility and applicability of linear food chain models in broad scale ecosystem planning.
> 4. Demonstrate the interfacing of nonlinear and linear modeling frameworks.

This modeling effort is directed specifically toward food chain modeling within the context of the phytoplanktonzooplankton model of Western Lake Erie. A seven spatial compartment, five system steady-state model was constructed for this purpose. Figure 98 illustrates the basic structure of the system.

As shown, the ecosystem is considered on five levels: the water column, phytoplankton, zooplankton, fish, and lake birds. The last two compartments are included as illustrations of higher trophic levels which can act as additional concentrators of the tracer substance. As such, they are not necessarily realistic representations and the results calculated should not be interpreted literally.

The geographical setting is Western Lake Erie which is divided into seven spatial compartments as shown in Figure 98. The basic model structure is linear - discussed more fully below - with a link from a more complex nonlinear eutrophication model.

Identification of a Tracer Substance. Identification of a suitable tracer substance is predicated on the following conditions:

1. The tracer must be a substance that concentrates in plant and animal tissue in measureable quantities.

FIGURE 98
INTERFACING OF EUTROPHICATION AND FOOD CHAIN MODELS
2. It should be significant to the public health or welfare of man.
3. It should be, preferably, a substance for which concentration factors or biomass concentrations have been determined.

On this basis, cadmium is selected as a tracer element in the food chain model. Cadmium occurs in combined forms in nature. No important ores of cadmium are known, but it invariably occurs as an impurity in zinc ores in a ratio of about 1 part cadmium to 200 parts zinc. It is typically a by-product of the zinc industry and is therefore prevalent in waste discharges associated with zinc plating processes.

Cadmium has a high toxic potential when ingested by humans. At concentrations of greater than $0.1 \mathrm{mg} / \mathrm{l}$, it accumulates in soft body tissue resulting in anemia, poor metabolism, possible adverse arterial changes in the liver, and at high concentrations, death. The Public Health Service Drinking Water Standard for cadmium is $0.01 \mathrm{mg} / 1$ for domestic supplies [27].

Relatively few studies have been made of cadmium concentrations and toxicities in the aquatic environment, but studies of mammals and fish have shown a considerable cumulative effect in the biomass. Concentrations of a few mg/l in food supplies have been known to cause sickness in man [28].

Some cadmium data for Western Lake Erie are summarized in Table 27. It should be noted that the lake water samples are representative of nearshore conditions whereas offshore samples have lower concentrations. Cadmium samples analyzed by the Canadian Centre for Inland Waters at offshore stations in Western Lake Erie confirm this as shown in Table 27. All fish samples for the data shown were collected from East Harbor, near Sandusky, Ohio.

## TABLE 27

## SUMMARY OF OBSERVED CADMIUM DATA ( $\left.\mu \mathrm{G} / L_{1}\right)$ FOR WESTERN LAKE ERIE

Average Maximum Minimum Number of Ref.
Tributaries:
a) Detroit River
b) Maumee River

Lake Erie (offshore)
$5.55 \quad 34$.
N. D.

140
12.5034 .547
[29]
N.D. N.D. N.D. 111
[29]

Lake Erie (nearshore)
a) Sandusky, O. water intake
$1.76 \quad 10$.
N. D.

17

1. 18 10. N.D. 17
[29]
b) Toledo, 0 .
water intake
$.59 \quad 10$.
N.D.

17
.59 10. N.D. 17
d) Port Clinton, $O$. water intake

## Fish:

a) Spottail shiner* $\quad 100$.
b) Gold Fish ${ }^{+} \quad \begin{array}{ll}1400 . \\ & 1100\end{array}$
c) White Bass+ 200.
$\begin{array}{lr}\text { d) Yellow Perch }\end{array}{ }^{+} \quad 500$.
e) Walleye ${ }^{+} \quad 200$.
$\begin{array}{llll}\text { Average in } & 8.34 & 34 . & \text { N.D. } \\ \text { Tributaries } & 87\end{array}$
Average in
Lake Water
NOTES:
${ }^{*}$ Whole Fish analysis ( $\mu \mathrm{g}$ Cd/gm tissue)
${ }^{+}$Fish liver analysis ( $\mu \mathrm{g} \mathrm{Cd} / \mathrm{gm}$ tissue)
N.D. Not Detectable

## Theory

A discussion of ecological and food chain modeling is given in Section VII, together with a review of the literature. A discussion of the generalized notion of a compartment in both physical and ecological space is presented. In general, the biological world is discretized into a series of trophic levels. A one dimensional trophic system can be considered in which each level is affected only by those levels above or below. A food web is then a logical extension of the one dimensional case for which interactions are more widespread among the trophic levels.

The equations of the theory are mass balances around each discrete trophic level positioned at some location in physical space. The relevant measure of toxicant mass in a trophic level is mg toxicant per unit biomass at that level. For example, for the phytoplankton, the measure is mg cadmium per mg chlorophyll while for fish, the measure might be mg cadmium per mg of fish carbon. Let:

$$
\begin{equation*}
N_{i j}=\left(\frac{\text { mass toxicant }}{\text { mass trophic level } i}\right) \tag{19}
\end{equation*}
$$

at location $j$ and

$$
\begin{equation*}
M_{i j}=\left(\frac{\text { mass trophic level } i}{\text { volume of water }}\right) \tag{20}
\end{equation*}
$$

at location $j$. Then $N_{i j} M_{i j}=C_{i j}$ is the mass of toxicant relative to volume of water at location $j$.

In one-dimensional trophic space for volume of water $V_{j}$, the rate of change of the mass of toxicant is given by?

$$
\begin{align*}
V_{j} \frac{d N_{i j} M_{i j}}{d t} & =\left(K_{i-1, i} M_{i-1} N_{i-1}\right)_{j} V_{j}-\left(K_{i . i} M_{i} N_{i}\right)_{j} V_{j}  \tag{21}\\
& + \text { Advection }+ \text { Dispersion }+ \text { Sources }- \text { Sinks }
\end{align*}
$$

where $K_{i-1, i, j}$ represents the rate of production of the toxicant in trophic level i due to the transfer from trophic level i-1, all located at position $j$ in geographical space and $K_{i}$ represents the transfer out of trophic level i. Note ${ }^{i}$ all K values have dimensions [1/T].
The advection terms between the $j^{\text {th }}$ location and all surrounding k locations will have the form:

$$
\begin{equation*}
-\sum_{k} Q_{j k} \frac{\left(C_{i j}+C_{i k}\right)}{2} \tag{22}
\end{equation*}
$$

for a central finite difference operator where $Q_{k}$ is the mass flow advected from $k$ to $j$ for positive outward flow. The dispersion terms will be of the form:

$$
\begin{equation*}
\sum_{k} E_{j k}^{\prime}\left(C_{i k}-C_{i j}\right) \tag{23}
\end{equation*}
$$

where $E_{j k}^{\prime}$ is the bulk dispersion $\left[L^{3} / T\right]$ between $j$ and all surrounding $k$ segments.

The complete mass balance equation for the mass of the tracer substance in trophic level $i$ at location $j$ is given by:

$$
\begin{gather*}
V_{j} \frac{d N_{i j} M_{i j}}{d t}=\left(K_{i-1, i} M_{i-1} N_{i-1}\right)_{j} V_{j}-\left(K_{i i} M_{i} N_{i}\right)_{j} V_{j} \\
-\sum_{k} O_{j k} \frac{\left(N_{i j} M_{i j}+N_{i k} M_{i k}\right)}{2}+\sum_{k}^{E} E_{j k}^{\prime}\left(N_{i k} M_{i k}-N_{i j} M_{i j}\right)+W_{i j} \tag{24}
\end{gather*}
$$

for $i=1 \ldots m$ trophic levels, $j=1 \ldots n$ spatial segments and $W_{i j}=$ direct input of tracer substance into trophic level $i$ at location $j$. The first term in Equation (24) represents the flux of material from trophic level i-l to level i while
the second term represents the flux out of level i; where the flux in Equation (24) is only "up the food chain." If other trophic levels interact with the $i^{\text {th }}$ level, this effect is a direct extension of Equation (24).

Most linear compartment models proceed by assuming a constant trophic level mass. It is interesting to explore this special case to draw some parallels to linear water quality models. For constant trophic level mass in space and time and a single volume (completely mixed), then Equation (24) becomes:

$$
\begin{equation*}
\mathrm{VM}_{i} \frac{d N_{i}}{d t}=S_{i-1, i} N_{i-1}-S_{i i} N_{i}+W_{i} \tag{25}
\end{equation*}
$$

where $S_{i-1, i}=\left(K_{i-1, i}{ }^{M}{ }_{i-1} V\right)$ and $S_{i i}=K_{i i} M_{i} V$ and no significant inflow or outflow of water is considered. The flux quantities, $S$, have dimensions of trophic level mass transfered per unit time. The quantity $M_{i} V$ has units of trophic level mass and is designated $\Gamma$, then:

$$
\begin{equation*}
\Gamma_{i} \frac{d N_{i}}{d t}=S_{i-1, i} N_{i-1}-S_{i i} N_{i}+W_{i} \tag{26}
\end{equation*}
$$

This equation is identical to the mass balance equation which results for a water quality variable in physical space. In the case of Equation (26), however, physical space is replaced by trophic space. Therefore Fi represents the volume of the $i^{\text {th }}$ trophic level, i.e., the mass of that level available for dilution of a toxicant, $N$, discharged into that $i^{\text {th }}$ level.

The quantity $S_{i-1, i}$ is analogous to the physical flow transport of water. It is clear, then, that in Equation (26) the size of the trophic level in terms of its mass is analogous to the size of a water body expressed in volumetric units.

Returning to Equation (24), one can show after some simplification that:

$$
\begin{equation*}
[A(d / d t)]_{i}\left(\mathbb{N}_{i} M_{i}\right)=\left[K_{i-1, i} V\right]\left(N_{i-1} M_{i-1}\right) \tag{27}
\end{equation*}
$$

where $[A(d / d t)] i_{i}$ represents an $n \times n$ matrix with a derivative operator on the main diagonal and with elements representing the spatial transport and dispersion of material, $\left(N_{i} M_{i}\right)$ is an $n \times 1$ vector of the tracer material in the $i^{\text {th }}$ trophic level, $\left[K_{i-1, i} V\right]$ is an $n x n$ diagonal matrix of transport between the $i^{1}-1^{s t}$ and $i^{\text {th }}$ trophic levels and $\left(N_{i-1} M_{i-1}\right)$ is an $n \times 1$ vector of tracer in the $i-1$ st level. Under steadystate conditions, the operator $d / d t$ is equal to zero and the matrix equation given by (27) represents a set of linear algebraic equations. An example will illustrate the model structure.

