PROMOTING PRODUCTIVE STRUGGLE IN MIDDLE SCHOOL MATH CLASSROOMS

by

Michael Reitemeyer

An education leadership portfolio submitted to the Faculty of the University of Delaware in partial fulfillment of the requirements for the degree of Doctor of Education in Educational Leadership.

Fall 2017

PROMOTING PRODUCTIVE STRUGGLE IN MIDDLE SCHOOL MATH CLASSROOMS

by

Michael Reitemeyer

Approved: ____

Chrystalla Mouza, Ed.D.
Interim Director of the School of Education

Approved: ____

Carol Vukelich, Ph.D.
Dean of the College of Education and Human Development

Approved: ____

Ann L. Ardis, Ph.D.
Senior Vice Provost for Graduate and Professional Education

I certify that I have read this education leadership portfolio and that in my opinion it meets the academic and professional standard required by the University as an education leadership portfolio for the degree of Doctor of Education.

Signed: ____

James Hiebert, Ph.D.
Professor in charge of thesis on behalf of Advisory Committee

I certify that I have read this education leadership portfolio and that in my opinion it meets the academic and professional standard required by the University as an education leadership portfolio for the degree of Doctor of Education.

Signed: ____

Amanda Jansen, Ph.D.
University of Delaware, School of Education

I certify that I have read this education leadership portfolio and that in my opinion it meets the academic and professional standard required by the University as an education leadership portfolio for the degree of Doctor of Education.

Signed: ____

Zoubeida Dagher, Ph.D.
University of Delaware, School of Education

I certify that I have read this education leadership portfolio and that in my opinion it meets the academic and professional standard required by the University as an education leadership portfolio for the degree of Doctor of Education.

Signed: ____

Deborah Panchisin, Ed.D.
Appoquinimink School District

ACKNOWLEDGMENTS

I wish to thank my adviser, Jim Hiebert, for inspiring me to be a better educator long before I met him, as well as my committee members for volunteering their own time to support me and for pushing me to my cognitive limits. I would like to thank the creators of IMP for opening my eyes both to mathematics and to teaching in ways I could have never possibly imagined—particularly Brian Lawler for his unwavering patience and thoughtfulness with me. Also, I would like to thank Jamila Riser for fostering and sustaining reform-based mathematics teaching in Delaware.

Lastly, I would like to thank my wife, Maddie, for her support and companionship through everything.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

A problem in my district where I am the math specialist is a lack of student productive struggle around the mathematics in middle school math classrooms. By productive struggle I mean students attempting to make sense of something that is not immediately apparent, working towards reconfiguring their understanding of facts, ideas, or procedures (Hiebert & Grouws, 2007). I visited each middle school math teacher at the beginning of this school year (September 2016) observing that 80% of middle level classrooms demonstrated lessons where students were mostly applying memorized or modeled procedures on routine exercises. I observed only 6% of teachers leading lessons where students were meaningfully engaged in making sense of the mathematics (the other 14% were providing problems for students to make sense of the mathematics but were routinizing or over-scaffolding problems to remove important opportunities for students to productively struggle). This lack of productive struggle is important as my district wants its students to understand mathematics conceptually and productive struggle is one of few research-supported ways to support this conceptual growth (Hiebert & Grouws, 2007).

My goal was to understand more about teachers' growth in creating a culture where students productively struggle with the mathematics. I also wanted to understand more about how teachers' beliefs about teaching changed as they began to shift some of

the mathematical authority to their students. Lastly, I wanted to examine how this transition towards a culture of productive struggle impacted students' beliefs about mathematics and themselves.

Professional development (PD) was the main avenue I pursued to support my goals—session based PD, continuous on-site support, and having teachers learn from teaching. This PD was designed to help teachers make this shift towards a classroom that promoted productive struggle. Then via anonymous teacher surveys, student interviews, and classroom observations I attempted to discern whether and how this PD affected both teachers and students.

The results were promising, particularly in terms of teachers' evolution in both practice and beliefs. Overall teacher improvement in terms of productive struggle was positive as the percentage of teachers at the lowest score on the rubric diminished from 80% to 49% over the course of the school year though there is still ample room for growth. A cohort of teachers found two PD interventions—co-planning lessons and discussions about math education—to be especially helpful in their growth in having their students take more ownership in doing the mathematics. Student attitudes were also primarily encouraging as they reflected on their classroom experiences compared to prior years.

As a next step I would recommend interviewing students immediately after the teacher performs an intervention that helps promote student productive struggle. The student interviews led to general feelings of satisfaction but were infrequently tied to specific teacher actions that created these feelings. I would also ask for more specific, immediate feedback from teachers related to the various professional development interventions to target what exactly about the professional development they found helpful.

Chapter 1

INTRODUCTION

Professional Development Approach

Professional development (PD) was the general approach I took in helping to promote classrooms that encouraged student productive struggle. The PD was approached from three avenues—session-based (traditional) PD, on-site continuous PD, teachers learning from teaching. The session-based PD consisted of me leading full- or half-day PD sessions to all middle school math teachers in my district. The on-site continuous PD involved me co-planning lessons, observing lessons, reflecting on observed lessons, as well as simply dialoguing with individual teachers or PLC's about math education topics (like wait-time, productive struggle, assessment, etc.). The teachers learning from teaching PD included me filming teachers and them analyzing the video with me and often other teachers in their PLC.

Contents of ELP Final Paper

Section	Description
Problem addressed	This section provides detailed information about the organizational context in which my problem exists, describes my role in this organization including my responsibility to address this problem, argues why this problem is worth addressing, and declares my improvement goal.
Improvement strategies	This section specifies what interventions I attempted detailing the overall improvement design with an explanation of why I decided on the interventions that I did. Details on implementation are included here.
Efficacy of improvement strategies	This section analyzes the recorded data from the improvement interventions commenting on their effectiveness as well as the fidelity of their implementation.
Reflections on improvement efforts	This section provides a conclusion as to whether or not the improvement goals were met while noting what interventions went well, which ones did not go well, and what should be changed for future iterations.
References	This section includes all references cited during this paper.
Appendices	Teacher beliefs worksheet and results, importance of listening to your students video, learning goals recapped, teacher survey response data, student interview responses, ELP proposal, artifacts, thoughts on teacher liberation

PROBLEM ADDRESSED

The problem I am addressing is that there appear to be very few middle school math classrooms in my district where students are productively struggling with key math ideas. Within the first six weeks of the 2016-2017 school year, I had observed all 35 middle school math teachers. On my first visit to their class I assessed the level of productive struggle in their room using the Cognitive Demand criteria adapted from Alan Schoenfeld's TRU-Math observation rubric (2014):

	Domain: Cognitive Demand
	To what extent are students supported in grappling with and making sense of math concepts?
1	Students are spending most of their time applying presented algorithms and/or are working on routine procedures.
2	Activities are presented that offer the possibility of conceptual richness but that richness is lessened by the teacher—often via over-scaffolding or proceduralizing presented tasks.
3	Students are productively struggling to make sense of the mathematics. Students are working towards building understandings; mathematical connections are at least attempted.

Figure 1 Rubric for analyzing level of opportunity for student productive struggle.

My observations resulted in the following:

Table 1 Observation results concerning level of productive struggle in middle school math classrooms in the district.

TRU-Math rubric score	Number of teachers	Percent of teachers
1	28	80.0
2	5	14.3
3	2	5.7

There is certainly a need for more productive struggle in the district's middle school math classrooms with over three-quarters of the teachers observed at the lowest level of the rubric and fewer than 6% of teachers at the highest level of the rubric.

The lack of productive struggle is problematic as research has supported its value in students learning mathematics conceptually (Bjork & Bjork, 2011; Hiebert & Grouws, 2007; Kapur, 2008; Kapur, 2009, 2011, 2014; Kapur & Bielaczyc, 2012; Lehman, D'Mello & Graesser, 2012; Warshauer, 2015a). Productive struggle is consistent with the district's vision for good instruction via the district beats: communication, collaboration, critical thinking, creativity. These beats are meant to foster student engagement and are expected to be at the heart of every lesson. Part of my responsibilities in my position as math specialist is to emphasize the district's vision by supporting teachers in their attempts to implement the beats. Fortunately, true collaboration and critical thinking happen when students are engaged in productive struggle—or are working on a task whose solution is not immediately apparent. Said another way, there is no need for students to think critically or engage in meaningful collaboration if the solution to a problem is immediately apparent. Similarly, students often must engage in creative thinking, using original ideas, when trying to solve a problem whose solution is not immediately apparent.

ORGANIZATIONAL CONTEXT

In August 2016, I became the math specialist for the Appoquinimink School District—a suburban, moderately high-achieving school district in Delaware.

Table 2 Students' math performance on two most recent Smarter Assessments.

	SY 2015 - 2016			SY 2016 - 2017		
Grade	District's percent proficient	State's percent proficient	+/-	District's percent proficient	State's percent proficient	+/-
6	53.6	37	+16.6	55	41.4	+13.6
7	49	39.6	+9.4	54.4	41.3	+13.1
8	51.2	37.7	+13.5	54.7	38.4	+16.3

While my responsibilities involve support instruction in grades 6 – 12, the adoption of a new middle school math curriculum led me to spend the majority of my time working with middle school teachers. Demographically that meant I was dealing with primarily white women who are overrepresented in our district's middle school math teaching population:

Table 3 Demographical information concerning race/ethnicity and gender of middle level math teachers.

Race	Number (n = 35)	Gender	Number (n = 35)
White	32	Women	33
Black	1	Men	2
Latin@	1		
Asian	1		

Note. The one Latina employee was not dual-identified ethnically/racially because she did not self-identify as anything other than Latina when asked.

IMPROVEMENT GOAL

I want to understand more about teachers' growth in creating a culture where students are productively struggling with the mathematics. I also want to understand more about how teachers' beliefs about teaching change as they begin to shift some of the mathematical authority to their students. Relatedly, I want to examine how this transition towards a culture of productive struggle impacted students' beliefs about mathematics and themselves. More than anything, I want teachers to feel liberated as they shift mathematical authority to the students and their students come to feel empowered in their classes (see Appendix H for more of my feelings on this topic).

Chapter 2

IMPROVEMENT STRATEGIES

To help me support teachers in fostering student productive struggle, I created the following Theory of Action to guide me:

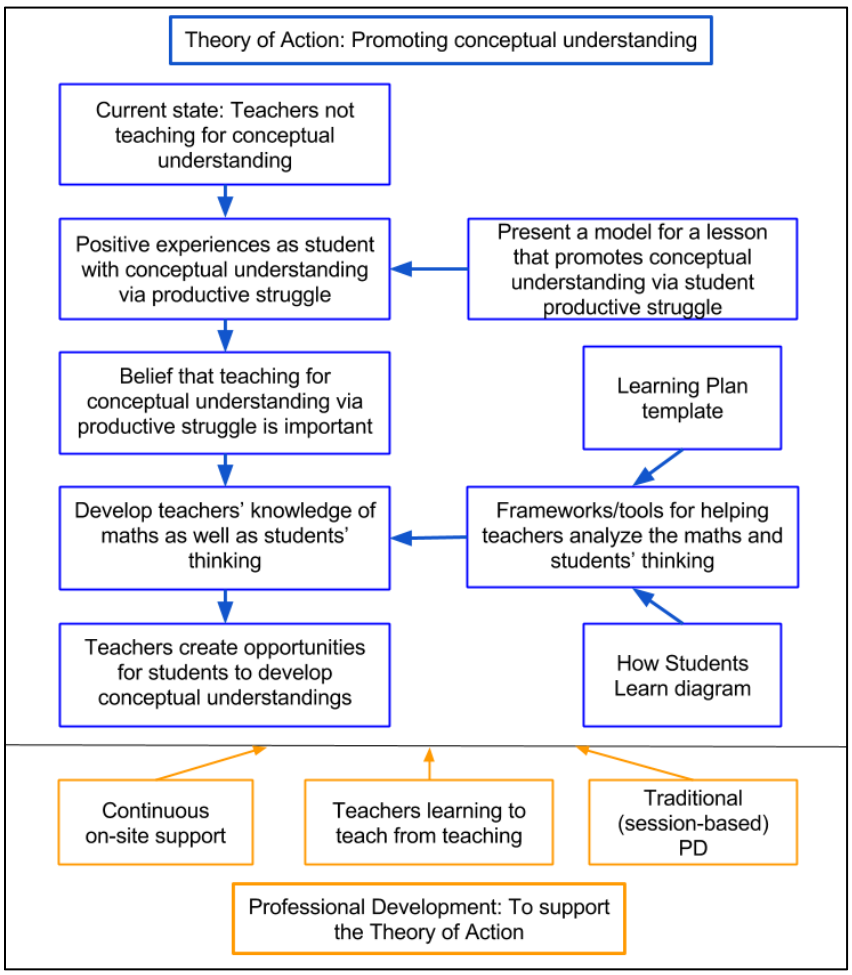


Figure 2 Theory of Action

Rationale

To support the Theory of Action I utilized three avenues of PD: traditional (session-based) PD, continuous on-site support, teachers learning to teach from teaching. Each of these three forms of PD were applied in an integrated sense with each district teacher receiving traditional PD during district in-service days, on-site support via classroom and PLC visits by me throughout the school year, and opportunities to learn from themselves via video review and from peers via department-level classroom observations and reflections. “There is a growing consensus that professional development yields the best results when it is long-term, school-based, collaborative, focused on students’ learning, and linked to curricula” (Hiebert, Gallimore & Stigler, 2002, p. 3). Traditional (session-based) PD allowed our teachers the opportunity to engage with the curriculum and uncover the mathematics therein while reflecting on how students will be making sense of the lesson’s prompts which are powerful PD opportunities (Hiebert, Carpenter, Fennema, Fuson, Human, Murray, Alwyn & Wearne, 1996; Kennedy, 1998). Iterative on-site support would encourage teachers to apply what they have learned from all the PD forms while prompting further reflection on their teaching practice (Ball, 1996; Hiebert et al., 2002). Teachers learning from teaching is another important avenue for improving teacher practice because the advice comes from peers, is clearly applicable, and is directly related to what they are doing in the classroom; furthermore, it moves teachers toward a professional atmosphere of debate, critique, and challenge which furthers teachers’ capacity to grow and refine their standards for what effective teaching looks like (Ball, 1996; Ball & Cohen, 1999).

A major motivation for promoting student productive struggle is the connection that productive struggle has to conceptual understanding. Teaching for conceptual understanding is a goal I have for my teachers and many teachers have for their students; productive struggle is one of the few research-grounded ways to support it:

> "If understanding is defined as the mental connections among mathematical facts, ideas, and procedures, then struggling is viewed as a process that reconfigures these things. Relationships among facts, ideas, and procedures are re-formed when new information cannot easily be assimilated or when the old relationships are found to be inadequate to make sense of a new problem (Piaget, 1960; Skemp, 1971). Struggle results in restructuring one's mental connections in more powerful ways." (Hiebert & Grouws, 2007, p. 388)

If teachers are going to teach for conceptual understanding, it would be helpful if they believed teaching for conceptual understanding is important. If you want to change a teacher's beliefs, one potentially effective approach is to give them positive experiences with a new system first (Guskey, 1989); changes in their personal beliefs may follow. If we are to expect teachers to offer their students learning opportunities that support conceptual understanding, then it makes sense to provide them experiences as learners in an environment that promotes conceptual understanding: "When teachers have opportunities to interact with their subject matter in ways that they aim for their own students to do (such as engaging in writing workshops, getting feedback on their own writing, giving critiques), they are more likely to engage in those practices in their classrooms" (Lieberman & Wood, 2003, as cited in Darling-Hammond, Hammerness, Grossman, Rust & Shulman, 2005, p. 396). Teachers need a model of what teaching for conceptual understanding looks like and be given the opportunity to evaluate how they can implement changes in their classroom to more resemble the practice they see modeled.

Teaching for conceptual understanding in mathematics requires two forms of knowledge: "…knowledge of the subject to select tasks that encourage students to wrestle with key ideas and knowledge of students' thinking to select tasks that link with students' experience and for which students can see the relevance of the ideas and skills they already possess". (Hiebert et al., 1996, p. 16). If teachers need knowledge of the subject and knowledge of students' thinking to effectively teach for conceptual understanding, then I needed to facilitate this knowledge acquisition. Kennedy (1998, p. 25) further validated this logical progression to focus on math knowledge and student thinking, "So teachers leave these [most effective] programs with very specific ideas about what the subject matter they will teach consists of, what students should be learning about that subject matter, and how to tell whether students are learning or not. This content makes the greatest difference in student learning". I have developed two frameworks or tools for helping teachers analyze both the mathematics as well as students' thinking: one is a "Learning Plan" template, the other a diagram concerning how students best learn mathematics conceptually (see artifacts 8 and 9). Also, a major part of the session-based PD days was spent investigating the mathematics from the curriculum in-depth.

Traditional professional development—session-based

I was able to offer one full day and five-half days of professional development to all of the middle school math teachers in the district; these teachers also had an initial half day without me acclimating themselves to the curriculum they would be piloting (Envisions 2.0). The first half-day was during the time we were implementing our piloted curriculum. I was tasked with demonstrating a lesson using our piloted resources, and then was given time to use at my discretion.

The lesson I chose was on the division of a fraction by a fraction. I chose this topic because after talking to over a dozen teachers I had learned that none of these teachers understood why the traditional algorithm of multiplying by the reciprocal was mathematically sound–other than it gave you the correct answer (“Keep-Change-Flip” was the de facto mnemonic I heard chanted in each class I had observed during this lesson). The lesson I demonstrated included a series of sub-claims, or sub-goals, that would lead to understanding of the division of fractions. Teachers definitely struggled productively as they sought to understand the mathematics involved. Feedback concerning the lesson was positive with many teachers claiming they had never seen a lesson like that before. The discretionary part of the PD was spent working on a common set of beliefs which continued during our next PD day (see Appendix A).

For the second half-day I modeled a lesson from the curricula I had just selected to be used during the next school year (2017-2018): Connected Mathematics Project 3 (CMP3). The lesson resulted in teachers grappling with ideas around polygon similarity. We continued our discussion about beliefs and teachers were given time to begin exploring the first unit of the curriculum.

The next PD session was for a full day. The first half of the day was focused on pedagogy where I had announced the pedagogical theme moving forward with our work in CMP3 being, “Building on student thinking.” In that regard, we watched a video then held a discussion on the importance of listening to what your students are saying rather than listening for the right answer (see Appendix B). The half day spent together also made the idea of planning for learning goals, or sub-goals underpinning a larger goal, a point of emphasis (Hiebert, Morris, Berk & Jansen, 2007). Planning for learning goals was a point emphasized during the second half of the day where we began diving into the CMP3 content in grade-level breakout groups (see Appendix C for a recap of our discussion about learning goals). I led the CMP3-centered PD for one grade, while two members of the Delaware Math Coalition led the PD for the other two grades. Again, we wanted to help develop teacher content knowledge in preparation for the teachers moving towards teaching for conceptual understanding.

The four remaining half days were all focused on CMP3 training with each grade continuing with its facilitator from the first day. The emphasis during those training days were two-fold: going through the problems and understanding how the mathematics developed over the course of the curriculum, and planning for potential student actions while including strategic teacher responses.

On-site support—continuous

For most district teachers on-site support consisted of me observing twenty minutes of a class plus providing feedback via an email or verbal dialogue. The verbal dialogue would take place that same day during the teacher's planning period. The dialogue would often include me pointing out something I noticed, and then I prompted further about how what I noticed supported a set of goals or norms which was possibly great—if those were your goals. Part of the intent of this dialogue was to draw attention to the fact that the teacher moves we make and the way we position ourselves as educators and the students as learners have consequences that impact student learning. This dialogue was part of a larger, simple goal—to get the district teachers to reflect on their practice.

I would also occasionally attend PLC's (an average of twice per building per grade). I also planned individual lessons with a handful of teachers outside of the cohort of six, though the majority of the on-site support was centered on that cohort of six teachers. The support included planning lessons and units together, co-writing assessments, video analysis (see next section), lesson study model (co-plan, observe, debrief), and discussions about math education.

Teachers learning from teaching

There are two ways I attempted to promote teachers learning from teaching—almost all of it was with the cohort of six teachers. One way teachers learned from teaching was through the traditional lesson study model. Here we (often two other teachers and me) planned a lesson together—focusing on a learning goal (Hiebert et al., 2007). Then we observed the lesson via me filming part of the lesson or via direct classroom visits (sometimes I would observe the first half of a lesson, then would go provide coverage for the other team member who would observe the second half of the lesson). Afterwards we would meet together and debrief the lesson focusing on how well we thought the learning goal was understood by the students, pointing to student artifacts as well as anecdotes during the observation to support our claims. Finally, we would talk about next steps focusing on the next couple days of instruction and/or ways to change the lesson for subsequent years to help better support the learning goal—the teachers would mark these changes in their lesson plan books.

The second way I attempted to promote teachers learning from teaching was through video analysis—either of the teachers themselves, district peers, or teaching video I found on the Internet. We would analyze the video with a single frame in mind—most typically that of unpacking the learning goal. We would then attempt to predict what the teacher's learning goal might have been and then offer critiques on how the lesson could have been altered to better support that learning goal. If our frame was on equitable participation we would examine the students' participation and brainstorm possible interventions the teacher could have implemented to foster more equitable engagement.

Regardless of the medium of analysis, the major message from my part was that your desired learning usually does not occur by happenstance. And whether your goal is content-focused or is centered on classroom culture, you need to intentionally plan for opportunities of meaningful engagement around your learning goal.

Analysis of Teacher Growth

As my goals were two-fold in nature—improve teachers promoting student productive struggle, and get a better understanding of what interventions helped initiate these changes—part of my implementation plan centered around how I was going to analyze and get feedback about teacher growth.

	Domain: Cognitive Demand
	To what extent are students supported in grappling with and making sense of math concepts?
1	Students are spending most of their time applying presented algorithms and/or are working on routine procedures.
2	Activities are presented that offer the possibility of conceptual richness but that richness is lessened by the teacher—often via over-scaffolding or proceduralizing presented tasks.
3	Students are productively struggling to make sense of the mathematics. Students are working towards building understandings; mathematical connections are at least attempted.

Figure 3 Rubric for analyzing level of opportunity for student productive struggle

The first analysis I did was to observe teachers both in September and then again in May, rating them on the adapted TRU-Math observation rubric (Schoenfeld, 2014; see Figure 3). The purpose of this was a simple measurement of whether our middle school math teachers had improved in supporting student productive struggle. Improvement here would be measured by an increase in score-level on the observation rubric (see Figure 3). Additionally, I analyzed the growth of the six cohort teachers. These data are essential because if teachers did not improve based on this rubric, then the results would provide an initial indication that my interventions were not very effective. But, these data are probably the least insightful because they do not answer why these teachers improved (or not).

In order to better understand why these teachers improved (if they did) I felt I needed feedback from the teachers—to ascertain if they felt they had improved and, if so, what they believe influenced the improvement. I also felt getting feedback from students was important. If I ultimately want students to experience this productive struggle, and to feel a sense of empowerment from the mathematical autonomy fostered by their teacher, then their thoughts on their experiences would be essential.

To gather feedback from teachers I chose to issue a digital anonymous survey that included both Likert-scale rating questions, extended response questions, and the opportunity for participants to add more open feedback at the end. I thought the Likert-scale questions would allow me to gain a level of objectivity devoid of my interpretation about some major feedback areas from the year, as well as grant me the opportunity to quantitatively analyze feedback about some broad areas. That said, considering my goal to better understand what stimulated their growth, I wanted some more in-depth feedback from them. And, considering how early I am in my understanding of how to help teachers improve, I thought qualitative data would be more helpful. Perhaps as I come to better understand teacher improvement I will be able to develop valuable, refined questions that do not require extended responses from teachers. I chose to offer the survey to the cohort of six teachers as well as six other, randomly selected, middle school math teachers to serve as an experiment-control study. Lastly, I wanted the survey to be completely anonymous (no demographic information was asked for) as I feel in-person interviews would have potentially positively-skewed the responses.

I chose to interview two randomly selected students from each of the six cohort teachers. I chose to interview these students as I wanted the flexibility to adapt my line of questioning, if need be, to reveal more relevant information. I also believed that in-person interviews would be the least disruptive form of data collection as I could not reliably hope that students would choose to write in-depth, extended responses voluntarily, and asking a teacher to give up thirty minutes of class time to have all their students write responses seemed unnecessarily invasive.

Chapter 3

EFFICACY OF IMPROVEMENT STRATEGIES

I would like to examine three separate areas of improvement in this section. The first being the improvement of the teachers, district-wide, in fostering student productive struggle. The second being an analysis of the six middle school teachers whom I worked most closely with. The third area of examination being my own improvement in understanding the consequences of my intervention strategies throughout the year.

Teacher Improvement—District-wide

Almost every middle school math teacher in the district received three half-days of PD from me, and another four half-days from either me (7^{th} grade) or a Delaware Math Coalition representative (grades 6 and 8) (I say almost every because nine of these thirty-five teachers teach two content areas and might have missed up to four of these seven half days—that said twenty-nine of the thirty-five teachers attended at least five of the seven trainings). I observed and gave feedback to almost every teacher an average of six times (sometimes the feedback was direct, other times I helped a building-level administrator craft the feedback). I sat in and participated in each building and grade-level PLC an average of two times. At each department chair meeting (eight times throughout the year) I gave a short PD session; one middle school building asked me twice to come in to their department meeting and offer the PD to their teachers.

Also, notably, nine of our middle school math teachers (including five of the six teachers in my smaller cohort) attended the Delaware Math Coalition's statewide PD (meeting for four full days during the school year). This PD's focus was on video analysis and learning goals. There was no direct PD on promoting productive struggle in classrooms, however, the presenters were all situated within a framework that values the importance of student productive struggle.

I observed each of our thirty-five district math teachers during both the months of September as well as eight months later in May. I used Alan's Schoenfeld's TRU-Math observation rubric (see Figure 3), focusing on the Cognitive Demand criteria to measure a teacher's level of fostering productive struggle (Schoenfeld, 2014). I chose the Cognitive Demand column because I felt it most accurately captured the heart of student productive struggle. And while I do believe student agency and identity are necessary for student productive struggle (see Appendix G, Artifact #9 for a flowchart demonstrating my conviction in this connection), I chose to only examine the Cognitive Demand section because I believe that if teachers were developing student agency and identity, then increased Cognitive Demand would have accompanied their teaching. Application of the three-point rubric could be open to a wide-range of interpretations, so I would like to explain how I decided to rate a teacher as either receiving a score of 1, 2, or 3.

To receive a rating of 1, the teacher either engaged in whole-class instruction where the content of that instruction was focused on a procedural algorithm for solving a specific problem type (this would include both use of mnemonics or not), or students were asked to solve large quantities of problems (at least ten) that required application of the same algorithm repeatedly.

To receive a rating of 2, the teacher offered students problems where the opportunity for productive struggle was possible via a more challenging problem or problems, but, in practice, the cognitive demand was lessened most typically by the routinization of the problematic aspects of these problems (as described in Stein, Grover, Henningsen, 1996). A memorable classroom example of a class receiving a rating of a 2 is where students were being introduced to exponential equations and were asked to choose the equation (multiple choice) that represented the exponential context and given table (a starting, single bacteria that then doubled every hour). The majority of students selected the equation $y = 2^x + 1$, assumingly thinking the "+1" represented the starting number of bacteria, a parallel to a linear (or any polynomial) equation. However, as a few students realized that their equation did not match the data, they changed their response to the equation: $y = 2^x$, claiming that their original equation had not matched the data in the table (descriptive modeling). Soon the entire class was convinced, and everyone had moved to the next question using the lens of finding the exponential equation that matched the given table of values. There was ample opportunity in this lesson to talk about the structure of an exponential equation and even to make connections to linear equations (or to explore analytical modeling of an exponential function). There was also room to question why students' original responses did not align to the equation that they ultimately selected. However, none of these opportunities were seized and the activity fell into the solely procedural application of an algorithm.

Other classrooms receiving a rating of a 2 included higher Depth of Knowledge questions but saw teachers reduce the cognitive demand of the task either before students began or as soon as a student asked for help. In this case, the teacher would say (or point to) the proper algorithm to use, often going as far as to say where each number from the problem should be placed in the algorithm.

To receive a rating of 3, the teacher needed to maintain a level of cognitive demand that kept students engaged in a cognitive place just beyond their current understanding. The focus in these classrooms was on making connections either between mathematical ideas or within the structure of the problems being attempted. However, the focus stayed on working towards mathematical connections even if the teacher ultimately made these connections explicit; importantly, the explicitness occurred *after* students had grappled with the problem themselves beyond their initial impasse (even if the student made no mathematical headway in making a connection, an attempt was made by the student to get themselves "unstuck").

I feel compelled to include the caveat that these observation scores are tentative. They only include a single beginning-of-the-year and end-of-the-year point of comparison, and I fully acknowledge that if I had observed the same teacher on multiple days their score would potentially have changed. In this sense, no individual teacher's score is fully reliable. That said, based on my multiple visits and experiences in these classrooms, I believe that the scores, as a whole, are representative of the teaching that I have seen both for the beginning and the end of the year.

Here are the results I observed:

Table 4 Results from walkthrough observations on middle school math teachers at the beginning and end of the 2016-2017 school year.

	September 2016		May 2017	
TRU-Math rubric score	Number of teachers	Percent of teachers	Number of teachers	Percent of teachers
1	28	80	17	48.6
2	5	14.3	15	42.9
3	2	5.7	3	8.6

The September 2016 data yield an average rating of 1.26 with a standard deviation of 0.55; the May 2017 data yield an average rating of 1.60 with a standard deviation of 0.64. A statistical t-test analysis (two-tailed, paired) demonstrates this shift to be statistically significant (p =

0.00018). It should be noted that this improvement shift was the result of teachers moving from a rating of 1 to a rating of 2, and that very few teachers made the shift to a rating of 3 (interestingly two of the three teachers rated a three in May 2017 were actually rated a one in September 2016—this also means one of the teachers rated a three in September fell in the May rating).

Teacher Improvement: Smaller Cohort

Each of these six teachers got all of the PD listed in the section prior as well as additional PD. This additional PD included lesson studies (planning, observing, debriefing a lesson), additional observation/feedback cycles, and non-targeted discussions about math education (in-person as well as via email with two of the teachers). I've labeled these six teachers below Teachers A – F with my overall level of involvement with them listed in descending order (i.e. I spent the least time working with Teacher F).

Here is their observation growth data:

Table 5 Observation data from beginning and end of school year measuring level of Cognitive Engagement of students in six teachers' classrooms.

	TRU-Math rubric score (September 2016)	TRU-Math rubric score (May 2017)
Teacher A	1	2
Teacher B	1	3
Teacher C	1	3
Teacher D	1	2
Teacher E	1	2
Teacher F	3	3

A few important notes about these teachers:

- Teacher A was a first-year teacher.
- Teachers B and C work together (same school, same grade), and they planned and pushed one another throughout the school year.
- I never fully planned a lesson with Teacher D (her choice), but I did on multiple occasions preview her lesson plan and offer my advice, then would observe the lesson and debrief with her. Most of my interventions with this teacher were centered on larger math education conversations. She and Teacher F work together, but don’t teach any of the same classes (Teacher F teaches Integrated Math I for advanced students) so my two visits to their PLC were not content-driven like they were with the other four teachers here.
- Teacher E asked for my help in January, which is when I started additional PD with her. She works with Teacher A.
- Teacher F and I did two lesson studies together, but I offered her essentially no other additional PD (I spent much time debating whether or not to include her in this cohort).

The improvement by these teachers as measured via classroom observations was significant and highly encouraging. Five of the six teachers had started at the lowest level in terms of engaging students in cognitively demanding tasks, but by the end of the year two had moved to the highest level and three moved to the middle level in terms of fostering productive struggle opportunities for their students.

Helping these teachers improve on this one Cognitive Demand metric was encouraging, but if I am to grow and understand why this growth took place, then I will need to explore what other growth took place and, more importantly, what did they think helped them grow as much as they had. Observation data alone would be insufficient for measuring growth in other areas, and it would not help me to understand what helped affect this growth. So now I am going to focus on the anonymous teacher survey I sent to these six teachers to identify further areas of growth as well as the causes of their growth.

Teacher survey response data

The data from the anonymous teacher surveys were very important for both soliciting evidence of teacher growth beyond their observation data as well as offering me information to help me better understand why these teachers had improved so much this year.

Each of these six teachers was asked to fill out an anonymous survey about their experiences and growth this year (see Appendix D). The survey was a mix of open response and Likert-scale ratings. The scale ranged from 1 (Not effective PD) to 4 (Highly effective PD). Here are the Likert-scale ratings from each teacher, listed chronologically by submission:

Table 6 Responses to Likert-scale responses from the six cohort teachers on anonymous survey.

	Whole day PD session	CMP3 training	Planning a lesson/unit with me	Discussions about math education with me	Observation feedback
Respondent 1	4	4	4	4	4
Respondent 2	3	4	4	4	3
Respondent 3	4	4	4	4	4
Respondent 4	3	4	4	2	2
Respondent 5	3	3	4	4	3
Respondent 6	3	2	4	4	4
Mean Avg. Response	3.3	3.5	4.0	3.7	3.3

These data show that overall the teachers at least felt that the PD I was offering them was helpful. Specifically, co-planning seems to be an intervention that teachers found highly valuable. Interestingly, discussions about math education were also very highly valued, except by one teacher, (recall that I never had a math education discussion with one of the teachers in the cohort (Teacher F) which very well might account for the one divergent rating).

I chose not to analyze the teachers from the control group. Only three of the six randomly selected teachers filled out the survey (see Appendix D), and one of those teachers I had actually worked quite a bit with (he was the inclusion teacher for Teacher A and frequently partook in our common planning sessions and discussions about math education). With only two teachers left, I did not feel I could draw any relevant conclusions about the differences in their experiences from those of the cohort group.

Selected responses from open-ended survey questions

I have chosen only to include responses (or parts of responses) that pertain to teachers' growth in terms of what I believe will help foster student productive struggle in their classrooms, or responses that indicate what a teacher believes helped them improve this year. This means that I do not include responses from all teachers to every question. (You can find their full responses in Appendix D.)

Question 1: In what ways have your teaching practices shifted this year? What sparked those changes? Please include a specific example if you can.

Respondent 1: "They have shifted quite a bit. Last year, I would do several problems in one class that were more skill based. This year, I may do one or two large problems to explore a concept."

Respondent 2: "I am striving to let students do the talking and thinking…. I have come to believe that students need to be doers and thinkers, not passengers or bystanders. I am not where I want to be yet, but I am on the path."

Respondent 3: "I have been able to shift my teaching practices to be an educator who allows students to take control of discussions and guide my instruction based on their conversations. I no longer feel the need to lead the majority of a class period. I have also noticed that I am getting better at allowing wait time (even if it seems like awkward pauses at first)."

Respondent 4: "Giving students control to problem solve and releasing some of the control in my classroom to be the one that explains. This has come through understanding the power and confidence that builds in students when they can discover something on their own."

Respondent 5: “This year I had my students discover the meaning behind the math. They were very hands on and love the projects they were given.”

Respondent 6: “This year I have given students more responsibility in their learning. Instead of me telling them how to do math and then having them apply this to application problems, I instead give them time to grapple with the math to truly understand how it works, opposed to being told how it works.”

Question 2: In what ways have your beliefs about education shifted this year? What sparked those changes? Please include a specific example if you can.

Respondent 3: “I am more confident in my belief that the teacher doesn't own the math, and neither does the student. I have been able to help my students see that I am not an ‘answer giver’ or a ‘hint giver,’ but someone who is able to help them get to a point where they can think more clearly about the math. This is because of conversations I have had with Mike, as well as conversations he had with my classes.”

Respondent 5: “Many of my beliefs are the same. However, one that has changed is that students should struggle. It's ok for them to have discourse and I allow them to think through it without my help.”

Respondent 6: “I have more faith that students can accomplish a task, my outlook is that I am not the keeper of math and that students (with guidance) can create their own theories about math. The changes were sparked by the first PD with Mike where we had to model a fraction divided by a fraction. This was a difficult task to do by adults that understood how to do the math and shortcuts, but not necessarily why the math works or why the shortcuts work. If I can gain a deeper understanding by learning that way, why wouldn't students gain a deeper understanding of math by learning that way as well?"