Consider the aquatic ecosystem as composed of three systems: water, phytoplankton, and zooplankton; and consider a direct input of the tracer substance into the water column. The steady-state matrix equations are then:

$$
\begin{equation*}
\text { Water: }[A]_{W}\left(C_{W}\right)=(W)_{i} \tag{28}
\end{equation*}
$$

Phytoplankton: $[\mathrm{A}]_{\mathrm{P}}\left(\mathrm{N}_{\mathrm{P}} \mathrm{M}_{\mathrm{P}}\right)=[\mathrm{KV}]_{\mathrm{wP}}\left(\mathrm{c}_{\mathrm{w}}\right)$
zooplankton: $\quad[\mathrm{A}]_{\mathrm{Z}}\left(\mathrm{N}_{\mathrm{Z}} \mathrm{M}_{\mathrm{Z}}\right)=[\mathrm{KV}]_{\mathrm{PZ}}\left(\mathrm{N}_{\mathrm{P}} \mathrm{M}_{\mathrm{P}}\right)$
Note that for the water column equations, the product term $\mathrm{N}_{\mathrm{i}} \mathrm{M}_{\mathrm{i}}$ does not appear and C represents the concentration of the ${ }^{i}$ toxicant in the water Wolumn. The solution for the phytoplankton system is then:

$$
\begin{equation*}
\left(N_{p}\right)=\left[M_{P}\right]^{-1}\left[A_{P}\right]^{-1}[\mathrm{KV}]_{W P}\left(C_{W}\right) \tag{29}
\end{equation*}
$$

where $\left[M_{P}\right]^{-1}=\operatorname{diag}\left(1 / M_{1} \ldots, 1 / M_{n}\right)$ the inverse of the phytoplankton biomass concentrations for the $n$ spatial segments. The concept of the trophic level mass acting as a diluting volume is indicated by this matrix.

If there is feedback between trophic levels or parallel interactions (food webs), the matrix equation is a general extension of Equation (27):

$$
\begin{equation*}
[A(d / d t)]_{i}\left(\mathbb{N}_{i} M_{i}\right)=\sum_{\substack{k \neq i}}\left[K_{k i} V\right]\left(\mathbb{N}_{k} M_{k}\right)_{i}, k=1 \ldots n \tag{30}
\end{equation*}
$$

The summation on the right hand side of the equation expresses all possible feedforward, feedback interactions. Under steadystate, Equation (30) represents a set of mn simultaneous linear equation which are readily solved.

In order to examine the behavior of a system such as Equations (28) and (30), consider a steady-state situation, a single spatial volume and a feedback loop from the phytoplankton and zooplankton to the water phase. The equations then are simply:

$$
\begin{gather*}
0=-K_{11} C_{w}+k_{21} C_{P}+k_{31} C_{Z}=w \\
0=K_{12} C_{w}-K_{22} C_{P}  \tag{31}\\
0=R_{23} C_{P}-K_{33} C_{Z}
\end{gather*}
$$

where $C_{p}$ and $C_{Z}$ represent the concentration of toxicant in the phytoplankton and zooplankton per liter of water. Note that if all mass is conserved, then $K_{i i}={ }^{\Sigma}{ }_{j} K_{i j}$. The ratio of the zooplankton toxicant concentration to the concentration in the next lowest trophic level is:

$$
\begin{equation*}
\frac{\mathrm{C}_{\mathrm{Z}}}{\mathrm{C}_{\mathrm{p}}}=\frac{\mathrm{K}_{23}}{\mathrm{~K}_{33}} \tag{32}
\end{equation*}
$$

or the mass of toxicant per mass of trophic level is given by:

$$
\begin{equation*}
\frac{N_{Z}}{N_{P}}=\frac{M_{P}}{M_{Z}} \frac{K_{23}}{K_{33}} \tag{33}
\end{equation*}
$$

The ratio of the concentration of toxicant in one trophic level to the preceding level is therefore inversely proportional to the mass of the levels and directly proportional to the ratio of the rates at which the toxicant is fed forward to a given level and the rate at which it leaves that level.

The Western Lake Erie Model

As indicated in Figure 98, a five system, seven spatial segment model has been constructed for Western Lake Erie, which results in a set of 35 simultaneous linear equations. This model accepts input from a nonlinear, non-steady-state eutrophication model. The system is assumed to be at temporal steady-state and all kinetic reactions are first order. The concentration of toxicant in the phytoplankton, zooplankton, fish, and lake bird systems is dependent on the selection of feedforward or growth coefficients, $K_{i-1, i}$,
from the previous trophic level. Likewise, decay of material is accomplished via self-decay terms, $K_{i j}$. By permitting the feedback coefficients to be some fraction of the difference ( $K_{i, i}-K_{i, i+1}$ ), the model permits resolubilizing of trace materials present in the $i^{\text {th }}$ trophic level. In the case where the sum of the feedforward and feedbackward rates does not equal the system decay rate, $\mathrm{K}_{\mathrm{i}, \mathrm{i}}$, allowances can be made for deposition of materials outside the influence of the water column.

The theoretical development of the food chain model given above indicates that estimates of both biomass and tracer substance concentration should be available in order to compute the theoretical concentrations of the tracer in a trophic level.

A significant feature of the eutrophication model is that it estimates the growth coefficients and biomass for phytoplankton and zooplankton systems. Since the time constants for the highest two trophic levels are appreciably longer than that of the phytoplankton and zooplankton, a steady-state assumption for these trophic levels is reasonable. Steady-state biomass concentrations for phytoplankton and zooplankton are obtained
by conducting a simulation for the entire year of 1970 , and average biomass data as well as average growth and death coefficient are extracted from the non-linear model output. The average annual system kinetics are incorporated into a linearized form of the eutrophication model to check for marked deviations in the biomass calculations, and no significant deviation is observed.

The phytoplankton-zooplankton eutrophication model is composed of two trophic levels and a water system. Thus it cannot be used to make estimates of the fish or lake bird biomass. In addition, there is a lack of available data on the magnitude of these populations. An absolute lower bound on the fish population is considered to be the annual conmercial catch. Estimates of the lake bird biomass are made on the basis of bird populations in Lakes Huron and Michigan. Although the final biomass estimates are constructed on tenuous grounds, they are considered reasonable. Future investigations can provide a better framework within which to make this type of biomass estimate. Again it should be recalled that the lake fish and lake bird trophic levels are included for illustrative purposes only.

As pointed out previously, it is possible to estimate average phytoplankton and zooplankton kinetics from the output of the nonlinear eutrophication model. The two higher trophic levels are considerably more difficult to define in terms of kinetic interactions. The populations are large and very diverse, necessitating an all inclusive definition of their behavior. Very limited data are available in this regard; therefore assumptions have been made to arrive at best estimates of the reaction rates $\mathrm{K}_{34}, \mathrm{~K}_{44}, \mathrm{~K}_{45}$, and $\mathrm{K}_{55}$. Studies on the growth rates of guppy populations, when converted to organic carbon, yield results comparable to those used in the food chain model. Although these rates do vary from species to species, they have been employed here simply to demonstrate the utility of food chain modeling.

Kinetic rates employed in the lake bird system are merely presented as demonstration values because they have no basis in experimental study. Bird populations are difficult to quantify in terms of growth and death rates, because the rates are subject to such unquantifiable effects as migration, feeding habits, and locations of feed grounds.

Figure 99 summarizes the kinetic reaction rates and the distribution of the net lake flows used for Western Lake Erie. As indicated previously, the kinetic rates for the fish and lake bird trophic levels are included solely as illustrated values to demonstrate the build-up of a tracer in higher trophic levels. The validity of the values can be estimated only by having data available on the tracer concentration in the biomass of the given trophic level.

Determination of Biomass Estimates. The food chain modeling results are most adequately interpreted when the concentration of the tracer substance can be quantified in terms of a biomass measurement common to each trophic level. Organic carbon has been selected as that basis. Physiological factors defining the carbon content of various trophic levels are readily available in the literature. For purposes of this model, the carbon content of the phytoplankton and zooplankton systems is taken as 50 percent of their dry weight. The carbon content of the fish and lake bird systems is considered to be 4 percent of the total weight.

Carbon concentrations of the phytoplankton and zooplankton are available from the average summer concentrations computed in the nonlinear eutrophication model and are tabulated in Table 28. For fish biomass, it was assumed that the carbon content would be approximately one twentieth of the phytoplankton biomass, i.e., 0.05 mg carbon/l. Total biomass of fish in all of Lake Erie on the basis of this carbon content is computed to be $1.25 \times 10^{9}$ pounds. The reported annual commercial catch for the lake is approximately 3.5 percent of this figure. Limited data are available on fish population of the Great Lakes, however, virtually no adequate measurements of Lake Erie's populations have been determined to date. For demonstration purposes, $1.25 \times 10^{9}$ pounds of total fish biomass as wet weight may be considered a reasonable number, although subject to possible order of magnitude variations.

For demonstration purposes, the bird populations are specified as $0.01 \mathrm{Mg} \mathrm{C} / \mathrm{l}$ of Lake Erie water. This figure is estimated on the basis of bird populations for Lake Huron and Lake Michigan [32]. The total biomass associated with these populations is about $0.2-0.5$ billion pounds (wet weight).