Question 3: How was the classroom experience different for your students this past year? Why do you think that is? Please include a specific example if you can.

Respondent 1: "They feel better. They smile more. They come up with the ideas rather than me giving them ideas. I think this was directly related to the changes that I have made in my teaching practices."

Respondent 2: "One thing that did seem to help my students was the use of large whiteboards…. I loved that students were making decisions about how they wanted to learn and do work together. I liked that this made the work very public in the group and it seemed to lead to really good conversations, much more than if everyone had been working on paper or in notebooks. I let go of some 'control' this year; in the past I might have said, 'no, we are not using whiteboards today' and I found that I liked the feel of the classroom better with students exercising more control."

Respondent 3: "My students shared with me that in math this year, they were able to understand the 'why' behind the math they were learning. They told me that it was different from the past because they were able to come to conclusions themselves. Sometimes this was frustrating for them at first, but they grew to expect it and would be disappointed if I ever 'gave something away' before they figured it out themselves. I think that the conversations I had with other math teachers in our district, my team members, and my district department head [me] influenced this change in my classroom."

Respondent 6: “I think that my students started to realize that it is the mistakes, wrong theories, pieces of ideas, and collaboration that allow people to really understand.”

Question 4: What professional development, formal or informal, did you find significant this year (can be more than one or can be none)? Please elaborate as to why certain PD was helpful to you or why it wasn't.

Respondent 1: “I think that the conversations I had with you (during PD, PLC's, and email) were the most beneficial.”

Respondent 2: “Every single time I have spoken with you formally or informally, I have learned something. I think when you helped us plan for a lesson in the beginning of the year you showed us how to get to the important ideas in a way that allowed the students to do the thinking and discovering. Each time you have observed our class and then talked to us afterwards I felt supported and challenged equally, never judged and found lacking as we usually do when other admin types observe. I would like to have more planning/PLC time with you as the year progresses. The sessions with Jamila [Delaware Math Coalition director—facilitated the CMP3 PD for one of the grades] have been invaluable, nothing is more necessary than the opportunity to work through the math together with Jamila there to point out how the understanding might unfold and how to facilitate it.”

Respondent 3: “I found that the most significant PD was when you came into our classes and helped us plan…. I also really enjoyed when you came in to observe a lesson (even if it was out of the blue) and gave feedback. I learned a lot when you took over some lessons because I was able to see exactly what you would do, and use that to adapt my own practices.”

Respondent 4: “Grade level and content specific math PD where we are doing the math or discussing the math is the best.”

Respondent 5: “The PLC PD's were helpful because they gave me a chance to ask questions I may not have in larger PDs. It also allowed you to walk us through how to teach a concept slowly.”

Respondent 6: “I think my PLC with one of my colleges ([Teacher B]) was very beneficial as we were on the same page and we did a really good job of giving and getting ideas from each other. I think having you (Mike) as a resource whenever needed was a really big help and really opened my eyes to ways to teach. I think full PD days working on the curriculum is helpful because it gives us a chance to really talk about how we can do the math in our room and it allows us to bounce ideas off of you and get your perspective.”

Question 5: What is one thing (or some things) you hope to get better at next year concerning teaching? Why that thing(s)?

Respondent 5: “I hope to get better at giving them more time. This way they learn that they will not be given the answer right away and that it is up to them to figure it out.”

Respondent 6: “Next, year I would like to do a better job in the beginning of the year to make it clear that mistakes, collaboration, and a desire to understand are things that will help them succeed in math—not whether they memorized their math facts or not.”

These responses are encouraging for me to read. I believe they speak highly to the effectiveness of the professional development these six teachers got this year. From these responses, I believe that it seems teachers are beginning to see the importance in letting their students do some of the mathematical thinking, and how empowering it can be to shift some of the mathematical authority to the students. However, although these teacher responses have given me some insight into what was helpful, many of these comments were too general for me to infer what precisely supported their change or growth. The responses were helpful enough though to guide me in a direction where I can refine these questions on future surveys to help me more acutely diagnose why certain PD was effective and what could be improved.

Student survey response data

It is one thing for teachers to feel they have changed their practices in meaningful way—it is more meaningful if their students' perceptions agree with those perceived changes. So I randomly interviewed two students in each of the six teacher's classrooms and asked six questions each about their experience this year. In this section, I go through each question and share what I learned from the students' responses to that question (see Appendix E).

Table 7 Demographic information from students surveyed (n = 12).

Gender		Race/Ethnicity		Math Track	
Male	4	White	8	Advanced math	5
Female	8	Black	2	Non-advanced	7
		Latin@	1		
		Asian	1		

Question 1: How important is getting the right answer in your math class? Is there a time when getting a right answer is especially important? Is there a time when getting a right answer is not so important? Can you give some examples?

Six students said it was not important to be right in their math class; I found it interesting that those six students came from only three teachers (Teachers B, C, and D), meaning that those teachers sent the message to their students that getting the correct answer was not important in their class. Two of these students (same teacher—Teacher D) drew from the language of Growth Mindset and spoke of the importance of making mistakes and learning from them—how you learn more when you make mistakes. Three students claimed getting the right answer was generally important (three said it was important sometimes but not all times); two of these students came from the same teacher (Teacher F; the other one came from Teacher E). I will talk more about these two students during Question 4a.

It seems clear to me from these data that teachers can create a culture where students think getting the correct answer is important or not. It also seems to me that the teachers I worked more with seemed to send that message to their students—that getting the right answer was not highly valuable. I was explicit with this language to this cohort of teachers, often telling these teachers, “You need to find a way to devalue the right answer if you want students to spend more time making sense of the math.” I believe that this de-valuing of correct answers is important because all too often the pursuit of the correct answer undermines sense-making. I also think that having students reflect on the importance (or non-importance) of correct solutions is epistemologically and meta-cognitively valuable.

I believe it can be soundly argued that getting the right answer is valuable—that we in fact focus on sense-making for the sake of leveraging that sense-making into getting the right answer. However, as an aside, I will say that as I get older the more I consider that getting the right answer (in a K-12 math class) may never be important (though it is often encouraging). This belief might be a remnant of teaching highly underserved high school students for ten years, but I would count myself successful if my students left my class curious, intellectually autonomous, and believed that they were smart and valuable, and that they *could* figure things out if they thought about them long enough—even if during my class they never once produced a correct answer to a math problem. I do recognize that this assertion is based on an entirely hypothetical case, as I have never had a student get every single problem wrong and still acquire these positive interpersonal feelings.

Question 2: In your math class where do most mathematical ideas come from? Meaning does the teacher usually introduce new concepts and formulas, or do you and your classmates usually make new connections and discover new formulas by yourselves? Can you give some examples?

There was not as much consistency between the two students and their teacher on this question (only Teacher B had both students say ideas consistently came from the students, and Teacher F had both students say students had the opportunity first but the teacher would move the class forward after a while). What I got most out of the question came from one of Teacher D’s students who said that typically *one* student would have an idea/formula and the teacher would see it and ask that one student to write their idea on the board. This student did not feel like ideas came from the class at large, but rather from the singularly successful student—and the teacher still clearly held authority by asking only correct students to go share at the board.

This interview response was particularly meaningful to me because I had just co-presented at NCSM (National Council of Supervisors of Mathematics) about the issue of status.

A major point of our presentation was that often when a teacher had started to cede mathematical authority to their class it would create a power vacuum for mathematical authority. If the teacher did not plan accordingly, a handful of students would become the mathematical authorities in the classroom leaving the vast majority of students with an impoverished view of themselves in terms of mathematical agency. This interview helped me realize that I probably should begin talking about status issues and ways to combat them with some of my teachers who have begun shifting the mathematical authority to their students.

Question 3: Do you believe the ideas you have in class are respected by your classmates? Follow-up: When you put forth an idea in class, what happens next? Like does the teacher tell you it's right/wrong, good/bad, does he/she ask the class to examine your idea? What happens?)

Eleven students said unequivocally yes, their ideas were respected (the other said sometimes). Almost every student said that sometimes the teacher says right/wrong, but most often the teacher will ask the rest of the class to weigh in about what they think about the answer. Only the two students from Teacher B were in agreement that their teacher routinely put student ideas on the board (right or wrong) and would pause the class and ask the other groups to stop and try to make sense of that one group's idea (I had actually coached this teacher on that very routine so was encouraged to hear these responses—the teacher had done a beautiful job of getting her students to see one another as havers of valuable mathematical ideas).

An honors student from Teacher C responded that, "It's not helpful for the teacher to tell us if it's right or wrong…" That response was also encouraging because the first time I observed Teacher C she would not let students move onto the next question on the worksheet until they came and checked each and every answer with her.

Question 4: I want you so to say if you agree or disagree with the following statement: I believe I sometimes have good mathematical ideas.

Follow-up if agree: Can you give an example of a good idea you had? Did you tell the teacher or classmates about the idea? What happened then? (If necessary--how does the teacher respond? If necessary--do you feel like the teacher hears your idea and then tells you if it's right or wrong, or do you feel like the teacher hears your idea and tries to have classmates build on your idea?)

Follow-up if disagree: Why don't you think you have good mathematical ideas?

Eleven students agreed with one disagreeing (a different student disagreeing from the last question). What I learned here is that there were two (advanced math) students who said they would not share their idea if they were not sure it was correct. These responses gave me insight into the need of creating a safe, risk-taking learning environment for some of these teachers. It has given me the insight that even if a teacher says it is okay to share incorrect answers and we learn from our mistakes—the culture of the classroom might not support that message. I will be recommending to teachers (particularly those whom I work with closely) to create an anonymous student survey to give in the late Fall to gleam from their students how safe they feel taking risks in their math class.

Question 5: Do you think you learned more in this math class compared to previous math classes? Why do you think so? Could you give some examples of things that you think helped you learn more or some things that you thought were maybe a waste of time?

Ten students said they believe they learned more this year with two students claiming they learned the same amount (both from Teacher D). Having a sample from a control group would have been helpful here to see generally what percentage of students typically feel that they learned more in their current math class compared to prior ones.

Three students claimed they liked the increased challenge two of whom positively referenced longer problems that made you think more. I will now examine some individual student responses, because I feel that most often responses to this question revealed the totality of student sentiment towards the year. And I want to know if students felt just better in general about their math class this year as they were asked to grapple with the mathematics more themselves.

Teacher A, Student 1: “It was the funnest [sic] math class because all my other math classes are just sitting and looking or just like going to stations and that's boring.”

Teacher A, Student 2: “My teacher last year she, so you know how like in college they show you videos, yeah I heard that, but that's what she did, and we were not at that level of learning. She just gave us videos and then a worksheet so I basically like learned nothing, and when I come here it all makes sense there's a reason for if I get something wrong, there's a reason for it.”

Teacher B, Student 1: “I do agree because it's not like a traditional classroom where the teacher just like tells you what to do; she actually let us figure it out on our own and elaborate on it together which helps me learn a lot better…. When we do group-work with the whiteboards that's very helpful. When we have to like figure out the formula on our own, because it helps me to understand it more rather than just knowing it and not understanding why.”

Teacher B, Student 2: “I do because [Teacher B] is always challenging us to see things from a different point of view, and she gives us challenges, and she lets us try to figure it out, and it's fun too.”

Teacher C, Student 2: “I liked that she was open to really like changing like what she wanted to do so she's like, ‘Oh I'll do this but I didn't want to do that so we're going to change the lesson for like a day or so and just focus on this’.… When we were walking out of class she would like how are you feeling give a thumb up middle or down of what you like feel toward what I am teaching. I really like that part.”

These student responses have communicated to me how much students appreciate both challenge and flexibility from their math teachers. I think these two ideas about classrooms—challenging and flexible—underpin a larger idea about successful math classes, primarily that they revolve around ideas. To put it another way, ideas (rather than solutions or structures) have become the currency of the classroom (Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray & Olivier, 1997). I do want to note here that students do not seem to mind where the ideas come from (teacher, classmates, groupmates, themselves), but they do seem to value the opportunity and time to grapple with those ideas—to see if they make sense and to ask questions about them. Students also seem to find it more valuable when an idea is put out there as simply that—an idea, rather than a fact about mathematics (if you recall from prior questions students did not like being told an answer was right or wrong and also did not like when classmates were called to the board to share their work only when the answer was right).

I plan on being explicit to teachers about this very concept—students value the time to wrestle with mathematical ideas and come to their own conclusions about whether or not the idea is right or wrong; students value the space to try out these new ideas and accept or discard them as they make sense of them. I plan on asking teachers to consider replacing the traditional Initiate-Response-Evaluate (IRE) trilogy of teaching with a new model of Thought Stimulus (often a problem), Idea(s) Explicitly Shared, those ideas are then Reflected on (individually) and

Discussed (as a class) (perhaps I can call this new format Problem-Idea-Discussion). I think we could simply re-visit and redefine the IRE cycle where the Initiate part needs to stimulate some perplexity, confusion, or doubt (Dewey, 1910); the Response part needs to put forth an idea (rather than a solution); and the Evaluate part needs to offer students (rather than the teacher) time to evaluate and discuss the idea put forth. The idea of students being motivated by having to make sense of something that has perplexed them is consistent with literature on cognitive impasses (Dewey, 1910; Vanlehn, Siler, Murray, Yamauchi & Baggett, 2003).

However, I wonder if it is prudent to primarily share this idea with teachers who have begun creating a classroom culture that promotes student agency (which is in the minority of our district classrooms unfortunately). I think having an agentive classroom is important because, as Solomon and Black (2008) have pointed out, "In order to 'try out' new ways of thinking, we need to perceive ourselves as having some agency in or control over what we are doing" (p. 75). The idea of connecting agency to productive struggle is a connection I believe to be vital as outlined in my artifact, *How Students Learn Conceptually* flowchart and paper. Here is the flowchart for reference:

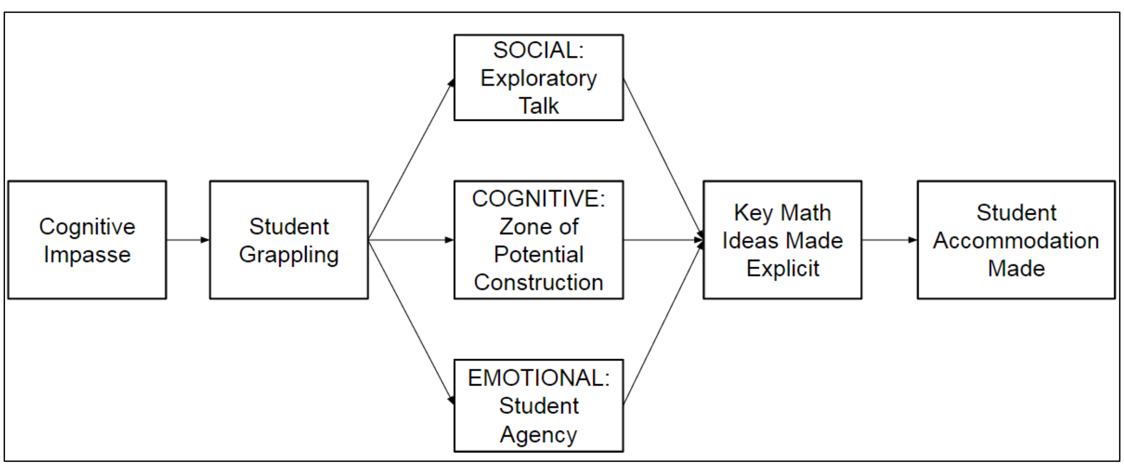


Figure 4 *How Students Learn Conceptually* flowchart

What I am unsure about, but believe to be true, is whether making ideas the currency of your classroom will naturally create an agentive culture, if true, this model should be recommended for all teachers.

Question 6: Is there anything you'd like to share with me about math class that you didn't get the opportunity to do so far?

There were no responses given here that I think helped deepen my understanding of teaching.

Chapter 4

REFLECTIONS ON IMPROVEMENT EFFORTS

In this section I will reflect on the overall success of my approach in meeting my improvement goals, why I think my goals were met, what I have learned as both a math specialist and as a researcher, and what changes I will make in the future.

Reflections on Overall Success

My initial goal was helping my district's middle school math teachers increase the opportunities for their students to productively struggle with the mathematics. I believe this goal was met as evidenced by the shift from 80.0% to 48.6% of the teachers scoring the lowest score on the TRU-Math's Cognitive Demand rubric. That said, success was limited as almost half the teachers continued to provide very little opportunity for their students to grapple with the mathematics, and still only 8.6% of the teachers scored the highest on the Cognitive Demand rubric (compared to 5.7% at the beginning of the year).

What was more encouraging was that, of the five teachers with whom I worked most closely, all had improved at least one full mark on the Cognitive Demand rubric with two teachers improving two marks (I have excluded the sixth teacher here as she both began and ended the year with the highest mark on the rubric; however, from my observations in her room this sixth teacher had improved noticeably in ceding more of the mathematical authority to her students as the year progressed).

While encouraged by the growth of the district teachers as a whole as well as the smaller cohort of teachers, I feel that this growth was probably the easiest growth to get. All six of these teachers reached out to me for help which meant they were more willing to dialogue with me and listen to suggestions I had for them. Similarly, the other district teachers who improved were using a very traditional textbook at the beginning of the year (Envisions 2.0) which leant itself to teaching where students did not struggle much to make sense of the mathematics, but by the end of the year the teachers were using either CMP3 (a reform-based curriculum) or even activities from the past (many of which were inspired by their last curriculum, Math in Context, another reform-based curriculum). Because of these advantages I had this year, I feel my data would not continue to look this good with the same approach in subsequent years. That is why I believe that if I really want to improve teaching for productive struggle in the district, I need to find out what helped move these teachers forward. Knowing this would help me refine and improve my own practice as I branch out my work to less amenable teachers.

I believe I was at least marginally successful in my goals of trying to understand teacher growth and its effects on the teacher as well as their students. I say marginally successful because I do believe that I have gained some insight into why teachers changed, but I still feel deficient in my knowledge of these very things. For example, if I was asked to help the six lowest scoring teachers in the realm of productive struggle I am skeptical that I would be highly successful. In a parallel to teaching, I think it takes a skilled educator to effectively teach any students, even those with the highest mathematical agency, and that is where I think I am in terms of helping teachers improve. I believe I know how to help the capable teacher who really wants to improve (in terms of student productive struggle). But just as it is more difficult to teach high-needs students, I think I am far from being able to reach the most challenging teachers.

Why I Think My Goals Were Met and What I Learned as a Math Specialist

Based on my interpretation of the information I gathered from teachers, I believe my goals were met, in part for several primary reasons. The first reason I believe teachers were able to move forward was the change in curriculum from a traditional to a reform-based material. This switch was coupled with four half-days of PD focused on working through the content like students while drawing attention to the way the mathematics progressed. Teachers had the opportunity to experience productive struggle and feel its effect firsthand—all while learning the mathematics more deeply—before being asked to change the way they teach.

Secondly, I learned the importance of having open math education conversations with teachers. I was not surprised to find that teachers rated them highly (five of the small cohort teachers rated math education conversations with the highest mark) because I had experienced those conversations and witnessed how they prompted teachers to reflect on their practices more deeply.

These conversations were often about large issues in education. Some examples of the questions we discussed are—why do we grade, how should we grade, why do we teach mathematics, are these math skills we are teaching useful or trivial, how do we know if a student knows something, what is at the heart of mathematics, who should hold authority over the mathematics, how can we make ideas the currency of the classroom? Talking about these topics with the cohort teachers (who again felt safe talking with me as they had invited me into their classrooms) let these teachers try on new beliefs and new ideas with me acting as a sounding board. Very often these teachers liked the way these ideas felt when we talked about them, and soon I would hear from them about how they had tried something completely new and different and how they loved it (I did not get any crash-and-burn emails this year, but that would have been fine too). I think these conversations helped tackle the adaptive challenge of having teachers believe they were doing more than simply conveying a set of math skills to a bunch of kids; rather that they were profoundly influencing the way students saw themselves and their place in this world.

These conversations moved beyond short-term teacher moves and towards a repositioning of the teacher-student dynamic along the lines of what Peter Johnston (2004) describes: "However, I must emphasize that the [highly-effective] teachers whose language we have explored in this book used that language mostly without conscious attention to it…. They can do this, in part, because of who they think they and the children are, but also *because of what they think they are doing*" (p. 80, italics mine). The teacher who thinks what they are doing is getting students ready for the next math class, or college, or the state test, is going to desire a very different set of practices than the teacher who thinks they are shaping a child's identity as a problem-solver and a person who figures stuff out. The importance of this idea of what a teacher thinks they are doing is beautifully represented by a quote that I shared at every PD session throughout the year:

> "The assumption that just being more explicit will make for better instruction assumes that language is simply a delivery system for information, a literal packaging of knowledge. It is not. Each utterance in a social interaction does much more work. For example, there are hidden costs in telling people things. If a student can figure something out for him- or herself, explicitly providing the information preempts the student's opportunity to build a sense of agency and independence, which, in turn, affects the relationship between teacher and student. Think about it. When you figure something out for yourself, there is a certain thrill in the figuring. After a few successful experiences, you might start to think that figuring things out is something that you can actually do. Maybe you are even a figuring-out kind of person, encouraging an agentive dimension to identity.

> When you are told what to do, particularly without asking, it feels different. Being told explicitly what to do and how to do it—over and over again—provides the foundation for a different set of feelings and a different story about what you can and can't do and who you are. The interpretation might be that you are the kind of person who cannot figure things out for yourself. This is doubtless one reason why recent research has shown that most accomplished teachers do not spend a lot of time in telling mode (Taylor et al., 2002 as cited in Johnston, 2004, p. 8)

Thirdly, the value teachers attributed to our conversations about math education I believe are also tied to the value they placed on co-planning lessons or units together. This connection was prevalent because often when we planned together, as we were making decisions about how to help students reach our learning goals, we had discussions about the larger goals of our lessons and how the activities we selected and the teacher moves we used would support (or undermine) those goals. In short, a major takeaway I got from this project was that having conversations that stimulated reflection in regards to the teacher's orientation about math education is extremely valuable. Relatedly, having conversations about the future—next week's lesson or a new plan for grading—I believe generated hope and optimism in these teachers; whereas, offering feedback after an observation while still valuable, did not tend to be as inspiring. This idea leads me to another lesson learned as I prepare to reach out to new teachers to work with—offering observation feedback positions me as a sole authority with them as the subordinate, whereas co-planning positions us as co-creators and peers in the education process. On a side note, I found it to be beneficial to plan for a lesson beyond tomorrow's as this removed the tension to simply get the lesson plan done, and it allowed for talks to diverge towards larger educational goals and ideas.

A belief of mine developed from interactions with my own students and further confirmed by these student interviews is that students enjoy being appropriately challenged.

When the classroom culture supports learning instead of performance, students do not want to be told the answer or formula and legitimately feel cheated when they are told.

Stepping back, I now can see that the second and third reasons are part of a larger issue of how I positioned myself relative to the teacher. As I reflected more about the role of our respective positions, I elaborated the theory of action I originally developed to examine the consequences of positioning. I came to understand that if I wanted to support teachers I needed to position myself as a co-creator in the educational process rather than as a punitive overseer. This change in position allows me to identity build with teachers. I now believe that the key to fostering change in teacher beliefs is to position the teacher to “try on” a new identity in a safe space and then encourage the teacher to “wear” this new identity into an authentic space (i.e. their classroom), finally asking the teacher to reflect on their experience using this new identity.

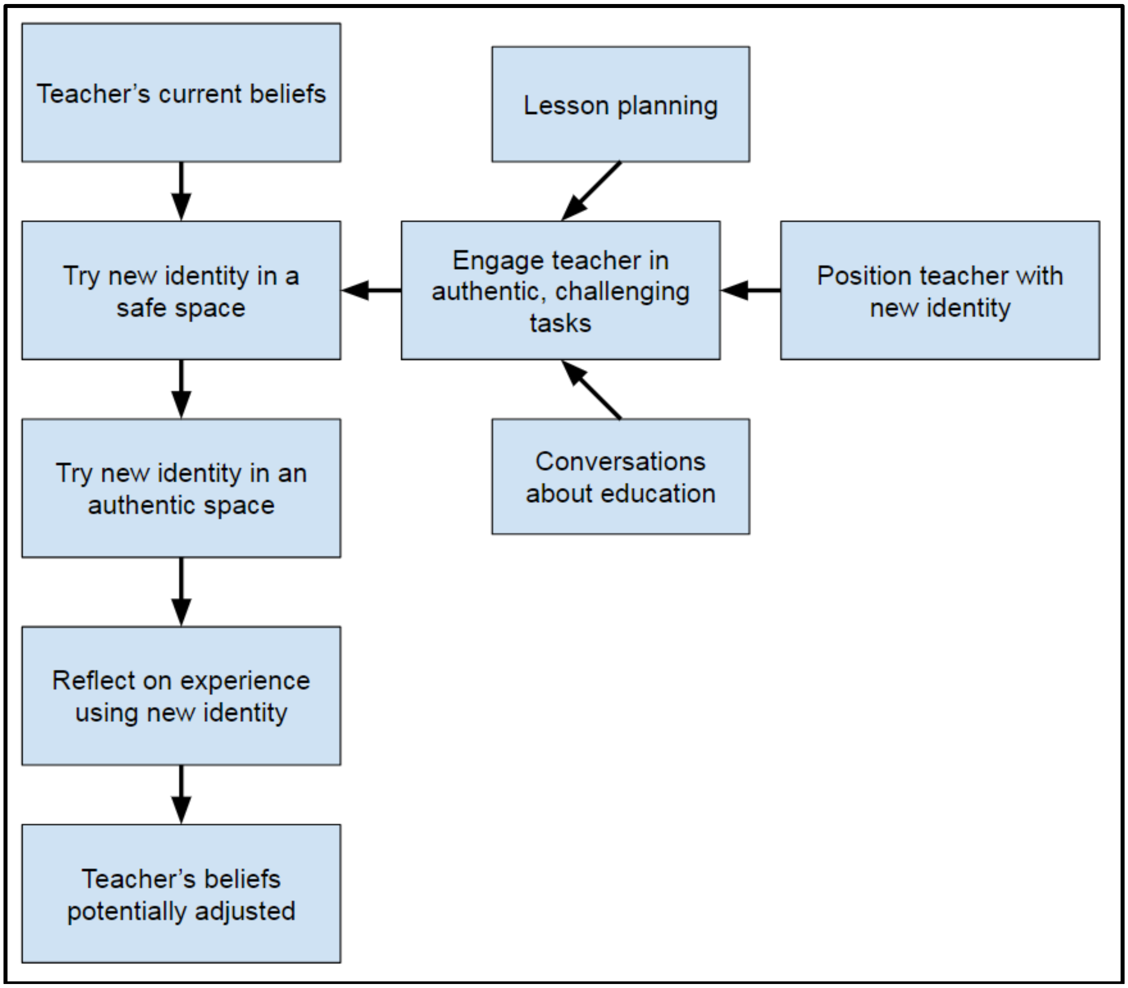


Figure 5 Theory of Action on changing teacher beliefs via an identity shift

In this theory of action, I see teacher authentic tasks being lesson planning and further developing systems of the profession like grading, management, grade reporting. Furthermore, this new theory of action complements my original, broader theory of action in terms of how to develop teacher beliefs. Note the highlighted section:

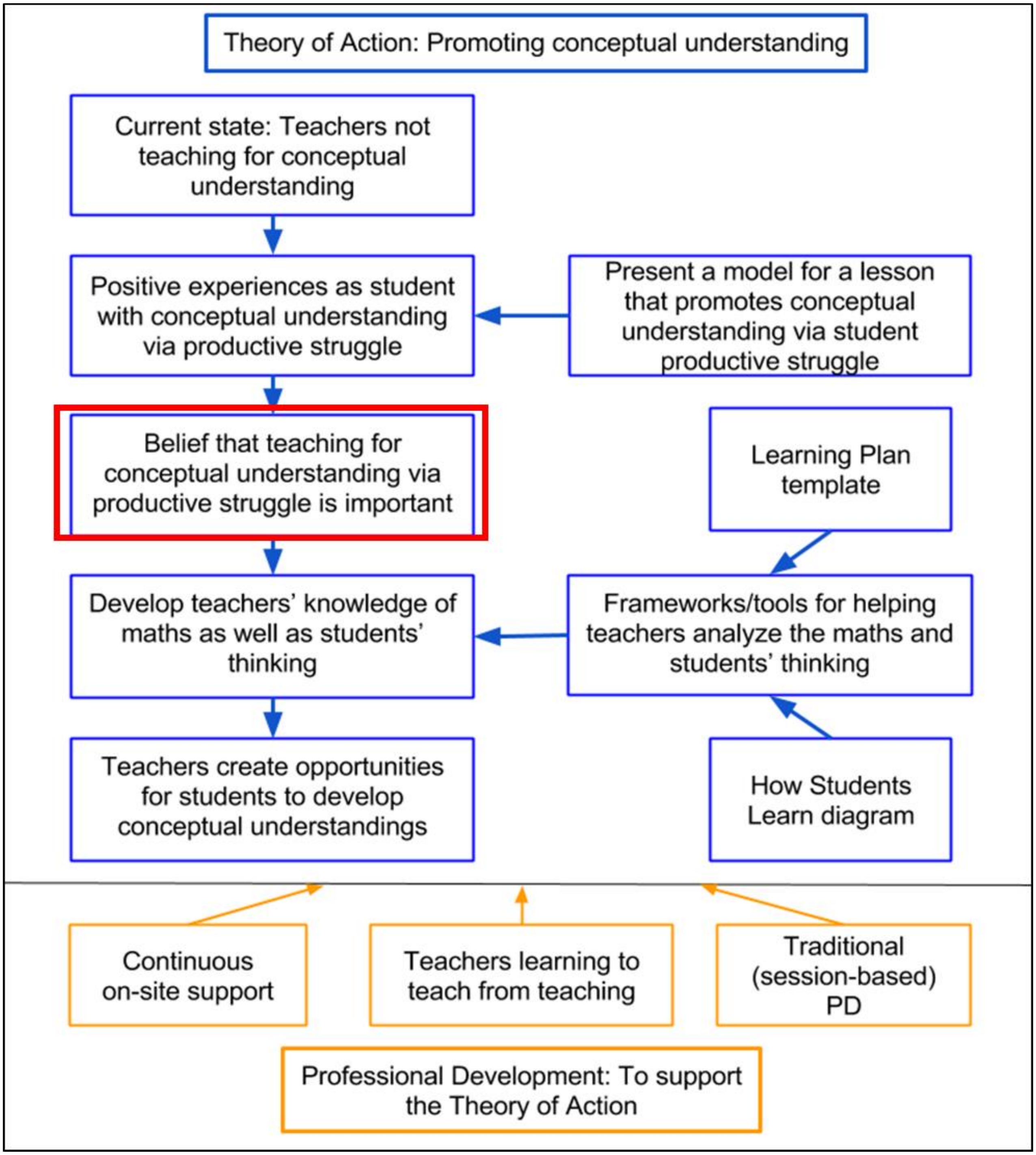


Figure 6 Adjusted Theory of Action

Something else worth noting from these co-planning sessions and math education conversations—I believe the teacher felt competent as an educator during these times where they had control (agency) of their future classroom. I believe these feelings of competency and agency cannot be overvalued.

I just want to be explicit here about what I perceive to be the dual-power of positioning. In one instance, someone (like the math specialist) can position the teacher, in a safe space, to try on a new identity which can result in a change of beliefs. In a second instance, someone can position themselves as a collaborator/co-creator with the teacher which can lead to the teacher feeling both competent and valued.

What I Would Have Done Differently

As already mentioned, I would have spent more time co-planning with teachers and having discussions about math education with them and less time giving them observations. During these times together, I would be more intentional about positioning these teachers with new identities to help foster a change in beliefs towards teaching in a way that further recognized student competency. I would also change the questions I asked my cohort teachers to try to find out what was it about co-planning or discussing math education that was particularly helpful to them—were there particular topics or themes that they found more compelling than others? I would also like to ask teachers directly whether or not they felt more liberated/euphoric as they began fostering student productive struggle. I would also go back and give student surveys at the beginning and end of the year to better quantify how students' feelings about math class changed throughout the year. One of those questions would explicitly ask students what they would do if they had an idea in math class but they were not sure if it was right—would they choose to share their idea with the whole class, a small group, the teacher, no one?

From a purely technical standpoint I would have sent out control group teacher surveys to a larger number of teachers recognizing that teachers whom I had not worked closely with would not be highly motivated to fill out the survey I had sent.

Next Steps

I believe for my district to move forward in student productive struggle we should continue our CMP3 content training this year as we roll out the new material—as already planned. I believe for the teachers to continue to sustain and continue to grow, one major shift must take place: teachers must change, “who they think they and the children are, but also because of what they think they are doing” (Johnston, 2004, p. 80). For me, this means they must change the way they view some major parts about math education. More teachers must come to see students as havers of wonderful mathematical ideas, must come to see themselves as student-empowers, and must come to see the mathematics as a means to an end, not the end itself, with that end being student intellectual and moral autonomy. I believe this shift in vision will only happen with the support of district office, building administrators, and influential teachers throughout the district. One reason I believe this idea can work, is because I have never seen a teacher who has come to see their role as student-empowerer, who has come to see their students as havers of wonderful mathematical ideas, to falter from teaching in a way where students productively struggle and where agency is fostered and identities are changed.

A shift in practice that I think would move our teaching forward would be to change our current administrator observation protocol. Currently in our middle schools, teachers are observed by one of their administrators approximately once per week and receive feedback that same day. Based off what I learned writing this paper, I would suggest cutting the observations in half, but coupling the remaining observations with a pre-observation check in. These check-ins could be relatively short (5 to 10 minutes) conversations where the teacher presents his or her learning goals coupled with their plan for supporting these goals. The administrator would have an opportunity to ask questions and offer feedback ahead of time which would, in my opinion, help position the administrator as a co-creator in the educational process rather than a punitive overseer. This would require PD for administrators about learning goals as well as informing them of the reasoning behind this practice change.

For anyone in a similar position to mine, I recommend focusing on co-planning and talking about larger math education ideas with individual or small groups of teachers. I recommend adopting a set of curricular materials that prioritize teaching for conceptual understanding. I recommend asking teachers to aim high—to truly envision their perfect classroom—then help them examine what cultural norms support that ideal classroom, then support them in building that classroom.

REFERENCES

Ball, D. L. (1996). Teacher learning and the mathematics reform: What we think we know and what we need to learn. *Phi Delta Kappan*, *77*(7), 500.

Ball, D. L., & Cohen, D. K. (1999). Developing practice, developing practitioners: Toward a practice-based theory of professional education. *Teaching as the learning profession: Handbook of policy and practice*, *1*, 3-22.

Bjork, E. L., & Bjork, R. (2011). Making things hard on yourself, but in a good way: Creating desirable difficulties to enhance learning. In *Psychology and the real world: Essays illustrating fundamental contributions to society* (pp. 55–64). New York, NY: Worth Publishers.

Costa, A., & Kallick, B. (2010). It takes some getting used to: Rethinking curriculum for the 21st century. In Jacobs, H. H. (Ed.). *Curriculum 21: Essential education for a changing world.* Alexandria, VA: Association for Supervision and Curriculum Development.

Darling-Hammond, L., Hammerness, K., Grossman, P., Rust, F., & Shulman, L. (2005). The design of teacher education programs. *Preparing teachers for a changing world: What teachers should learn and be able to do*, 390-441.

Dewey, J. (1910). *How we think*. Boston: Heath.

Gallimore, R., Ermeling, B. A., Saunders, W. M., & Goldenberg, C. (2009). Moving the learning of teaching closer to practice: Teacher education implications of school-based inquiry teams. *The Elementary School Journal*, *109*(5), 537-553.

Garet, M. S., Porter, A. C., Desimone, L., Birman, B. F., & Yoon, K. S. (2001). What makes professional development effective? Results from a national sample of teachers. *American educational research journal*, *38*(4), 915-945.

Guskey, T. R. (1989). Attitude and perceptual change in teachers. *International journal of educational research*, *13*(4), 439-453.