(4) EDETROIT RINER

## TABLE 28

## ASSUMED AVERAGE BIOMASS CONCENTRATIONS FOR FOUR TROPHIC LEVELS IN mg CARBON/LITER - SPRING 1970 <br> WESTERN LAKE ERIE

| Sys tem | Spatial Segment |  |  |  | Figure 2) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7. |
| Phytoplankton ${ }^{1}$ | 0.95 | 0.90 | 1. 45 | 1.00 | 0.65 | 0.95 | 1.55 |
| Zooplankton ${ }^{1}$ | 0.10 | 0.10 | 0.10 | 0.10 | 0.20 | 0.40 | 0.10 |
| Lake Fish ${ }^{2}$ | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| Lake Birds ${ }^{2}$ | . 01 | . 01 | . 01 | .01 | . 01 | . 01 | . 01 |

NOTES :
${ }^{\text {² Average Values - Spring 1970; Source - non-linear }}$ phytoplankton-zooplankton model.
${ }^{2}$ Estimates of Average Annual Concentrations

Concentrations of tracer substance are presented as the concentration in the water column in $\mathrm{mg} / 1$ corresponding to $C_{i j}=N_{i j} M_{i j}$ in Equation (21).
The distribution of flows is shown in Figure 99 and is obtained from other hydrodynamic information and a verification of a chloride tracer model which also provides estimates of $E^{\prime}$ as discussed previously. With the transport structure, kinetics, and biomass estimates at hand, the solution of the thirty-five equations provides the distribution of cadmium in trophic and physical space.

Model Results

Figure 100 summaries the results of the application of the model to Western Lake Erie. The general increase in concentration as the cadmium proceeds up the food chain results from two factors: the decrease in total biomass at higher trophic levels and the decrease in uptake rate. Note that under the conditions assumed in Figure 99 and Table 28, the concentration of cadmium in the fish trophic level is about $20-30 \mu \mathrm{~g} \mathrm{~cd} / \mathrm{mg}$ carbon which compares favorably with the reported data as indicated on Figure l00. Also, for the reaction rates indicated in Figure 99, the cadmium concentration in the water phase does correspond to the order of magnitude of observed concentrations.

If cadmium had been considered as a conservative variable, and therefore not subject to uptake by the biological system, a value of about $5 \mu \mathrm{~g} / 1$ cadmium in the water column is calculated due to the input loads of the Detroit and Maumee Rivers (see Table 27). This is 1 to 2 orders of magnitude greater than that calculated by the food chain model. It is also interesting to note that $5 \mu \mathrm{~g} / \mathrm{I}$ is 50 percent of the U.S. Public Health Service standard. At least from the point of view of drinking water quality, it is the biological uptake phenomenon which has kept the cadmium concentrations low in the water phase.


FIGURE 100
COMPARISON OF FOOD CHAIN MODEL OUTPUT WITH SOME OBSERVED DATA IN WESTERN LAKE ERIE

It is also informative to examine the distribution of the mass of cadmium over the seven spatial segments and over the five systems. The results of the total mass computations appear in Table 29. As shown, about 54 percent of the cadraium resides in the three upper trophic levels, the remainder being distributed in the water and a separate bottom or sediment compartment. The values in the latter compartment were not computed directly but obtained by difference. It should be stressed again that the results apply only to the set of uptake coefficients used in the model. The percent mass distribution in the spatial segments reflects the effect of the flow transport in the structure. It should be recognized, however, that the concentrations will vary depending on the volume of the spatial segment.

Figures 101 and 102 show the model results as a function of distance along the south shore, i.e., along segments 3, 7, and 6. A smooth curve has been drawn between the three concentrations (see Figure 100 for the actual levels). It can be seen that the concentration in the water decreases rapidly with distance because of the uptake by the aquatic ecosystem. The degree of uptake depends on how long the water takes to travel along the south shore, i.e., how long the water is exposed to the predatory effect of the phytoplankton.

This residence time-spatial effect can also be seen by examining Figure 102 which shows concentration factors (relative to phytoplankton) of each of the upper trophic levels. Note that by being in an interactive phase longer, i.e., by traveling through segments 3, 7, and 6, the concentration factors increase by about one order of magnitude.

The simple food chain model illustrated here could prove useful in large scale planning applications provided additional data are collected on the various trophic levels. The model demonstrates the increase in concentration of potentially toxic substances, such as cadmium, as one proceeds up the food chain. The food chain demonstration model also illustrates how interactive modeling between complex nonlinear and linear compartment models can be accomplished.

PERCENT OF MASS OF CADMIUM BY SPATIAL SEGMENT AND SYSTEM LEVEL AS CALCULATED FOR FOOD CHAIN MODEL

| Segment No. 1 <br> (See Figure 98) | \% Total Cadmíum Mass | System <br> (See Figure 98) | \% Total Cadmium Mass |
| :---: | :---: | :---: | :---: |
| 1 | 2 | 1 - Water | 37 |
| 2 | 23 | 2 - Phyto. | 24 |
| 3 | 3 | 3 - Zoo. | 13 |
| 4 | 16 | 4 - Lake Fish | 15 |
| 5 | 44 | 5 - Lake Birds | 1 |
| 6 | 7 | Bottom | 10 |
| 7 | $\frac{5}{100}$ |  | $\overline{100}$ |



FIGURE IOI
COMPUTED CADMIUM CONCENTRATION ALONG SOUTH SHORE OF WESTERN LAKE ERIE


FIGURE 102
COMPUTED CADMIUM CONCENTRATION FACTORS ALONG SOUTH SHORE OF WESTERN LAKE ERIE

## Summary

The demonstration modeling framework illustrates the processes of model selection, synthesis, verification, and application of mathematical models to various Great Lakes water resource problems. The Great Lakes basin-wide model of chlorides and Total Dissolved Solids demonstrates the utility of such analysis for large space scale and long time scale problems. The regional analyses as exemplified by the models of Western Lake Erie indicate the applicability of systems analysis techniques to a variety of important water resource problems. The coliform bacteria model for Western Lake Erie indicates the role that the hydrodynamic transport structure plays and illustrates the use of chlorides as a tracer for verification purposes. A model of a non-conservative variable, such as coliform bacteria, can be used to evaluate a variety of wastewater control actions.

The eutrophication model of Western Lake Erie demonstrates the feasibility of constructing a model that approximately verifies observed data. The validity of the basic model structure is further enhanced by a favorable hindcast using 1930 data. The eutrohpication model, in its preliminary form given in the demonstration modeling framework, illustrates the major interactions that exist in the eutrophication phenomenon. Although some applications of the model are given for projected future conditions, additional detailed analyses are required before firm conclusions could be drawn on future water quality levels.

The food chain model illustrates the interaction between the eutrophication model and a model of the build-up of potentially toxic substances. Problems of verification and data availability are demonstrated by the food chain model. This model is an example of a modeling structure that is still in an early stage of development and is in need of additional research and development effort before it can be utilized in a planning context.

The overall demonstration modeling framework, therefore, illustrates the various levels of model availability and
feasibility. It is concluded, in general, that the use of mathematical models in a Limnological Systems Analysis is feasible and can, at the present state of the art, provide important quantitative information for the decision-making and planning processes.

## REFERENCES

[1] Beeton, A.M., "Eutrophication of the St. Lawrence Great Lakes," Limnology and Oceanography, Volume`l0, No. 2, (April 1965).
[2] O'Connor, D.F., Mueller, J.A., "A Water Quality Model of Chlorides in Great Lakes," Journal of the Sanitary Engineering Division, A.S.C.E., Volume $\overline{96}$, NO. SA4, pp 955-975 (August, 1970).
[3] Rainey, R.H., "Natural Displacement of Pollution from the Great Lakes," Science, Volume 155, No. 3767 (March 10, 1967).
[4] Brunk, I.W., "Hydrology of Lakes Erie and Ontario," Proc. 8th Conf. Great Lakes Research, Publication No. ll, GLRD, University of Michigan (1964).
[5] Carlson, G.T., and Persoage, N.P., "Development and Coordination of Basic Hydrologic Data for International Joint Commission Study of the Great Lakes," Proc. loth Conf. on Great Lakes Research (1967).
${ }^{[6]}$ DeCooke, B.G., "Regulation of Great Lakes Levels and Flows," Proc. of the Great Lakes Water Resources Conf., Toronto, Canada (June 1968).
[7] Ownbey, C.E., and Kee, D.A., "Chlorides in Lake Erie," Proc. 10th Conf. Great Lakes Research (1967).
[8] U.S. Department of Health, Education, and Welfare, Public Health Service, Division of Water Supply and Pollution Control, "Report on Pollution of Lake Erie and its Tributaries, Part l, Lake Erie," (July 1965).
[9] U.S. Department of Health, Education, and Welfare, Public Health Control, "Great Lakes - Illinoi.s River Basins Comprehensive Study, Interim Report - Illinois River Basin," (August 1961).

## REFERENCES

(continued)
[26] Great Lakes Basin Commission Framework Study, personal communication.
[27]U.S. Department of Health, Education, and Welfare, Public Health Service Drinking Water Standards Revised, Publication No. 956, U.S. Government Printing Office, p 61, (1969).
[28] Water Quality Criteria, Report of National-Technology Advisory Committee, to Secretary of the Interior, F.W.P. C.A., Washington, D.C., (April 1968).
[29] Environmental Protection Agency, "STORET Data Inventories," Washington D.C..
[30] Canadian Centre for Inland Waters, Limnological Data Reports, Lake Erie, Canadian Oceanographyic Data Centre, Burlington, Ontario.
[31] Lucas, H.F., Edgington, D.N., Colby, P., "Concentrations of Trace Elements in Great Lakes Fishes," Jour. Fishery Res. Bd. of Canada, Volume 27, No. 4, pp 677-684 (1970).
[32] Ludwig, J.P., "Herring and Ring-Billed Bull Populations of the Great Lakes, 1960-1965," Proc. 9th Conf. G.L.R.D., University of Michigan, Publication No. 15 ; pp $80-88$ (1966).

## Problems Proposed for Study in Phase II

In order to arrive at a recommended course of action in Phase II of the Limnological Systems Analysis, the procedure presented in Figure 1 was followed. The methodology generally consists of two broad areas of evaluation and analysis: first, the administrative determination of problems, water uses, and associated variables; and second, the scientific engineering analysis of modeling frameworks associated with these problem areas. The alternate programs are essentially based on a synthesis of these factors, a product of the priority of the problems, and the status of the associated modeling area. This section carries the analysis to the point of delineation of the general problem categories which are to be considered for inclusion in Phase II.