Heifetz, R. A., Grashow, A., & Linsky, M. (2009). *The practice of adaptive leadership: Tools and tactics for changing your organization and the world*. Harvard Business Press.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K., Wearne, D., Murray, H., Olivier, A., Human, P. (1997). *Making sense: Teaching and learning mathematics with understanding*. Heinemann, 361 Hanover Street, Portsmouth, NH 03801-3912.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K., Human, P., Murray, H., Alwyn, O., & Wearne, D. (1996). Problem solving as a basis for reform in curriculum and instruction: The case of mathematics. *Educational researcher*, *25*(4), 12-21.

Hiebert, J., Gallimore, R., & Stigler, J. W. (2002). A knowledge base for the teaching profession: What would it look like and how can we get one? *Educational researcher*, *31*(5), 3-15.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. *Second handbook of research on mathematics teaching and learning*, *1*, 371-404.

Hiebert, J., Morris, A. K., Berk, D., & Jansen, A. (2007). Preparing teachers to learn from teaching. *Journal of teacher education*, *58*(1), 47-61.

Johnston, P. H. (2004). *Choice words: How our language affects children's learning*. Stenhouse Publishers.

Kapur, M. (2008). Productive Failure. *Cognition and Instruction*, *26*(3), 379–424.

Kapur, M. (2009). Productive failure in mathematical problem solving. *Instructional Science*, *38*(6), 523–550.

Kapur, M. (2011). A further study of productive failure in mathematical problem solving: unpacking the design components. *Instructional Science*, *39*(4), 561–579.

Kapur, M. (2014). Productive failure in learning math. *Cognitive Science*, *38*(5), 1008–22.

Kapur, M., & Bielaczyc, K. (2012). Designing for productive failure. *Journal of the Learning Sciences*, *21*(1), 45–83.

Kennedy, M. (1998). Form and substance in inservice teacher education. Research monograph.

Lehman, B., D'Mello, S., & Graesser, A. (2012). Confusion and complex learning during interactions with computer learning environments. *The Internet and Higher Education*, *15*(3), 184–194.

Meikle, E. M. (2016). Selecting and Sequencing Students' Solution Strategies: Reflect and Discuss. *Teaching Children Mathematics*, *23*(4), 226-234.

National Council of Teachers of Mathematics (NCTM). (2014). Principles to actions: Ensuring mathematical success for all.

Norton, A., & D'Ambrosio, B. (2008). ZPC and ZPD : Zones of teaching and learning. *Journal for Research in Mathematics Education*, *39*(3), 220–246.

Solomon, Y., & Black, L. (2008). Talking to learn and learning to talk in the mathematics classroom. *Exploring talk in schools*, 73-90.

Schoenfeld, A. H. (2014). What makes for powerful classrooms, and how can we support teachers in creating them? A story of research and practice, productively intertwined. Educational Researcher, 43(8), 404-412.

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. American Educational Research Journal, 33(2), 455–488.

VanLehn, K., Siler, S., Murray, C., Yamauchi, T., & Baggett, W. B. (2003). Why do only some events cause learning during human tutoring? *Cognition and Instruction*, *21*(3), 209-249.

Warshauer, H. K. (2015). Productive struggle in middle school mathematics classrooms. *Journal of Mathematics Teacher Education*.

Warshauer, H. K. (2015). Strategies to Support Productive Struggle. *Mathematics Teaching in the Middle School*, *20*(7), 390-393.

Wiggins, G. P., & McTighe, J. (2011). *The understanding by design guide to creating high-quality units*. ASCD.

Appendix A

BELIEFS WORKSHEET AND RESULTS

Talking Points

- **Round 1**: State whether you agree with Column A, Column B, or are unsure. Give your reason why. Everyone goes in turn with no discussion.
- **Round 2**: After listening to what others say, state whether you agree, disagree, or are unsure. Include some of what you heard as you talk about why.
- **Round 3**: Tally the opinions in your group
- *Move to the next prompt*

From NCTM's *Principles to Action*:

Prompt	Column A	Column B
1	Students can learn to apply mathematics only after they have mastered the basic skills.	Students can learn mathematics through exploring and solving contextual and mathematical problems.
2	The role of the teacher is to tell students exactly what definitions, formulas, and rules they should know and demonstrate how to use this information to solve math problems.	The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.
3	The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests.	The role of the students is to be actively involved in making sense of math tasks by using varied stratgies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and consider the reasoning of others.
4	An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.	An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.

Also from NCTM's *Principles to Action*:

Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Read the above teaching practices. Then discuss them with your group—things you liked, didn't like, questions you have, etc..

Here were the voted upon results from the above assignment, each number represents a group of approximately four teachers (these were made available to all participating teachers via Google Doc link):

B = we agree with column B with reservations
SB = we strongly agree with column B
A = we agree with column A
SA = we strongly agree with column A
M = we are mixed on agreement between columns A and B
1 = we want the Math Teaching Practices included in our beliefs
2 = we would like to consider the Math Teaching Practices but don't want them included yet
3 = we do not want the Math Teaching Practices included in our beliefs

	1	2	3	4	5	6	7	8	9
Q1	B	B	B	A	M	B	M	M	B
Q2	SB	B	B	B	B	M	B	B	B
Q3	B	B	SB	B	B	B	B	B	SB
Q4	B	M	SB	SB	M	M	B	B	M
MTP	1*	1	1*	3	1**	1**	1*	1	1

- *-For #2, there is a district piece to supporting us w/resources and training that give us that access
- **-What do you expect to see in my classroom using all of these?

(Prompts presented were from Principles to Action*'s productive vs. unproductive beliefs and the Math Teaching Practices.)*

(These were made available to all participating teachers via Google Doc link after it had been created during a PD session):

This was our brainstorm list (not agreed upon or anything formal like that) on what the ideal classroom looks like (1/20/2017 PD):

The Ideal Classroom

- Students
  - Have positive identities as thinkers and learners
  - Engaged, working collaboratively, asking ea other q's, leading the discussion
  - Not afraid to be wrong, confident that they will arrive at the answer, don't mind struggling
  - Not just engaged but interested in their task--they find value in what they're doing
  - Persevering through problems, productive struggle, they aren't giving up
  - Free to be creative thinkers
- Teacher
  - Providing guiding questions to push thinking to the next level
  - Giving them feedback on their ideas
  - Has a good understanding of how kids are understanding the mathematics
- Task
  - Problem solvers not solving problems
  - Student inquiry and data drive instruction
- Ideas
  - All ideas and opinions are valued
  - Different strategies and perspectives are encouraged and respected
  - Key mathematical ideas are made explicit
- Other
  - Reasonable class size

From the discussion and results shown above, this sheet was distributed at the next PD session:

Here are our beliefs about good math instruction. I've added some points of emphasis. To see the unedited version click here.

APPO Middle School Teachers tentative, non-permanent, non-binding beliefs about education (as talked about on 10/7/2016 and 10/10/2016):
There seemed to be consensus around both the role of the student and the role of the teacher:

- The role of the teacher is to **engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves toward shared understanding of mathematics**.

- The role of the student it to be actively involved in **making sense of math tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and consider the reasoning of others**.

- An effective teacher **provides students with appropriate challenge, encourages perseverance in problem solving, and supports productive struggle in learning mathematics**.
    - *we don't believe this fully captures all that encompasses an effective teacher, but we do agree an effective teacher does these three things*

- We agree that NCTM's Math Teaching Practices represent good teaching practices with the following caveats:
    - For #2, there is district support concerning resources and training
    - What do you expect to see in my classroom using all of these?

Appendix B

THE IMPORTANCE OF LISTENING TO YOUR STUDENTS VIDEO

Presentation by Max Ray-Riek about the importance of listening to students.

Title. Why 2 is Greater than 4: A Proof by Induction

Video URL. https://www.youtube.com/watch?v=h00Ux1qx2zw

Appendix C

LEARNING GOALS RECAPPED

The following sheet was passed out to new teachers at the beginning of this school year (August 2017) to help orient them in what had been discussed during secondary PD in the spring of 2017.

Learning Goals, PD recap

"Without explicit learning goals, it is difficult to know what counts as evidence of students' learning, how students' learning can be linked to particular instructional activities, and how to revise instruction to facilitate students' learning more effectively in future lessons. Formulating clear, explicit learning goals sets the stage for everything else." (Hiebert et al., 2007)

- Our grain size for learning goals is usually too big. We need to unpack our larger, usually procedural, learning goals into conceptual sub-goals.

TABLE 1

Uncover underlying concepts of a mathematical learning goal that is related to a mathematical procedure by working out the procedure in detail, thinking at each step about what knowledge is needed to understand why the step works the way it does.

Learning goals

Nonunpacked mathematical learning goal	Unpacked mathematical learning goal
Students will understand why the standard algorithm for addition of multidigit whole numbers works according to the joining meaning of addition. Example: $\begin{array}{r} {}^{1} \\ 57 \\ +\ \underline{36} \\ 93 \end{array}$	Students will understand *why* the standard algorithm for addition of multidigit whole numbers works according to the joining meaning of addition. a. Addition can be interpreted as joining two or more quantities to find a missing whole. b. The answer in the ones column represents the number of ones that we have after joining all the ones in the ones column and after we have exchanged 10 ones for 1 ten. c. The little 1 above the tens column represents the one group of ten that was exchanged for 10 ones. d. The answer in the tens column represents the number of tens that we have after joining all the tens in the tens column.

(taken from Meikle, 2016)

"Teachers often analyze their practice in terms of a smooth implementation of activities rather than an anticipated change in students' thinking." (Hiebert et al., 2007)

- We need to measure the success of our lessons by analyzing whether or not students made sense of our conceptual sub-goals.
- We need to avoid the cycle of--showing procedural algorithm, students practice said algorithm, we assess on algorithm fluency, we move onto the next algorithm.

Planning a lesson--learning goals are important because they help us…

- Select tasks
- Select solution strategies to be shared (see Meikle, 2016)
- Construct questions
- Construct assessments (formal or informal)

Reflecting on a lesson--learning goals help us diagnose student understanding so we can…

- Modify future implementations of lessons
- Modify subsequent instruction based on students' current understanding
- Identify students' current understandings rather than just quantifying their performance (i.e. My students don't understand that the numerator of a fraction represents the number of pieces, rather than my students can't add fractions)

Appendix D

TEACHER SURVEY RESPONSE DATA

Responses from all nine teacher respondents. Respondents 1 – 6 represent the cohort of six teachers I worked most closely with. Responsdents 7 – 9 represent the responses of the attempted control group.

Question 1: In what ways have your teaching practices shifted this year? What sparked those changes? Please include a specific example if you can.

Respondent 1
They have shifted quite a bit. Last year, I would do several problems in one class that were more skill based. This year, I may do one or two large problems to explore a concept.

There were a few causes for the change.

1) I am no longer in a toxic coteaching relationship. I can try new things without fear of a crying session or passive aggressive email.

2) A curriculum. Before, we didn't have a curriculum and were coming up with our own things. I wasn't able to put the time into making the problems open ended because was too busy making he problems! The curriculum gave me a starting point and allowed me to play around and have fun with the already existing problems. I learned how to open them up and make the kids really delve into the mathematics.

3) Ed pushed me to work on my warm ups. I made them more open ended and played around with creating problems that would allow any student to respond in some way. I feel like this really changed he way that kids entered my class. They didn't come in feeling defeated already. By connecting the warm ups to our goal for the day, it also allowed them to make better connections.

4) you! Before, I didn't really have a good resource. Now I know that I have someone to go to for guidance and ideas. You have been really helpful!

5) Jackie. She has come up with some pretty cool ideas for the year (pool problem, number trick problem)

Respondent 2
I am striving to let students do the talking and thinking. It is a process and does not always follow a completely upward path.....but being mindful of the students' role in their own learning

as I plan and during instruction has been my focus. I think the more I have read and learned about growth mindset and Jo Boaler's ideas, and the more I have observed classrooms in our district (classrooms that operate this way and ones that do not) the more I have come to believe that students need to be doers and thinkers, not passengers or bystanders. I am not where I want to be yet, but I am on the path.

Respondent 3
With the support and guidance of Mike Reitemeyer, I have been able to shift my teaching practices to be an educator who allows students to take control of discussions and guide my instruction based on their conversations. I no longer feel the need to lead the majority of a class period. I have also noticed that I am getting better at allowing wait time (even if it seems like awkward pauses at first). This is due to Mike giving us wait time at PD - I noticed what a difference it made.

Respondent 4
Giving students control to problem solve and releasing some of the control in my classroom to be the one that explains. This has come through understanding the power and confidence that builds in students when they can discover something on their own. This was sparked by giving more control and seeing the benefits. There are still kids who need help getting there, but most kids can get there with less help than you think.

Respondent 5
This year I had my students discover the meaning behind the math. They were every hands on and love the projects they were given.

Respondent 6
This year I have given students more responsibility in their learning. Instead of me telling them how to do math and then having them apply this to application problems, I instead give them time to grapple with the math to truly understand how it works, opposed to being told how it works. An example of this was teaching multiplying negative numbers this year. This was the very first activity I tried when I fully gave students the responsibility of working through the math together. How I would have taught it in previous years is to talk about patterns and how a negative x negative = positive then I would give cute pictures to help them "remember" this. This year, after a discussion about positive x positive and positive x negative, I gave the problem -2 x -3 and asked them to figure out what it meant and what it equaled. Many students came up with cook ideas and situations that allowed them to understand why it would make a positive. One specific student example that comes to mind actually comes from a collegues regular math classroom. The example was as follows: "If 2 students go to the movies and for 3 dollars a piece they would spend 6 dollars total, 2 x -3 = -6. If those two people do not go to the movies for 3 dollars a piece then they save 6 dollars (don't spend the $6), -2 x -3 = 6.

Respondent 7
I've become more comfortable letting my students productively struggle through a question. Sometimes, it's easy to see a lot of your kids struggling and wanting to help them, but allowing them to struggle before figuring it out has been beneficial. Allowing areas where the students can respond with their own thinking, helps the students to engage the problem.

Respondent 8

I have tried to incorporate more "real world" problems/higher order thinking problems into my classroom. The smarter balance test sparked this change.

Respondent 9
I am trying to implement more discovery learning, hands on activities and more Kagan strategies to improve student engagement.

Question 2: In what ways have your beliefs about education shifted this year? What sparked those changes? Please include a specific example if you can.

Respondent 1
I don't know that my beliefs have shifted.

Respondent 2
I think my shifting ideas about education have taken place over several years. I feel strongly about culture and communication in the classroom. Students who are learning are those who are talking and questioning, as well as listening. Of course, they also need to be diving in and doing the work. So I want to be less in the front of the room and more of a facilitator. There are times I do this fairly well, and other times that I fall back into the habit of being the traditional teacher. I think that for me I need to develop a stronger culture that supports and encourages all students to be fully participative daily. Talking to other teachers, and being involved in PD's in our district has been helpful for me. Also having a curriculum specialist who prioritizes and values the kind of math instruction that leads to longterm understanding of the concepts is of great value.

Respondent 3
I am more confident in my belief that the teacher doesn't own the math, and neither does the student. I have been able to help my students see that I am not an "answer giver" or a "hint giver", but someone who is able to help them get to a point where they can think more clearly about the math. This is because of conversations I have had with Mike, as well as conversations he had with my classes.

Respondent 4
Taking time to build relationships with students is critical to helping them believe in themselves, getting them to trust and respect you and those around them, and motivating them to try harder. Without homeroom this year it was harder to build relationships with some kids, and it negatively impacted them. There were kids that I couldn't motivate, which usually is a strength of mine. It wasn't a shift, but rather reinforcing a belief I had.

Respondent 5
Many of my beliefs are the same. However, one that has changed is that students should struggle. It's ok for them to have discourse and I aloe them to think through it without my help.

Respondent 6
I have more faith that students can accomplish a task, my outlook is that I am not the keeper of math and that students (with guidance) can create their own theories about math. The changes were sparked by the first PD with Mike where we had to model a fraction divided by a fraction. This was a difficult task to do by adults that understood how to do the math and shortcuts, but not necessarily why the math works or why the shortcuts work. If I can gain a deeper

understanding by learning that way, why wouldn't students gain a deeper understanding of math by learning that way as well?

Respondent 7
I think it's helped to lead to a more student led learning experience, where the student discovers why behind answers as compared to just repeating answers and formulas.

Respondent 8
No changes.

Respondent 9
I feel as though we are requiring less from students and placing more onus on the teachers to ensure that students are successful. Students are no longer required to complete homework or hand in class assignments; they can complete partial or no work yet receive a 50% in HAC sending the message that these things are devalued. In essence, a student can perform well on summatives only, with little effort on other assignments and still earn an A or B or C.

Question 3: How was the classroom experience different for your students this past year? Why do you think that is? Please include a specific example if you can.

Respondent 1
They feel better. They smile more. They come up with the ideas rather than me giving them ideas. I think this was directly related to the changes that I have made in my teaching practices.

One thing that I tried this year was to tell the kids I refuse to help them with nothing. In order to get feedback with me, they have to have at least tried something. I think that this has helped decrease he learned helplessness that had been plaguing my classroom.

Respondent 2
Ipads certainly had an impact and not in a good way. But one thing that did seem to help my students was the use of large whiteboards. After designing lessons specifically to use them a few times it sort of became a natural thing that students would just go get whiteboards and do their work on the boards together. I loved that students were making decisions about how they wanted to learn and do work together. I liked that this made the work very public in the group and it seemed to lead to really good conversations.....much more than if everyone had been working on paper or in notebooks. I let go of some "control" this year....in the past I might have said, "no, we are not using whiteboards today" and I found that I liked the feel of the classroom better with students exercising more control. I think that also the idea of changing seats more was good, I did not do it daily, but I did do it more often at your suggestion and I think it gave the room a different feel, students did not get too settled in or complacent in one group.

Respondent 3
My students shared with me that in math this year, they were able to understand the "why" behind the math they were learning. They told me that it was different from the past because they were able to come to conclusions themselves. Sometimes this was frustrating for them at first, but they grew to expect it and would be disappointed if I ever "gave something away" before they figured it out themselves. I think that the conversations I had with other math teachers in our

district, my team members, and my district department head influenced this change in my classroom.

Respondent 4
Getting kids up and moving and feeling empowered to solve their own problems was exciting. I did less practice of skills they have learned since they were doing more learning for understanding, but at the end of the year, their overall skill set was not a strong. This year I want to do even more conceptual learning with kids really discovering and teaching each other, but also some practice throughout the year to keep concepts fresh.

Respondent 5
It was different due to the discovery I mentioned above.

Respondent 6
I think that my students started to realize that it is the mistakes, wrong theories, pieces of ideas, and collaboration that allow people to really understand.

Respondent 7
I think the new curriculum that I started using at the end of the year gave the kids an opportunity try to develop their own answers to the problems. When the kids were asked to sort polygons into groups and then sub groups, it was interesting to see the debates as to why some groups sorted them one way and other groups sorted another way.

Respondent 8
Not very different.

Respondent 9
Because they are taking on less responsibility to perform they seem more in need of assistance. However I have gotten better at letting them struggle through a problem with minimal assistance.

Question 4: What professional development, formal or informal, did you find significant this year (can be more than one or can be none)? Please elaborate as to why certain PD was helpful to you or why it wasn't.

Respondent 1
i think that the conversations I had with you (during pd, PLC's, and email) were the most beneficial. You were very visible and easy to reach. Getting feedback on my ideas helped me to better facilitate them in the classroom!

Respondent 2
Every single time I have spoken with you formally or informally, I have learned something. I think when you helped us plan for a lesson in the beginning of the year you showed us how to get to the important ideas in a way that allowed the students to do the thinking and discovering. Each time you have observed our class and then talked to us afterwards I felt supported and challenged equally, never judged and found lacking as we usually do when other admin types observe. I would like to have more planning/plc time with you as the year progresses. The sessions with Jamila have been invaluable, nothing is more necessary than the opportunity to work through the math together with Jamila there to point out how the understanding might unfold and how to facilitate it.

Respondent 3
I found that the most significant PD was when you came into our classes and helped us plan. Working with the CMP curriculum was definitely helpful, but I'd also like to know how other teachers plan on adapting this curriculum for their classrooms. In other words, how are they taking the problems in the curriculum and creating lessons around them that engage students and meet the expectations of administration? I also really enjoyed when you came in to observe a lesson (even if it was out of the blue) and gave feedback. I learned a lot when you took over some lessons because I was able to see exactly what you would do, and use that to adapt my own practices. I would also really enjoy a PLC about general math education topics (or education in general). For example, how do other teachers grade? What strategies do they use in inclusion classrooms? What do their expectations look like for mathematical conversations? These conversations would be helpful to me.

Respondent 4
Full day trainings are helpful, half days are harder. Grade level and content specific math pd where we are doing the math or discussing the math is the best. If I never write another learning map again I will be beyond thrilled- what a waste of time.

Meeting with PLC's is good, there is always too much to discuss and not enough time. We might move ours to after school next year so that we have enough time without feeling rushed.

I think ways to bring the buildings together would be good. We really used to be a united group, but there seems to be more frustration between buildings than ever before. I think this started when we had to write common maps, formatives, and summatives. Planning together and sharing was what held us together - an attitude that we were 'all in it together' but when we had to sit and formalize it caused anger. I think if teachers recognize that we are moving forward and we need to make the most/best of it, the situation would be better. Also, when we use an hour or more at the beginning of a PD to talk about the same things, values or the way kids learn, it just eats up time that we could use for curriculum discussion. A short talking points (20 mins or less) or an inspirational video, or one problem we do and share out by grade level might be better.

I would rather take short videos using the swivvl and talk through what could have been better than be observed, because when I am teaching I forget half of what was said/done. Maybe instead of being observed as often, we could do some kind of video feedback group/system where we focus on a specific area of growth to target or be able to discuss how we could plan a better lesson to teach the same concept? Just a thought, but it would be nice to change it up.

Respondent 5
The PLC PD's were helpful because they gave me a chance to ask questions I may not have in larger PDs. It also allowed you to walk us through how to teach a concept slowly.

Respondent 6
I think my PLC with one of my colleges (Crawford) was very beneficial as we were on the same page and we did a really good job of giving and getting ideas from each other. I think having you (Mike) as a resource whenever needed was a really big help and really opened my eyes to ways to teach. I think full PD days working on the curriculum is helpful because it gives us a chance to

really talk about how we can do the math in our room and it allows us to bounce ideas off of you and get your perspecitve.

Respondent 7
I liked the opportunity to work through problems. I think we could benefit more from working through problems and then trying to go straight from that to putting together our lesson plan, then moving on to the next one.

Respondent 8
I found the CMP PD helpful with Jamila. She walked us through each problem and helped us explore different ways to solve/answer. We weren't left to figure it out on our own. She told what was important and what wasn't. It feels like the PD will be beneficial when planning our lessons next year.

Respondent 9
I prefer half days diving into CMP, planning with building math teachers/ PLC

Question 5: What is one thing (or some things) you hope to get better at next year concerning teaching? Why that thing(s)?

Respondent 1
I'm not sure. Usually, I set goals for myself to improve each year. Since I won't be starting out the year at school, I haven't thought about it. I guess my goal is to figure out to still be a good teacher while being a sleep deprived working mom with two toddlers!

Respondent 2
I would like to get better at sort of managing the purposeful share out of the whiteboard work. I need to get better at summarizing in general. I think that if I can get more purposeful in sharing out student work/ideas and summarizing more cohesively with students then they will solidify their understandings and will be able to rely more strongly on them when they need to make connections down the road.

Respondent 3
Next year I hope to get better at planning mathematical conversations ahead of time. This year I was able to start asking questions that made students think about the math they were doing, but I would like to get better at planning secondary questions, and also think about what questions may arise from students during these conversations. I went to a TRU Math PD where we discussed in our groups the different ways we thought students would think about a certain problem. This really helped me see how many ways you can approach a problem, and it helped us all plan our instruction to prepare for these different approaches.

Respondent 4
The group I had was harder to motivate and get interested in the math and activities. I tried to add more activities using the iPad or that were 'engaging' but really I need to incorporate more challenging math that helps them be engaged through challenge or discovery. I want them to be so excited they are looking stuff up at home and asking more questions.

Respondent 5

I hope to get better at giving them mor time. This way they learn that they will not be given the answer right away and that it is up to them to figure it out.

Respondent 6
Next, year I would like to do a better job in the beginning of the year to make it clear that mistakes, collaboration, and a desire to understand are things that will help them succeed in math - not whether they memorized their math facts or not.

Respondent 7
Finding ways for the students to buy into the idea of productively struggling. It's easy for kids to sit back and struggle until a teacher or another student provides them with an answer.

Respondent 8
I am hoping that the new curriculum will allow the students to be more willing to try/persevere when solving problems. I am hoping that they will be experiencing real world problems more often in class.

Respondent 9
I hope to create student driven activities more frequently and allow students to struggle more.

Question 6: This last prompt is just your opportunity to communicate with me anything you feel is important that you didn't get to write about yet. (Optional)

Respondent 1
I like that you give your true opinion and make decisions.

Respondent 2
Thank you for your leadership this year. I know it has not been easy. Please continue to stand up for what you know is best for our students. Please also continue to work with teachers and students. Students get so much out of having you in the classroom!

Respondent 3
Thank you for your guidance this year. I know that I have improved as an educator, and I am confident that I will continue to grow with your support. Your advice and help with planning, ideas, and understanding the content have been so valuable to my coworkers and I. I appreciate that you take the time to work with us individually, and with our students. You have pushed me to think more about the way that I teach, and I think that in turn, my students are gaining a deeper understanding of the math, and growing to love math the way that I do. Thank you!

Respondent 4
Just a few thoughts, but take it or leave it, I won't be offended....
I think that you are doing a really great job, continue to communicate when you see growth or when a teacher is trying. I know that we can be tough sometimes, but if you show that effort and a willingness to learn are things you value, then others might start to shift.
Charlie often picked favorites and that really frustrated people, so pull teachers that are sometimes not highlighted out to be models for others - even new teachers or special education teachers.

Constant walkthroughs, the teacher evaluation system, and our administrators are all pushing for excellence at every moment of every day, so sometimes we just need time to breathe when we are all together - I am wondering if happy hour or just some 'motivational/funny moments' might help curb some of the pressure.
Constantly supporting teachers is hard, especially when some are negative, but you recognizing the hard work and constantly valuing the job we do each day is helpful, even when we don't tell you, just know it means a lot.

Respondents 5 – 9 left this blank

Questions 10 – 14—Likert-scale responses (4 = highly effective PD; 1 = not effective PD; responses of N/A indicate the teacher did not experience that type of intervention from me)

Respondent	Whole day PD session	CMP training (even if this was with Val or Jamila, still rate this as it will give me valuable feedback about planning for next year)	Planning a lesson/unit with me (individually or within a PLC)	Discussions about math education with me (individually or in PLC)	Observation feedback
1	4	4	4	4	4
2	3	4	4	4	3
3	4	4	4	4	4
4	3	4	4	2	2
5	3	3	4	4	3
6	3	2	4	4	4
7	3	3	3	3	4
8	2	4	N/A	2	N/A
9	2	3	N/A	1	N/A

Question 15: This is your chance to elaborate on any of the ratings you gave above. (Optional--in particular if you feel like you already talked about this in a prior response feel free to skip this)

Respondent 2
The 3 for Full Day is because at the RMS PD days there was a lot of confusion about what was expected, through no fault of yours. I think half days seem to be very productive. Full days can be a bit too brain intensive, I feel that half days maximize our time and our ability to do productive work.

Respondent 4
Jamila has been really helpful with our trainings, and I think that for us to move forward together as a grade level keeping her is going to be important. There are a few loud voices (some negative) that will not go against Jamila and I think she was the perfect person for our trainings. She also moves a good pace, realizing that we want exposure to most of the content but

I haven't really gotten feedback after an observation, just a short conversation once I think. As I stated earlier, I think video might be a good way to go if we created goals and then were able to see if changes put in place led to improvement. This would take more time, but would prevent you seeing only this part of the lesson, or coming in on a day when the schedule is strange. I don't love to see myself teach, but know that the practice of analyzing video is extremely helpful.

Respondent 6
I think the PD with you and 7th grade was great. I think the 6th grade PD I attended was not wonderful and I am not sure if that is because of the other teachers not buying into the student agency or if it was the trainer. There was not much time in the 6th grade PD to dive into the curriculum, more time was placed on whole group discussions that were not very beneficial (due to the teachers lack of enthusiasm)

All non-listed respondents left this response blank.

Appendix E

STUDENT INTERVIEW RESPONSES

The following is a template for the questions I asked students during their interviews. For my notes from individual student interviews, download them here:
https://www.dropbox.com/s/wua8s02o1bbqrac/ELPstudentinterviewnotesfull.pdf?dl=0

Student Interview Questions

1. How important is getting the right answer in your math class? Is there a time when getting a right answer is especially important? Is there a time when getting a right answer is not so important? Can you give some examples?

2. In your math class where do most mathematical ideas come from? Meaning does the teacher usually introduce new concepts and formulas, or do you and your classmates usually make new connections and discover new formulas by yourselves? Can you give some examples?

3. Do you believe the ideas you have in class are respected by your classmates? By the teacher? Can you give an example from class? (or potentially to be replaced with—When you put forth an idea in class, what happens next? Like does the teacher tell you it's right/wrong, good/bad, does he/she ask the class to examine your idea? What happens?)

4. I want you so to say if you agree or disagree with the following statement: I believe I sometimes have good mathematical ideas.

4a. Follow-up if agree: Can you give an example of a good idea you had? Did you tell the teacher or classmates about the idea? What happened then? (If necessary--how does the teacher respond? If necessary--do you feel like the teacher hears your idea and then tells you if it's right or wrong, or do you feel like the teacher hears your idea and tries to have classmates build on your idea?)

4b. Follow-up if disagree: Why don't you think you have good mathematical ideas?

5. Do you think you learned more in this math class compared to previous math classes? Why do you think so?

6. Is there anything you'd like to share with me about math class that you didn't get the opportunity to do so far?

Appendix F

ELP PROPOSAL

University of Delaware

ELP Proposal

Promoting Productive Struggle in Middle Level Math Classrooms

Michael Reitemeyer

Overview

My role in the Appoquinimink School District is that of District Mathematics Specialist. Operationally this means that I am directly providing district-wide professional development for all secondary math teachers as well as observing and coaching teachers on a one-on-one basis. The problem I am tackling is that most middle school (6-8) math teachers are not actively teaching for conceptual understanding. Teaching for conceptual understanding means that students need to be productively struggling with the mathematics as well as key mathematical ideas need to be made explicit. My work with district teachers will focus on supporting student productive struggle in their classrooms as a way to promote conceptual understanding. I chose to focus on productive struggle as opposed to making key mathematical ideas explicit as I believe that the former is harder to implement and requires more nuance in activating (though making key mathematical ideas explicit will also be a part of my professional development with these teachers it will not be the focus of my research here).

My goal is to guide all middle level math teachers in the district to promote a classroom environment where their students productively struggle with the mathematics to help develop their conceptual understanding of the mathematics. And while I will have the opportunity to work with all secondary math teachers, I will focus on middle level math teachers and will work closely with six of these middle level teachers towards the same end of promoting productive struggle in their math classes.

The purpose of this proposal is to assess the problem of a lack of teaching for conceptual understanding in the district and to move forward with a plan to attempt to address the problem. In this proposal I describe some of the context of the district where I am working as well as describing my role within the district, lay out the problem I am addressing, put forth a plan for addressing the problem, and describe ten artifacts I have already created.

Organizational Context

While I am technically a K – 12 District Math Specialist my charge and focus has primarily on secondary (6 – 12) schools, that said due to the adoption of a new middle school curriculum (to be implemented next school year) as well as advocating for my time from middle school teachers and their principals, I have spent most of my time working with our middle level teachers. Here is the district's middle school performance data in mathematics as compared to the state:

Table 1

Students' mathematics performance on most recent Smarter Assessment.

Grade	Appoquinimink's percent proficient	Delaware's percent proficient
6	53.6	37.0
7	49.0	39.6
8	51.2	37.7

The data indicate that Appoquinimink School District is a high-performing district in Delaware.

Concerning the district's middle level math educators, a demographic breakdown shows the following:

Table 2

Demographical information concerning race/ethnicity and gender of middle level math teachers.

Race	Number (n = 35)	Gender	Number (n = 35)
White	32	Women	33
Black	1	Men	2
Latin@	1		
Asian	1		

Note. The one Latin@ employees was not dual-identified ethnically/racially because they did not self-identify as anything other than Latin@ when asked.

Appoquinimink School District explicit states, “Our mission is to provide a world-class education where each of our students gains the knowledge, understanding, skills and attitudes needed to contribute and flourish in a global society” (Appoquinimink School District’s website, retrieved 11/2/16). Relatedly, Appoquinimink School District has a set of four “district beats” that should be at the heart of instruction in every classroom. These beats are: communication, collaboration, critical thinking, creativity—all of which lead to student engagement.

This district has yet to examine the ideas of teaching for conceptual understanding and what productive struggle might look like in their classrooms. By productive struggle I mean “…that students expend effort to make sense of mathematics, to figure something out that is not immediately apparent” (Hiebert and Grouws, 2007, p. 387). However, four middle level teachers (11.4%) have attended state-wide professional development where the phrase “productively grappling with the mathematics” has been commonly used though not explicitly explored. In my observations of middle school teachers throughout the district (explored in the next section) there is a clear and collective lack of productive struggle happening in these classrooms where students are routinely given highly procedural tasks to carry out and where mathematical authority almost exclusively rests on the teacher.

Problem Statement

The problem I am addressing is that there are very few middle school math classrooms in the district where students are productively struggling with key mathematical ideas. Within the first six weeks of the 2016-2017 school year I had observed all 35 middle school math teachers (six inclusion teachers were observed with their mainstream counterpart and shared the rating of the room as both teachers had planned and were enacting the lesson together; one teacher was not counted as he only teaches one math class and does not attend math PD). On my first visit to their class I assessed the level of productive struggle in their room using the Cognitive Demand criteria from Alan Schoenfeld's TRU-Math observation rubric:

Summary Rubric

	The Mathematics	Cognitive Demand	Access to Mathematical Content	Agency, Authority, and Identity	Uses of Assessment
	How accurate, coherent, and well justified is the mathematical content?	*To what extent are students supported in grappling with and making sense of mathematical concepts?*	*To what extent does the teacher support access to the content of the lesson for all students?*	*To what extent are students the source of ideas and discussion of them? How are student contributions framed?*	*To what extent is students' mathematical thinking surfaced; to what extent does instruction build on student ideas when potentially valuable or address misunderstandings when they arise?*
1	Classroom activities are unfocused or skills-oriented, lacking opportunities for engagement with key grade level content (as specified in the Common Core Standards)	Classroom activities are structured so that students mostly apply memorized procedures and/or work routine exercises.	There is differential access to or participation in the mathematical content, and no apparent efforts to address this issue.	The teacher initiates conversations. Students' speech turns are short (one sentence or less), and constrained by what the teacher says or does.	Student reasoning is not actively surfaced or pursued. Teacher actions are limited to corrective feedback or encouragement.
2	Activities are at grade level but are primarily skills-oriented, with few opportunities for making connections (e.g., between procedures and concepts) or for mathematical coherence (see glossary).	Classroom activities offer possibilities of conceptual richness or problem solving challenge, but teaching interactions tend to "scaffold away" the challenges, removing opportunities for productive struggle.	There is uneven access or participation but the teacher makes some efforts to provide mathematical access to a wide range of students.	Students have a chance to explain some of their thinking, but the teacher is the primary driver of conversations and arbiter of correctness. In class discussions, student ideas are not explored or built upon.	The teacher refers to student thinking, perhaps even to common mistakes, but specific students' ideas are not built on (when potentially valuable) or used to address challenges (when problematic).
3	Classroom activities support meaningful connections between procedures, concepts and contexts (where appropriate) and provide opportunities for building a coherent view of mathematics.	The teacher's hints or scaffolds support students in productive struggle in building understandings and engaging in mathematical practices.	The teacher actively supports and to some degree achieves broad and meaningful mathematical participation; **OR** what appear to be established participation structures result in such engagement.	Students explain their ideas and reasoning. The teacher may ascribe ownership for students' ideas in exposition, **AND/OR** students respond to and build on each other's ideas.	The teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings.

My observations resulted in the following:

Table 3

Observation results concerning level of productive struggle in middle school mathematics classrooms in the district.

TRU-Math rubric score	Number of teachers	Percent of teachers
1	28	80.0
2	5	14.3
3	2	5.7

There is certainly a need for more productive struggle in the district's middle school math classrooms with over three-quarters of the teachers observed at the lowest level of the rubric and fewer than 6% of teachers at the highest level of the rubric.

The lack of productive struggle is problematic as research has supported its value in students learning mathematics conceptually (Hiebert and Grouws, 2007; Warshauer, 2015a; Bjork and Bjork, 2011; Lehman et. al, 2012; Kapur, 2008; Kapur, 2009; Kapur, 2011; Kapur and Bielaczyc, 2012; Kapur, 2014). Productive struggle is consistent with the district's vision for good instruction via the district beats: communication, collaboration, critical thinking, creativity, engagement. True collaboration and critical thinking happen when students are engaged in productive struggle—or are working on a task whose solution is not immediately apparent. Said another way, there is no need for students to think critically or engage in meaningful collaboration if the solution to a problem is immediately apparent. Similarly students often must engage in creative thinking, using original ideas, when trying to solve a problem whose solution is not immediately apparent.

Improvement Goal

I want to understand more about teachers' growth in creating a culture where students are productively struggling with the mathematics. I also want to understand more about how teachers' beliefs about teaching change as they begin to shift some of the mathematical authority to their students. Relatedly, I want to examine how this transition towards a culture of productive struggle impacted students' beliefs about mathematics and themselves.

To measure whether my hypotheses about improving student productive struggle are accurate (see Theory of Action later in this paper), I will re-collect data using the TRU-Math rubric's Cognitive Demand key and will compare scores from the beginning of the year to the end of the year to determine the degree of growth in productive struggle in middle school math classrooms. My goal for improvement is to observe four middle school math teachers scoring a three on the TRU-Math rubric (up from the two initially observed), and to observe at least twelve teachers scoring a two on the TRU-Math rubric (up from the five initially observed). If I reach both sub-goals, more than half the middle school math teachers will be above a one on the scoring rubric—a significant improvement from the original 20%.

My goal, post-graduation, is to observe classrooms throughout the district with an inverted distribution on the TRU-Math rubric than what I originally observed meaning over three-fourths of teachers should be scaffolding lessons that support student productive struggle and under 5% of observations should witness students mainly applying memorized procedures.

Concerning my goal of understanding more about teachers' beliefs changing as they begin to shift mathematical authority to their students, I'd like to collect data from teachers who have made or have begun to make the shift of transferring mathematical authority to their students and see how it has affected their beliefs about teaching and/or students. Personally, as I shifted more and more of the mathematical authority to my students I began to see teaching not as something I did unto students but as something we explored together where every student brought legitimate mathematical value into the classroom. I began to see (almost all of) my students as competent and capable; I legitimately came to see and appreciate their strengths as opposed to focusing on what mathematical “skills” they had lacked. I came to value them as mathematicians. It was the single most powerful transition of my career. I felt liberated as my students began to feel empowered as they came to identify as powerful thinkers and doers of mathematics.

Anecdotally through conversations with other teachers I know I am not alone in this experience. I want to collect data to see if to what extent is this outcome—feeling liberated as a teacher as your students identify as doers of mathematics—tied to the shift of mathematical authority and promotion of productive struggle in teachers’ classrooms.

I plan on collecting anonymous written responses from teachers. I will select six middle school teachers I have worked one-on-one with as well as randomly selecting six middle school teachers I haven’t worked one-on-one with (the response will ask a demographic question identifying the teacher as someone who has or has not worked extensively with me). The teacher responses will be evaluated to see what disparities arise between the two sets of responses. The hope here being that there will be some connection between teachers’ experiences with transferring mathematical authority to their students and engaging them in productive struggle experience to teachers’ feelings of elation towards these changes.

Table 4

Summary of my personal improvement goals.

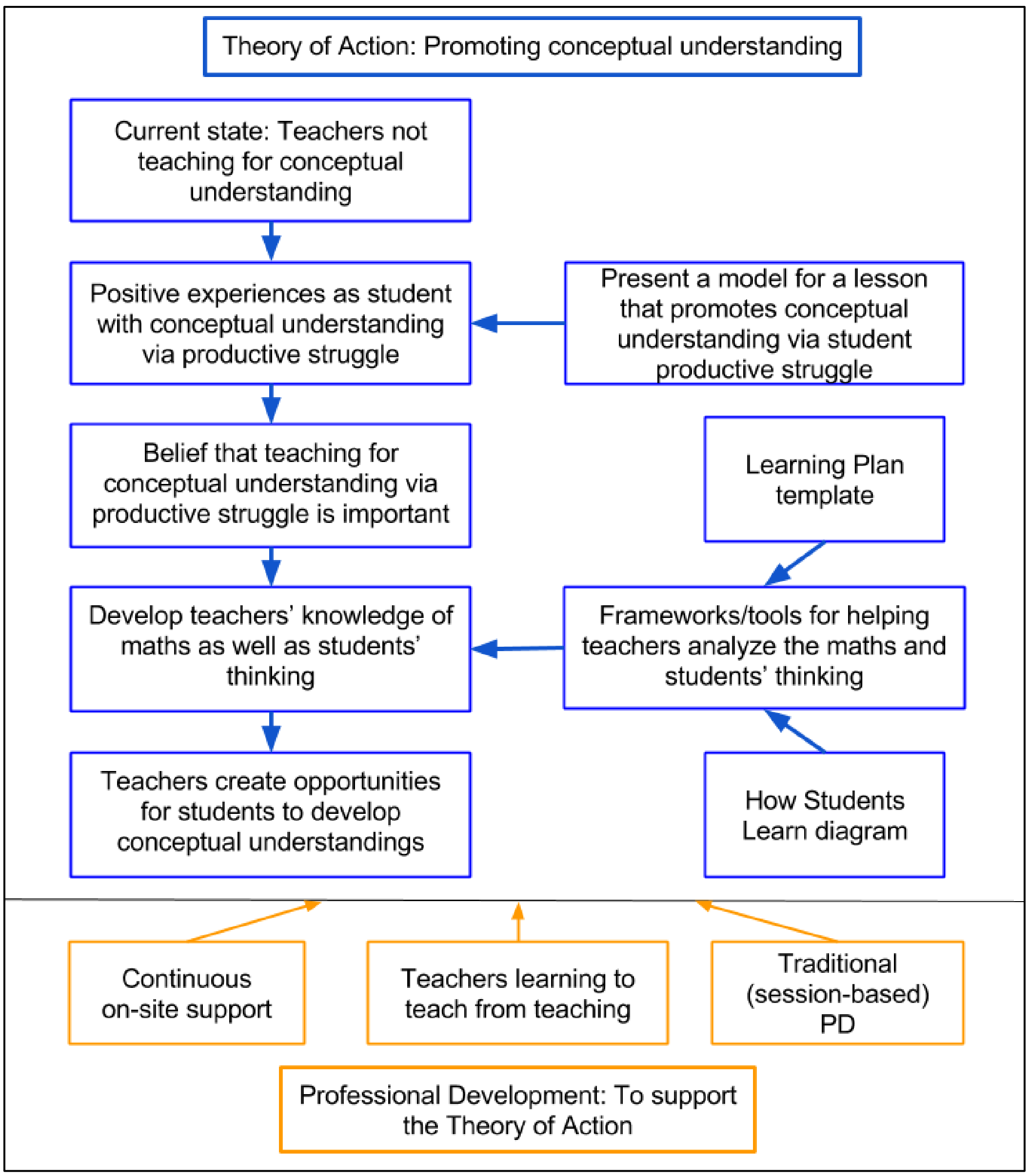
What I hope to understand better	What data will enable me to gain these insights	From whom I will gather this data
Teachers growth in creating a culture where students routinely productively struggle with the mathematics	Pre- and post-intervention observation data rating the general level of productive struggle in classrooms via rubric	All middle level teachers in the district; the six teachers I have worked with most closely will also have their data disaggregated
How teachers' beliefs about teaching change as they begin to shift some of the mathematical authority to their students	Anonymously written long-form responses to questions concerning teaching beliefs	The six teachers whom I have worked most closely with this year coupled with six other middle level teachers
How students have felt about their math class as they have engaged routinely in struggling productively with the mathematics	Student interviews comparing their current year's math experience to those from prior years	Six students from among the six teachers whom I have worked most closely with this year

My hope in disaggregating the data from my most worked with teachers is to see whether there has been a positive trend upwards in the district even without lots of direct intervention on my part. Similar data between the groups would lead me to believe that teacher growth towards a productive struggle culture can happen without much on-sight intervention but simply through traditional professional development structures coupled with infrequent, unannounced walkthroughs and debriefs. Similarly I am collecting written responses from six teachers whom I have not worked closely with to act as an experimental control group which will once again help me to decide what impact my close working with teachers has had this first year.

Lastly, I want to discuss the idea of identifying why teachers grew (or didn't) rather than just identifying if they did grow. I think the teacher written teacher responses will give me the best access to the why of teacher growth. One thing I plan on asking teachers is which PD activities, in their opinion, affecting their growth the most meaningfully; I then plan on asking them to elaborate on explaining why those highly ranked PD activities influenced them so much. My hope is to tap into teachers who did grow this year and find out specifically what helped them to grow. If I can find out certain activities were overwhelmingly more influential than others it will determine how I allot my time in future years' PD activities.

Improvement Approach

The move from where teaching is now to where I would like to see it, will be both complicated and challenging. To guide my work, I have created a theory of action. In this section I will describe and elaborate my theory of action.



If teachers are going to teach for conceptual understanding, it would be helpful if they believed teaching for conceptual understanding is important. If you want to change a teacher's beliefs, one potentially effective approach is to give them positive experiences with a new system first (Guskey, 1989); changes in their personal beliefs may follow. If we are to expect teachers to offer their students learning opportunities that support conceptual understanding, then it makes sense to provide them experiences as learners in an environment that promotes conceptual understanding: "When teachers have opportunities to interact with their subject matter in ways that they aim for their own students to do (such as engaging in writing workshops, getting feedback on their own writing, giving critiques), they are more likely to engage in those practices in their classrooms" (Lieberman and Wood, 2003, as cited in Darling-Hammond et al., 2005, p. 396). Teachers need a model of what teaching for conceptual understanding looks like and be given the opportunity to evaluate how they can implement changes in their classroom to more resemble the practice they see modeled.

Teaching for conceptual understanding in mathematics requires two forms of knowledge: "*knowledge of the subject* to select tasks that encourage students to wrestle with key ideas and *knowledge of students' thinking* to select tasks that link with students' experience and for which students can see the relevance of the ideas and skills they already possess". (Hiebert et al., 1996, p. 16; emphasis mine). If teachers need knowledge of the subject and knowledge of students' thinking to effectively teach for conceptual understanding, then I need to facilitate this knowledge acquisition. Further validating this logical progression to focus on math knowledge and student thinking, "So teachers leave these [most effective] programs with very specific ideas about what the subject matter they will teach consists of, what students should be learning about that subject matter, and how to tell whether students are learning or not. This content makes the greatest difference in student learning" (Kennedy, 1998, p. 25). I have developed two frameworks or tools for helping teachers analyze both the mathematics as well as students' thinking: one is a "Learning Plan" template, the other a diagram concerning how students best learn mathematics conceptually (see artifacts 8 and 9).

To support the aforementioned Theory of Action I will utilize three avenues of professional development: traditional (session-based) PD, continuous on-site support, teachers learning to teach from teaching. Each of these three forms of professional development will be applied in an integrated sense with each district teacher receiving traditional PD during district in-service days, on-site support via classroom and PLC visits by me throughout the school year, and opportunities to learn from themselves via video review and from peers via department-level classroom observations and reflections. "There is a growing consensus that professional development yields the best results when it is long-term, school-based, collaborative, focused on students' learning, and linked to curricula" (Hiebert et al., 2002, p. 3). Traditional (session-based) PD will allow our teachers the opportunity to engage with the curriculum and uncover the mathematics therein while reflecting on how students will be making sense of the lesson's prompts which are powerful PD opportunities (Kennedy, 1998; Hiebert et al., 1996). Iterative on-site support will encourage teachers to apply what they have learned from all the PD forms while prompting further reflection on their teaching practice (Ball, 1996; Hiebert et al., 2002). Teachers learning from teaching is another important avenue for improving teacher practice the advice comes from peers, is clearly applicable, and is directly related to what they are doing in the classroom; furthermore, it moves teachers toward a professional atmosphere of debate, critique, and challenge which furthers teachers' capacity to grow and refine their standards for what effective teaching looks like (Ball and Cohen, 1998; Ball, 1996).

Traditional PD will be utilized to model what teaching for conceptual understanding looks like (modeling lessons with teachers as students), analyze these lessons, and build consensus around what good math teaching looks like for our districts' students. Whole group sessions present opportunities for shared experiences in these lessons and analyses as well as time to engage in discussions that will ultimately lead towards district consensus. This type of PD gives me the opportunity to focus on district-wide curriculum training as well as a platform for introducing common teaching moves. This PD will also give the opportunity for teachers from one school to see teachers from other schools practicing the craft as teachers will become part of the lesson presentation as well as will be the subject for video analysis. Research by Garet and colleagues (2001) supports the decisions we have made concerning our traditional professional development listed above as well as the other two types of professional development about to be discussed:

> "On the basis of this emerging evidence, we view the degree of content focus as a central dimension of high-quality professional development... Opportunities for active learning can take a number of forms, including the opportunity to observe expert teachers and to be observed teaching; to plan how new curriculum materials and new teaching methods will be used in the classroom; to review student work in the topic areas being covered; and to lead discussions and engage in written work." (p. 925)

Continuous on-site support by me will be utilized to offer multiple iterations of observation-feedback that focus on the content, the pedagogy, and the content-specific pedagogy, all important for improving instruction. On-site support will allow me to record teachers to analyze teaching in a one-on-one setting as well as collecting footage to be shared by that building's math department for department-wide professional development (or possibly grade-wide, rather than department-wide PD). This on-site, iterative support is important in helping teachers grow as, "Repeated experiences with a set of conceptual ideas, along with repeated opportunities to practice skills and modes of analysis, support deeper learning and the development of expertise" (Darling-Hammond et al., 2005, p. 393).

As teaching is an ever-evolving and highly complex practice teachers can learn to teach from teaching. Some effective ways that teachers can learn from teaching include reflection, analyzing video of themselves or other teachers, and analyzing student work. Research by Ball and Cohen (1999) showed that teachers could learn from authentic classroom materials and did not necessarily have to learn from teaching itself. Garet and colleagues (2001) go more in-depth on where active learning for teachers can come from:

> "One element of active learning is the opportunity for teachers to observe expert teachers, be observed teaching in their own classroom, and obtain feedback. These opportunities can take a variety of forms, including providing feedback on videotaped lessons, having teachers visit each other's classrooms to observe lessons, and having activity leaders, lead teachers, mentors, and coaches observe classroom teachers and engage in reflective discussions about the goals of a lesson, the tasks employed, teaching strategies, and student learning." (p. 925)

I plan heeding Garet and colleagues' (2001) advice on incorporating videos and lessons from other district teachers in ongoing professional development as well as supporting teachers observing each other within their buildings. Teachers are more likely to listen to other teachers as they share similar experiences, are not threatening, and possess the domain-specific content and pedagogical knowledge needed to offer relevant and specific feedback. A note here about teaching teams, “Teams are more effective with peers leading rather than administrators or content experts in the facilitator role for several reasons. Peer-facilitators are uniquely positioned to model ‘a leap of faith,’ frame the work as an investigation, help the group ‘stick with it,’ and guide protocol use as a full participant in the inquiry process” (Gallimore et al., 2009). Related to teachers learning from other teachers, we need to create an environment that breaks the “teaching is an island” perception in order to break the cycle of teachers teaching how they were taught—which was often lacking in conceptual understanding.

In my school district collaborative planning during common planning periods was already the standard when I took this position which means that I do not need to put the work into creating teacher teams but rather leveraging these teacher teams towards teaching for conceptual understanding and towards looking towards one another to push themselves forward in their teaching.

I will have the opportunity to engage every middle level math teacher in the district in the professional development described above via district in-service days, building-level walkthroughs, and PLC visits. That said, a small contingent of teachers (5 – 6) will get additional support from me via additional walkthroughs, co-planning sessions and, occasionally, video analysis of their lessons. These teachers will be self-selected through an electronic survey sent to all secondary math teachers at the beginning of the year simply asking them, on a scale of 1 – 10, how amenable they are to my helping them. Any teacher responding to the survey with an eight or above I would schedule an observation for. After an observation debriefing, I’d ask the teacher if they wanted to co-plan a lesson in the next week or two. Teachers responding affirmatively would start a cycle of intervention from me. Furthermore, teachers who simply asked me to come to their rooms to give feedback or co-plan a lesson would also initiative this same cycle of intervention from me. My reasoning behind this selection process was the overwhelming advice I got from other math specialists and coaches who said things along the lines of, "In your first year on the job, only work closely with teachers who have explicitly invited you into their rooms. Give other teachers the time needed to be skeptical and to warm up to your help."

Organizational Role

My current responsibilities related to the goal of promoting teaching for conceptual understanding include: developing and implementing secondary math PD for district-wide PD days, supporting instruction through planning-observation-analysis/feedback iterations, introducing curricula and tasks that lend themselves towards conceptual understanding and student productive struggle, cooperating with participating teachers to turnkey out-of-building PD back to their teaching peers, overseeing assessment development which, if assessing for conceptual understanding, will serve as a potential motivator for teachers to teach for conceptual understanding. As a new employee to the district this year, I have not been involved in past efforts to address the problem of the lack of teaching for conceptual understanding.

As is often the case, my responsibilities are shared by those above me, specifically by my direct supervisor, Charlotte Webb, and her supervisor, Deborah Panchisin. That said, their responsibilities lie in the larger picture of curriculum and instruction within the district and the implementation of the professional development outlined above falls on me alone. I am the one who will be leading the district-wide PD, offering ongoing on-site support to teachers, and working to develop teaching teams that will push each other forward towards improved teaching practice.

Description of Artifacts

Table 5

A description of my ELP artifacts.

#	Artifact	Type	Audience	Description	Timeline	Status
1	Literature review	Paper	Math education scholars	A literature review on productive struggle focusing primarily on the literature post Hiebert & Grouws (2007).	Fall 2014 (EDUC833)	Complete
2	Unit on integrals	Paper	Calculus teachers; curriculum designers	A unit on the development of the definition of the integral for a calculus class. This paper looks examines the research on the calculus as well as student learning ultimately focusing on a genetic decomposition of the definition of the integral as the focus of the unit.	Spring 2015 (EDUC897)	Complete
3	Grant proposal	Paper	The National Science Foundation grant department	A grant proposal to secure funding for district-wide professional development around the topic of professional development.	Spring 2014 (EDUC836)	Complete
4	Professional development	PowerPoint; videos; handout	Secondary math teachers	The PowerPoint and accompanying materials I used for my district math PD at the beginning of the school year.	Fall 2015 (EDUC879)	Complete
5	Evaluation	Paper	Policy makers and analysts	An evaluation of my classroom compared to my math teaching peers concerning student perceptions of their autonomy.	Spring 2016 (EDUC863)	Complete
6	Professional development	Video	Secondary math teachers	A video used for district PD concerning the Zone of Potential Construction (ZPC) and the importance of cognitively appropriate tasks.	Spring 2016	Complete
7	Mentor sheet	Questionnaire	My mentee	A questionnaire designed for my mentee for the purpose of evaluating her self-perceived growth in teaching for productive struggle.	Summer 2016	Complete
8	Learning plan template	Paper	Math educators	A planning template that focuses teachers' attention towards more macro issues of the unit rather than the micro decisions typically addressed in a conventional lesson plan.	Fall 2016	Complete
9	How students learn conceptually flowchart and paper	Paper	Math educators	A flowchart and explanation that attempts to establish a simple framework for teachers to understand how students learn conceptually.	Fall 2016	Complete
10	Classroom footage from my Integrated II inclusion class	Video	Math educators	An impromptu video taken from my class that I think demonstrates my own growth from my doctoral work and offers an example of student productive struggle in a secondary math classroom.	Spring 2016	Complete

1. Literature Review

The first artifact is a literature on the concept of productive struggle focusing primarily on the literature post Hiebert and Grouws (2007). This artifact ties to my goal by establishing a research base outlining the importance of productive struggle in math classrooms as well as gaining insight into what types of interventions have been tried and published. This artifact is at the heart of my work and is a crux of my ELP.

2. Unit on Integrals

This artifact examines my development of a unit on the definition of the integral using an *Understanding by Design* curricular framework coupled with a Habits of Mind framework for greater, non-mathematical learning goals. The purpose of this project was to develop a unit on the definition of the integral that would support students' conceptual understanding of the topic such that they could transfer the definition to novel situations. This artifact connects with my goals of promoting conceptual understanding by offering a model of what a unit designed to promote conceptual understanding might look like at the secondary level. In the artifact I engage with the concept of genetic decomposition, or breaking a mathematical concept down to all of its sub-claims, which is a tool I can use while co-planning with teachers who are trying to teach for conceptual understanding. This co-planning aspect ties this artifact to my mentee sheet (artifact #7) as ideas and concepts from this unit were used routinely while co-planning with my mentee.

3. Grant Proposal

This artifact attempts to make progress on the research question: how can we, as a district, further new math teachers' teaching practice to better promote students' conceptual understanding of key mathematical concepts? The grant puts forth a professional development plan for promoting the teaching of conceptual understanding throughout the district. The grant focuses on two different styles of professional development—traditional and reform. These two styles represent both intensive workshop PD coupled with ongoing support via lesson studies during the school year. This artifact clearly connects with my project's goals of promoting conceptual understanding throughout the district. This artifact is deeply connected with my professional development artifacts later on the list.

4. Professional Development Materials for August 2015

This artifact represents my first opportunity in my former school district to address the idea of teaching for conceptual understanding. The focus of the PD and corresponding video is bringing up the idea of "desirable difficulties" (Bjork and Bjork, 2011) and posing its importance in teaching for conceptual understanding. One activity in the PD focuses on asking questions to shift more of the mathematical thinking to the student.

This artifact relates to my project because if we can get teachers to believe students learn better when engaged in desirable difficulties, then buying into the idea of student productive struggle is a logical next step (and student productive struggle will help lead to conceptual understanding). Using an activity that has teachers thinking about ways to re-examine questions they might ask students with the purpose of shifting more of the mathematics to the student is absolutely in-line with the concept of productive struggle.

These professional development artifacts were influenced by my grant proposal (artifact #3) as well as by my literature review (artifact #1). My grant proposal specifically looked at what should be accomplished during professional development workshops (which was the format where these PD materials were used) which tied directly to why these particular materials were created. My literature review introduced me to Bjork and Bjork (2011) and the concept of desirable difficulties which I thought was a great segue into student productive struggle and ultimately teaching for conceptual understanding. My literature review also led me to the researcher Hiroko Warshauer via her article "Productive struggle in middle school mathematics classrooms" (2015a) which led me to a summary article by Warshauer (2015b) that I shared with my teachers during this PD session.

5. Evaluation

This artifact presents an evaluation of my own classroom studied under the lens of--whether or not a teacher (me in this case) who intentionally tried to effect intellectual autonomy in his students could be successful. To reach my goal of effecting intellectual autonomy I attempted to leverage peer-to-peer interactions on rich, challenging tasks (with multiple entry and exit points) to change the way students view school and specifically math class ultimately leading to students self-identifying as doers and developers of mathematics rather than passive agents in the classroom. Interventions attempting in the classroom included the promotion of exploratory (or rough draft) talk, public posting of student wonderments, groups randomized daily, class discussions about autonomy, tasks that promote student productive struggle, utilization of non-permanent shared writing surfaces, and listening then posing questions to groups based on their current understanding to prompt them to think more deeply about the mathematics. I included this artifact not just in light of the evaluation itself, but because the evaluation focused on my own classroom and interventions I utilized to try to promote intellectual autonomy. I believe promotion of intellectual autonomy, which is in line with my project goals as an important part of teaching for conceptual understanding, requires having students do some of the actual mathematical thinking or cognitive lifting in the class. Promoting intellectual autonomy has students doing much of their own thinking and coming to individual and class decisions about the mathematics.

The interventions attempted in my classroom are related to my literature review (artifact #1) as many of the ideas implemented came as a result of reading the research on productive struggle post Hiebert and Grouws (2007). Researchers engage in this work more closely analyzed what makes for effective (or ineffective) student productive struggle.

6. Professional Development Video for April 2016

This video artifact attempts to delineate the differences between the Zone of Proximal Development (ZPD) and the Zone of Potential Construction (ZPC). The video is based primarily on the work of Norton and D'Ambrosio (2008). The authors highlight the power of teaching within a student's ZPC, not their ZPD, for the development of a student's conceptual understanding of the mathematics. This artifact ties to my project's goal as it demonstrates that simply presenting students with challenging mathematical tasks will not lead to conceptual understanding—the mathematics being prompted by the task must reside within a student's ZPC if we hope for the student's schema to be modified (for the student to gain conceptual understanding of the mathematics).

Furthermore, this video ties to my project's goal by attempting to answer three questions most teachers really want to understand: how can student struggle be unproductive? Why do students struggle to retain some ideas but not others? Why do many students think some people are born good at math (but not them)? Having reasonable explanations to these questions will help my own status with teachers which will grant me greater influence in their classrooms where I will be advocating for teaching for conceptual understanding.

This video was informed by my literature review (artifact #1) which is where I first read Norton and D'Ambrosio (2008). The prior professional development (artifact #4) also influenced this video as I originally needed to affect teachers' beliefs about the importance of desirable difficulties/productive struggle before addressing the topic—when is a student's struggle with mathematics not productive (when it's not in his or her ZPC).

7. Mentor Sheet

This artifact presents feedback from my mentee where she answered whether my mentoring led her to support student productive struggle in her classes. This artifact supports my project's goal by analyzing the impact I had in offering a new teacher continuous advice throughout the school year in terms of supporting student productive struggle. This artifact is tied to my grant proposal (artifact #3) as the proposal addresses what teacher professional development should look like during the school year in a supporting nature. I also had the opportunity with my mentee to spend large blocks of time with her at the end of the summer to engage in long, intensive conversations about mathematics; these long sessions are in line with the grant proposal's call for summer-based workshops. Both professional development artifacts (#4 and #6) are also relevant here as my mentee was a participant in both of these sessions. Furthermore, my unit on integrals (artifact #2) was valuable while mentoring as we would co-plan lessons together and used ideas from my integral unit—genetic decomposition, planning for big picture (non-mathematical) goals, placing the content in the larger mathematical timeline, introducing the Understanding by Design framework. Lastly, my mentee benefited from my evaluation (artifact #5) as I shared frequently with her the intervention strategies I was using in my classroom. My hope was that she would take back these strategies to her classroom.

8. Learning Plan Template

This artifact presents a template for teachers to use while planning for a particular math concept (for example—division of fractions). The grain size for this template is probably longer than one day but probably shorter than one unit; the focus is on a particular concept which often might take about a week to unpack in a class. The goal of this template is to change the way teachers plan.

In my opinion teachers spend most of their planning time focused around a math concept (like division of fractions) and think about what warm-up questions they will ask, what slides they'll show to communicate how to address the concept (what visuals to include, etc.), what pages in the book we will look at, how they will have students practice the concept (in groups, pairs, jig-saw, Kagan structures, etc.), how they will assess if students understood the concept (often via an exit ticket or something similar), and what homework they will assign. In my opinion teachers make the mistake of not breaking down their desired mathematical learning goal into smaller, sub-goals that explore how students develop on understanding of the final mathematical concept. Continuing with my example of the mathematical concept of division of fractions, teachers know their students come in never having been asked to divide fractions before, often teachers do not plan for a series of claims or sub-understandings a student will typically go through to make sense of how to divide one fraction by another. Most teachers simply start with the algorithm of multiplication of the reciprocal (often using the mnemonic "keep-flip-change") rather than considering the cognitive progression needed to make sense of the algorithm.

This template attempts to help situate teachers' planning at a more conceptual level by prompting them with the following questions:

- What do students need to already understand to access the lesson's learning objective?
    - What math lead up to this concept and what follows it?
- How are you going to get the students interested in the learning objective?
    - I.e. What perplexity, confusion, or doubt are you going to leverage to draw students into the mathematics?

- What potential misconceptions do you predict arising while students are working towards the lesson's objective?
    - How do you plan on addressing these misconceptions when/if they do arise?
- For students who have already grasped the learning objective, how do you plan on extending/deepening/challenging their thinking about the topic?
- How do you plan to move towards bigger picture goals this unit?
    - What classroom norms or specific activities do you plan on introducing to move you towards these goals?

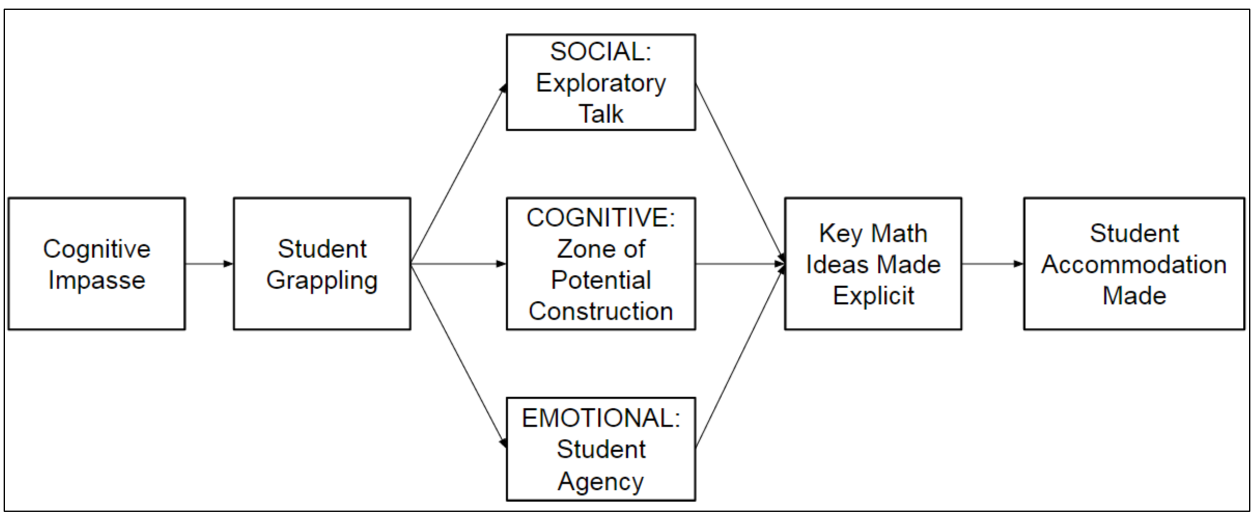
This artifact fits into my goals of having teachers teach for conceptual understanding by prompting teachers to think about how the mathematical concepts logically develop as well as prompting teachers to think about how students' conceptions about the mathematics might reasonably develop. It also prompts teachers to consider creating some sort of cognitive impasse in students which is beneficial in developing students' conceptual understanding (Lehman et al., 2012).

Planning for misconceptions, and treating these as learning opportunities, allows students to continue in their productive struggle. Directly "correcting" student misconceptions can short-circuit these opportunities. In my observations, this often happens when teachers have not planned what to do when these misconceptions arise. This idea of teacher responses to students that keep the students struggling is supported by Warshauer (2015a): "A teacher response that allows more time on task acknowledges the students' effort and competence particularly in the face of difficulty and increases the quality of engagement without lowering the cognitive demand and is more apt to encourage the student to persist despite the student struggles" (p. 6).

This artifact is connected to my literature review (artifact #1) where I read about the importance of cognitive impasses in developing conceptual understanding. It is also connected to my unit on integrals (artifact #2) where I explored planning for student goals beyond the mathematics (Costa and Kallick, 2010; Wiggins and McTighe, 2011). This template is also tied to my professional development on the ZPC (artifact #6) because teachers are asked to consider what prior knowledge a student needs to access. This, in turn, might suggest some pre-teaching is needed to help bring the learning objective into students' ZPCs.

9. How Students Learn Conceptually Flowchart and Paper

This artifact presents one suggestion for how students learn mathematics. It offers this flowchart:



Then a brief paper describes each item in the flowchart coupled with some research supporting their inclusion. The purpose of the flowchart and paper is to help teachers examine their own understanding of how students learn conceptually. The flowchart and paper were kept brief to be less intimidating to teachers while still incorporating major ideas and teaching practices that, in my opinion, if implemented, would have a profoundly positive impact on math classrooms in my district.

This artifact directly links to a video I created about the ZPC (artifact #6) and most of the research cited in the paper comes from my literature review (artifact #1). This artifact also ties to my Learning Plan template (artifact #8) as that template asks teachers to leverage cognitive impasses in their lessons as well as sequencing the mathematically claims for a complex topic that will help keep a topic within a student's ZPC.

10. Classroom Video—Integrated Math II, inclusion

My final artifact is an unedited video from my Integrated Math II class. I included this final artifact because I believe it shows how I have internalized and implemented what I have learned about productive struggle and teaching for conceptual understanding. The video shows an inclusion class (every student in the video save one had an IEP) of traditionally underperforming students (the average grade level performance was second grade as reported by the Scholastic Math Inventory assessment given at the beginning of the year with no student scoring above the fourth grade) who perform meaningful work on a linear programming problem. In my opinion, students are engaged in Exploratory Talk and are seen working on a problem just beyond their comfort level (but within many students' ZPCs); I also believe this level of committed interaction would not have been possible without the students believing they could do the work (i.e. the students showed mathematical agency).

This video is linked to my How Students Learn Conceptually flowchart and paper (artifact #9) as elements of the social, cognitive, and emotional components of productive struggle are demonstrated throughout the video. This video is linked to my evaluation (artifact #5) as it serves to support the claims reported by my students concerning their mathematical agency and autonomy in my class. This video is also linked to my literature review (artifact #1) as the litany of research motivated me to develop a classroom where students were productively grappling with the mathematics.

Works Cited

Appoquinimink School District's website. Strategic plan. Retrieved from http://apposchooldistrict.com/apps/pages/index.jsp?uREC_ID=177066&type=d on 11/2/16.

Ball, D. L. (1996). Teacher learning and the mathematics reform: What we think we know and what we need to learn. *Phi Delta Kappan*, *77*(7), 500.

Ball, D. L., & Cohen, D. K. (1999). Developing practice, developing practitioners: Toward a practice-based theory of professional education. *Teaching as the learning profession: Handbook of policy and practice*, *1*, 3-22.

Bjork, E. L., & Bjork, R. (2011). Making things hard on yourself, but in a good way: Creating desirable difficulties to enhance learning. In *Psychology and the real world: Essays illustrating fundamental contributions to society* (pp. 55–64). New York, NY: Worth Publishers.

Costa, A., & Kallick, B. (2010). It takes some getting used to: Rethinking curriculum for the 21st century. In Jacobs, H. H. (Ed.). *Curriculum 21: Essential education for a changing world.* Alexandria, VA: Association for Supervision and Curriculum Development.

Darling-Hammond, L., Hammerness, K., Grossman, P., Rust, F., & Shulman, L. (2005). The design of teacher education programs. *Preparing teachers for a changing world: What teachers should learn and be able to do*, 390-441.

Gallimore, R., Ermeling, B. A., Saunders, W. M., & Goldenberg, C. (2009). Moving the learning of teaching closer to practice: Teacher education implications of school-based inquiry teams. *The Elementary School Journal*, *109*(5), 537-553.

Garet, M. S., Porter, A. C., Desimone, L., Birman, B. F., & Yoon, K. S. (2001). What makes professional development effective? Results from a national sample of teachers. *American educational research journal*, *38*(4), 915-945.

Guskey, T. R. (1989). Attitude and perceptual change in teachers. *International journal of educational research*, *13*(4), 439-453.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K., Human, P., Murray, H., ... & Wearne, D. (1996). Problem solving as a basis for reform in curriculum and instruction: The case of mathematics. *Educational researcher*, *25*(4), 12-21.

Hiebert, J., Gallimore, R., & Stigler, J. W. (2002). A knowledge base for the teaching profession: What would it look like and how can we get one? *Educational researcher*, *31*(5), 3-15.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. *Second handbook of research on mathematics teaching and learning, 1*, 371-404.