The procedure shown in Figure 1 consists of developing a list of categories which cover the general water resource problems of the Great Lakes. These problems basically arise from the various water uses of the area and are summarized in Table 2. For each of the problem areas, pertinent water variables are identified (Table 3). The step of assigning a priority to each of these problem areas is the essential one in the administrative definition of the project.

An identification of the various modeling areas, as they apply generally to water resource problems, was simultaneously and independently performed. These areas were then grouped into eleven modeling frameworks which were determined by the problems and variables enumerated in the above-mentioned steps relating to the administrative definition. This step was followed by an evaluation and a ranking of the various modeling frameworks, as summarized in Figure 41.

An interactive analysis of the eleven modeling frameworks and seven problem categories led to the listing shown in Table 17 and discussed in Section VIII.

Of the seven problem categories, those to be included in the Phase II study were selected on the basis of the following criteria:

1. Existing models and programs directly related to or operational on Great Lakes problems would not be duplicated.
2. Problem categories included in the Phase II effort would deal with broad scale interactive Type II planning, not with local problems or operational control of specific problems.
3. Adequate technical knowledge should be available for development and application of preliminary models.
4. Adequate data and information on system conditions and inputs should be available to provide at least minimum verification of models proposed for Phase II.

A summary of this step is presented in Table 30. For each problem category, this table includes a general assessment of the modeling status and of the previous modeling efforts on the Great Lakes. In addition, an evaluation of the data availability and the extent to which the category fits a Type II planning problem are indicated.

The first criterion indicates that existing models for water balance, erosion, and lake circulation should be used in the Phase II study. Funds should be allocated to interfacing with these existing models but should not be expended to duplicate them. Major planning efforts associated with lake levels and erosion would therefore be retained by the Corps of Engineers. Interfacing of the results of these ongoing planning activities with other water resource areas would be part of the Phase II study.

| $\qquad$ TABLE 30 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| RELATION OF MODEL PROBLEMS |  |  |  |  |
| Problem Category | Existing Modeling Effort on the Great Lakes | General State of knowledge and modeling | Data Availability | Type II <br> Planning <br> Problem |
| 1. Mean monthly water levels and flow rates of the Great Lakes | Adequate for planning and model interaction needs | $\begin{gathered} \overline{\text { Well }} \text { advanced } \\ (70) \end{gathered}$ | Adequate for planning needs | Generally yes |
| 2. Erosion-sediment | Meets near term planning needs | Generally empirical (58) | Inadequate for model verification | Yes |
| 3. Ice | None for planning | Inadequate for modeling (not Type II planning problems) (35) | Available | ```Generally no``` |
| 4. Concentrations of toxic or harmful substances | None for planning | Adequate for <br> lst cut modeling (47) | Adequate for lst cut modeling needs | Yes |
| 5. Concentrations of organic and inorganic chemicals which exceed water quality standards | None for planning | Adequate for <br> lst cut model- <br> ing (50) | Adequate for lst cut modeling needs | Yes |
| 6. Eutrophication | None for planning | Adequate for <br> lst cut model- <br> ing (50) | Adequate for lst modeling needs in problem areas of the lakes | Yes |
| 7. Bacteria and virus concentrations in the water body | None for planning (except in near shore area) | Adequate for <br> lst cut modeling (68) | Inadequate for model verification | No near shore local Yes, lake zone |
| NOTE: Numbers in brackets are average for modeling frameworks associated with each problem category. |  |  |  |  |

Application of the second criterion, which requires broad scale interactive Type II planning, eliminates local water resource modeling and predictive models for specific operational problems. Therefore Phase II funds should not be devoted to either modeling of ice formation for prediction of the opening and closing of the navigation season or improvement of lake level forecasting models for the six month projections. These two modeling efforts, while important in the Great Lakes, are essentially associated with operation and management of a specific element of the system. In addition, beach closings and harbor pollution, which are generally of a local nature, do not conform to the criteria of broad interactive water resource problems. These water resource problems, while significant, are therefore excluded from the Phase II study.

Application of the first and second criteria has therefore resulted in elimination of water levels and flows, erosionsediment, ice, and local aspects of the Public Health problem categories from consideration in Phase II. The four problem areas which should be considered for inclusion in the Phase II effort are:

1. Eutrophication
2. Water quality (excluding harbor scale problems)
3. Public Health (excluding local beach scale problems)
4. Accumulation of harmful or toxic substances in the food chain.

Examination of these remaining problem categories is made based on the third and fourth criteria: namely, the levels of modeling knowledge and data availability. With respect to problems of accumulation of toxic and harmful substances in the food chain, the available information and knowledge are only marginally adequate for modeling in this area. The
problem priority, however, appears to be reasonably high and it is anticipated that the general public and governmental agencies will tend to increase the priority given to this problem category in the future. It is therefore recommended that the Phase II study address this problem. The lack of scientific understanding and data indicates that this area should have a relatively modest funding level in the Phase II study, sufficient to provide a preliminary step in analysis. This recommendation is made acknowledging the possible risks of not obtaining reliable model output which will be useful in planning and decision making processes.

The remaining problem categories all have an adequate data base and model status to satisfy the third and fourth criteria. The water resource problems associated with pathogens and other indicator bacteria have low priority when the local beach scale is excluded from consideration. It is therefore recommended that only moderate funding or regional scale modeling of the pathogen problem be included in the Phase II study.

The remaining problem categories of eutrophication and water quality are interrelated. Based on discussions with agency, state, and commission staffs, coupled with the general public awareness of the water resource problems, it is concluded that eutrophication has a slightly higher problem priority in the Great Lakes than do large scale water quality problems. However, the assessment is not sufficient to justify a difference in allocation of funds and, since the two areas are closely related, it is recommended that each receives approximately equal funding in the Phase II Limnological Systems Analysis.

There are two additional benefits which may be derived from the four modeling frameworks recommended in the Phase II study. The output may be used in the process of establishing standards for these water resource problem categories. The insight and knowledge gained from modeling should be used to guide data collection problems in terms of variables to be measured with their associated spatial and temporal scales. This modeling effort may be employed to increase the effectiveness of ongoing sampling and data collection programs in the Great Lakes.

Finally, it is recognized that landbased regional and metropolitan activities have a profound impact on the four problem areas delineated above. Consequently, planning alternatives of this nature must be interfaced with the proposed Phase II effort. It is anticipated that these planning efforts will be a part of the continuing Great Lakes programs and no funds have been proposed in the Phase II Limnological Systems Analysis program for these activities.

In summary, the problem categories and models recommended for inclusion in the Phase II study are based on a qualitative estimate of the product of problem priorities, the available data, and modeling status. Problem priorities have not, as of the writing of this report, been assigned by authorities in the Great Lakes region to the extent that the Phase II study can be directed by priorities. It is possible and desirable for representatives of the public to establish problem priorities to which the Phase II study could directly respond. Modifications in the specific models, problems, and funding level of the Phase II study may then be made to insure that the study is responsive to public needs.

## Alternate Programs

A range of approaches to the overall Limnological System Analysis is explored in this section. The alternate programs are used as a basis for the selection of the recommended program for the Phase II study. Four alternates are presented which vary in philosophy and structure with respect to the direction of the Phase II program.

The following factors are considered in developing these alternates:

1. The model evaluation results, summarized in Section VIII and presented in Figure 41, which include an evaluation of the basic knowledge and understanding of the phenomena involved, the level of previous models and their verification, and the degree of successful application
2. A subjective evaluation of the relative importance of each water resource problem
3. The extent of availability of trained personnel to carry out the Limnological Systems Analysis

The common basic assumptions underlying the alternate programs are:

1. University and other government sponsored research and data collection on the Great Lakes will continue at approximately present levels.
2. The International Field Year on the Great Lakes (IFYGL) will provide increased understanding of lake processes and will act as a stimulus, at least through 1975, for research and field effort in Lake Ontario.
3. Hydrodynamic circulation models and output will be available from existing ongoing programs. No effort in this direction (except for provision for suitable interfacing) is therefore included in the alternates.

Alternate A

This alternate is predicated on the assumptions that understanding of many biological, chemical, and physical phenomena with respect to water resource problems in the Great Lakes is far from complete and requires substantially more effort. Field observations have generally been insufficient to describe adequately the changes which are
apparently taking place so that meaningful predictions cannot be made. These assumptions are coupled with the fact that the development or application of a limnological systems analysis approach in the Great Lakes area has been extremely limited, specifically with respect to those problem contexts which have been modeled elsewhere and are known to be adaptable to the Great Lakes setting. A clear distinction is drawn between the ability to construct equations of phenomena and the existence of reliable meaningful predictive tools for answering planning questions. This alternate thus reflects the position that much more must be done to understand the interactions in the limnological system and that more effort in data analysis and model verification is required.

There are, however, several selected problem contexts which can be modeled to produce useful results. The methodology proposed for this alternate is to select the most competent individual specialists available in each problem area and to define a series of special studies for the purpose of verifying existing or readily developed models for which sufficient data are available. For those areas where this is not the case, an estimate should be made of what can and should be done with further data and model development.

Given a reasonable assessment of the problems and their interactions, funds should be allocated to a specific organization to coordinate and interact the models produced by the individual specialties. Having thus initiated the framework for a limnological systems analysis, in cooperation with the individual projects mentioned, the basis for completion of this analysis would be established.

Accordingly, this alternate highlights the development of necessary information, verification of available models, and the extension of useful predictions which relate to meaningful problems in conjunction with the initial steps in the development of an overall analysis. The applications proposed, therefore, for Alternate A are directed to those problems that are both practical and can be modeled with existing analyses. Primary among these are the water quality and eutrophication problems of the lakes which are given high priority in the alternate.

Alternate B

This alternate retains substantial reservations about the feasibility and utility of constructing limnological models for certain problem contexts, as for example, the construction of models of the broad scale ecological impact of water resources activities. However, in contrast to Alternate $A$, this alternate begins with a specific centralized activity devoted to the development of computational software with applications to specific problems. The applications suggested in this alternate generally include those of Alternate A. Furthermore, recognizing the need for interactive model development, the initiation of such a development is recommended at a restricted level of funding.

A generalized linear steady-state feedback system would be programmed under this alternate to address problems such as water quality analysis, food chain, and broad scale ecological models. The program would have provision for about 2,000 spatial compartments; and programming effort would be devoted to development of computational efficiency.