Kapur, M. (2008). Productive Failure. *Cognition and Instruction*, *26*(3), 379–424.

Kapur, M. (2009). Productive failure in mathematical problem solving. *Instructional Science*, *38*(6), 523–550.

Kapur, M. (2011). A further study of productive failure in mathematical problem solving: unpacking the design components. *Instructional Science, 39*(4), 561–579.

Kapur, M. (2014). Productive failure in learning math. *Cognitive Science*, *38*(5), 1008–22.

Kapur, M., & Bielaczyc, K. (2012). Designing for productive failure. *Journal of the Learning Sciences*, *21*(1), 45–83.

Kennedy, M. (1998). Form and substance in inservice teacher education. Research monograph.

Lehman, B., D'Mello, S., & Graesser, A. (2012). Confusion and complex learning during interactions with computer learning environments. *The Internet and Higher Education*, *15*(3), 184–194.

Norton, A., & D'Ambrosio, B. (2008). ZPC and ZPD : Zones of teaching and learning. *Journal for Research in Mathematics Education*, *39*(3), 220–246.

Warshauer, H. K. (2015). Productive struggle in middle school mathematics classrooms. *Journal of Mathematics Teacher Education*.

Warshauer, H. K. (2015). Strategies to Support Productive Struggle. *Mathematics Teaching in the Middle School*, *20*(7), 390-393.

Wiggins, G. P., & McTighe, J. (2011). *The understanding by design guide to creating high-quality units*. ASCD.

Appendix G

ARTIFACTS

1. Literature Review
2. Unit on Integrals
3. Grant Proposal
4. Professional Development Materials (PowerPoint, Video)—Fall 2015
5. Program Evaluation
6. Professional Development Video—Spring 2016
7. Mentor Sheet Questionnaire
8. Learning Plan Template
9. How Students Learn Conceptually Flowchart and Paper
10. Classroom Footage from my Integrated II inclusion class

1. Literature Review

Abstract

Having students struggle productively with key mathematical ideas is one of the few conditions research has supported that leads to growth in students' conceptual understanding (Hiebert and Grouws, 2007). This paper presents a review of the literature on the analysis of productive struggle, emphasizing the literature written after Hiebert and Grouws's (2007) handbook chapter that shed light on the important role of productive struggle in promoting students' conceptual understanding. This paper analyzes how research since 2007 has addressed productive struggle and what this means for education. This paper investigates some possible precursors that might promote or stymie a student's engagement in a productive struggle episode. This paper presents a theory of learning emphasizing the importance of productive struggle as well as how a student might be led to engage in productive struggle. This paper also discusses why we might not see students engaged in productive struggle in many U.S. classrooms.

Keywords: productive struggle, productive failure, conceptual understanding, theory of learning

1. **Introduction and background**

The *Common Core State Standards* (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) describes their eight Standards of Mathematical Practice as "varieties of expertise that mathematics educators at all levels should seek to develop in their students;" the authors prioritized these Standards by beginning with "1. Students will make sense of problems and persevere in solving them" (p. 6). In order for this practice to be realized students must, "engage in mathematical sense-making activities which necessarily foster dissonance and is often unnerving" (Frykholm, 2004, p. 125). Said another way, students must *productively struggle* to make sense of mathematical ideas. Hiebert and Grouws (2007) described such struggle as one of only two features of teaching that "consistently facilitates students' conceptual understanding" (p. 387). So if we, as a mathematics education community, value conceptual understanding we need to better understand what productive struggle is, what conditions promote it, and what conditions subdue it.

Hiebert and Grouws (2007) described this struggle as:

> "We use the word *struggle* to mean that students expend effort to make sense of mathematics, to figure something out that is not immediately apparent…. The struggle we have in mind comes from solving problems that are within reach and grappling with key mathematical ideas that are comprehendible but not yet well formed (Hiebert et al., 1996). By struggling with important mathematics we mean the opposite of simply being presented information to be memorized or being asked only to practice what has been demonstrated." (p. 387-388, italics in original)

In the wake of the Hiebert and Grouws (2007) chapter, attempts to measure a student's productive struggles have emerged but are still few. "Prior empirical research on student struggles was limited to a focus on examining the occurrence of struggle in whole-class discussion settings and did not examine in detail the nature of individual students' struggles" (Warshauer, 2014, p. 18). Kapur (2014) used a self-report survey that used a 5-point Likert-scale that asked students to report their own level of engagement while these same students estimated their amount of mental effort through a nine-point rating scale using questions from previous literature on cognitive load (Paas, 1992; as cited in Kapur 2014). Warshauer (2014) attempted to observe and take note of productive struggle while separate coding by colleagues offered some measure of validity; however, Warshauer focused his observations on whether or not the struggles were productive, assuming that a level of struggle could be presumed on cognitively-demanding tasks. Empirical studies attempting to measure individual students' productive struggles have been scant.

I, like many other mathematics teachers, began my teaching career relying on direct instruction as my main method of teaching. But like many other mathematics teachers I was not satisfied with the level of learning my students were accomplishing and was losing confidence that my teaching was helping students learn the mathematics. As a result, I began to explore different methodologies; "…the approach itself [direct instruction] was not broadly challenged until relatively recently. Although this approach is still probably the most frequently used, it has been discredited as a primary approach to mathematics instruction, because the results have not been good. With the loss of confidence in direct instruction came a loss of clarity about the teacher's role in promoting student learning" (Simon, 2013, p. 96). I think it is also important to note, in my personal story, I teach at a high-poverty school, and these students were by and large not successful in the traditional ways of learning and achieving in math classes. Perhaps if I had taught at a school where the majority my students were achieving at high levels on standardized assessments I would not have been so convinced of their learning deficit nor would I have been as eager to find an alternative method for teaching. Nonetheless, given my teaching circumstances when my school had the chance to pilot a problem-based curriculum, I was willing to try a change. However, while I was more satisfied with the new curriculum to a large degree, I was still not content with the level of learning I perceived was occurring in my classes. I had faith in this problem-based curriculum but knew that I could be teaching better; I knew my students were capable of far more than what was currently happening in my classroom.

After originally reading the Hiebert and Grouws (2007) chapter, my own classroom practice changed dramatically. I began reflecting on whether or not the problem-based activities I was using in class were truly leading to my personal goal--the development of my students' conceptual understanding of mathematics. I had witnessed students easily, quickly, and correctly work through their assignments, but on subsequent assessments the students were not demonstrating understanding of the conceptual points that I thought they should have been learning from the previously completed activities. After reading the Hiebert and Grouws (2007) chapter, I began to push my students more deeply through questioning, to limit my use of scaffolding (when appropriate) I had been using, and to prize the struggle my students were engaging in. I quickly noticed a significant improvement in their conceptual understanding of the mathematics. My initial success personally embracing the pedagogy of productive struggle, I began this literature review wanting to know more about productive struggle wondering what others may have learned as well. Specifically I made the decision to focus on the following:

1. What does the literature tell us about productive struggle, particularly since Hiebert and Grouws's 2007 article?
2. How can educators foster learning opportunities for students to engage in productive struggle?

2. Methodology

Reading

I begin my inquiry by re-reading the Hiebert and Grouws (2007) chapter, "The Effects of Classroom Mathematics Teaching on Students' Learning," as this was the chapter that had a profound impact on my own teaching practice and got me interested in productive struggle initially. I focused specifically on their section entitled, "Feature 2: Students Struggle with

Important Mathematics," and I found every citation in that section and began my literature review by reading each article written in the past 30 years. I chose 30 years because much of the literature cited before then were the seminal architects of modern education (Dewey, Piaget, Vygotsky, Festinger) and not research studies. (I wound up reading Dewey's (1910) *How We Think* because it had been cited in many of the articles I had read, and I believed reading it myself would help my depth of understanding about the subject; I also read Piaget's (1947/2002) chapter "Intelligence and Biological Adaptation" from his book *The Psychology of Intelligence* which I cite once in this paper because it was required reading for a course I took this semester.)

Beyond the citations included in the Hiebert and Grouws (2007) chapter, I made the decision to focus on articles written in 2007 onwards in order to determine if any new research had emerged to corroborate, contradict, or add clarity to Hiebert and Grouws' (2007) claims. An electron search of ERIC database using the keywords "productive struggle" yielded only one result—Warshauer, 2014, which I promptly read. I used the ERIC database because it has been the most recommended database to use by each of my professors. Then, with the advice from two colleagues (Kristin S. McKenney and Joseph DiNapoli), I expanded my search in ERIC for "productive failure," "confusion AND mathematics," and "complex AND achievement." I was hoping to find more in the literature that was in essence concerning productive struggle but perhaps labeled differently. After reading the abstracts of each article on the first page of hits I read any article whose focus seemed to be on anything like productive struggle (e.g. Kapur; 2014, 2009, 2012). If an article came up repeatedly (at least twice) in those readings I would then read that article too (e.g. Pekrun et al., 2002; Pekrun and Stephens, 2012). After that I stopped reading.

Writing

While reading, I made a table representing key claims made by the authors as well as evidence they used to support their claims. I also kept track of definitions used by the authors for future use in both my paper and as a lens to understand other papers I would read on the topic. I submitted small pieces of the paper in subsequent weeks to my professor, Dr. Amanda Jansen, that consisted of my motivation, a paper outline, a list of literature readings, potential research questions, and a list of assumptions that I would be making. Dr. Jansen, along with colleagues from my class (specifically Kristin S. McKenney), offered suggestions and critiques that helped form my final paper. I began writing this paper by first stating my research questions then writing the results sections. I then continued on to write the methodology section, introduction and motivation section, the abstract, and lastly the discussion section. Concerning the writing of this paper, an important note to make is that there are several instances while writing where I drew conclusions from the literature based off a constructivist framework of learning.

3. Results

Feature 1: What does the literature tell us about productive struggle?

The research done before 2007 seems to validate Hiebert and Grouws's (2007) conclusion that when students struggle productively with important mathematical ideas students may attain greater conceptual understanding of the mathematics. Hiebert and Grouws (2007) may have popularized the term "productive struggle," but the ideas behind it go back at least as far as Dewey's (1910) and Piaget's (1947) conceptualization of cognitive disequilibrium which prompts the need to resolve it. More recently Confrey (1990) similarly declared, "for students to modify and adapt their constructions, they must: (1) encounter a situation that they experience as personally problematic, as a roadblock to where they wish to be" (p. 116); while Brown (1993) in a similar fashion purported, "It may be helpful, in an effort to find significant ways of relating the two [epistemology and pedagogy], to take seriously the concept of confusion--not as something to be avoided, but rather as a force to be embraced" (p. 115). Rereading the articles cited by Hiebert and Grouws (2007) that were published within the past 30 years confirmed to me that the conclusions reached by Hiebert and Grouws were based on a sound review of the literature.

Since 2007, further studies have seemed to support Hiebert and Grouws's conclusion concerning productive struggle. For example, Kapur (2008, 2009, 2011, 2012 with Bielaczyc, 2014) focused his five studies almost exclusively on whether or not "productive failure," was more effective than traditional ways of teaching mathematics to promote students' conceptual understanding (traditional here meaning a regimen of teacher lecture followed by student practice). In the literature Kapur (2014) defines productive failure as, "the sequence of problem solving followed by instruction" (p. 1008); however simply defined, Kapur's operationalized definition had students working without instructional support on problems that were almost always beyond the current grasp of the students (as evidenced by the students' complete lack of correct solutions generated to the given problems). Essentially Kapur had the students struggle with mathematics not immediately apparent to the students before being "taught" it. Furthermore, relating it more acutely to productive struggle, Kapur had students take a self-reflection survey where they gave input on their own mental effort (which showed higher scores during the productive failure treatment). These conditions align Kapur's "productive failure" with Hiebert and Grouws's "productive struggle," and Kapur's findings about the positive effectiveness of productive failure concerning students' conceptual understandings of key mathematical ideas lends additional credence to Hiebert and Grouws's (2007) claim about the effectiveness of students engaging in episodes of productive struggle. The one difference was that in Kapur’s studies students *always* failed first--appropriate solutions were never reached. Whereas, Hiebert and Grouws did not insist on failure as a prerequisite to reaping the benefits of productive struggle, but simply grappling with mathematical tasks where the solutions were not immediately apparent.

Moving to a more detailed look at what facilitates productive failure, Kapur (2011, 2014) ran two separate tests to examine if (1) giving students the problem before instruction first, but with teacher support would yield the same results as just having the students struggle with the problem first without support; and (2) could the gains from productive failure be the result of examining failure—learning from others’ mistakes as has been alluded to by other researchers (Borasi, 1994)—or did the failures have to come from the students themselves. If the theory behind productive struggle was correct then learning from another’s mistakes without struggling with the key mathematical ideas personally would not yield as strong of results as if the student had personally struggled with the mathematics himself/herself. Both cases showed that gains, as measured on the pre and post-tests, from only the productive failure treatment (no teacher support, no external mistakes reviewed) were significantly more beneficial than either of the alternative treatments (or the lecture-practice treatment). This evidence seems to strongly support Hiebert and Grouws’s (2007) claim about the effectiveness of students engaging in productive struggle with key mathematical ideas, and Kapur seems to have enhanced this claim by researching possible confounding variables not addressed by the research before 2007. These clarifications are important as American teachers have a hard time watching their students struggle (Stein et al., 1996; Hiebert et al., 2003; Warshauer, 2014; Philipp, 2007) and might both try to “help” or “support” their students or show a mistake made by a peer hoping to mitigate some of the struggle that would need to be occur within each individual student.

Other researchers continued to be observing success in productive struggle situations even when their research designs were very different from one another. As students continued to work to make sense of mathematics that was not immediate apparent to them--productively struggle--students continued to outperform their control group counterparts. Similar to Kapur, Lehman et al. (2012) created struggle-dependent situations in order to compare achievement gains of the experimental groups to that of the control group where no confusion was (intentionally) induced. None of the subjects were offered help from a teacher but rather had to struggle through the confusion themselves if they were to be successful. This study differed from Kapur's studies as there was no direct instruction component--only the struggle component (the "productive failure" component in Kapur's terms). Lehman et al. (2012) found, "Learners who partially-resolved their confusion performed better on the comprehension test" (p. 189). This research by Lehman et al. (2012) corroborates the work done by other researchers where the cognizing agent had to work to remove confusion during a learning opportunity: "…confusion was significantly correlated with learning gains in three studies involving tutoring sessions with AutoTutor, an ITS with conversational dialogues (Craig et al., 2004; D'Mello and Graesser, 2011; Graesser, Chipman, et al., 2007)," (Lehman et al., 2012, p. 185) leading the authors to conclude that, "Thus, inducing confusion not only created opportunities for learning, it also resulted in increased learning, despite the fact that there was not any explicit intervention to help learners regulate their confusion" (Lehman et al., 2012, p. 190). This study once again reinforces the claim centered around the definition of productive struggle—the need to struggle to make sense of something not immediately apparent. A synthesis of these studies shows that the immediate resolution of the confusion is not where the value behind productive struggle lies, but rather the value lies in the struggle to make sense of the ideas itself.

The literature analyzed thus far seems to show that intentional periods of student struggle seemed to lead to increased achievement. However, what has not been addressed yet is whether or not productive struggle is effective while occurring in smaller doses, not in a separate session, but embedded directly within a more traditional instructional framework. Bjork and Bjork (2011) examined prior research done on interleaved versus blocked instruction where interleaved instruction means that multiple topics were taught during a single lesson rather than the more typical blocked lesson where a singular topic is focused upon. The interleaved method often left students frustrated, confused, and uncertain about conclusions they thought they were supposed to be reaching. Despite students feeling more confident in their understandings after undergoing blocked instruction the post-tests revealed that the students scored higher when they were taught via the interleaved lessons even though they felt more confused and had predicted they had learned more during the blocked instruction. Bjork and Bjork (2011) suggest a theory, "that having to resolve the interference" led to improved understanding (p. 61); later on that same page they add to their argument, "Said differently, when some skill or knowledge is maximally accessible from memory, little or no learning results from additional instruction or practice…. Basically, any time that you, as a learner, look up an answer or have somebody tell or show you something that you could, drawing on current cues and your past knowledge, generate instead, you rob yourself of a powerful learning opportunity" (p. 61). This idea is very much in line with the thoughts of resolving disequilibrium that Dewey and Piaget have talked about decades before. Students who were faced with smaller, non-separate instances of productive struggle still seemed to learn more than when they did not get these opportunities to engage in productive struggle. The literature seems to suggest once again that increased cognitive struggle leads to increased understanding.

To summarize, no matter how productive struggle is worded the research both before and since Hiebert and Grouws's 2007 chapter seem to suggest that when a student engages in productive struggle conceptual learning is significantly increased. I expanded my search to look at similar themes such as student confusion and confusion resolution, and productive failure, which supported the underlying tenets of productive struggle--students grappling with important mathematical ideas to make sense out of something not immediately apparent seems to lead to increased conceptual learning and long-term retention. This conclusion corroborates the observations of education pioneers--John Dewey and Jean Piaget who declared the importance of inducing a student to a state of perplexity or creating some cognitive disequilibrium within a student in order to effect deep learning.

Feature 2: How can educators foster learning opportunities for students to engage in productive struggle?

It is one thing to claim that productive struggle is an important element in developing conceptual understanding, but it is quite another to implement mathematics instruction so that students are productively struggling on a daily basis. Teachers cannot simply continue doing what they are currently doing and merely hope or plead with students to start struggling with key mathematical ideas. As Dewey (1910) notes, "There is something specific which occasions and evokes it [reflective thinking]. General appeals to a child (or to a grown-up) to think, irrespective of the existence in his own experience of some difficulty that troubles him and disturbs his equilibrium, are as futile as advice to lift himself by his boot-straps" (p. 12).

Before embarking on this literature review I personally never held to nor ever thought about a theory of learning—or how a teacher should try to create a lesson to give improved opportunities for their students to learn key mathematical ideas and concepts. After reading the

Hiebert and Grouws (2007) chapter years ago and attempting to implement activities that promoted productive struggle in my students, I became convinced that students grappling with mathematical ideas not immediately apparent to them was a key part in students learning complex mathematical concepts. While I did notice several students for the first time really understood and retained these mathematical concepts (as students brought them up unprompted in future classes), many students still were not engaged in productive struggle despite being given the same prompts as their struggling peers; this discrepancy existed despite the fact that my classes were ability-level grouped so all the students were at roughly similar academic achievement levels as measured by achievement in prior math classes as well as similar achievement on our state's standardized tests (and sometimes students with lower prior achievement scores productively struggled while their higher-achieving peers did not). This discrepancy in struggling is what lead me to this research question, how can educators foster learning opportunities for students to engage in productive struggle? When researching productive struggle I found some seemingly important prerequisites for students to be likely to engage in a productive struggle episode. Figures 1 and 2 below demonstrate what I believe the literature suggests about how students get to engage in an episode of productive struggle and ultimately go through a potential cycle of deep learning. The first figure shows several paths a lesson might take whereas the second figure focuses on the prerequisite steps towards getting to deep learning.

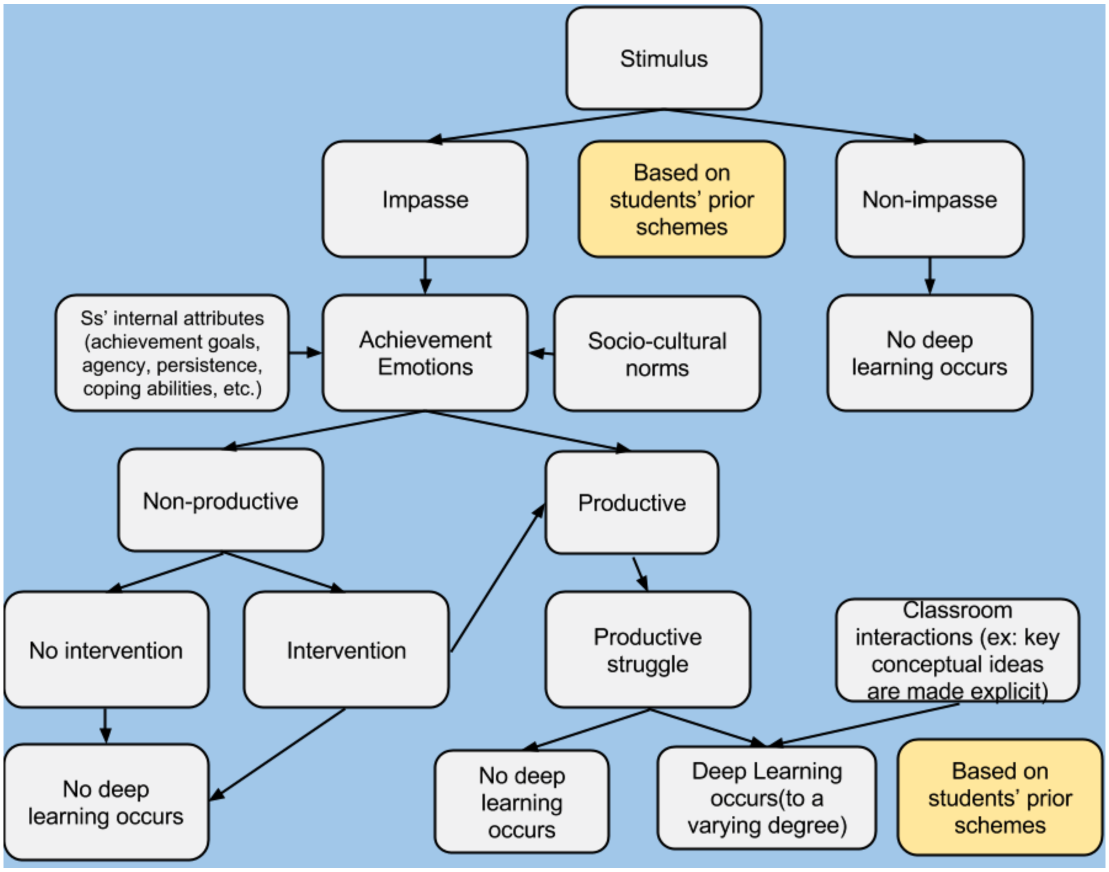


Figure 1. A flowchart representing several possibilities of where a lesson might go.

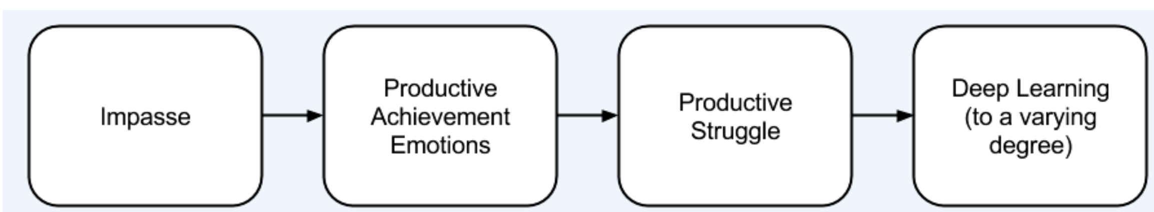


Figure 2. A flowchart representing some potentially necessary precursors of deep learning.

Impasse. As already cited in the preceding paragraph, Dewey (1910), when discussing what leads to deep learning, refers to the importance of the student to confront, "some difficulty that troubles him and disturbs his equilibrium" (p. 12). Dewey (1910) claims, "…the origin of thinking is some perplexity, confusion, or doubt" (p. 12). I believe that the literature concurs with Dewey and points to the need of an impasse to lead to deep learning. "A nonoperational definition of an impasse is that it occurs when a student realizes that he or she lacks a complete understanding of a specific piece of knowledge; a more operational definition is that an impasse occurs when a student gets stuck, detects an error, or does an action correctly but expresses uncertainty about it" (Vanlehn et al., 2003, p. 220). Dewey (1910) later emphasized, "Demand for the solution of a perplexity is the steadying and guiding factor in the entire process of reflection" (p. 11). Dewey (1910) also addresses the counter-statement—if confronting a cognitive impasse might lead to reflective thinking, what about when no impasse is reached: "As long as our activity glides smoothly along from one thing to another, or as long as we permit our imagination to entertain fancies at pleasure, there is no call for reflection" (p. 11). Lehman et al. (2012) seemed to agree, "…impasses that trigger states of cognitive disequilibrium and confusion can create opportunities for deep learning of conceptually difficult content" (p. 184). Vanlehn et al. (2003) hypothesizes why a cognitive impasse will lead to an accommodation or learning, "The basic idea is that an impasse motivates a student to take an active role in constructing a better understanding of the principle" (p. 220). These authors all seem to reiterate that a student reaching a cognitive impasse is an important step in the deep learning process.

Another way to view a cognitive impasse is as being in a state of confusion: "Confusion is triggered when learners are confronted with information that is inconsistent with existing knowledge and learners are unsure about how to proceed" (Lehman et al., 2012, p. 186). The "unsure how to proceed" is a learner reaching a cognitive impasse. This promotion of confusion goes against what many teachers believe and have been taught about what education should look like. In 1986 a report consisting of the opinions of fourteen deans of education at major research institutions entitled, *Tomorrow's Teachers*: *A Report of the Holmes Group* (who would arguably be the teachers currently teaching), described competent teachers as "careful not to bore, *confuse*, or demean students" (p. 29; as cited in Brown, 1993, emphasis mine). Brown also cited the report, “Good teachers explain to their students the problem, the solution, and when to use a specific process…. Most expert teachers cover at least 40 problems a day through games, drills, or written work. Novice teachers on the other hand, may cover only six or seven” (p. 109). Certainly these descriptions from the report carry on today where according to the TIMSS 1999 video study in U.S. classrooms 78.1% of the time new mathematical concepts were simply stated as opposed to 17% of the time in Japanese math classrooms (Stigler and Hiebert, 1999; as cited in Frykholm, 2004). So despite a plethora of research literature suggesting the importance of the learner reaching a cognitive impasse or reaching some confusion, there is practical literature promoting, and researching describing, classrooms in the U.S. that effectively avoid these cognitive perturbations.

Achievement Emotions. Once reaching a cognitive impasse or state of confusion a student does not immediately engage in productive struggle. Rather, prior to the productive struggle, when one becomes confused or perplexed emotions will flare. Lehman et al. (2012) asserted, “These events that trigger impasses place learners in a state of cognitive disequilibrium, which is ostensibly associated with heightened physiological arousal and more intense thought as learners attempt to resolve impasses” (p. 186). These authors defined cognitive disequilibrium as, “a state of uncertainty that occurs when an individual is confronted with obstacles to goals, interruptions of organized action sequences, impasses, contradictions, anomalous events, dissonance, incongruities, unexpected feedback, uncertainty, deviations from norms, and novelty” (p. 186). This connection from a cognitive impasse to an emotional response is not surprising for the constructivist as Piaget (1947) pointed out, “Affective life and cognitive life, then, are inseparable although distinct.”… “…aspects which are in fact associated and in no way represent individual faculties” (p. 6, 7). In modern research Lehman et al. (2012) confirm, “Importantly, there is considerable overlap in the neural circuitry that supports cognitive and emotional processes” (p. 184). If the cognitive and emotional processes are linked, educators must take note of the emotional responses their students demonstrate after being confronted with a cognitive impasse.

The literature suggests that impasses create cognitive disequilibrium which in turn activates an emotional response. I am referring to the definition of emotions as the phenomena that, “involve sets of coordinated psychological processes including affective, cognitive, physiological, motivational, and expressive components” (Pekrun and Stephens, 2012, p. 4). Pekrun and Stephens (2012) identified four classifications of emotions that affect student performance: achievement emotions, emotions pertain to achievement or outcomes; epistemic emotions, emotions that are associated with the knowledge-generating aspects of learning; topic emotions, emotions triggered by the specific content of what is studied; social emotions, emotions triggered through social interactions and cultural expectations in the learning environment. Emotions play an important role in engagement, agency, and overall achievement, specifically affecting—attentional resources, learning strategies, activation of memory networks, motivation to learn, and self-regulation of learning (Pekrun and Stephens, 2012). The case of emotions in learning is extremely complex because each individual learner may internalize the same emotions differently with some emotions (such as anger) motivating some students while disengaging others. However, Artino and Jones (2012) found both frustration and enjoyment to be positive predictors of students’ engagement in metacognition in a study they did evaluating learners self-report surveys and pre and post-test assessments after taking an online-course. This apparent contradiction between two these two studies sheds light on the need for further research into not just what emotions are being demonstrated by engaged and disengaged students, but also how exactly are we measuring success after these emotional responses.

Before moving on I would like to define "deep learning." I am defining deep learning to be the results of "reflective thought." Reflective thought, as defined by Dewey (1910) is: "Active, persistent, and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it, and the further conclusions to which it tends" (p. 6). The subsequent accommodations resulting from this, "active, persistent, and careful consideration," is how I am defining deep learning.

4. Discussion

Limitations

If the literature both before and after 2007 seem to indicate that productive struggle is an effective means of promoting conceptual understanding, an obvious question emerges: why don't we see more students engaged in productive struggle in their classes? One reason is that giving teachers a curriculum with a large portion of cognitively demanding tasks—tasks that might be well-suited for student struggle—does not ensure that students are struggling with the mathematics at the intended cognitive demand; as Stein et al. (1996) reported, task set up—the task announced by the teacher, is often very different from task implementation—the manner in which students actually work on the task. In the Stein et al. (1996) study teachers were trained in implementing these potentially struggle-inducing, cognitively demanding tasks. Yet in only 38% of these tasks were students observed engaging in, "sustained thinking, reasoning, and 'the doing of mathematics" (p. 476). So even when a curriculum promotes cognitively demanding behavior from students struggle is not assured.

In a similar vein, another problem is that teachers can easily be deceived by students' performance granting the illusion of understanding and perhaps productive struggle. Many struggle-permitting, cognitively-demanding tasks are done in groups or as a whole class. If the teacher gives an assignment that is within their students' zone of proximal development (ZPD) many individual students might be assessed that day to understand the mathematics; however, if the task was not within the individual student's zone of potential construction (ZPC) than the student's scheme may not have been accommodated and understanding may be an illusion (Norton and D'Ambrosio, 2008). Bjork and Bjork (2011) also talk about the illusion of understanding warning about the pitfalls of conflating achievement with learning; the authors state, ''…prior exposures create a sense of familiarity that can easily be confused with understanding'' (p. 62). So our assessments might not be measuring understanding and learning but simply familiarity and achievement.

Another reason we might not see productive struggle as a common theme in U.S. classrooms is due to the nature of teaching as a cultural activity (Stigler and Hiebert, 2009; Hiebert, 2013; Williams, 2013).

> "Traditionally, teaching and curriculum development were based on showing and telling students what they were to learn. The assumption, often implicit, was that motivated students would take in the knowledge shared by the teacher and incorporate it into their mathematical knowledge, perhaps not after one teaching session, but certainly with sufficient repetition and practice. Not only was the show-and-tell approach clear, it was in alignment with people's natural instincts about how to 'share' knowledge. The model was fundamentally unproblematic." (Simon, 2013, p. 96)

This explanation makes sense of why historically teaching was rooted in teaching-is-telling. The entrenchment continues because: “Methods of teaching are handed down from one generation to the next. Cultural activities are learned by growing up in a culture, watching how others do things, and following their lead” (Hiebert, 2013, p. 52). One finding by research that does seem to contradict our historical intuition is that, ‘‘…conditions that create challenges and slow the rate of apparent learning often optimize long-term retention and transfer’’ (Bjork and Bjork, 2011, p. 57). Bjork and Bjork (2011) take their challenge of direct instruction even further, “Basically, any time that you, as a learner, look up an answer or have somebody tell or show you something that you could, drawing on current cues and your past knowledge, generate instead, you rob yourself of a powerful learning opportunity” (p. 61). Bjork and Bjork here are promoting the importance of the learners need to struggle productively going so far as to note that when someone makes apparent some idea that a learner could have struggled to produce that the learner has been actually “robbed of a powerful learning opportunity;” this potent language demonstrates just how strongly these authors may feel about the value of individual discovery as compared to the simple teacher-telling that is so frequent in U.S. mathematics classrooms.

There are many other systemic reasons why teaching does not change (Hiebert, 2013), but I would like to examine two more that directly are related to the psyche of individual teachers. One is, "Telling students how to perform procedures also supports teachers' sense of efficacy, because the conventional nature of procedures is such that students cannot be expected to know them until the teacher shows them, and so students' successes in mastering the procedures can be attributed to the teacher” (Philipp, 2007, p. 281). This sense of efficacy may be why in the Stein et al. (1996) study, in 64% of the tasks—tasks implemented by teachers trained in reform-based mathematics pedagogy—the researchers observed, “challenges became nonproblems," due to overt teacher intervention (p. 479). This lack of results from trained teachers segues into my second point supported elsewhere in the literature: expert teachers struggle and fail while attempting more complex, student-centered methodologies of teaching so the expectation that all teachers seamlessly engage in these behaviors is not realistic (Philipp, 2007; Chazan and Ball, 1999; Stein et al., 1996).

Calls for future research

As noted earlier, both Figure 1 and Figure 2 above represent what I believe the literature implies for a cycle of deep learning to take place. However, what both figures fail to communicate are the difficulties in the steps between "impasse" and "productive struggle." Even the more detailed flowchart of Figure 1 follows "impasse" with "achievement emotions" which I believe to be informed by the literature. That said this flowchart claims, "socio-cultural norms," as well as "student's internal attributes (achievement goals, agency, persistence, coping abilities, etc.)," both the socio-cultural norms and student's internal attributes play a huge part in determining whether or not a student goes from an impasse to struggle productively with the task or whether the student chooses a number of other non-productive behaviors where the student chooses not to struggle productively with the task. While prior studies have addressed both socio-cultural norms (in my readings for this paper the following studies addressed socio cultural norms: Warshauer, 2014; Borasi, 1996; Kapur, 2011; Stein et al., 1996) and student's internal attributes (see from my bibliography: Pekrun and Stephens, 2012; Pekrun et al., 2002; Brown, 1993; Bjork and Bjork, 2011; Piaget, 1947; Vanlehn et al., 2003; Harackiewicz et al., 2008; Hatano, 1988; Inagaki et al., 1998; Shors, 2014; Zimmerman, 1989), more research in the area is needed, particularly on a student's internal attributes and how they affect whether or not the student engages in productive struggle, because even though the research surrounding a student's internal attributes seems vast the topic is extremely difficult to gain understanding in, and the inner-workings of a human are so complex that continued research is vital—specifically in regards to how these attributes affect a student's level of engagement in productive struggle. A specific example could be research into the relationship between specific student-held epistemological beliefs and the resulting achievement emotions displayed when confronted with a cognitive impasse. Similarly, while research has been done on teachers and the difficulties in changing their practices (see in my bibliography: Bjork and Bjork, 2011; Brown, 1993; Chazan and Ball, 1999; Frykholm 2004; Hiebert, 2013; Hiebert et al., 2003; Philipp, 2007; Stein et al., 1996), continued research must be done on how our educational system can change teachers' epistemological beliefs about teaching such that they embark on creating classrooms where students are given the opportunities to productively struggle with key mathematical concepts.

As I personally move forward with my Educational Leadership Portfolio (ELP) I now have a reinforced belief in the importance of engaging students in productive struggle, a recognition of the importance of the role a cognitive impasse plays in fostering productive struggle and hence deep learning, and I have a broader understanding of the difficulties and nuances that lie between establishing a student reaching a cognitive impasse and that student engaging in productive struggle. My own ELP will attempt to address some of the “student's internal attributes” that I have read about during this literature review while training the rest of the Ninth Grade Academy teachers at my school in how to present students with opportunities to engage in productive struggle. This team of teachers will also film and analyze their students working on cognitively-demanding tasks and will use a rubric to determine if students are in fact engaged in productive struggle. Students and teachers will be interviewed and the transcripts will be analyzed from a qualitative standpoint to determine successes, failures, and needed modifications.

To conclude, 100 years after Dewey observed that students needed to be perplexed and to resolve the resulting disequilibrium, we are finally proving that his observations held merit and that there is a great deal of complexity behind his initial purporting. Students seem to need to struggle individually to help build conceptual understanding but not surprisingly, one size does not fit all once again in education and a variety of inputs will determine what causes a student to be perplexed and to be motivated to resolve that disequilibrium. In my opinion, it would prove fruitful if the mathematics education community spent time focusing on how to create an effective cognitive impasse for students, how to measure what a successful engagement in productive struggle looks like, as well as spend time and energy analyzing the socio-cultural norms and students' internal attributes that currently lead some students to engage in productive struggle while others choose not to.