A generalized non-linear, non-steady-state system is also suggested. It should have provision for about l,000 compartments and should be expandable so that 4,000 to 10,000 compartments can be considered to address such problems as seasonal eutrophication, food chain, and broad scale ecological modeling. Significant programming effort would be devoted to development of computational efficiency. A detailed user's manual (of both the linear and non-linear programs) which addresses the program and the problems which are to be analyzed would be prepared. The programming effort devoted to the input-output would be minimized with card, disk, or tape input and output, and digital graphical displays as options. The program would be constructed to facilitate significant future expansion and embellishment of inputoutput routines and options as well as interlinking of this program with the other programs, such as lake level, chemical equilibrium, and hydrodynamic models.

The applications in this alternate generally include those of Alternate A and are expanded to include some applications on the impact of consumptive use of water and a beginning effort in the development of an interactive ecological model. In addition, supporting output from existing models is to be provided. Specifically, lake level models and hydrodynamic models would be operated by other agencies and individuals, and the results would provide input to the above activities.

Alternate C

This alternate is centered around a broad scale interactive planning model of limnological systems, and is based on the notion that the implementing agency is not concerned primarily with the detailed solutions of problems but with exposing the ramifications of proposed actions and in the planning of water resource projects. The nature of the applications in this alternate is concerned more with broadly interactive problems, rather than the detailed applications suggested in Alternate $B$. The planning model itself consists of two major sections - a linear steady-state compartment model and a non-linear, non-steady-state model. The essence of the alternate lies in the belief that suitable information exists today to provide at least a first basis for estimating the impact of various proposed water resource projects. Thus, the approach in this alternate is to gather existing subsystem mechanistic models, parameter values, and input forcing function levels into a broad interactive modeling framework capable of accommodating new information as it is developed from ongoing research effort in other areas.

All environmental mechanisms may not be understood; indeed, some may be quite poorly understood, while other systems may be understood in detail. In this alternate, the fact that a mechanism or phenomenon is poorly understood does not preclude its entry into the modeling framework at the level of known information. This feature necessarily introduces an unknown level of risk into the system framework, a risk that at any given point in time may produce a planning result that is subsequently shown to be wrong in some sense.

The essence of Alternate $C$ is the willingness to take the risks necessary to answer long range planning problems.

The development of computational software at this level is similar to Alternate $B$ and consists of synthesizing linear and non-linear modeling frameworks. The application of this alternate, however, has a direction different from that in Alternates $A$ and $B$. The emphasis in the applications is on the broad scale limnological effects of water resource activities, yet with some applications devoted to regional and local problems.

Alternate D

This alternate consisting of a large interactive Limnological Systems Analysis recognizes that an essential component for successful long term Type II planning is the ability to identify the consequences of alternate planning decisions. The interactive nature of the processes which control the water resources of the Great Lakes makes the elucidation of these consequences a difficult exercise. However, this alternate, like Alternate $C$, assumes that the amount of scientific laws and information available for the construction of a Limnological Systems Analysis which can respond to a large class of planning problems is sufficient to warrant a concerted effort at synthesis in the form of large computer models.

To perform these calculations on a lake-wide basis using a three-dimensional grid with a density of grid points sufficient to resolve medium scale phenomena is a task which strains even the largest currently available third generation computers. (See for example the discussion given in Section VIII). With large computations of this sort, the amount of output generated requires graphical display for interpretations of the result. Interactive computation can be of enormous aid in the construction, verification, and understanding phase of model building.

The observations indicate that development of an interactive Limnological Systems Analysis computer capability, which can utilize the power of currently available third generation computers (and soon to be available fourth generation computers), is a necessary adjunct to the construction of lake-wide or regional models with both reasonable spatial detail and a sufficient number of physical, biological, and chemical variables to achieve the capability of exposing subtle and unexpected consequences of planning decisions.

The applications envisioned in this alternative are those proposed in Alternate $C$. The major difference in approach is that significant effort will be expended to achieve efficient and flexible software which produces output that is useful both for planning purposes and for model understanding and verification. The larger computational capacity available in this alternative ( 50,000 to perhaps 100,000 compartments), the flexibility of input-output, model structuring, and model linking will enable the addressing of a wide series of planning and modeling questions.

It is estimated that this alternate will require a minimum of five years for completion. This alternate strongly supports the approach to a sequential development of a large computer structure, rather than the immediately beginning development of a 100,000 compartment framework.

In the development and design of this alternate, three steps are proposed:

1. Initially, emphasis would be on the bulk of the numerical algorithms and programming, together with preparation of rudimentary input and output facilities for a 5,000 compartment capacity. This would occur during the first year of effort; and output from this activity would be used early to aid in solutions of some of the problem applications indicated for alternate C.
2. During the second-third year of the Phase II work, effort would be devoted toward improvement of execution speed and the increase of system capacity to 20,000 compartments. This would include a major effort at achievement of flexibility of model structuring and interactive capabilities.
3. The fourth and fifth years would involve evaluation of the ultimate capability of a 50,000-100,000 compartment modeling structure. External efforts would be directed toward interactive computational models via cathode ray tube terminals. Interaction would take place directly with planning personnel and large scale planning questions; and a variety of interactions would be addressed with the computational framework. Coincident with the effort would be the final applications of Alternate $C$.

The four preceding alternates present a range of viewpoints of the nature of a Phase II study for Limnological Systems Analysis. In summary, Alternate A generally considers large scale interactive modeling as marginally feasible. Alternate $B$ exhibits an increasing trend toward feasibility of large scale systems analysis, but with substantial reservations about the actual utility of the results, while Alternate $C$ assumes that sufficient information is available to construct useful limnological models. Alternate D extends the concept of Alternate $C$ on a larger scale particularly with respect to computer software and input-output developments. It can also be noted that there is a variety of views expressed in the alternates regarding the priorities of the various problems.

The recommended alternate which is aimed at extracting the best elements of the four described above within certain
funding levels, in essence, combines Alternates $B$ and $C$. Alternate A is rejected on the grounds that in spite of certain risks, the Great Lakes region cannot afford to wait any longer in structuring a framework that can respond to broad planning problems. On the other hand, there are a sufficient number of poorly understood phenomena to warrant a negative vote on Alternate $D$, which would probably result in only marginal advances for significant greater expenditures.

## Funding Levels

Based on the above analysis and discussion, three levels of funding can be identified (see Table 31):

Level l. The cost at this level is $\$ 0.7$ million; and a two year completion time is reasonable. This funding represents the lowest level at which a meaningful Limnological Systems Analysis can be carried out and provides the basis for future expansion to a broad scale interactive Limnological Systems Analysis effort. It is proposed that at this level of funding, the eutrophication model be developed to the fullest practical extent. This level is therefore associated primarily with Alternate $B$ and, to a limited degree, with Alternate A. This effort would include the development of necessary non-linear non-steady-state computer programs, verification analysis, and application all specifically directed to Lake Erie. The development of an LSA is not considered feasible below the $\$ 0.7 \mathrm{million}$ level. The most useful results of funding below this level can be obtained through support of existing efforts in individual problem and modeling contexts as represented in Alternate A.

Level 2. The cost at this level is estimated at $\$ 2.0$ million with a three year completion time. The level represents a favorable balance between the priority problem contexts that can be approached rapidly, given present modeling status, and those which have high priority, but for which modeling frameworks must be significantly advanced. This level is therefore associated with Alternates $B$ and $C$. It is proposed that this level of funding cover spatial scales on a basinwide lake and regional basis and investigate the effects of various uses on water quality, chemical levels, and food chains.

## SUMMARY OF FUNDING LEVELS



TABLE 31
(continued)

## SUMMARY OF FUNDING LEVELS

| Estimated Costs |
| :---: |
| (millions of dollars) |
| Funding Level |
| II III |

c. Regional Scale

Water quality (bacterial and dissolved oxygen distributions) and eutrophication. As in funding level 2 , with additional problem areas included - 0.55 0.7
d. Other lake wide and regional applications, fishery model development, links to other sub-models, erosion sediment model development, development of ice formation, and movement forecasting model
Applications subtotal
$\frac{-}{0.4} \quad \frac{-}{1.4} \quad \frac{0.5}{2.8}$

Funding Level Total
0.72 .0
3.9

Level 3. In addition to the problem addressed in Level 2, problem areas of lower priority are also included. This level is estimated at $\$ 3.9$ million, with a three year completion time. Level 3 therefore represents a more intensive effort than Level 2. It is felt that this level of funding is the maximum amount that could be prudently spent for Phase II of the Limnological Systems Analysis. It is proposed that this level cover the elements of funding Level 2 and, in addition, a broader range of water resource problems, such as fisheries, erosion and sediment, and an increased effort in the food chain modeling.

A summary of these funding levels is presented in Table 31 These estimates include only modeling studies using readily available existing and anticipated data on the Great Lakes. The estimates further assume that the agency or organization staff performing the analysis for Phase II represents an experienced group well-versed in the techniques of mathematical modeling and the specific applications to the aquatic limnological setting. They do not include provision for field data collection, program, administration, agency or program coordination, or data retrieval facilities.

## Recommended Phase II Study

It is recommended that the Commission fund Phase II efforts at the $\$ 2.0 \mathrm{million}$ level.

At this funding level, a balance is obtained by taking an initial step to develop and apply broad scale interactive models providing a framework for testing the adequacy of technical knowledge, evolving an administrative structure, and training technical personnel in the Great Lakes problemoriented setting.

On the basis of the analysis given in Section VIII, it is concluded that the Phase II program be specifically directed to the four priority problem categories of (1) Eutrophication, (2) Water Quality, (3) Public Health, and (4) Toxic and Harmful Substances. The results of the model synthesis step
in the overall methodology were given in Table 17 which relates each problem category to synthesized model
frameworks. Inspection of this table indicates that seven modeling structures are associated in varying degrees with the four high priority problem categories.

Therefore the formal modeling structure proposed for Phase II is composed of a broad scale framework consisting of seven integrated modeling subsystems: water balance, lake circulation and mixing, chemical, eutrophication, dissolved oxygen, pathogens and ecological as shown in Figure 103. These modeling frameworks are divided into two major components by virtue of their common computational frameworks: (1) linear steady-state systems, and (2) non-linear non-steady-state systems.