Works Cited

Artino, A. R., Jones, K. D. (2012). Exploring the complex relations between achievement emotions and self-regulated learning behaviors in online learning. *The Internet and Higher Education*, *15*(3), 170–175. doi:10.1016/j.iheduc.2012.01.006

Bjork, E. L., & Bjork, R. (2011). Making things hard on yourself, but in a good way: Creating desirable difficulties to enhance learning. In *Psychology and the real world: Essays illustrating fundamental contributions to society* (pp. 55–64). New York, NY: Worth Publishers.

Borasi, R. (1994). Capitalizing on errors as “Springboards for inquiry”: A teaching experiment. *Journal for Research in Mathematics Education*, *25*(2), 166–208.

Brown, S. I. (1993). Towards a pedagogy of confusion. In A. M. White (Ed.), *Essays in humanistic mathematics* (pp. 107-121). Washington, DC: Mathematical Association of America.

Chazan, D., & Ball, D. (1999). Beyond being told not to tell. *For the Learning of Mathematics*, *19*(2), 2-10.

Common Core State Standards Initiative. (2010). Common Core State Standards for Mathematics. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

Dewey, J. (1910). *How we think*. Boston: Heath.

Frykholm, J. (2004). Teachers' tolerance for discomfort: Implications for curricular reform in mathematics. *Journal of Curriculum and Supervision*, *19*(2), 125–149.

Hiebert, J. (2013). The constantly underestimated challenge of improving mathematics instruction. In *Vital directions for mathematics education research*(pp. 45-56). Springer New York.

Hiebert, J., Gallimore, R., Gamier, H., Givvin, K. B., Hollingsworth, H., Jacobs, J., et al. (2003). Teaching mathematics in seven countries: Results from the TIMSS 1999 Video Study (NCES 2003-013). Washington, DC: U.S. Department of Education, National Center for Education Statistics.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. *Second handbook of research on mathematics teaching and learning*, *1*, 371-404.

Kapur, M. (2008). Productive Failure. *Cognition and Instruction*, *26*(3), 379–424. doi:10.1080/07370000802212669

Kapur, M. (2009). Productive failure in mathematical problem solving. *Instructional Science*, *38*(6), 523–550.

Kapur, M. (2011). A further study of productive failure in mathematical problem solving: unpacking the design components. *Instructional Science*, *39*(4), 561–579. doi:10.1007/s11251-010-9144-3

Kapur, M. (2014). Productive failure in learning math. *Cognitive Science*, *38*(5), 1008–22. doi:10.1111/cogs.12107

Kapur, M., & Bielaczyc, K. (2012). Designing for productive failure. *Journal of the Learning Sciences*, *21*(1), 45–83. doi:10.1080/10508406.2011.591717

Lehman, B., D'Mello, S., & Graesser, A. (2012). Confusion and complex learning during interactions with computer learning environments. *The Internet and Higher Education*, *15*(3), 184–194. doi:10.1016/j.iheduc.2012.01.002

Norton, A., & D'Ambrosio, B. (2008). ZPC and ZPD : Zones of teaching and learning. *Journal for Research in Mathematics Education*, *39*(3), 220–246.

Pekrun, R., Goetz, T., Titz, W., & Perry, R. P. (2002). Academic emotions in students' self-regulated learning and achievement : A program of qualitative and quantitative research. *Educational Psychologist*, *37*(2), 37–41. doi:10.1207/S15326985EP3702

Pekrun, R., & Stephens, E. (2012). Academic emotions. In *APA educational psychology handbook* (2nd ed., pp. 3–31).

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In *Second handbook of research on mathematics teaching and learning* (Vol. 4, pp. 257–315).

Piaget, J. (1947, 2002). Intelligence and biological adaptation. *The psychology of intelligence* (pp. 3-12). New York: Routledge.

Simon, M. A. (2013). The need for theories of conceptual learning and teaching of mathematics. In *Vital directions for mathematics education research* (pp. 95-118). Springer New York.

Stein, M. K., Grover, B. W., Henningsen, M., & Grover, W. (1996). Building student capacity for mathematical thinking and reasoning : An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal*, *33*(2), 455–488.

Vanlehn, K., Siler, S., Murray, C., Yamauchi, T., & Baggett, W. B. (2003). Why do only some events cause learning during human tutoring? *Cognition and Instruction*, *21*(3), 209–249.

Warshauer, H. K. (2014). Productive struggle in middle school mathematics classrooms. *Journal of Mathematics Teacher Education*. doi:10.1007/s10857-014-9286-3

Williams, S. R. (2013). Reflections on a portrait of our field. In *Vital directions for mathematics education research* (pp. 1-14). Springer New York.

Bibliography

Capon, N., & Kuhn, D. (2004). What's so good about problem-based learning? *Cognition and Instruction*, 22, 61-79.

Harackiewicz, J. M., Durik, A. M., Barron, K. E., Linnenbrink-Garcia, L., & Tauer, J. M. (2008). The role of achievement goals in the development of interest: Reciprocal relations between achievement goals, interest, and performance. *Journal of Educational Psychology*, *100*(1), 105–122. doi:10.1037/0022-0663.100.1.105

Hatano, G. (1988). Social and motivational bases for mathematical understanding. In G. B. Saxe & M. Gearhart (Eds.), *Children's mathematics* (pp. 55-70). San Francisco: Jossey-Bass.

Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65-97). New York: Macmillan.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K., Human, P., Murray, H., et al. (1996). Problem solving as a basis for reform in curriculum and instruction: The case of mathematics. *Educational Researcher*, 25(4), 12-21.

Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic. *American Educational Research Journal*, 30, 393-425.

Inagaki, K, Hatano, G., & Morita, E. (1998). Construction of mathematical knowledge through whole-class discussion. *Learning and Instruction*, 8, 503-526.

Kapur, M., & Kinzer, C. K. (2008). Productive failure in CSCL groups. *International Journal of Computer-Supported Collaborative Learning*, *4*(1), 21–46. doi:10.1007/s11412-008-9059-z

Rakes, C. R., Valentine, J. C., McGatha, M. B., & Ronau, R. N. (2010). Methods of instructional improvement in algebra: A systematic review and meta-analysis. *Review of Educational Research*, *80*(3), 372–400. doi:10.3102/0034654310374880

Schwartz, D. L., & Martin, T. (2004). Inventing to prepare for future learning: The hidden efficiency of encouraging original student production in statistics instruction. *Cognition and Instruction*, *22*(2), 129-184.

Shors, T. J. (2014). The adult brain makes new neurons, and effortful learning keeps them alive. *Current Directions in Psychological Science*, *23*(5), 311–318. doi:10.1177/0963721414540167

Stigler, J. W., & Hiebert, J. (1999). The teaching gap: Best ideas from the world's teachers for improving education in the classroom. New York: Free Press.

Zimmerman, B. J. (1989). A social cognitive view of self-regulated academic learning. *Journal of Educational Psychology*, *81*(3), 329–339. doi:10.1037//0022-0663.81.3.329

2. Unit on Integrals

Abstract

This paper examines my development of a unit on the definition of the integral using an *Understanding by Design* curricular framework coupled with a Habits of Mind framework for greater, non-mathematical learning goals. The purpose of this project was to develop a unit on the definition of the integral that would support students' conceptual understanding of the topic such that they could transfer the definition to novel situations. While utilizing the *Understanding by Design* ideal of working backwards to create a unit, I chose to do a genetic decomposition of the definition of the integral and to structure the activities within the unit on promoting student understanding up levels of the decomposition. The unit ends with an activity assessment that asks students to transfer their knowledge of the integral to a novel situation that requires utilizing the definition of the integral in the three-dimensions where students' prior experience has solo been with two-dimensions.

Keywords: integral, Calculus curriculum, genetic decomposition

Introduction

The aim of this project is to update my AP Calculus unit on integration. The problem I am trying to fix is that many of my students finish my current unit on integration with a depraved understanding of the concept. Specifically students do not show mastery of its definition and henceforth cannot transfer the concept to novel situations. Similarly I believe that my current unit on integration does not address any of the larger, non-content goals I have for my students so I would like my updated unit to address these larger goals as well. The project is significant to me in the sense that I love and find beauty in the Calculus and want all of my students to at least appreciate it which requires understanding it. Concerning the professional community, almost every AP Calculus teacher I know laments the lack of resources and good curricula available for the course. I agree that at least in my district and state there is a tremendous focus on high school math only through Algebra II which in many ways makes sense as the SATs and other standardized tests only assess through that course and if a major goal is to close the achievement gap than focusing on developing rich curricula for courses taken primarily by those already in the upper echelon of schooling probably is not the best way to close this gap. Nonetheless, rather than continually search for good lessons/curricula concerning integration, I have decided to develop it myself and will share the unit with the professional community of AP Calculus teachers in Delaware.

Theoretical Frameworks

I am using design principles described by Wiggins and McTighe in their *The Understanding by Design Guide to Creating High-quality Units* (2011) as well as the non-content framework laid out by Costa and Kallick in Chapter 13 of Jacobs's *Curriculum 21: Essential Education for a Changing World* (2010).

I start by following Wiggins and McTighe's (2011) suggestion of starting with a big idea and designing backwards with a theory of action on how students are going to get from their current state to a state that includes the understanding of this big idea. My specific big idea for this unit is for students to understand the definition of the integral. As mentioned earlier I do not believe that all of my students leave my AP Calculus class with a clear understanding of the definition of the integral, and furthermore, I have probably surveyed over fifty AP Calculus graduates (many of whom performed very well on their AP exam) who were taught by other Calculus teachers—not a single one of them have been able to give me anything close to a clear definition of the integral/integration. I want to deliberately develop this unit using Understanding by Design (UbD) principles to hopefully have more of my students understand this integral definition within the Calculus.

The major parts I want to use from Wiggins and McTighe (2011) are: identifying a rich content area to develop—an end goal to work towards, use a theory of action to get students to that conceptual goal, use assessments to require student understanding to be successful—authentic, novel problems, use essential questions to help from debates and students' curiosities.

Concerning the non-content development of the unit I am seek to align the unit with the major aims laid forth by Costa and Kallick (2010) and to foster: "creativity and innovation, critical thinking and problem solving, communication and collaboration" (p. 212). Relatedly, the prompts I plan on using with my students reflect the Costa and Kallick's (2010) idea: "It is a curriculum that gives students practice engaging with complex problems, dilemmas, and conflicts whose resolutions are not immediately apparent" (p. 212). Throughout all of my courses for the entire year I plan on emphasizing Costa and Kallick's (2010) Habits of Mind, and I plan on continuing that emphasis during this unit. The Habits of Mind that I believe will come across clearly in my planning will be the following: persisting, thinking flexibly, questioning and problem posing, striving for accuracy and precision, applying past knowledge to novel situations, thinking and communicating with clarity and precision, and thinking interdependently (p. 212-213).

Strategy

I will now attempt to briefly describe the planning of my unit via UbD's three stages of development.

Stage 1—Identify Desired Results

As previously stated I want my students to come to an understanding of the definition of the integral. And by understanding I mean it in the way Wiggins and McTighe (2011) defined it: "…an important generalization, a new insight, a useful realization…. An understanding is not a fact (though it may sound like one) but a "theory" in the broadest sense; it is the result of inference…" (p. 14). I want students to be able to apply the definition of the integral to novel situations—see Stage 2 for examples. Students will be able to use the definition of the integral (and technology) to find the area underneath known curves in contextual or totally abstract problems.

Some essential questions that I plan on using to guide this unit are: How can I most accurately calculate the area of a curved, irregular shape (one that cannot be dissected into known shapes)? How close is close enough? What is the relationship between rate and area? How can I tell if a calculation is an over- or under-estimate? How are calculating these estimates of area related to calculating derivatives—a concept devoid of area/geometry?

I think that utilizing the UbD's backwards design means that as a curriculum designer you need to think about the steps or prerequisite knowledge needed before arriving at the desired understanding and make a theory of action or some type of map to help lead the students from where they are currently to a deep understanding of the concept. When discussing how a student comes to truly understand the concept of the limit, Cottrill et al. (1996) talked about the concept's *genetic decomposition* or the possible steps that describe what it could mean to understand the limit. The genetic decomposition described the developing mental constructions that students might be making en route to understanding the concept of the limit more completely. The following is the genetic decomposition that I have come up with for the understanding of the integral:

1. Students need to know how to calculate the area of known 2D shapes given their dimensions.
2. Students need to know how to calculate the area of unknown 2D shapes given their dimensions by dissecting the unknown shape into known shapes.
3. Students need to know how to approximate the area of an unknown 2D shape given approximate dimensions that cannot be dissected into known shapes.
4. Students need to know how to create a system for approximating the area of an unknown 2D shape that behaves predictably given the dimensions of the shape entirely or at specified intervals (introduction to Riemann Sums). This system must include multiple shapes of equal width.
5. Students must understand that the smaller the width-interval the more accurate a calculation you can attain.
6. Students need to be able to create a system per the prior step that gives both a known over-estimate and a known under-estimate of the shapes actual area (left-bound and right-bound Riemann sums).
7. Students need to examine and determine which system will consistently provide the most accurate area calculation: left-bound rectangles, right-bound rectangles, mid-aligned rectangles, or trapezoids.
8. Students need to be able to create a system of calculating the area of a 2D shape that includes the summation of (using summation notation) one of the

above shapes (aligned rectangles or trapezoids). Students do not need to be able to calculate this summation by hand.

9. Students need to be able to create a system like that in the prior step that uses a width interval of a non-whole number (typically a decimal or fraction 0.1, 0.01, or 0.001 units wide). Students do not need to be able to calculate this summation by hand.

10. Students recognize that when the width-interval is small enough the chosen shape to calculate the area will not matter as each used shape approaches the same calculation. Students will gravitate to either left-bound or right-bound rectangles.

11. Students must be able to apply the implications of the definition of the limit when dealing with infinity.

12. Students can successfully create a system like that in step 10 that uses infinitely many rectangles each with an infinitesimally small width-interval. Students must recognize the need for new notation as our prior summation notation (sigma) is not equipped to calculate infinitely many objects of infinitesimally small size.

13. Students are now ready to reinvent or follow with understanding the formal definition of the integral.

Having a genetic decomposition is a valuable tool for a curriculum designer to work with when attempting to engage students in understanding a complex topic (such as integrals). These steps can help curriculum designers target very specific sub-ideas and can develop activities that bring these nuanced understandings to light.

Stage 2—Determine Acceptable Evidence

Some more formal formative assessments will be used along the way to see if students are grasping the concepts of approximating area as well as the skills of calculating the approximate area underneath a curve using Riemann Sums (see Appendix). Some less formal formative assessments will be used as well which besides the everyday conversations with groups, will be group presentations of what they have uncovered so far about approximating area. These presentations will serve as a better assessment, in my opinion, for measuring conceptual understanding and will allow for prompting questions and wonderments by the class. Certainly this assessment is also in line with Costa and Kallick's (2010) three goals of learning mentioned earlier: "creativity and innovation, critical thinking and problem solving, communication and collaboration" (p. 212).

Certainly transfer of learning is an important goal of education (UbD cites it as a primary goal for learning (Wiggins and McTighe, 2011, p. 3)), and so assessing for transfer is an important part in assessing understanding, not simply memorization—"Knowledge learned at the level of rote memory rarely transfers; transfer most likely occurs when the learner understands underlying concepts and principles that can be applied to problems in new contexts. Learning with understanding is far more likely to promote transfer than simply memorizing information from a text or a lecture" (Wiggins and McTighe, 2011, p. 5). So giving students activities or assessments for which transfer is necessary to be successful, should be included in any good unit, and is included in mine. Between various stages of the genetic decomposition activities will have specific questions that target that exact nuanced evolution of this likely sequence of development. For example one final question in an activity asks students to come up with the most accurate system they can for approximating the area under a given curve. Student responses to this question will inform me as to where there are in steps 8 through 10 in the genetic decomposition process. That feedback will allow me to plan appropriately for the next lesson as I will have a solid grasp of student Zones of Potential Construction (ZPC) for that next activity. These activities will be structured such that a student must transfer what he/she learned from a prior activity and a prior stage within the genetic decomposition to find full success in the next activity.

The ultimate assessment in determining whether or not students understand the definition of the integral will be in a final activity of the unit where students are asked to calculate the volume of an actual lemon that I give each group. Students can only be completely successful in this activity if they truly understand the definition of the integral, because they need to apply that definition to a completely new situation working with 3-dimensions for the first time. Similarly, students will not be summing rectangles like they have been, but rather will have to sum cylinders, so if they do not understand what the *dx* in the integral means they will not be successful; similarly if they do not understand that the integral means to sum infinitely many objects they will not be successful. This activity is very much in line with UbD's call for authentic assessments that require transfer to measure learning.

Stage 3—Plan Learning Experiences and Instruction Accordingly

My plan stems from the genetic decomposition cited above and structuring activities to help stimulate thoughts in students that may lead them from one level to the next. Students will be offered multiple learning opportunities at each of these levels within the genetic decomposition as research supports the importance of multiple learning opportunities for students to grasp a new concept (Hiebert and Grouws, 2007; Nuthall, 2005).

Primarily the daily tasks will revolve around a few questions that I pose to each of the groups often coupled with a given function shown on the projector or given to them as a handout. Students will be expected to productively struggle with the questions asked of them and to persist in making sense of the mathematics. A year-long theme that will remain present this unit is the importance in promoting student productive struggle to increase conceptual understanding of complex mathematics (Hiebert and Grouws, 2007; Hiebert and Wearne, 1993; Warshauer, 2014). Engaging students in this productive struggle during the unit will effect several of the Habits of Mind listed as goals for learning by Costa and Kallick (2010): persisting, thinking flexibly, questioning and problem posing, and thinking interdependently. Costa and Kallick's (2010) other Habits of Mind mentioned earlier—striving for accuracy and precision, applying past knowledge to novel situations, and thinking and communicating with clarity and precision—will be overtly brought about in this unit as students are asked to come up with the most precise calculation for area under a given curve, are asked to apply learnings from prior levels of the genetic decomposition to help understand new levels, and are asked to present their thoughts openly to the entire class.

Concerning instruction and algorithms I want to point out here that I do not plan on telling students about Riemann Sums or the definition of the integral before they have created their own methods of approximating the calculation of an unknown area. Considering the Habits of Mind goals listed previously, coupled with my own ultimate goal in education (and Piaget's) of developing moral autonomy, sharing algorithms with students before they have had the opportunity to develop some algorithm themselves can actually undermine many of my larger educational goals (Kamii and Dominick, 1997).

Furthermore, concerning specifically Calculus, Swinyard and Larsen (2012) studied how students came to understand the concept of the limit, and they found out some things that I believe will relate directly to why I do not want to give my students the definition of the integral before they come up with their own definition. Citing Fernandez (2004), “students may reason more coherently about the formal definition when allowed to build on their spontaneous conceptions…. when students in her study were not forced to use notation traditionally associated with the definition, they were more likely to reason coherently about the formal limit concept” (Swinyard and Larsen, 2012, p. 467). So as we can see in a very similar context to integrals, students were more likely to reason coherently about the concept when they were not forced to use the traditional definition. Fortunately this research is in line with not only with my mathematical goals for my students but my overarching learning goals for them as well.

Product/Outcome

My integral unit is now one that intentionally aims at prompting student through a structured system of levels within a genetic decomposition of the integral. Students are explicitly the creators of and owners of the mathematics they will be using. I will merely be offering prompts to help (hopefully) stimulate their thoughts towards unknown areas and ultimately integration. So my role as teacher is new, and I think vastly improved particularly if you are looking at this unit through the lens of these larger learning goals (like the Habits of Mind and moral autonomy) as well if you are valuing conceptual understanding over procedural fluency.

Many of the lessons/prompts are new as I had never thought of or used the genetic decomposition before. I usually just walked through my textbook's activities on Riemann Sums and integration. And not only the lessons but my role of teacher will be different and a lot more in line with asking questions than providing answers. Using the idea of a genetic decomposition is entirely new to me with this unit being the first time I have used it. Then taking the genetic decomposition and applying UbD principals to perform a backwards design of the unit is something that is completely new to me, and I think a significant improvement over the lack of system I was using before—just going through the textbook and hoping students learned the material.

The UbD framework got me thinking about formative assessment in a new light as well, where I now plan on using the last question in an activity or reflection to, one that revolves around transfer to a novel situation, to assess my students' understanding of the concept or sub-concept. Relatedly, my final assessment, the volume of the lemon task will be a completely new activity but also a completely new lens to view assessment (with a focus on using transfer activities to measure understanding—this is a huge shift from what I have been doing which has been more traditional in nature, i.e. tests, quizzes, essays). So the way I will assess this unit is new and vastly different and improved from prior years.

For sample products from the unit please see the Appendix.

Limitations

A serious limitation to the rework of my unit is my personal lack of understanding on how students come to an understanding of the integral. My genetic decomposition makes sense to me and fits into my own anecdotal experience teaching and learning the Calculus, but the plan is far from research-based or even tested in any way (we will have our first test using this new unit plan next winter). Secondly the assessments that I will be using will also be simply assessments that I have made up and not tested themselves for being effective assessments. Thirdly to access many of the levels the genetic decomposition requires that students have many mathematical understandings that they may or may not have. The current genetic decomposition does not address these shortcomings, but hopefully through testing the unit the decomposition can be adjusted for common student misconceptions coming into the unit on integrals.

Reflection

From creating this unit I have learned a lot about dissecting a complex mathematical concept such that you can scaffold learning opportunities for students to ultimately come to an understanding of the concept—I never had thought about these ideas before. I also learned about reflecting on greater learning objectives and how sometimes our content objectives can undermine our greater learning objectives.

This project will be useful at the school/district level as I plan on distributing this unit to the other teachers in the district who teach the Calculus. I also plan on distributing the unit to teachers outside of the district, but I do not know all of them. I think newer teachers of Calculus will find the unit particularly helpful and will implement it accordingly—particularly if their beliefs about greater learning objectives align with my own.

This project has raised new questions for me concerning curriculum design. Particularly, what research do most curriculum developers bring into their curricula? Do these designers ever go out and do research on important issues when no scholarly work is available? How would I go about more scientifically measuring the success (or failure) of various facets of my own unit?

This final question is one that I will reflect upon at great length as I teach the unit this upcoming school year.

References

Costa, A., & Kallick, B. (2010). It takes some getting used to: Rethinking curriculum for the 21st century. In Jacobs, H. H. (Ed.). *Curriculum 21: Essential education for a changing world.* Alexandria, VA: Association for Supervision and Curriculum Development.

Cottrill, J., Dubinsky, E., Nichols, D., Schwingendorf, K., Thomas, K., & Vidakovic, D. (1996). Understanding the limit concept: Beginning with a coordinated process scheme. *Journal of Mathematical Behavior*, 15, 167–192. doi:10.1016/S0732-3123(96)90015-2

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In J. Frank & K. Lester (Eds.), Second handbook of research on mathematics teaching and learning (pp. 371–404). Charlotte: Information Age.

Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic. *American educational research journal*, *30*(2), 393-425.

Kamii, C., & Dominick, A. (1997). To teach or not to teach algorithms. *The Journal of Mathematical Behavior*, *16*(1), 51-61.

Nuthall, G. (2005). The cultural myths and realities of classroom teaching and learning: A personal journey. *The Teachers College Record*, *107*(5), 895-934.

Swinyard, C., & Larsen, S. (2012). Coming to understand the formal definition of limit: Insights gained from engaging students in reinvention. *Journal for Research in Mathematics Education*, *43*(4), 465–493. doi:10.5951/jresematheduc.43.4.0465

Warshauer, H. K. Productive struggle in middle school mathematics classrooms. *Journal of Mathematics Teacher Education*, 1-26.

Wiggins, G. P., & McTighe, J. (Eds.). (2011). *The understanding by design guide to creating high-quality units*. ASCD.

Appendix

Formative Assessment

Here is an example of a formal formative assessment that I mentioned in the paper.

Calculus, Learning Target #9: Evaluate Riemann sums and the definition of the integral.

Form B

1. What estimate will give you the most accurate approximation of the area under the curve: $f(x) = x^2$, from 1 to 3 with 4 subintervals of equal length. Circle one of the following.
    a. Right Riemann sums
    b. Left Riemann sums
    c. Midpoint Riemann sums
    d. Trapezoidal sums

2. Find the area under the curve of the function: $g(x) = -x^2 + 9$, from 0 to 3, using Right Riemann sum approximation with 6 subintervals of equal length. ____________________
Is your answer an over-estimate or under-estimate of the actual area? ______________________

Formative Assessment Based Around Transfer

Here is the handout that the students will be given along with the lemon. All classroom materials will be available as well—unit grid paper will be very useful here.

The Lemon Activity

Calculate the volume of the lemon I just handed your group. Get as accurate a volume as you can. Good luck.

Lesson Plan

Here is what a sample lesson plan from the unit looks like.

Table 1 PART 1 – LESSON SUMMARY

Table 2

5**LESSON TITLE/DESCRIPTION** and **SOURCE**
6Approximating Areas of Unknown Shapes; IMP4—*Another Trip* activity; handout with graph of 1st quadrant—$f(x) = 0.2x^2+1$; x:[0,6].

Table 3

7**MATHEMATICAL TRAJECTORY** 8*How is the lesson situated in the bigger mathematical storyline that is being developed across the unit? year? beyond?*
9Students have recently grappled with identifying the area under a speed-time graph as distance travelled and have calculated these distances/areas already by decomposing the graphs into known geometric shapes (rectangles, triangles, trapezoids). In this lesson students will approximate the areas of unknown shapes (with curves). 0This leads students very much towards formal Riemann Sums which ultimately leads to integrals.

Table 4

Table 5 LESSON GOALS

11**Mathematical Content** 12*What is the mathematical relationship* 13*or "big idea" being developed?*	14**Practices and/or Habits of Mind** 15*In what ways will students being engaged in mathematical thinking and doing?*	16**Socio-Mathematical Norms** 17*What norms for interaction are being established or maintained?*
8The big idea here is that we can use shapes we know how to get the area of to approximate the area of an unknown shape. Also, the smaller you make your pieces the more accurate your approximation is. Students will hopefully begin to think about how to systematize their summations when the pattern of the graph is known (i.e. a function).	9Ss will grapple with a seemingly impossible activity but will use mathematical reasoning to make it almost possible. Ss will reflect on, how close is close enough? how close can I get my answer to an impossible answer? 0 1	2Students will develop and check their approximations with one another and will make value determinations themselves on whose approximations were better than others. Ss will learn to actively engage with abstract conceptual ideas where they discuss the mathematics and where the teacher does not provide the "right" answer. Ss provide daily summer at end of class.

Table 6

PART 2 - LESSON FLOW

3**LAUNCH and EXPLORE—Activity 1** 4*How will you introduce the task in a way that establishes HIGH cognitive demand?*

5*Will you introduce the task in writing, using a picture, verbally, other? Why?*
6*How will you launch to foster accountable, private think time and access for ALL students?*
7*How will your launch support your content and socio-mathematical lesson goals?*

8**Grouping:** Random **Time:** 35
9**Materials:** IMP4, *Another Trip*
0**Actions:**
1Ask students to look at the graph from the activity *Another Trip*. Ask students to share anything they notice or wonder about the graph. If not made explicit by students, move towards asking students, Do you think you can get the perfect area under this graph? Why or why not? Verbally launch class to find the best approximation they can.
2Students work on and complete their approximation. Students compare their approximations to their groupmates' and the consensus best approximation from each group shares their work under the document camera for the whole class. Students then are asked to reflect on whether or not anyone's area was perfect. Possible teacher prompts: How can we get it as close to perfect as possible? What additional information might help us get a better approximation than we currently have? What strategies might help us in finding out the approximate area easier?
3
4Summarize:
5I will make no mathematical connections or conclusions at the end of this activity, but might ask additional prompting questions if I do not believe students are thinking about how to deal with this seemingly impossible task of finding the area under this curve.

Table 7

6**LAUNCH and EXPLORE—Activity 2**
7*How will you introduce the task in a way that establishes HIGH cognitive demand?*
8*Will you introduce the task in writing, using a picture, verbally, other? Why?*
9*How will you launch to foster accountable, private think time and access for ALL students?*
0*How will your launch support your content and socio-mathematical lesson goals?*

1**Grouping:** Random **Time:** 50
2**Materials:** handout of function's graph, $f(x) = 0.2x^2 + 1$, x:[0,5] (with gridlines)
3**Actions:**
4Verbally launch: This function and accompanying graph represent Sarah's travelling speed in miles per day and the number of days she was travelling. I want you to figure out how far Sarah travelled for the first five days of her trip. Work on it silently for 7 minutes, and then I will notify you that you may work as a group afterwards. Begin.
5Note: The graph does not indicate the labels I just verbally alluded to before as I want to move my students up on the ladder of abstraction towards the more abstract. I included labels in my verbal prompt for those students who still need to tie the mathematics to a context.
6Students will often originally create one shape to try to approximate the area, if groups are not holding their members who do this accountable, then step in and ask them for a better approximation. Students often create multiple known shapes with uneven x-intervals (I intentionally used an interval of 5 for this purpose).
7If students are struggling/making good progress then do not intervene much. If students are quickly making good progress consider offering groups the following prompts:
8How can I make your strategy more generalizable for other problems/functions that we might see?
9How can you get a more accurate approximation?
0Can you calculate a relatively close area approximation that you know is an under- (and/or over-) estimate of the perfect area? How can you be confident that your answer is in fact an under-/over- estimate?
1Purpose of whole class share out: make sure everyone sees a student who broke their interval into equal parts; this will help students develop a system for calculating areas of more complex functions/intervals later. So if each group has already done this then skip the whole class share out.

Table 8

2**REFLECTION**

3*How will you engage students in reflecting on their own thinking/activity in relation to your lesson goals? What questions will you ask?*

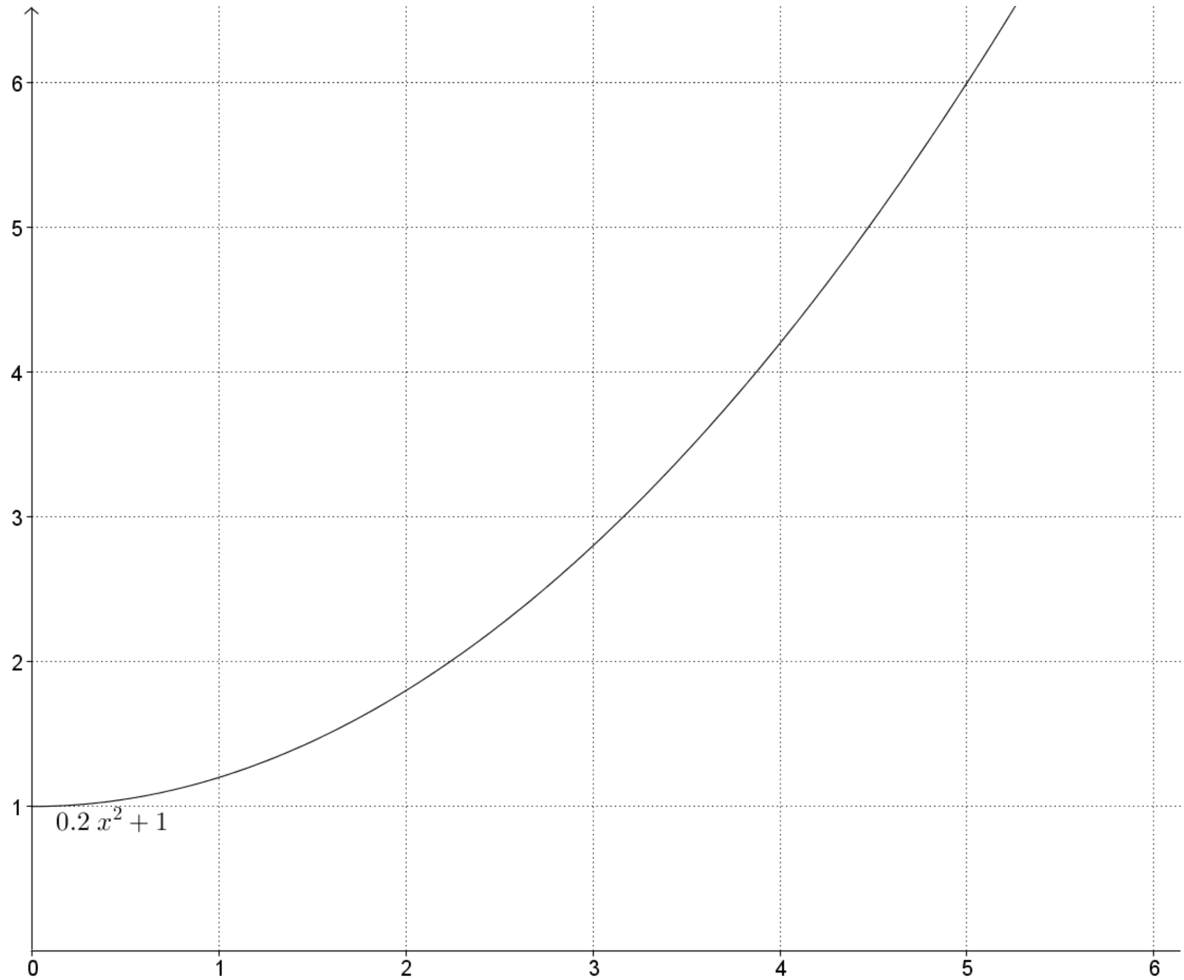
4Students will write down and have group discussions on the following prompts, written on board:

5How did our graphs today differ from prior days? How did their differences change the way we calculated their areas?

6As of now, do we have an exact method for calculating areas of unknown shapes/curves? How close have we gotten?

Handout Discussed In Paper

Here is the simple handout of the graph discussed during the lesson plan above in all of its simplicity.



3. Grant Proposal

Project Summary

This proposed project falls under type 3: Workshop/Conference/Synthesis and primarily addresses the strand: Teaching. Secondary mathematics (grades 6 – 12) is the discipline being addressed.

Overview

The proposed project is driven by the research question: how can we, as a district, further new math teachers' teaching practice to better promote students' conceptual understanding of key mathematical concepts? The goal is to move from the current state where the vast majority of U.S. teachers do not teach for conceptual understanding to a desired state where the majority of teachers are teaching more for conceptual understanding.

The proposed project will offer both conventional and reform types of professional development to non-experienced math teachers in the Red Clay Consolidated School District (Wilmington, Delaware) over a two-year period. Each non-experienced teacher will pair with an experienced teacher from their building in both forms of professional development. The conventional professional development will be a weeklong summer workshop focused on improving both non-experienced teachers' as well as experienced teachers' curriculum-specific mathematics content knowledge as well as focusing on students' thinking about the curriculum.

The reform-style professional development will happen during the school year when the experienced partner as well as district math specialists will observed the non-experienced teacher using a teacher evaluation system (TRU-math). The purpose of the evaluations in non-evaluatory but rather is intended to focus the non-experienced teachers instruction and environment on teaching for conceptual understanding.

Statement on the intellectual merit of the proposed activity

From this project we are going to learn whether or not a multi-structured professional development program can support non-experienced teachers to teach more for conceptual understanding. We might learn that non-experienced teachers will continue to teach in the ways in which they were taught despite our best efforts, and we should therefore focus our resources elsewhere. We might learn that mentor pairs that we recommend using are not effective for creating professional growth in either and that perhaps similarly experienced teachers would develop more quickly if working together.

Statement on the broader impacts of the proposed activity

If successful this activity could have a huge impact nationally on how teachers are trained once teaching (both non-experienced and experienced). If shown to be successful mass replication of the program could substantially impact the teaching of mathematics (and possibly other subjects) in this country.

Project description

Research Question: How can we, as a district, further new math teachers' teaching practice to better promote students' conceptual understanding of key mathematical concepts?

Current state: Math teachers in the U.S. do not teach for conceptual understanding.

Desired state: Average observation score on rubric by end of second year of participating teachers to be over the middle score on the rubric.