Three broad activities are suggested for the implementation of the Phase II program:

1. Generalized computer program development and modifications to accommodate recently evolved numerical and software techniques.
2. Application of existing systems technology to those classes of problems for which the model evaluation ranking indicates that a reasonable degree of "success" for the application is assured.
3. Gathering of existing sub-system models, parameter values, and inputs to a broad interactive modeling framework capable of accommodating new information as it is developed from ongoing research in other areas.

The last step incorporates the fact that if a mechanism or phenomenon is poorly understood, its entry into the modeling framework is not precluded, but incorporated at the level of known information. The models to be developed, tested, and applied under the third activity are those that have not

generally been successfully applied to problem analysis and prediction of future conditions. The ecological and food chain modeling framework is an example of this situation.

No data collection program in addition to those already in existence is proposed; the feasibility analysis indicates that sufficient data are available for Phase II effort and, in addition, it is difficult to determine additional data requirements until that which is presently available is thoroughly and adequately analyzed.

Applications for Recommended Program

In choosing applications for inclusion in the recommended Phase II program, consideration is given to the points outlined earlier, especially the priority of the problem and the model status. The applications at this funding level reflect the problem priorities that are weighted in the direction of the effects of man's activities on the chemical and biological quality of the lakes. At the end of the three year effort, it is estimated that a modeling framework will be available that can answer a broad array of planning problems. During the Phase II study, of course, some planning associated with the various problem areas will be directly addressed.

Three space scales are included in the computer program development (Great Lakes scale, lake wide, and regional). Each scale is then applied to each problem context, together with the nature of the modeling framework (i.e., linear and non-linear, non-steady-state). It is further assumed that the results of ongoing land based regional planning activities will provide the information on systems inputs associated with various planning alternatives, levels of population, and industrial growth.

The three levels of spatial detail that are recommended for incorporation in the Limnological Systems Analysis are therefore:

1. Great Lakes wide scale (approximately 100-500 spatial compartments)
2. Lake wide scale (250-1,000 spatial compartments)
3. Regional scale (250-1,000 spatial compartments)

For each of these spatial scales, approximately 5-20 interactive variables would be considered, leading to a maximum number of compartments of approximately 5,000. As discussed previously, it is not considered feasible at this time to structure an LSA for a number of compartments greater than 50,000. Since the present state of the art of Limnological Systems Analysis is at a considerably lower level (on the order of 500 to 1,000 compartments), this appears to be a prudent level for Phase II effort.

The applications and associated costs are summarized in Table 32 and Table 33. On a Great Lakes space scale, two broad areas of application assume particular importance. The first area includes the effects on the limnological system of the direct use of water from the lakes for municipal, industrial, and other related activities. Specifically, applications to be considered are: basin wide use of evaporative cooling control methods; water diversions for municipal and industrial water supply; and other uses of water such as diversion of wastewater for land reclamation projects. In each of these cases, the linear systems framework appears appropriate. Both time variable and steady-state analyses would be conducted. The applications would consider the effect of the various components of water use on concentrations of indicator conservative constituents, such as chlorides and total dissolved solids, changes in lake level; and associated changes in overall quality of the lake. The time scale would be years to decades.

The second area of application on the Great Lakes space scale is concerned with the problem of water quality and effects of increased lake fertilization. Therefore, this grouping of applications deals with changes, on a Great Lakes scale,
PROBLEM TIME AND SPACE SCALES FOR RECOMMENDED APPLICATIONS

| Problem Category | Great Lakes Scale | Lake Wide Scale |  | Regional Scale |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Lake Erie | Lake Ontario | Duluth Area of Superior | $\begin{aligned} & \text { Superior } \\ & \text { Lake } \\ & \text { Michigan } \\ & \hline \end{aligned}$ | $\begin{gathered} \text { Saginaw } \\ \text { Bay } \\ \hline \end{gathered}$ | Green Bay | $\begin{gathered} \text { Lake } \\ \text { St. Clair } \\ \hline \end{gathered}$ |
| I. Water Quality |  |  |  |  |  |  |  |  |
| a. Dissolved oxygen | - | seasonal ${ }^{2}$ | - | - | - | seasonal ${ }^{1}$ | seasonal ${ }^{1}$ | - |
| b. Chemical | annual | annual ${ }^{1}$ | - | - | - | - | - | - |
|  | decade ${ }^{2}$ | - | - | - | - | - | - | - |
| II. Public Health | - | - | - | - | monthly ${ }^{1}$ | monthly ${ }^{1}$ | monthly ${ }^{1}$ |  |
| III. Eutrophication <br> a. Biomass | annual ${ }^{2}$ | weekly ${ }^{2}$ | - | week $1 \mathrm{y}^{2}$ | week $1 y^{2}$ | - | - | - |
| IV. Food Chain | - | - | annual ${ }^{1}$ | - | - | - | - | annual ${ }^{1+}$ |

[^9]| Problem Category | Great <br> Lakes <br> Scale | TABLE 33 <br> ESTIMATED COSTS OF RECOMMENDED APPLICATIONS <br> Lake Wide <br> Scale <br> Regional Scale |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Lake Erie | Lake ontario | Duluth Area of Superior | $\begin{gathered} \text { Southern } \\ \text { Lake } \\ \text { Michigan } \\ \hline \end{gathered}$ | $\begin{gathered} \text { Saginaw } \\ \quad \text { Bay } \\ \hline \end{gathered}$ | Green <br> Bay | Lake <br> St. Clair | Total Cost |
| I. Water Quality <br> a. Dissolved Oxygen | - | 0.15 | - | - | - | 0.05 | . 05 | - | 0.25 |
| b. Chemical | 0.15 | 0.10 | - | - | - | - | - | - | 0.25 |
| II. Public Health | - | - | - | - | 0.05 | 0.05 | 0.10 | - | 0.20 |
| III. Eutrohpication <br> a. Biomass | 0.10 | 0.25 | - | 0.05 | 0.15 | - | - | - | 0.55 |
| IV. Food Chain | - | - | 0.10 | - | - | - | - | 0.05 | 0.15 |

NOTE: All Costs reported in millions of dollars.
Total Estimated cost of applications........................................................................................... 40

in nutrient levels and plant biomass. The modeling framework is generally non-linear and non-steady-state, although linear sectors can be included. A non-linear chemical equilibrium model would be included, as well as a seasonal and yearly non-Iinear model of algal biomass as developed in the Demonstration Model.

On the lake wide and regional scale, primary emphasis is given to the effects on water quality and the general ecosystem of present and future discharges of municipal and industrial wastes, agricultural runoff, and other natural and man-made pollutants. Both linear and non-linear model frameworks are employed, and the time scale is generally seasonal or steady-state. Problem variables would include biomass and chemical, biochemical, and bacterial indicators of water quality.

Geographically, emphasis on the lake wide scale is directed toward Lake Erie, an area associated with water quality and eutrophication problems. In addition, it is recommendedd that the results of the International Field Year on the Great Lakes effort on Lake Ontario be used on a lake-wide basis as input data for an analysis of the ecological and food chain problem category for that lake. Therefore, it is suggested that a broadly based interactive model be developed which would provide a basis for decision making on the planned or inadvertent introduction of food chain accumulants such as pesticides and heavy metals.

On a regional scale, as shown in Tables 32 and 33 , five areas are recommended for problem analysis and model application:

1. Duluth area of Lake Superior
2. Southern Lake Michigan
3. Saginaw Bay
4. Green Bay
5. Lake St. Clair

The applications thus cover a wide geographical area and span problem locations in all Great Lakes.

The applications as described are approximately evenly distributed between specific problem contexts that have a reasonable degree of assurance of being successfully analyzed by the Limnological Systems Analysis and problem contexts that are more vaguely stated and require incorporation of less well-understood phenomena. It is believed that pursuit of the Phase II program as outlined above will provide a meaningful structure to the decision making processes on the water resources of the Great Lakes.

# SECTION XI 

## ACKNOWLEDGEMENTS

## SECTION XI

## ACKNOWLEDGEMEN'TS

The information and results presented in this report are an outgrowth of contributions by numerous individuals interested in the Great Lakes water resource problems and management. Foremost among those who contributed to this project are Mr. Frederick O. Rouse, Chairman of the Great Lakes Basin Commission; Mr. Leonard T. Crook and Mr. David C.N. Robb, staff members of the Commission.

In addition, significant inputs to the study were provided by the Board of Technical Advisors convened by the Great Lakes Basin Commission. This board provided technical input and project review periodically during the study. The members of the board, Dr. John M. Armstrong, Dr. Alfred M. Beeton, Dr. G.E. Birchfield, Dr. Carl W. Chen, Dr. Leo J. Hetling, Dr. Norbert Jaworski, Mr. Edwin L. Johnson, Dr. Ronald T. McLaughlin, Dr. Arthur P. Pinsak, Mr. Howard L. Potter, and Dr. Sam B. Upchurch contributed in their areas of technical speciality with a candor and generosity that is greatly appreciated.

The Plan and Program Formulation Committee composed of staff members from the various federal agencies and states represented on the Commission furnished valuable input and review, particularly with respect to definition of administrative and planning problems in the Great Lakes Basin. The encouragement of this group assisted in insuring that the study results were directed towards solution of planning problems on the Great Lakes.

Battelle Northwest performed subcontracting services within the project for problem definition and state of the art technical evaluation and analysis in the areas of hydrological balance, ice and lake-wide temperature, and erosion-sediment.

Hydroscience's Board of Technical Consultants consisting of Dr. Donald R.F. Harleman, Dr. James R. Kramer, Dr. Clifford H. Mortimer, Dr. Gerald A. Rohlich, and Dr. George W. Saunders provided guidance and initial input to the study. Their contribution is both appreciated and acknowledged.

Several individual consultants gave assistance in specialized technological areas. In particular, Drs. Bernard C. Patten, Gerald J. Paulik, and G.T. Csanady provided valuable contributions in the areas of ecological, fisheries, and hydrodynamic modeling, Dr. Raymond Canale joined our staff on a temporary basis and provided significant assistance in development and application of demonstration models. Dr. R.T. Gedney kindly supplied us with detailed hydrodynamic calculations for the demonstration model.