Hypothesis: The pathway to get us from current state to desired state is professional development, specifically mixed-type professional development centered on curriculum-specific content knowledge and students' thinking about the curriculum.

The proposed project will assess the current state of inexperienced math teachers (fewer than 3 years teaching math) in the district via observations using the TRU-math teacher evaluation framework. It is predicted that the initial average evaluation score will be rated in the lowest 1/3 range on rubric (an average score below 1.3); it is also predicted that by the end of the two-year project the average evaluation score will be rated above the midpoint on the evaluation rubric (an average score above a 2.5).

Importance

If students are going to advance in STEM careers they will need a strong foundation in mathematics—specifically they will need deep-rooted conceptual understanding of numbers, problems, and other key mathematical concepts. The procedural focus of most U.S. math classrooms is insufficient in preparing students to meet high levels of success in STEM fields. If we want students to understand the mathematics more deeply and richly than they often do now, we need to focus teachers' energies on teaching for conceptual understanding, especially when research shows there is often no drop in procedural fluency while conceptual understanding is the focus (Hiebert and Grouws, 2007).

My hypothesis is important because research supports the potential transformation influence of effective professional development. My theory of action (see Research and development design below) paves a logical pathway for why this project proposal has a high probability of being effective. Starting with the current state of where most teachers currently lie pedagogically in the U.S.—focusing on procedures and computation—to how we can get these teachers to teach more effectively for conceptual understanding. And if this project shows success for this one district, we can disseminate the vary concrete steps we took to create this project which can easily be replicated as the project is not resource-intensive beyond a district's typical personnel and cost for professional development.

The proposed project differs from most professional development in the U.S. by offering both conventional and reform-styled professional development opportunities that is specifically aimed at non-experienced teachers. Currently most districts offer conventional professional development only that is aimed at all secondary math teachers in the district (if not aimed at all secondary teachers in the district). Furthermore, this proposed project focuses on two research-supported facets of professional development—curriculum specific content knowledge and students' thinking about the curriculum, versus the potpourri of non-research-supported professional development trainings often offered in districts across the country.

Perhaps the most appealing part of this proposed project is how easily replicated it can be if it shows to be successful. The cost of implementation will not be prohibitive for many districts and many districts already spend significant money on professional development that could merely be reallocated towards a system similar to the one proposed here. The fact that the only personnel used in the proposed project are local parties and any other resources are most likely already in the possession of the school districts (curriculum materials, student work samples, low-grade video recording devices) makes replicated this professional development system very simple and appealing. Couple these points with the growing environment of high stakes testing in mathematics, paired with tests that aim to assess conceptual understanding more than high stakes did in the past, make the timing of this proposed project, and the probably of its replication highly likely if successful.

Research and development design: Theory of Action

	Type of PD	What teachers gain	
Current State: Teachers not teaching for conceptual understanding. Skills and experiences not present to transition teachers away from the methods from which they were taught.	Traditional	teaching for conceptual understanding modeled	**Desired State**: Teachers are teaching more effectively for conceptual understanding.
		content knowledge	
		knowledge of student thinking	
		pertinent research shared and discussed	
	Reform-minded	quick iterations, multiple opportunities, feedback, many new ideas attempted and modified	
		curriculum specific--increased curricular content knowledge	
		gain knowledge of student thinking about curriculum	
		teachers learn from teaching and from other teachers--effective collaboration	
		typically long-term, sustained, collobarative	

“The success of ambitious education reform initiatives hinges, in large part, on the qualifications and effectiveness of teachers. As a result, teacher professional development is a major focus of systemic reform initiatives” (Garet, 2001). Specifically addressing the grant’s research question of teachers promoting conceptual understanding, research by Berk & Hiebert (2009) suggests that professional development can promote teachers teaching more for conceptual understanding. My theory of action uses both conventional professional development (workshop) and reformed professional development (an iterative teacher evaluation/feedback system).

If teachers are going to teach for conceptual understanding they need to believe teaching for conceptual understanding is important[1]. If you want to change a teacher’s beliefs you need to give them positive experiences with a new system first (Guskey, 1989)[2], their personal beliefs on teaching change afterwards. They need to be motivated to change their practices which can come from internal sources such as the joy of watching students learn or external sources such as conforming to district or state policy for job security and advancement purposes[3]. Teachers need a model of what teaching for conceptual understanding looks like and be given the opportunity to evaluate how they can implement changes in their classroom to more resemble the practice they see modeled[4]. Teachers are more likely to listen to other teachers as they share similar experiences, are not threatening, and possess the domain-specific content and pedagogical knowledge needed to offer relevant and specific feedback[5]; furthermore, “Teams are more effective with peers leading rather than administrators or content experts in the facilitator role for several reasons. Peer-facilitators are uniquely positioned to model 'a leap of faith,' frame the work as an investigation, help the group 'stick with it,' and guide protocol use as a full participant in the inquiry process" (Gallimore, 2012). Related to teachers learning from other teachers, we need to create an environment that breaks the "teaching is an island" perception in order to break the cycle of teachers teaching how they were taught[6]—which was often lacking in conceptual understanding.

Teaching for conceptual understanding in mathematics requires two forms of knowledge: "*knowledge of the subject* to select tasks that encourage students to wrestle with key ideas and *knowledge of students' thinking* to select tasks that link with students' experience and for which students can see the relevance of the ideas and skills they already possess". (Hiebert, 1996, p. 16; emphasis mine). If teachers need knowledge of the subject and knowledge of students' thinking to effective teach for conceptual understanding than we need our professional development to support those two areas. Further validating this logical progression to focus professional development is extant research on the very topic: "So teachers leave these [most effective] programs with very specific ideas about what the subject matter they will teach consists of, what students should be learning about that subject matter, and how to tell whether students are learning or not. This content makes the greatest difference in student learning" (Kennedy, 1998, p. 25). To reiterate, teachers need these two types of knowledge to effectively teach for conceptual understanding and research has already shown that professional development focused on these two types of knowledge has shown to be effective in teaching for conceptual understanding. "There is a growing consensus that professional development yields the best results when it is long-term[7], school-based[8], collaborative[9], focused on students' learning[10], and linked to curricula[11]" (Hiebert et al., 2002, p. 3).

If we are to expect teachers to offer their students learning opportunities that support conceptual understanding then they have had to have an experience as learners in an environment that supports conceptual understanding[12]; the rationale behind this need for teacher's prior experience comes from research by Lieberman and Wood

(2003): “When teachers have opportunities to interact with their subject matter in ways that they aim for their own students to do (such as engaging in writing workshops, getting feedback on their own writing, giving critiques), they are more likely to engage in those practices in their classrooms” (as cited in Darling-Hammond, 2005).

As teaching is an ever-evolving and highly complex practice teachers must learn to teach from teaching[13]. Some effective ways that teachers can learn from teaching includes reflection, analyzing video of other themselves or other teachers, analyzing student work. Research by Ball and Cohen (1999) showed that teachers could learn from “authentic classroom materials” and did not necessarily have to learn from teaching itself. Garet (2001) goes more in-depth on where active learning for teachers can come from:

> One element of active learning is the opportunity for teachers to observe expert teachers, be observed teaching in their own classroom, and obtain feedback. These opportunities can take a variety of forms, including providing feedback on videotaped lessons, having teachers visit each others' classrooms to observe lessons, and having activity leaders, lead teachers, mentors, and coaches observe classroom teachers and engage in reflective discussions about the goals of a lesson, the tasks employed, teaching strategies, and student learning. (p. 925)

With the primary goal of this proposed project being the teaching of conceptual understanding by math teachers, we should not neglect the impact that this professional development will have in positively affecting teaching changes of the experienced teachers[14]. Research by Kunzman (2002) of experienced teachers who took courses in Stanford’s Teacher Education Program (STEP), a program designed for pre-service (non-experienced) teachers, has shown there to be significant improvements of the experienced teachers who partook in the program.

A tertiary goal of the project that supports the primary goal of teaching for conceptual understanding is to familiarize both non-experienced and experienced teachers with teaching in a way that aligns with the Common Core State Standards for Math[15]. The document aims to steer teachers towards teaching for conceptual understanding; early on it states, "These Standards endeavor to follow such a design, *not only by stressing conceptual understanding of key ideas*, but also by continually returning to organizing principles…" (p. 4, emphasis mine).

Research and development design: Implementation

The first summer of the proposal will include a 35-hour week-long professional development workshop for all non-experienced math teachers in the district (fewer than 4 years math teaching experience) along with an experienced math teacher from each secondary school in the district; each non-experienced participant will be paired up with an experienced teacher from their building (if one is available, if not an experienced teacher from another secondary school will be paired)[5, 8]. The experienced teachers will also partake in the workshop's activities[14]. An experienced teacher here will be defined as a teacher who is currently teaching multiple secondary mathematics courses and has been doing so continuously for at least the past five years; each of the experienced teachers will be recommended by the building principals for their high level of teaching expertise. To ensure a higher likelihood of effectiveness, in the workshop there will be a focus on attributes of professional development proven to effectively impact the teaching/learning cycle, namely: a focus on curriculum-specific mathematical content knowledge[11] coupled with knowledge about students' thinking about the content presented[10] with opportunities for teachers to engage in active learning[4] (Cohen & Hill, 2001; Garet, 2001).

> There have also been a series of studies that suggest that professional development focused upon how students learn specific content within subject matter areas is helpful for teachers, *particularly if the instruction is focused upon assisting students toward deeper conceptual understandings* (Darling-Hammond, 2005, p. 404; emphasis mine).

The expectation is that these summer workshops (two in subsequent years)[7] will improve teachers' curriculum-specific mathematical content knowledge along with their knowledge of student thinking about the same mathematical content which will in turn lead to improved teaching and hence improved student learning. These workshops will also have teachers doing the mathematics together[9] structured in a way that models an environment supportive of learning for conceptual understanding [2, 4, 12].

During the summer workshops teachers will collaboratively[9] analyze teaching artifacts such as classroom video clips and student work samples[13]. The goals of these activities will be to show teachers models of effective environments that support conceptual understanding[4] , to show the benefits and importance of teaching for conceptual understanding[1], to challenge their beliefs about what teaching should look like[2], and also to teach teachers how to learn from teaching[13].

There will also be sessions during the summer workshop for presentations and discussions on research, pedagogy, and what effective teaching and learning look like[1, 6]. "Harris and Sass (2007a) provide evidence that certain types of teacher professional development (those providing pedagogical content knowledge) lead to improvement in teacher effectiveness" (as cited in Harris, 2009). Furthermore, research has shown that times of teacher reflection connected to research and theory lead to improved teacher education (Darling-Hammond, 2005).

Another theme that will be imbedded into these workshops will be connecting the Common Core State Standards in Mathematics (CCSS-M) to teaching for conceptual understanding[15]. Specifically the language of the authors in the introduction to the Standards, the authors' descriptions of each mathematical practice, the authors' introduction to high school (or middle school) mathematics, as well as the authors' description of each domain at the secondary level will be analyzed and discussed as a possible catalyst for teacher change. The rationale behind the connection to the CCSS-M is for inexperienced teachers to better understand the charge of the CCSS-M which lay a foundation for including conceptual understanding in math education in our country (which unfortunately has been absent from most K-

12 mathematics classrooms for decades now). Furthermore, the text brings up great question prompts that will be discussed during the workshop such as, "What does mathematical understanding look like?" (p. 4)[1]. And the eight Standards for Mathematical Practice will be referenced repeatedly during times of lesson planning as well as video critiquing. Furthermore, right now, in most states, math teachers must learn to adapt to the Common Core if they expect to retain and advance in their teaching positions, so understanding and implementing the common core becomes a huge motivational factor, especially for non-tenured teachers[3].

Research by Garet (2001) supports the decisions we have made concerning our traditional professional development listed above as well as our reform-minded professional development about to be discussed:

> "On the basis of this emerging evidence, we view the degree of content focus as a central dimension of high-quality professional development... Opportunities for active learning can take a number of forms, including the opportunity to observe expert teachers and to be observed teaching; to plan how new curriculum materials and new teaching methods will be used in the classroom; to review student work in the topic areas being covered; and to lead discussions and engage in written work." (p. 925)

The reformed professional development will take the shape of a teacher evaluation system (TES) that will offer at least four iterations of a feedback-improvement cycle each year (at least eight total[7]). The participating teacher will be observed or will be video recorded by their teaching mentor, where the observer will analyze the lesson through the TRU-math framework (Schoenfeld, 2014) that focuses on teaching for conceptual understanding based off of what the literature says are best practices for teaching for conceptual understanding. Experienced teaching peers will be used[5, 8] rather than administrators or content experts because, "Teacher-facilitators are trying out in their classrooms the same lessons as everyone else in the group… this significantly lessens the chances the setting is converted from inquiry-focused to a more conventional professional development (PD) 'presentation' structure that puts teachers in a passive rather than active role" (Gallimore, 2012). Furthermore, using teaching peers rather than an administrator allows the observer to address curriculum concerns[11] as well as student thinking about the curriculum[10]—two features Cohen & Hill (2001) make for effective professional development. And these iterative TES cycles will focus specifically on mathematical content and turning lessons into experiments[13] "making each phase of teaching deliberate and intentional" (Hiebert, et al., 2003). Additionally, work by Taylor and Tyler (2012) suggest that using a TES can improve teaching.

Experienced teachers will work with non-experienced teachers to offer suggestions while planning and offer feedback after observing—this allows the non-experienced teacher to try new ideas right away and get near-immediate, expert feedback on how it went which can be a powerful experience for the new teacher if the experience is a positive one which can lead to a transformation of beliefs about teaching and learning[2] (Guskey, 1989). Both teachers will essentially become part of a professional learning community with an iterative lesson study focus[13]. "[E]merging research suggests that opportunities to engage in 'lesson study,' where groups of teachers are engaged in joint observation, analysis, and evaluation of lessons, may have particular promise as a learning environment in which teachers engage in learning with their peers (Darling-Hammond, 2005, p. 405). This iterative cycle constantly scrutinized under the lens of the TES (TRU-math) will help improve both sets of teachers: "Repeated experiences with a set of conceptual ideas, along with repeated opportunities to practice skills and modes of analysis, support deeper learning and the development of expertise" (Darling-Hammond, 2005, p. 393). It is important to note here that these repeated experiences and opportunities are only possible in this reform-minded professional development system which is why having mixed types of professional development is so important.

New teachers were selected to participate in this professional development because their practices are not as well-developed as more experienced teachers.

Furthermore, the support offered through the summer workshops along with the TES can help combat the “teaching is an island”[6] mentally that pervades much of the profession. If we can change the mindset of our youngest teachers that teaching does not have to be an isolating practice, as well is influence their belief that teaching is something that improves with cycles of critiques[13], then we can make lasting, systemic changes to the profession.

Measurement

The initial data will be collected during the Spring of the first year and will be collected by both district math personnel and a teaching peer using the same TRU-math framework that the teaching peers will be using for the TES cycle; both district personnel and teaching peers will have been trained in using this framework prior to their first observation of the participating teachers.

Other data sets will be collected after both one year in the program and two years after the initial data has been collected; the same district personnel and teaching peer will once again evaluate the participating teachers using the TRU-math framework. Then the initial data, the intermediate data, and final data sets will be compared using two paired sample t-test to determine statistical significance (or lack thereof) in the difference between observation evaluations—one t-test will compare the district personnel’s differences and the second t-test will compare the teaching peer’s differences. The rationale for having an intermediate evaluation is to analyze when most of the teacher growth took place (if any did). These results will help future professional development agents decide whether a one-year program of this sort will be sufficient for the growth of inexperienced teachers, or perhaps this data will suggest to professional development agents that the second year is where most of the teacher growth happens and should only commit to a program like this if they are investing for at least two years.

Furthermore, each participating teacher will fill out a survey at the beginning of the program asking them to self-evaluate in terms of their own teaching—

specifically geared towards teaching for conceptual understanding. The teachers will fill out the same survey again after participating in the two-year program. A paired sample t-test will be used to determine statistical significance in the differences between each participating teacher's two surveys. There will also be a section on the final survey for each participating teacher to leave feedback on their perceived strengths and weaknesses of the program both related to teaching to conceptual understanding and other side-effects that might have resulted from their participation in this program.

For quality assurance district math personnel will overlap 10% of their caseloads and these overlapping scores will be compared. If these case scorings are found to differ more than 15% of the time then both district math personnel will meet, discuss and come to an agreed upon score for the disagreed upon observations, and restart the process again for two more overlapping candidates, until both personnel create similar marks on their compared evaluations. If disagreements perpetuate and no agreed-upon score can be found, then an average score for each candidate will be used in the final assessment score analysis.

Timeline:

- May 2015: district personnel and teacher peers trained on TES
- May 2015: collect initial observation data
- June 2015: first summer PD institute
- August 2015 – May 2016: observation/improvement system in process
- May 2016: collect intermediate observation data
- June 2016: second summer PD institute
- August 2016 – April 2017: observation/improvement system in process
- May 2017: final observation data collected; teacher surveys collected
- May 2017: observation data analyzed; teacher survey data analyzed

Dissemination

The results of the proposed project will be first written up and applied to be published in the Journal for Research in Mathematics Education (JRME) and if denied other peer-reviewed math education journals will be contacted. The results will be submitted to JRME whether the project indicates positive or negative results, because it is important for researchers interested in professional development to see why this project may have failed.

However, if the project shows statistically significant successful results than the results and project detail summary will be submitted to the National Association of Secondary School Principals (NASSP) in hopes that it will be published in their monthly magazine as well as their weekly e-newsletter—both of which are read by tens of thousands of principals and school leaders from across the country. The results will also be published online and will be promoted through a network of math educators using social media to increase its readership and relevance. The overarching goal of this project is to test an easily replicable, research-based, professional development framework in one district, and if successful, see it replicated nationwide. So the dissemination of the results and procedures is highly important, particularly if the project is successful. If the results of this project are positive and the study does get published by the NASSP then I predict dozens of other school districts across the country will immediately move towards trying something similar with more to follow if this second generation of districts also find success.

Expertise

The following educators have agreed to advise on the project, all of whom have extensive experience in teacher training and professional development and all of whom have worked on multiple NSF-funded grant projects.

- Dr. James Hiebert – University of Delaware
- Dr. Jon Manon – University of Delaware

- Dr. Brian Lawler – San Diego State University
- Dr. Jamila Riser – Delaware Math Coalition, Executive Director
- Sherry Fraser – co-author of the Interactive Mathematics Program (IMP) (a secondary curriculum used already in the district and to be a focal point of the professional development)

Broader impact

Other than the intended impact this proposed project will have on non-experienced teachers in the Red Clay Consolidated School District and hopefully non-experienced teachers in replicating school districts, the proposed project has a great opportunity to impact the experienced teachers who they themselves are probably not teaching for conceptual understanding as well as they could if research is any indication. Furthermore, this proposed project, if successful, can impact the conversation districts and states around the country are having about what math teaching should look like and what teacher training should look like. It is all too frequent for states and districts to have no framework for evaluating math teaching nor have a theory of action for what the development of in-service teachers should look like. This proposed project can change that. It can break math teacher observations out and away from the mold of the generic observation, discipline non-specific that most teachers are evaluated from. And this of course can lead a charge to bring about change in other subjects and how those teachers are evaluated, and a whole slew of new frameworks can be brought up, and discussed, and we can reinvigorate the conversation about what good teaching looks like in any particular subject. This may have the effect of calling into question whether a generic, multi-discipline teacher evaluation framework is what is best for student learning in this country.

References

Ball, D. L., & Cohen, D. K. (1999). Developing practice, developing practitioners: Toward a practice-based theory of professional education. *Teaching as the learning profession: Handbook of policy and practice*, *1*, 3-22.

Berk, D., & Hiebert, J. (2009). Improving the mathematics preparation of elementary teachers, one lesson at a time. *Teachers and Teaching: Theory and Practice, 15*, 337-356.

Cohen, D., & Hill, H. (2001). *Learning Policy: When State Education Reform Works*. Hartford, CT: Yale University Press.

Darling-Hammond, L., & Hammerness, K. (2005). The Design of Teacher Education Programs. In *Preparing Teachers for a Changing World: What Teachers Should Learn and Be Able to Do* (pp. 391–441).

Gallimore, R., Ermeling, B., Saunders, W., Goldenberg, C., & Saunders, W. M. (2012). Moving the Learning of Teaching Closer to Practice : Teacher Education Implications of School-Based Inquiry Teams. *The Elementary School Journal*, *109*(5), 537–553.

Garet, M. S., Porter, A. C., Desimone, L., & Birman, B. F. (2001). What Makes Professional Development Effective? Results From a National Sample of Teachers. *American Educational Research Journal*, *38*(4), 915–945.

Guskey, T. (1989). Attitude and Perceptual Change in Teachers. *International Journal of Educational Research*, *13(4),* 439–453.

Harris, D. N. (2009). Would Accountability Based on Teacher Value Added Be Smart Policy? An Examination of the Statistical Properties and Policy Alternatives. *Education Finance and Policy*, *4*(4), 319–350. doi:10.1162/edfp.2009.4.4.319

Hiebert, J., Gallimore, R., & Stigler, J. W. (2002). A Knowledge Base for the Teaching Profession : What Would It Look Like and How Can We Get One ? *Educational Researcher, 31*(July), 3–15.

Hiebert, J., & Grouws, D. A. (2007).The effects of classroom mathematics teaching on student learning. In F. K. Lester, Jr. (Ed.), *The Second Handbook of Research on Mathematics Teaching and Learning* (pp. 371-404). Charlotte, NC: Information Age Publishing.

Hiebert, J., Morris, A.K., & Glass, B. (2003). Learning to learn to teach: An ‘experiment’ model for teaching and teacher preparation in mathematics. Journal of Mathematics Teacher Education, 6, 201–222.

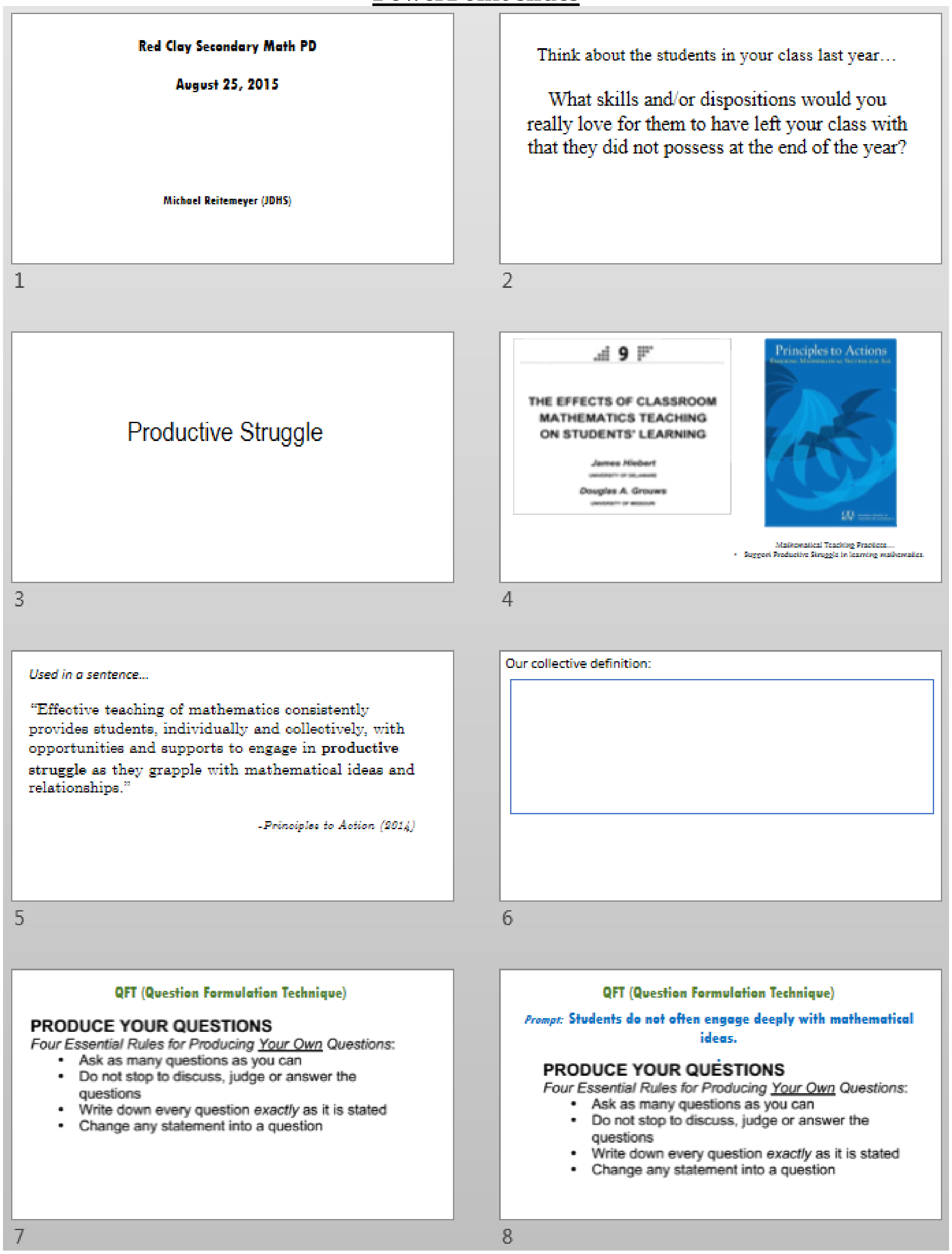
Kunzman, R. (2002). Pre-Service Education for Experienced Teachers: What STEP Teaches Those Who Have Already Taught. *Issues in Teacher Education, 11(1)*, 99-112.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics.* Washington, DC: Authors.

Taylor, E., & Tyler, J. (2012). Can Teacher Evaluation Improve Teaching? *Education Next*, *Fall*, 78–84.

4. Professional Development Materials—Fall 2015

PowerPoint slides



QFT (Question Formulation Technique)

***Prompt:* Students do not often engage deeply with mathematical ideas.**

PRIORITIZE YOUR QUESTIONS

Choose your three most important questions:

-
-
-

Why did you choose these three as the most important?

9

Desirable Difficulties Video

LINK

10

Our goals for you from this PD

- Recognize the potential positive impact of letting students struggle productively with the mathematics
- Reflect on your own practice in ways you might be over-helping your students

Let's be clear…

- This is not a call to revamp any of your lessons
- It is a call to consider occasionally shifting more of the cognitive load to your students by asking more purposeful questions or by giving more purposeful comments

11

Let's be clear… (cont.)

- This is not about privileging one learning goal over others, however, the goal of having students understand mathematics conceptually is a common and very difficult goal which is why we are spending some time here working on it
- Productive struggle is not a silver bullet, but it is a very practical adjustment to make modest gains with students
- Students do not need to engage in 45 min. sessions of productive struggle; smaller bursts are fine/appropriate

12

"Productive struggle doesn't necessarily mean presenting a cognitively-demanding task that students work on for 30 min. It could take place in much smaller bursts when a teacher poses a short task or asks a puzzling question.

(cont.)

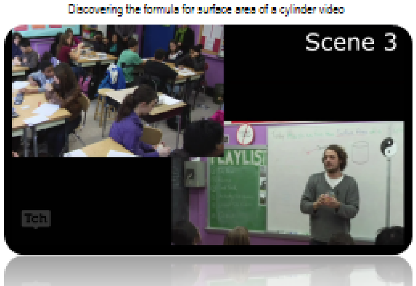
13

One nice rule of thumb to check whether the question is one that might trigger productive struggle is whether a specific answer is needed for the teacher to continue the lesson. Questions that require specific answers to move on are questions that are likely part of a recitation--the much more common form of teaching. Recitation usually includes breaking down the task or discussion into small pieces and asking students to fill in the blanks, fill in the pieces that allow the teacher to move to the next piece or step."

–Jim Hiebert

14

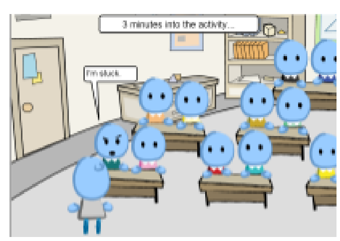
Discovering the formula for surface area of a cylinder video



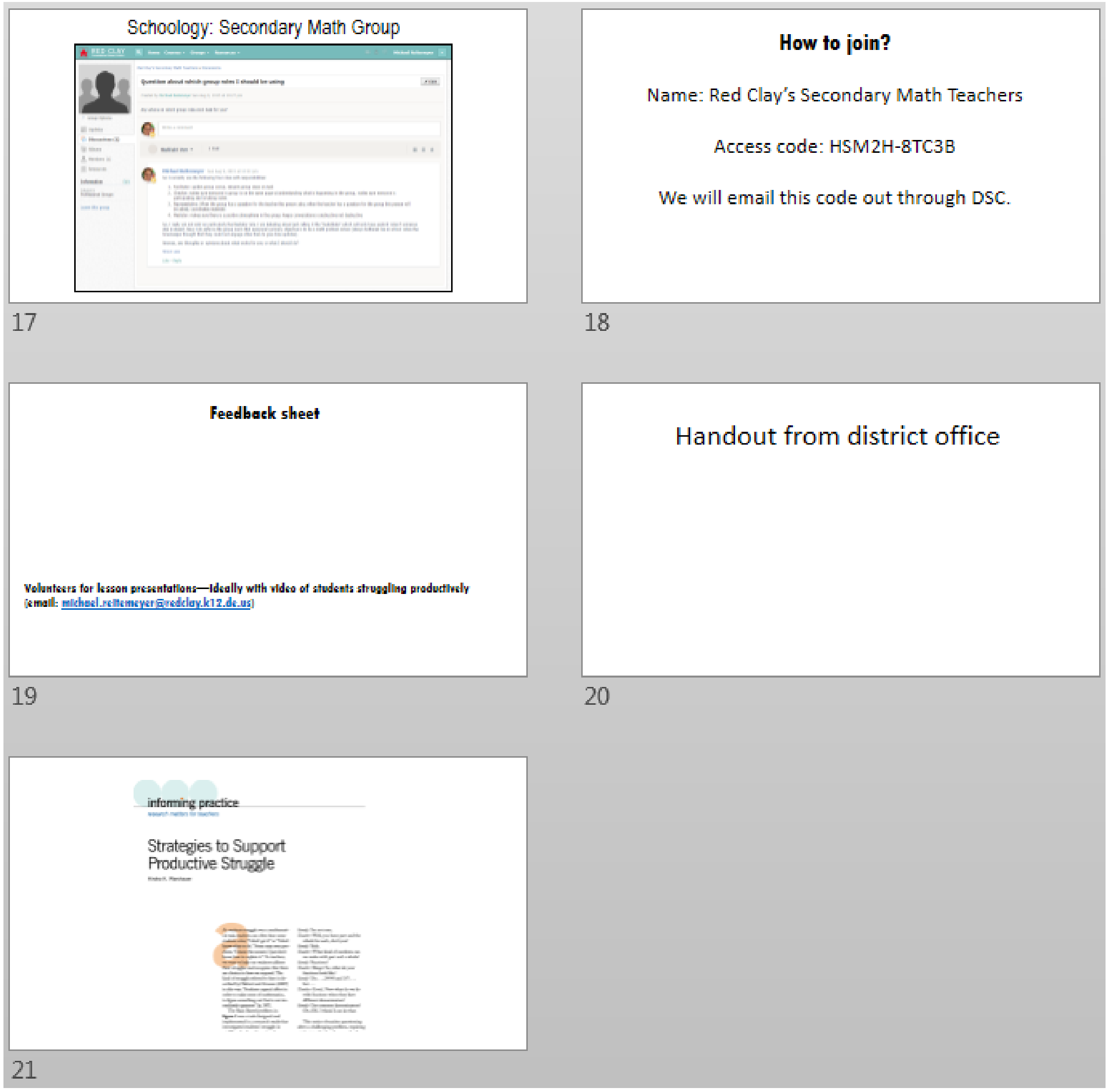
link

15

Teacher Response handout



16



Professional development video

The video can be found at the following URL: https://youtu.be/-UBWuZgd-jc

The video is based off of the work of Robert Bjork and heeds his call to incorporate desirable difficulties into lessons. The video uses several examples from Bjork as well as other studies where students (or participants) to make the claim that often what we think is best for us is completely wrong. And that often when we feel we are confused—that's when we do our best learning.

5. Program Evaluation

EXECUTIVE SUMMARY

The purpose of this evaluation was to determine whether or not a teacher (me in this case) who intentionally tried to effect intellectual autonomy in his students could be successful. The program being evaluated in this paper is simply my own classroom—located in a high-needs public high school in the mid-Atlantic region with a focus on first-time ninth grade students. That said there are a number of interventions or teacher moves I have used throughout the course of the year that I think will help foster students' self-perception of intellectual autonomy; two primary interventions being the promotion of exploratory talk among the students, and class discussions where I stated explicitly my goal for them of intellectual autonomy coupled with their thoughts on the value of intellectual autonomy.

The conclusions from the evaluation seem to support that my students both engaged more in exploratory talk and had higher self-perceptions of intellectual autonomy. However, the sample size was small, students were not randomly assigned classes, and the massive variance between classrooms makes finding the precise reasons for this increased self-perception very difficult to pinpoint.

Introduction

Program description.

The goal of this program was to help students develop intellectual autonomy. To reach my goal I hoped to leverage peer-to-peer interactions on rich, challenging tasks (with multiple entry and exit points) to change the way students view school and specifically math class; the new goal has students self-identifying as doers and developers of mathematics rather than passive agents in the classroom. My goal involved a shift in mindset coupled with legitimate interactions with classmates concerning mathematics that will ultimately increase students' intellectual autonomy. I hoped to couple this intellectual autonomy with tasks and discussions to help them develop further moral autonomy as well; however, the focus of this evaluation will be on the development of intellectual autonomy.

While the process of initiating students in exploratory or rough draft talk is the only intervention being evaluated, there are other interventions that I have initiated that I believe have helped students become more intellectually autonomous. These other interventions include: public posting of student wonderments, groups randomized daily, class discussions about autonomy, tasks that promote student productive struggle, utilization of non-permanent shared writing surfaces, and listening then posing questions to groups based on their current understanding to prompt them to think more deeply about the mathematics.

The findings from this evaluation will help me in determining whether or not the interventions and teacher moves I have tried this year are making a difference in terms of student autonomy. Positive, statistically significant results will help me help other teachers in pursuing some of these activities I have tried. Data from surveys and interviews will also help me better identify which activities and teacher moves were more powerful in the students' eyes in promoting their sense of autonomy.

Purpose of evaluation.

The purpose of this evaluation was to assess whether or not students in my classes believed they were developing intellectual autonomy to a greater degree than students from other math teachers' classes at the same school. A secondary purpose of the evaluation was to see if one of the interventions that might be leading to increased autonomy, student engagement in exploratory talk, was happening more in my classroom than in the control teachers' classrooms.

Evaluation questions.

The process question I analyzed is: are students in the program engaging in exploratory talk and writing? In this study exploratory talk means talk where ideas are still being formed or pondered upon; resolution still hasn't solidified in the talker's mind; the talker's ideas are still highly flexible during this period and forward feedback is actively sought. Exploratory talk is to be contrasted to presentational talk where ideas have been solidified (at least tentatively) and backwards feedback is actively sought [I picture forward feedback being like putting a seedling of a thought out there and asking your audience to help the idea grow or to move the idea further or try it in several different directions (sometimes to the trash); I picture backwards feedback as feedback where you look at someone's final product and look to strengthen its foundation (which often means trying to poke a hole in it) by asking critical questions. Much of the talk asks for by teachers in most U.S. classrooms is presentational, not exploratory, in nature (Mercer and Hodgkinson, 2008). Student engagement is fortunately measurable via observing how often students are putting forth their unfinished ideas about the mathematics.

The outcome question I analyzed is: are students self-identifying as more autonomous at the end of the program? Measuring an actual growth in student autonomy is very difficult and arguably its results might be seen as subjective. But in the particular case of autonomy perception closely mirrors reality in the sense that if a student feels more autonomous they are likely to have developed the sense of agency needed to act more autonomously which will often lead to actual autonomy. And fortunately student perceptions about their own intellectual autonomy are more easily measurable by asking them about their beliefs regarding how often they think for themselves in math class.