Members of the Canada Centre for Inland Waters, in particular, Dr. James P. Bruce, were most helpful in making Western Lake Erie water quality data available for use in the Demonstration Model effort. Their assistance is sincerely appreciated.

Members of the Hydroscience staff who participated in this study are: Dr. Robert V. Thomann, Dr. Dominic M. Di Toro, Dr. John A. Mueller, Mr. Daniel S. Szumski, Mr. Henry J. Salas, Mr. Joseph A. Nusser, Mr. James J. Fitzpatrick, Mr. William F. Lederle, Miss Viola J. Whartenby, and Mrs. Leslie A. Fitzpatrick. Their contributions to the project were enormous and are sincerely appreciated. Special acknowledgement is expressed to Mrs. Anne Detroyer for her patient and considerate efforts in synthesizing and editing the final project report.

The cooperation and input of all the above individuals and the assistance and cooperation of innumerable federal, state, and university personnel involved in Great Lakes Water Management is acknowledged with sincere thanks.

## SECTION XII

## NOMENCLATURE

## SECTION XII

NOMENCLATURE

## From Section VII


Page
$F_{i c}=$ ice weight force ..... 122
$\mathrm{F}_{\mathrm{m}}=$ wind force on ice surface ..... 122
$\mathrm{F}_{\mathrm{r}}=$ froude number ..... 120
$\mathrm{F}_{\mathrm{S}}=$ shore resistance force ..... 122
$\mathrm{F}_{\mathrm{w}}=$ frictional force of water on ice ..... 122
g = acceleration due to gravity ..... 120
$\mathrm{k}_{\mathrm{b}}=$ bending coefficient ..... 122
$\mathrm{k}_{\mathrm{C}}=$ crushing coefficient ..... 122
$k_{s}=k_{b}+k_{s r} R_{s r} / R_{b}+k_{c} R_{c} / R_{b}$ ..... 122
$k_{s r}=$ shearing coefficient ..... 122
$\mathrm{P}_{\mathrm{a}}=$ atmospheric pressure ..... 117
Q $=$ net gain or loss of heat ..... 116
$Q_{b}=$ effective back radiation ..... 116
$Q_{C}=$ heat gain by condensation ..... 116
$Q_{\mathrm{e}}=$ heat loss owing to evaporation ..... 116
$Q_{h}=$ heat conduction across interface ..... 116
$Q_{r}=$ reflected radiation ..... 116
$Q_{S}=$ insolation heat source ..... 116
$\mathrm{R}=$ Bowen ratio ..... 117
$R_{b}=$ ultimate strength of ice in bending ..... 122
$R_{C}=$ ultimate strength of ice in crushing ..... 122
$R_{\text {sr }}=$ ultimate strength of ice in shearing ..... 122
Page
$t=$ thickness of the upstream edge at equilibrium ..... 120
$\mathrm{T}_{\mathrm{a}}=$ air temperature ..... 117
$T_{W}=$ water temperature ..... 117
$\mathrm{V}=$ velocity of upstream edge ..... 120
$\mathrm{W} \quad=$ wind speed ..... 118
$y=$ water depth in front of upstream edge ..... 120
$\xi=$ porosity ..... 120
$\rho_{,} \rho_{1}=$ specific masses of water and solid ice ..... 120
Thermal Models
$A(z, t)=$ cross sectional area ..... 128
$E(z, t)=$ vertical dispersion coefficient ..... 128
$Q(z, t)=$ vertical net flow rate ..... 128
$S_{j}=$ inputs of thermal energy ..... 128
$S_{o}=$ outputs of thermal energy ..... 128
$t=$ time ..... 128
$T(z, t)=$ water temperature ..... 128
$z=$ depth ..... 128
Lake Circulation and Mixing Models
$A_{z}=$ coefficient of vertical eddy viscosity ..... 144
f = coriolis parameter ..... 140

|  |  |  | Page |
| :---: | :---: | :---: | :---: |
| $\mathrm{F}_{\mathrm{X}}$ | $=$ | force in x direction | 140 |
| $\mathrm{F}_{\mathrm{y}}$ | $=$ | force in $y$ direction | 140 |
| $\mathrm{F}_{\mathrm{Z}}$ | $=$ | force in $z$ direction | 140 |
| 9 | $=$ | acceleration of gravity | 140 |
| h | $=$ | bottom depth | 144 |
| H | $=$ | bottom depth | 145 |
| $\mathrm{K}_{\mathrm{X}}$ | $=$ | $x$ component of the heat dispersion coefficient | 141 |
| $\mathrm{K}_{\mathrm{Y}}$ | $=$ | $y$ component of the heat dispersion coefficient | 141 |
| $\mathrm{K}_{\mathrm{z}}$ | $=$ | $z$ component of the heat dispersion coefficient | 141 |
| p | $=$ | pressure | 140 |
| $Q^{\text {T }}$ | $=$ | net heat input | 141 |
| t | $=$ | time | 140 |
| T | $=$ | water temperature | 141 |
| u | $=$ | $x$ component of the velocity | 140 |
| U | $=$ | depth averaged velocity in $x$ direction | 145 |
| V | $=$ | $y$ components of the velocity | 140 |
| V | $=$ | depth averaged velocity in $y$ direction | 145 |
| w | $=$ | $z$ components of the velocity | 140 |
| X | $=$ | spatial coordinate | 140 |
| Y | $=$ | spatial coordinate | 140 |
| z | $=$ | spatial coordinate | 140 |

Page
$\rho \quad=$ fluid density ..... 140
$\eta \quad=\quad$ height of the free surface ..... 144
${ }^{\tau} \mathrm{x}=\mathrm{x}$ component of the wind stress ..... 144
$\tau_{y}=y$ component of the wind stress ..... 144
$\tau_{b x}=x$ component of bottom frictional stress ..... 145
${ }^{\tau}$ by $=y$ component of bottom frictional stress ..... 145
$\tau_{s x}=\mathrm{x}$ component of surface wind stress ..... 145
${ }^{\tau}$ sy $=y$ component of surface wind stress ..... 145
Erosion and Sediment Models
$\mathrm{H}=$ peak to valley wave height ..... 176
M $\quad=\quad$ beach slope ..... 178
$R \quad=\quad$ maximum wave run-up ..... 176
T = wave period ..... 176
$\alpha \quad=\quad$ beach slope ..... 176
Chemical Models
$a_{i e}=$ number of models of element e contained in ..... 185 component i.
$A_{i}=$ molecular formula of $i^{\text {th }}$ chemical component ..... 186
$b_{e}=$ total mole fraction concentration of element $e$ ..... 185
$\mathrm{E}=\quad$ number of elements ..... 185
Page
G $\quad=\quad$ Gibbs free energy ..... 185
[H] $=$ molar concentrations of $\mathrm{H}^{+}$ ..... 186
$\left[\mathrm{HCO}_{3}\right]=$ molar concentrations of $\mathrm{HCO}_{3}{ }^{-}$ ..... 186
$\left[\mathrm{H}_{2} \mathrm{CO}_{3}\right]=$ molar concentrations of $\mathrm{H}_{2} \mathrm{CO}_{3}$ ..... 186
[i] = activity of component i ..... 185
$K_{C}=$ solubility product for condensed species ..... 187
$\mathrm{K}_{\mathrm{j}}=$ equilibrium coefficient ..... 788
in $=$ natural logarithm ..... 185
$n_{i}=$ mole fraction concentration of component $i$ ..... 185
$\mathrm{N}=$ number of components ..... 185
$\mathrm{R}=$ universal gas constant ..... 185
$\mathrm{T}=$ temperature in ${ }^{\circ} \mathrm{K}$ ..... 185
$\nu_{i}=$ stoichiometric coefficient ..... 186
$\mu_{i}=$ chemical potential ..... 185
$\mu_{i}{ }^{\circ}=$ standard free energy ..... 186
Eutrophication Models
A $=$ zooplankton growth rate ..... 215
C $=$ carnivore predation rate ..... 215
D $\quad=$ natural death rate ..... 215
g $=$ predation rate/unit zooplankton concentration ..... 218
Page
G $=$ zooplankton grazing rate ..... 214
$\mathrm{H}=$ zooplankton biomass ..... 215
$\mathrm{K}_{1}=$ growth rate/unit nutrient concentration ..... 218
$K_{2}=$ respiration rate ..... 218
$K_{5}=$ phytoplankton uptake rate ..... 218
$\mathrm{K}_{6}=$ regeneration rate ..... 218
$M=$ exchange rate ..... 218
$\mathrm{N}=$ nutrient concentration ..... 218
$\mathrm{N}_{\mathrm{B}}=$ nutrient concentration in hypolimnion ..... 218
$\mathrm{P} \quad=\quad$ phytoplankton biomass ..... 214
$P_{h}=$ phytosynthesis rate ..... 214
$\mathrm{R}=\mathrm{respiration} \mathrm{rate}$ ..... 214
$R_{p}=\quad$ respiration rate ..... 217
$\mathrm{V}=$ sinking velocity ..... 217
$z \quad=\quad$ vertical spatial coordinate ..... 217
Dissolved Oxygen Models
B $=$ benthic uptake rate ..... 234
c $=$ concentration of dissolved oxygen ..... 232
$c_{0}=$ dissolved oxygen concentration at $z=0$ ..... 234
$c_{s}=$ saturation concentration of dissolved oxygen ..... 234
$\mathrm{E}=$ diagonal matrix of dispersion coefficients ..... 232
$\mathrm{E}_{\mathrm{h}}=$ dispersion coefficient at $\mathrm{z}=\mathrm{h}$ ..... 234
$\mathrm{E}_{\mathrm{O}}=$ dispersion coefficient at $\mathrm{z}=0$ ..... 234
$\mathrm{E}_{\mathrm{z}}=$ dispersion coefficient in z direction ..... 233
$\mathrm{K}=$ temperature dependent reaction rate ..... 233
$\mathrm{K}_{\mathrm{L}}=$ surface transfer rate ..... 234
$L(z, t)=$ concentration of dissolved organic matter ..... 233
$P(z, t)=$ rate of photosynthetic contribution of dissolved ..... 233 oxygen
$R(z, t)=$ rate of respiratory sink of dissolved oxygen ..... 233
$\overrightarrow{\mathrm{U}} \quad=$ velocity vector ..... 232
$\Sigma_{i} S_{i}=$ sum of all sources and sinks of dissolved oxygen ..... 232
$\Delta \quad=\quad \frac{\partial}{\partial X} \vec{i}+\frac{\partial}{\partial y} \vec{j}+\frac{\partial}{\partial z} \vec{k}$ ..... 232
Pathogens and Indicator Bacteria Models
$c_{b}=$ concentration of bacteria ..... 246
$\mathrm{K}_{\mathrm{b}}=$ rate of die-off of the bacteria ..... 246
$W_{b}=$ direct discharge of bacteria ..... 246
Fishery Models
$a_{i i}={\underset{\text { interaction }}{ } \begin{array}{l}\text { species }\end{array}}$ ..... 262
$b_{i}=$ growth constant for $i^{\text {th }}$ fish species ..... 262
$\mathrm{c}=\mathrm{ratio}$ of F to x ..... 259
$c_{i}=$ fishing intensity for the $i^{\text {th }}$ fish species ..... 262
$\mathrm{F}=$ fishing mortality rate ..... 259
g = growth rate ..... 260
$\mathrm{k}=$ logistic model constant ..... 259
K $=$ empirical coefficient ..... 260
L = asymptotic length of fish ..... 260
$\mathrm{M} \quad=\quad$ rate of natural mortality ..... 259
$\mathrm{n}=$ effect of external environment ..... 259
$\mathrm{N}=\mathrm{fish}$ numbers ..... 260
(N) = vector of numbers of fish in the year classes ..... 264
p $=$ fish biomass ..... 259
$p_{i}=$ population of the $i^{\text {th }}$ fish species ..... 262
P = asympotic equilibrium fish population ..... 259
$P_{i}=$ fraction of year class $i$ that survives from ..... 264 t to t + l
r $=$ rate of recruitment ..... 259
$\mathrm{R}=$ number of recruits ..... 260
$R_{i}=$ fraction of year class i that is newborn at ..... 264
Page
$t_{c}=$ age of fish retained by fishing gear ..... 260
$t_{0}=$ time zero ..... 260
$t_{r}=$ age of fish entering exploitation area ..... 260
W $=$ fish weight ..... 260
$x \quad=$ fishing effort ..... 259
$x_{i}=$ fishing intensity of the $i^{\text {th }}$ species ..... 262
y $=$ yield ..... 259
$\delta \quad=\quad$ empirical coefficient ..... 260
Ecological and Food Chain Models
A $=$ matrix of competition coefficients $\alpha_{i j}$ ..... 290
$B_{i}=$ niche bredth of specie $i$ ..... 289
c $=$ concentration of tracer in the organism ..... 283
c' $=$ concentration of tracer in the water ..... 283
$\mathrm{C}=\mathrm{mn} \times \mathrm{I}$ vector of concentrations ..... 281
$C_{i r}=$ concentration of variable $i$ at location $r$ ..... 277
$D_{h}=$ niche diversity of environment $h$ ..... 289
$f_{i h}=$ proportion of individuals of species i in ..... 289
$\begin{aligned} \mathrm{F}_{\mathrm{i}, r s}= & \text { bulk transport of variable } i \\ & \text { location } s\end{aligned}$ ..... 277
Page
g $=$ vector of sources ..... 281
$g_{\text {ir }}=$ source of variable $i$ at location $r$ ..... 277
[K] = mn x mn matrix of interaction coefficients ..... 281
$\mathrm{K}=$ column vectors of carrying capacities $K_{i}$ ..... 290
$\mathrm{K}_{\mathrm{i}}=$ equilibrium population of specie $i$ ..... 289
$K_{i j, r}=\quad$ causal transformation of variable $i$ to variable ..... 277 $j$ at location $r$
$P_{i h}=$ proportion of specie i in environment $h$ ..... 289
Q $=$ flow rate ..... 284
$r \quad=\quad$ growth rate of specie $i$ ..... 289
$\mathrm{V}=$ single homogeneous volume ..... 284
W = mass input rate ..... 284
$x=$ column vectors of species $X_{i}$ ..... 290
$x_{i}=i^{\text {th }}$ specie concentration ..... 289
$\alpha_{i j}=$ competition coefficient between species $i$ and $j$ ..... 290
$\lambda=$ half-life (base e) ..... 284
From Section IX
Introduction
C $=$ concentration ..... 318
$Q_{n}=$ parameters ..... 319
$t_{0}=$ detention time ..... 318
$\mathrm{V}=$ lake volume 318
$\mathrm{V}_{\mathrm{n}}=$ volume of $\mathrm{n}^{\text {th }}$ lake $\quad 318$
W = wastewater input . 318
$W_{n}=n^{\text {th }}$ wastewater input $\quad 319$

Lake Erie Western Basin Chlorides and Coliform Models
$c_{j}=\quad \begin{aligned} & \text { concentration of the water quality variable in } \\ & \text { segment } j\end{aligned}$
$\mathrm{E}_{\mathrm{kj}}^{\prime}=\quad$ bulk dispersion coefficient between segments $k \quad 332$
$Q_{k j}=$ net advective flow from segment $k$ to segment $j 332$
$W_{j}=\quad$ mass input rate to segment $j \quad 332$
$\begin{array}{lll}\alpha_{k j}= & \begin{array}{l}\text { weighting factor for the finite difference } \\ \text { approximation used }\end{array} & 332\end{array}$
$\beta_{k j}=1-\alpha_{k j} \quad 332$

Lake Eutrophication Model
$a_{1}=$ conversion efficiency of zooplankton 349
$a_{Z P}=$ carbon/chlorophyll ratio - zooplankton 349
$c_{p}=$ inorganic phosphorus 348
$\mathrm{C}_{\mathrm{g}}(\mathrm{T})=\begin{aligned} & \text { temperature dependent grazing rate of the } \\ & \text { zooplankton biomass }\end{aligned} \quad 341$
$\begin{array}{lll}C_{N}= & \text { total inorganic nitrogen } & 348\end{array}$
$D_{z j}=$ death rate - zooplankton ..... 349
$D_{p}=$ phytoplankton death rate ..... 346
$D_{Z}=$ zooplankton death rate ..... 346
$\mathrm{e}=2.718$ ..... 347
f $=$ photoperiod ..... 347
$G_{p}=$ phytoplankton growth rate ..... 346
$G_{Z}=$ zooplankton growth rate ..... 349
$\mathrm{H}=\mathrm{depth}$ of segment ..... 347
I $=$ light intensity ..... 346
$I_{a}=$ mean daily light intensity ..... 347
$I_{s}=$ optimal light intensity ..... 346
$I_{o}=$ surface light intensity ..... 346
$K_{e}=$ extinction coefficient ..... 346
$\mathrm{K}_{\mathrm{mN}}=$ half-saturation constant - total inorganic ..... 348 nitrogen
$K_{m p}=\quad \begin{aligned} & \text { half-saturation constant }- \text { total inorganic } \\ & \text { phosphorus }\end{aligned}$ ..... 348$K_{4}=$ empirical mortality constant350
$K_{2}(T)=$ temperature dependent grazing rate of the zoo- ..... 341 plankton biomass
$\mathrm{K}_{3}(T)=$ temperature dependent endogenous respiration ..... 350rate$\mathrm{P}=$ phytoplankton chlorophylla346
$r=$ time averaged growth rate reduction factor 347

$S_{i k j}=\quad$| the $k^{\text {th }}$ source or sink of substance $i$ in |
| :--- |
| segment $j$ | 341

$V_{j}=$ segment volume 341
$z=$ depth 346
$z=z o p l a n k t o n$ biomass concentration 349

A Food Chain Model of Cadmium in Western Lake Erie
$[A(d / d t)]_{i}=n x \operatorname{n~matrix} \quad 413$
$C_{p} \quad=\begin{aligned} & \text { concentration of toxicant in the phyto- } \\ & \text { plankton }\end{aligned} \quad 415$
$\mathrm{C}_{\mathrm{W}} \quad=\begin{gathered}\text { concentration of } \\ \text { column }\end{gathered} \quad 414$
$C_{z} \quad=\begin{aligned} & \text { concentration of toxicant in the zoo- } \\ & \text { plankton }\end{aligned} \quad 415$
$\begin{aligned} K_{i-1, i, j}= & \begin{array}{l}\text { toxicant production from } i \\ \\ \text { trophic level at location } j\end{array} \text { to } i^{\text {th }} \quad 412\end{aligned}$
$\left[K_{i-1, i} V\right]=n \times n$ diagonal matrix 413
M $\quad=$ mass trophic level/volume water 411
$\mathrm{N} \quad=$ mass toxicant/mass trophic level 411
$\left(N_{i} M_{i}\right) \quad=n \times 1$ vector of the tracer material $\quad 413$
$\Gamma_{i} \quad=i^{\text {th }}$ trophic level mass 413


[^0]:    *Collecting (Reporting). See Legend of Table 8.

[^1]:    * See Legend at end of Table.

[^2]:    *See legend at end of Table.

[^3]:    *See legend at end of Table.

[^4]:    *See Legend at end of Table 10.

[^5]:    Horizontal velocities at a constant $14.9 \mathrm{M}(49.2 \mathrm{ft})$ from surface. Wind direction,W5OS; wind magnitude, 10.1 meters per second ( 22.7 mph ); friction depth, 27.4 meters ( 90.0 ff ); rivers: Detroit, Niagara.

[^6]:    EXIST CONCENTRATIONS FIGURE 26
    WITH HIGH BACTERIAL

[^7]:    ERIE LAKE

    FIGURE 65 N4ヨSㅋM NI NOI $\perp \forall 18 \forall \wedge$

    THROUGH OCTOBER, 1970
    $\exists \Varangle \cap \perp \forall \forall \exists d W \exists \perp$ ㅂ $\exists \perp \forall M$

[^8]:    FIGURE 89
    O\＆6！OL 1 OVOONIH NJSOXLIN OINVOYO
    G1VO QヨAY3S80 ONV SLInSヨy 7コ00W to NOSIyVdWOO

[^9]:    ${ }^{1}$ Steady-state model application
    ${ }^{2}$ Time variable model application