The connection between my process and outcome questions is based on the idea that when students try out their own, burgeoning ideas they are taking that initial necessary risk towards intellectual and moral autonomy. As Douglas Barnes points out (as cited by Mercer and Hodgkinson, 2008), “At the centre of working on understanding is the idea of ‘trying out’ new ways of thinking and understanding some aspect of the world: this trying out enables us to see how far a new idea will take us, what it will or will not explain, where it contradicts our other beliefs, and where it opens up new possibilities.” So these exploratory ideas are the ones that represent that student outreach towards autonomy, so if the data from my process question is positive it should promote positive results in my outcome question otherwise my theory of action (or data collection) is misguided and needs revision.

Evaluation Plan

Evaluation design.

Evaluation Question		Sample	Variables/ Instruments	Data Collection Procedures	Data Analysis Procedures
Process	Are students engaging in rough draft talk and writing?	All first time ninth graders at Wehnimers High School in Mr. R’s classes	Classroom observations; seating chart to record tally marks on	Have another teacher(s) observe my classes and observe the number of students who engaged in exploratory talk and/or writing. I will do the same for the other freshmen math classes.	Do a chi-square analysis comparing numbers of students who did and did not engage in exploratory talk and/or writing in Mr. R’s class versus control classes.

Outcome	Are students self-identifying as more autonomous at the end of the program?	All first time ninth graders at Wehnimers High School in Mr. R's classes	Student self-perception about their own autonomy; survey and interviews.	Hand out survey to 2 of Mr. R's classes (his two freshmen classes) and to the other 4 freshmen math classes at the school. Quasi-randomly select students to interview from Mr. R's classes	Surveys only. Do a chi-square analysis of each survey question comparing Mr. R's classes to control classes reporting on statistically significant differences.

Sample.

All first time ninth grade students taught by me, 31 students, will be the sample for both of my evaluation questions. All first time ninth grade students save the 31 students I teach and complex special education students at Wehnimers High School will be analyzed as part of the program and control groups for both of my evaluation questions. This sample represents 130 students.

This sample will serve to represent the larger population of first time ninth graders in non-affluent public schools in the United States' mid-Atlantic region. As every Wehnimers first time ninth grader will be analyzed in this evaluation a wide variance in mathematical ability will be represented. Relatedly, Wehnimers' high level of ethnic and socio-economic diversity will also strengthen its status as a representative sample. The 31 students I teach span an honors class as well as an inclusion general-level class; there is significant ethnic, socio-economic, and gender diversity in these 31 students. Specifically in my classes there are 14 females, 17 males; 11 white non-Hispanic, 10 white Hispanic, 9 black, and 1 Asian; 8 students have I.E.P.s; 4 students have at least one parent (biological or step) with a college degree. From a regional perspective we know that math classrooms across the country are remarkably similar in structure, student participation, and content, so using a geographically constrained sample as I am should not be as problematic in an analysis of math classrooms as it would be for other populations.

Instruments and variables.

The variable being evaluated concerning my process question is student engagement in exploratory talk and/or writing. The instrument I used to capture data is a simple form to be used during a classroom observation. The form had the following prompt on it: Student put forth an unfinished idea concerned with mathematics towards his peers (group or whole class) orally or in writing. The form also had the seating chart of the class on it and whenever a student puts forth an unfinished idea concerned with mathematics towards his peers the observer noted it and put a tally mark by the student. The initial goal was to see what percent of students are engaging in exploratory talk but keeping a frequency count will be recorded as there might be a worthwhile comparison in frequency but my initial hypotheses was that in many other classrooms students will not be engaging in exploratory talk at all so a frequency count would not be needed. However, if my hypothesis was wrong than having how frequently exploratory talk is engaged in will be very useful data.

The variable being evaluated concerning my outcome question is student beliefs about their own level of intellectual and moral autonomy. The primary instrument used to collect data on this question was a student survey administered towards the end of the program (end of the school year) (see Appendix B). This survey will be given to all Wehnimers first time ninth graders. The survey did not includeany demographic questions because of the small sample sizes even a few seemingly innocuous demographic questions could identify respondents (one teacher only had thirteen respondents so even simply asking for gender and race would have identified the majority of those students).

The secondary instrument used to collect data about my outcome question was student interviews using a specific set of questions probed students more deeply about their views on their self-autonomy and its development during the program (see Appendix C).

Data collection procedures.

Students in the program (my classes) as well as students in other Wehnimers math teachers' classes were observed to see if they were engaging in exploratory talk. These same students were then surveyed towards the end of the year to see if their reports of self-identified autonomy are significantly different. Only students in the program were eligible to be interviewed, however.

Observations were be done by me on the other two control group teachers and by one of the control teachers on me. We each observed one another from the 15 – 45 minute mark of class (90 minute classes); a teacher with a free period or counselor covered the observer's class as needed. This specific timeframe allowed the teacher to set up class first to hopefully see more instances of exploratory talk while not being a significant drain on the observer as a resource. There was a potential for observer bias here; fortunately these students have seen each of the three math teachers throughout the year as we have a secluded ninth grade academy at Wehnimers and all the teachers often interact with students whom we do not teach.

Data was collected via the student survey concerning the outcome question gauging student self-identified autonomy. This survey was given to all Wehnimers first time ninth graders on the same day by their math teachers and was gathered by having students place their surveys in a manila envelope placed away from the teacher to discourage indirect teacher interference and promote more honest answers from the students. The surveys in the folders were collected by each teacher and turned in to an assistant of mine who entered the data into an Excel spreadsheet file where the data will then be analyzed by me.

Data was collected via the student interview concerning the outcome question gauging student self-identified autonomy. The three students were selected using a matched random framework as an endpoint meaning I want the students to be randomly chosen, but I also want the students to represent a diverse range of subcultures as well. I used an Excel program (see Appendix D) that will randomly select three student from a list of the 31 students in the program; I will run the program until the three students chosen met the requirements: both genders; students from both honors and from general-level courses; at least two races/ethnicities needed be represented. The randomization process selected the following students: a Hispanic female student with an IEP from my inclusion class, a white female student from my honors class, and a black male student with an IEP from my inclusion class. A colleague conducted the interviews using the interview questionnaire but due to a limitation on resources only students in the program were interviewed. The interviews were voice-recorded with permission from the interviewees. The interviews were transcribed by an assistant of mine for analysis.

Data analysis procedures.

The classroom observation data were analyzed using a chi-square analysis test. The test analyzed the number of students who both did and did not put forth an unfinished idea (i.e. engaged in exploratory talk), comparing my students to the students from the other freshmen math classes. I examined my students' data versus each other teachers' data separately to ensure that my data is statistically significant from both other teachers rather than simply one of the other teachers skewing the other's data. The null hypothesis was that the teacher the student has makes no difference in relation to the degree to which unfinished ideas are put forward by students. The critical value for the test's p-value was set to less than 5% to claim statistical significance.

The survey data also was analyzed using a chi-square analysis test. In this case responses of strongly agree and agree were lumped together likewise for strongly disagree and disagree; this combining was done to enable a chi-square analysis which analyzes binary choices very well. A chi-square analysis had to be done to establish statistical significance because this sample was too small for a t-test to detect significance. Each question on the survey was analyzed independently via separate chi-square tests to see which questions had statistically significant different answers from students. Each question was analyzed between my students' responses and each other teachers' separately to ensure that my data is significant from both other teachers rather than simply one of the control teachers skewing the other's data. Once again the critical value for the test's p-value was set to less 5% to claim statistical significance.

The interview data was not be analyzed using a statistical analysis tool. Rather the interviews were reviewed by me to help understand why some of the potential differences between math teachers may have existed. Particular attention was paid to specific experiences or activities that the students reference in helping them to raise their self-evaluation of their autonomy; particular attention was also made to any teacher moves made that the students remember as having an impact on their personal growth regarding autonomy.

Results

Results from observations.

Concerning the classroom observations that I made in my peers' classes and that they made in mine where we recorded the number of students who engaged in exploratory talk during the 30 minute timeframe we were observing, the results are shown in Table 1:

Table 1

Observation Data of Students Engaging in Exploratory Talk During Math Class

Teacher	Number of students observed	Number of students who engaged in exploratory talk	Percent of students who engaged in exploratory talk
Mr. Reitemeyer	28	14	50.0
Ms. Alistar	15	3*	20.0
Ms. Braum	51	13*	25.5

Note. * represents a p-value of < 0.05 when compared to Mr. Reitemeyer's data using a chi-square analysis.

The data show that even with these small sample sizes there was a statistically significant difference in the amount of students engaging in exploratory talk in my room compared to each of the two control classes.

Results from survey.

The results from the twelve-question student survey (see Appendix B) are shown in Table 2. All questions were asked on a four-point Likert scale where 1 = Strongly agree, 2 = Agree, 3 = Disagree, and 4 = Strongly disagree.

Table 2

Student Survey Concerning Student Self-perceptions About Their Intellectually Autonomy in Math Class

		Mr. Reitemeyer's students' responses			Ms. Alistar's students' responses			Ms. Braum's students' responses		
#	Question	N	Mean	SD	N	Mean	SD	N	Mean	SD
1	I like it best when the teacher shows how to solve a problem, so I can do that on similar problems in the future.	25	1.92	0.86	13	1.77	1.01	49	1.78	1.07
2	I like to make sense of and try to solve problems myself.	25	1.88	0.78	13	2.31	1.03	49	1.96	0.93
3	I get to be creative in math class.	25	2.24	1.01	13	2.85	0.90	49	2.45	0.89
4	It is frustrating when we have to figure out a problem the teacher hasn't shown us how to solve yet.	25	2.16	1.03	13	1.92	1.26	49	2.18	1.17
5	Mathematics is a mental tool that I can use if I want to.	25	1.75	0.79	12	2.08	1.00	48	2.1	0.86
6	Math problems can be interpreted in many different ways.	25	1.88	0.93	13	2.08	1.12	48	1.9	0.90
7	I usually have trouble thinking of good ideas in math class and wait for my peers to show me what to do.	25	2.28	0.79	13	2.42	0.79	49	2.53	0.89
8	The teacher is the mathematical authority in our math class.	25	2.68	1.03	13	2.23	0.83	49	2.02*	0.90
9	I really like it in math class when the students debate what is the correct solution to a problem.	25	1.92	0.88	13	2.15	0.80	48	2.08	0.85
10	I depend on the teacher to make sense of the mathematics for me.	25	2.4	0.96	13	2.54	0.97	49	2.5	0.85
11	I've learned to think more for myself this year in math class as compared to prior years in math class.	25	1.71	0.69	13	2.08**	1.19	49	2.12**	0.99
12	I think for myself in this math class.	25	1.76	0.52	13	2.15**	1.14	49	1.94*	0.88

Note. *,** represent a p-value of < 0.05, < 0.01, respectively, when compared to Mr. Reitemeyer's data using a chi-square analysis.

The data show statistically significant results between me and the two control teachers on questions eleven and twelve. I believe these two questions are the two questions that most acutely measure student self-perception of intellectual autonomy as question twelve asks them directly about intellectual autonomy and question eleven asks them to compare their experience with intellectual autonomy to prior years. It is noteworthy that ten of the twelve responses did not yield statistically significant results when comparing my students' responses to those of the two control teachers. However, in every question save question four, my students' mean response was in the more autonomous direction than the two control teachers.

There was a chance for students to write in at the end of the survey (again see Appendix B), however, no student wrote anything meaningful.

The data also show that students from all three teachers seem to think they think for themselves in math class particularly when compared to prior years. Students across the board seem to like it when students debate the correct solution to a problem, and the majority of students from each teacher agree that math can be interpreted in many different ways.

Results from interview.

There were some common generalizations from all three students. When asked, "How has math class this year been different from prior years?" All students mentioned the use of groups every day, the use of the group whiteboards, and the new curriculum on how this year was different. Carla, the Hispanic female student with an IEP, said, "You never told us how to do anything or gave us formulas or anything like that; you actually expected us to figure all this stuff out for ourselves…. It used to be just that right away the teacher would give you the formula and you'd maybe memorize the formula and then you'd use the formula to get the right answers on the next test, but we never had to memorize a formula in this class." Theresa, the white female honors student, talked about what her math education was like in prior years:

> "I read a lesson in my textbook, immediately after finishing reading the lesson I did a bunch of problems that were often the same exact ones gone over in the lesson with a figure values tweaked. Then I practiced 30 problems dealing with everything I'd learned so far. This was pages of overwhelming equations without a practical application. I know some of my friend here at Wehnimers really dislike [the math series used], but I love learning this way. I love figuring things out and problem solving and working together with kids when my thinking comes to a standstill. My brain hurts after every math lesson, and it's the best feeling in the world: it's like being sore after a long hike, bike ride, or work out. It means I'm getting stronger. Even complicated problems are within my reach now if I have some way to bounce ideas around and attempt different methods."

All three respondents agreed that they have grown as a mathematician this year, that they have grown as independent thinkers this year, and none had hoped the interviewer had asked a question that was not asked.

Jermandre, the black male student with an IEP, said that, "I've always liked math before this year. But this year was different. It was really a challenge. I really like solving all these type of problems. To me it was like a challenge every day; like every problem was like a puzzle that we had to try to solve. I don't know. I never had a class like this before. I liked it." Concerning her view of her growth as a mathematician, Theresa added, "Overall, I've learned that math is enjoyable, a collaborative activity that if I put the effort into, I can excel in. I've always shied away from science careers because of the math aspect, but now I feel like I could pursue something to do with environmental studies. Once I understand a concept in math, I don't have trouble with it. I actually understand instead of just following steps."

Conclusions and Recommendations

I believe the data from the observation shows that compared to their peers more of my students are engaging in exploratory talk. Not only were a statistically significant higher percentage of my students engaged in exploratory talk, but the level of interactions were much stronger. In the control classes examples of exploratory talk were only students asking groupmates if they were doing the problem correctly. In my class students were putting out ideas like, "What do you think would happen if the base was negative?" To be responded with, "Can the base be negative? How would that make sense in terms of Alice [the context for which they were attempting to learn about exponents]?" And rather than ask their groupmates simply if they were correct students would say things like, "Hey I got here then tried square-rooting. I don't know. That's the only thing I can think of to take out an exponent. Do you have any ideas?" Again we weren't seeing these academic risks being taken in the control teachers' classes.

Furthermore, the data suggested that my students might have a somewhat higher perception of their own intellectual autonomy. The two questions that most directly had students reflect on their own intellectual autonomy saw statistically significant results from my students. Additionally on every question save the mean from my students was more autonomous than either other teacher—which might not have made responses for those individual questions statistically significant but would have significance over the span of all the questions. However, considering the very small sample size of the study I do not believe this evaluation to be conclusive. Similarly, even if students in my class are developing a greater sense of intellectual autonomy there are too many variables at play between teachers to identify exactly why my students are developing this autonomy more so than the control group teachers. Furthermore students here were not randomly distributed, a common problem in education research, with Ms. Braum and I teaching an honors class each where students may have higher perceptions of autonomy coming in, and where I teach the only special education section which might have students coming in with lower perceptions of autonomy as compared to their peers.

I do think the data is promising enough to suggest that my students are developing a great sense of autonomy than those in the control group. I recommend further study into what teaching practices vary between me and the control teachers that seem to be leading to this increased sense of autonomy. I also recommend further analysis into other aspects of education that might be worth measuring—for example, are my students scoring lower on standardized tests because he is taking more time to work on student autonomy? Setting up a new series of trials where teachers model steps from my Logic Model (see Appendix A) who did not engage in those practice beforehand, and afterwards comparing their students' perceptions of intellectual autonomy to a control group would be a study worth investigating.

The data suggest that all three teachers have students that are identifying as at least somewhat intellectually autonomous. Research should focus on other schools to see if there is something at Wehnimers that is fostering this perceived autonomy, for instance their highly reform-based mathematics curriculum, or is this data normal for ninth graders from similar school environments?

Asking a question on the student survey that attempted to link students' perception about using white boards and their autonomy would also be a recommended to change to help strengthen (or weaken) the tie I made earlier about the connection between exploratory talk and intellectual autonomy. Further analysis on this connection, regardless of white board usage, is also worth investigation. If math classrooms want to develop independent thinkers than finding ways to promote intellectual autonomy needs to be researched. I think this is a good starting point for some ideas on how student intellectual autonomy might be developed, but this evaluation is merely the tip of the proverbial iceberg.

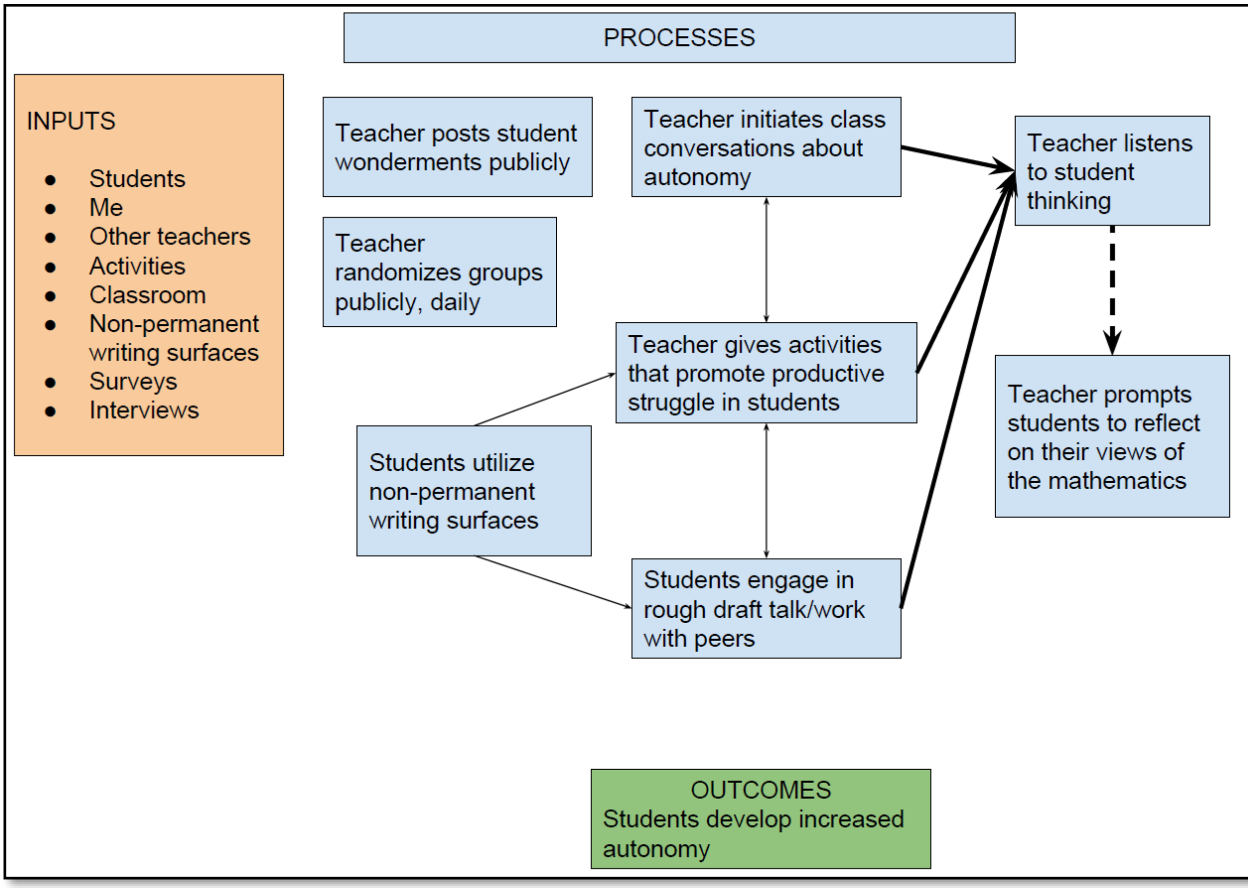
References

Mercer, N., & Hodgkinson, S. (Eds.). (2008). *Exploring talk in school: Inspired by the work of Douglas Barnes*. Sage.

Appendices

Appendix A.

Logic map for promoting student intellectual autonomy in the classroom.



Appendix B

Ninth Grade Math Class Student Survey

Directions: There are 12 questions. All are opinion questions. Please answer honestly.

Your responses are anonymous.

For every question 1 = Strongly Agree, 2 = Agree, 3 = Disagree, 4 = Strongly

Disagree

[There were bubbles to fill in on the actual forms the students received.]

1. I like it best when the teacher shows how to solve a problem, so I can do that on similar problems in the future.
2. I like to make sense of and try to solve problems myself.
3. I get to be creative in math class.
4. It is frustrating when we have to figure out a problem the teacher hasn't shown us how to solve yet.
5. Mathematics is a mental tool that I can use if I want to.
6. Math problems can be interpreted in many different ways.
7. I usually have trouble thinking of good ideas in math class and wait for my peers to show me what to do.
8. The teacher is the mathematical authority in our math class.
9. I really like it in math class when the students debate what is the correct solution to a problem.
10. I depend on the teacher to make sense of the mathematics for me.
11. I've learned to think more for myself this year in math class as compared to prior years in math class.
12. I think for myself in this math class.

Is there anything you'd like to add related to this survey? (Optional.) [This question had space for students to write in a free response.]

Appendix C

Student Interview Questions

1. How has math class this year been different from prior years?
2. Do you think you've grown this year as a mathematician?
3. Do you think you've become more of an independent thinker this year?
4. Is there anything I didn't ask you that you think I should have?

Appendix D

Randomization tool for selecting three students at random to be interviewed.

	A	B	C	D	E	F	G	H
1	Class Roster			Randomized				
2	Student 1		**1**	Student 2				
3	Student 2		**2**	Student 26				
4	Student 3		**3**	Student 8				
5	Student 4			Student 24				
6	Student 5			Student 4			RANDOMIZE	
7	Student 6			Student 9				
8	Student 7			Student 3				
9	Student 8			Student 17				
10	Student 9			Student 30				
11	Student 10			Student 28				
12	Student 11			Student 15				
13	Student 12			Student 1				
14	Student 13			Student 12				
15	Student 14			Student 16				
16	Student 15			Student 19				
17	Student 16			Student 11				
18	Student 17			Student 14				
19	Student 18			Student 5				
20	Student 19			Student 6				
21	Student 20			Student 25				
22	Student 21			Student 13				
23	Student 22			Student 31				
24	Student 23			Student 21				
25	Student 24			Student 7				
26	Student 25			Student 29				
27	Student 26			Student 27				
28	Student 27			Student 18				
29	Student 28			Student 23				
30	Student 29			Student 10				
31	Student 30			Student 20				
32	Student 31			Student 22				
33								

6. Professional Development Video—Spring 2016

The video can be found at the following URL: https://youtu.be/sqrMbiy5nBA

This video defines then compares the Zone of Proximal Development (ZPD) and the Zone of Potential Construction (ZPC). The video synopsizes the work of Norton and D'Ambrosio (2008) claiming that for long-term retention to occur, activities need to be within a student's ZPC not their ZPD. The research shares a case study of a boy who is offered a task within his ZPD but outside of his ZPC, and while the boy meets with some immediate success, he fails to retain the knowledge demonstrating a lack of reorganization of his schema. However, when that same child is retaught with activities structured within his ZPC he is able to demonstrate long-term retention. The implications of this research are made clear in the video—that students who rely on support from the teacher or a more capable peer often do not learn the material despite some evidence that they have. Rather, as educators, we need to care to place activities closer to where students' understandings lie and must be cautious about offering too much help directly or through high-achieving peers.

7.Mentor Sheet Questionnaire

Mentee name: ******** ********

Mentor name: Michael Reitemeyer

School: John Dickinson High School

School year: 2015 – 2016

Mentor feedback form

1. *In what ways do you feel like your mentor helped guide you this year with regard to classroom instruction?*

My mentor supported my classroom instruction in several critical ways this school year. First, I received a significant amount of guidance in curricular decisions and implementation. Since the curriculum is very problem-based, my mentor helped me tremendously in figuring out how and in what ways to support students as they worked through the activities. My mentor helped me to highlight students' ideas as opposed to relying upon my own explanations of how to solve problems.

Additionally, my mentor supported me to consider my messaging to students around mathematics, the curriculum, agency, and productive struggle. When I first began to implement the ideas we discussed, I experienced resistance from students who were unaccustomed to the challenge, rigor, and independence that came along with these practices. My mentor and I worked carefully on strategizing how to build student buy-in to the value of these practices, emphasizing the importance of clear, direct conversations with students as well as relating these practices to other familiar contexts where productive struggle is expected and known to be beneficial. I learned the importance of taking the time to cultivate students to view themselves as doers of mathematics before jumping directly in to these practices which, to many students, are novel.

Even beyond the curriculum, the relationship between agency and motivation were fundamental undertones of my conversations and observations with my mentor around classroom management practices. Together, we analyzed the ways that my classroom could be crafted to respect student agency and grappling, sometimes necessitating less structure and routine on my part as a teacher. We also focused on talk-time within the classroom, consistently searching for ways to minimize teacher lecturing and prioritize student ideas.

Through the support of my mentor this school year, I feel that, most importantly, I was guided to find ways to cohesively integrate values, such as student autonomy and struggle, into all of the many facets that make up classroom instruction. With this united approach toward teaching and learning, I feel that my classroom instruction has become more effective, targeted, and empowering to the students who enter my classroom.

2. *Do you think, as a result of this mentorship, you personally tried to have your students productively struggle with the mathematics more this year than you would have if you had not had this mentorship?*

Yes, I believe that this mentorship did encourage me to implement productive struggle more this year than I would have on my own. Though I have frequently employed socio-cultural lenses to my classroom practices, prioritizing group work, feedback, and negotiating mathematical understanding, productive struggle is a perspective I was not actively considering when the year had started. Combined particularly with the emphases of our problem-based curriculum, my mentor's notions about the importance of student-generated solutions prior to teacher intervention resonated with and complemented many of the practices I had already employed. Importantly, my mentor not only spoke of his beliefs about the importance of productive struggle, but also helped me to think critically about my own discursive moves within the classroom that served to undermine or support productive struggle when he came to observe. Not only did I come to believe in the power of productive struggle through conversation with my mentor, but I felt supported in implementing it effectively within my classroom, which I believe led to my effort and interest in working with this idea.

3. *As a result of this mentorship are you more likely, less likely, or equally likely to have your students productively struggle with the mathematics in your future years of teaching?*

I believe that, as a result of this mentorship, I am more likely to implement productive struggle in my future years of teaching, primarily because this year I saw and experienced the power of productive struggle when it is effectively implemented within the classroom. I watched students create their own impressive representations to conceptualize challenging problems, taking ownership of their models and applying them to solve future problems. I saw students retain material throughout the school year by leaning on their own understanding as opposed to my explanations, which I know will serve them far better in future years of mathematics. Releasing some control of the ideas circulating within my classroom for students to construct built student agency and empowerment within my classroom, as well, creating an even more positive, engaging culture. I am excited to refine and grow my own ideas about productive struggle in my future years of teaching, and I feel that, supported by this mentorship, I recognize the necessity of this element within mathematics teaching and learning and have come away with concrete strategies about how to effectively employ it.

8. Learning Plan Template

~~Lesson~~ *Learning* Plan Template

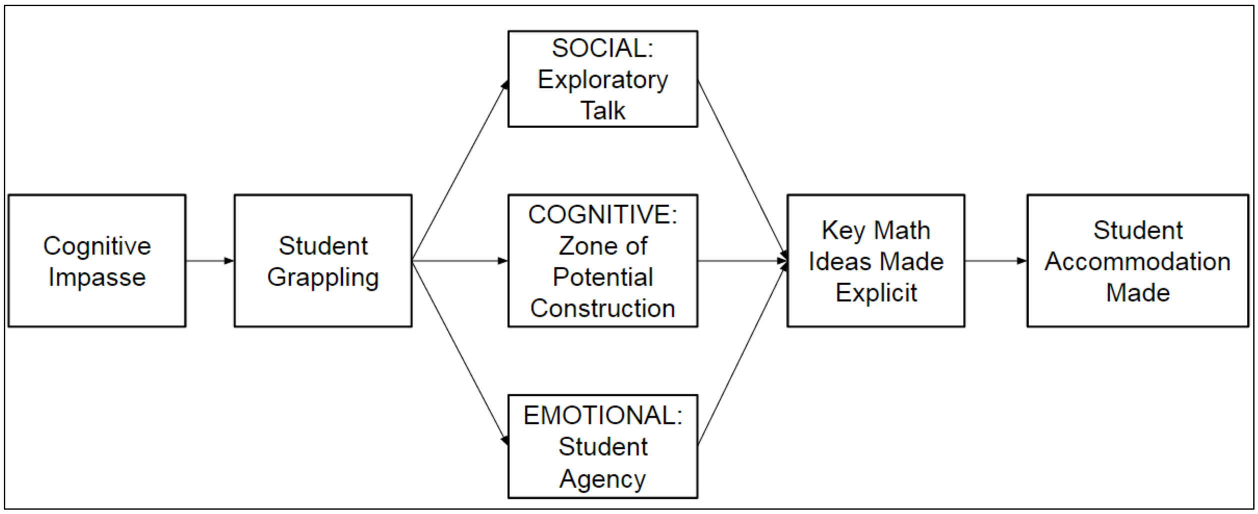
Prerequisite Understandings *What do students need to already understand to access the lesson's learning objective?* *It might help to think about where the mathematics is situated on the larger mathematical continuum—i.e. what math lead up to this concept and what follows it?*
Cognitive Impasse *How are you going to get the students interested in the learning objective?* *i.e. What perplexity, confusion, or doubt are you going to leverage to draw students into the mathematics?*
Claims *What potential claims do you want to draw out from students to get them from their current understanding to understanding the lesson's objective?* *What justifications are you predicting students will use to support these claims?*

Misconceptions *What potential misconceptions do you predict arising while students are working towards the lesson's objective? How do you plan on addressing these misconceptions when/if they do arise?*
Extensions *For students who have already grasped the learning objective, how do you plan on extending/deepening/challenging their thinking about the topic?*

Non-mathematical goals (Optional) *How do you plan to move towards bigger picture goals this unit? What classroom norms or specific activities do you plan on introducing to move you towards these goals?*

9. How Students Learn Conceptually Flowchart and Paper

How Students Learn Conceptually



Cognitive Impasse

John Dewey (1910) famously wrote, "…the origin of thinking is some perplexity, confusion, or doubt" (p. 4). Modern research has supported the importance of instantiating some confusion or cognitive dissonance in a student to help prompt an accommodation in their schema. Defining an impasse as something that, "occurs when a student gets stuck, detects an error, or does an action correctly but expresses uncertainty about it. The basic idea is that an impasse motives a student to take an active role in constructing a better understanding of the principle" (Vanlehn et al., 2003, p. 220). Vanlehn et al. (2003) went so far as to say, "Successful learning appears to require that the student reach an impasse" (p. 209); "The useable data suggest that impasses increase the likelihood of learning but do not guarantee it" (p. 239).

Lehman et al. (2012) explains why they believe impasses lead to greater learning: “These events that trigger impasses place learners in a state of cognitive disequilibrium, which is ostensibly associated with heightened physiological arousal and more intense thought as learners attempt to resolve impasses” (p. 186). The study continued to claim, “Recent research has shown that confusion is both a prevalent emotion during learning and is positively correlated with learning, particularly at deeper levels of comprehension” (Lehman et al., 2012, p. 186, citing Baker et al., 2010; Craig et al., 2004; D'Mello & Graesser, 2011; D'Mello et al., 2010; Graesser, Chipman, et al., 2007; Lehman et al., 2008). A final reference from Lehman et al. (2012): “Thus, inducing confusion not only created opportunities for learning, it also resulted in increased learning, despite the fact that there was not any explicit intervention to help learners regulate their confusion” (p. 190).

Student Grappling

The importance of student grappling in conceptual learning comes from Hiebert and Grouws (2007): “Our interpretation of the literature on teaching for conceptual understanding points to a second feature of teaching that consistently facilitates students’ conceptual understanding: the engagement of students in struggling or wrestling with important mathematical ideas” (p. 387). Hiebert and Grouws (2007) refine what student struggling should and should not look like: “The struggle we have in mind comes from solving problems that are within reach and grappling with key mathematical ideas that are comprehendible but not yet well formed (Hiebert et al., 1996). By struggling with important mathematics we mean the opposite of simply being presented information to be memorized or being asked only to practice what has been demonstrated" (p. 387 – 388). (For the importance of student grappling/productive struggle see also: Lampert, 2001; Schoenfeld, 1994; Henningsen and Stein, 1997; Stein et al., 1996).

This concept of student grappling or productive struggle needs to be further examined. The concept seems to be immensely important in developing students' conceptual understanding, however, simply telling students to "grapple with the mathematics" or to "productively struggle with your assignment" probably will not be very effective. So next I would like to explore how to engage students in this productive struggle.

Social: Exploratory Talk

> "A complex thought system requires a great deal of shared experience and conversation. It is in talking about what we have done and observed, and in arguing about what we make of our experiences, that ideas multiply, become refined, and finally produce new questions and further explorations." (Mary Budd Rowe as cited in Cazden, 2001, p. 61)

I like this above quotation not only because it describes the importance of student talk in learning but because it points out, specifically, that students who dialogue together "produce new questions and further explorations." As we think more deeply about what productive struggle should look like there is an element of perseverance or tenacity in the concept. No one would argue a student productively struggled when he or she spent five seconds on a problem. But, more acutely, what does perseverance on a problem look like in practice? I think we almost always see perseverance on a problem manifest itself in terms of an evolution of student thinking about the problem. We see students grappling with or exploring new questions they produced while working on the original prompt. And Mary Budd Rowe so neatly tied this evolution of new questions and explorations back to students dialoguing.

When Hatano (1988) explored the factors that motivated students to comprehend classroom content, he found "dialogical interaction" to be one of four supporting conditions.

> "When asked to find why a sewing machine can make stitches, Miyake found that pairs of subjects spent as long as sixty to ninety minutes trying to integrate different perspectives and knowledge bases through discussion. One of the pair claimed to understand the device before long, but criticism by the partner created once again the state of nonunderstanding (cognitive incongruity) that motivated the pursuit of deeper levels of understanding." (Hatano, 1988, p. 61)

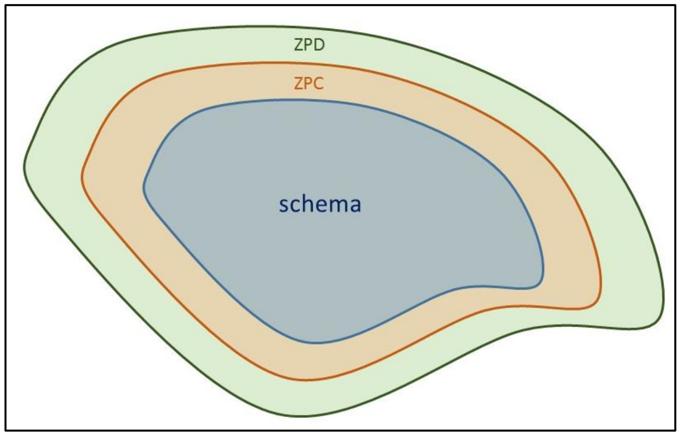
Relating to the previously mentioned importance of perplexity, "Perplexity is induced when one finds different ideas among fellow participants in dialogical interaction. The presence of others expressing different ideas is especially advantageous for amplifying perplexity, because one has to confront them. It is harder to maintain as plausible those ideas one merely reads or is exposed to passively" (Hatano, 1988, p. 61). "In fact, a number of investigators with differing theoretical orientations have found that peer discussion and decision making facilitate meaningful learning, understanding, and cognitive growth (Hatano, 1988, p. 62, citing Inagaki, 1986; Perret-Clermont, 1980; Smith, Johnson, and Johnson, 1981). Hatano (1988) tersely concludes, “One is likely to seek justifications and explanations much more often in dialogical interaction than in solitary activity” (p. 62).

I would like to explore Hatano’s (1988) “dialogical interaction” further, and be more explicit about the type of talk Hatano, I believe, was referring to. I feel this distinction is important because during classroom observations I frequently see groups of students talking with one another, but I do not believe there is the vetting and refining of ideas that Hatano (1988) described. Exploratory Talk (also referred to as Rough Draft Talk (Jansen et. al, 2016)) is described by Barnes (2008) as dialogue where students are “trying out new ways of understanding” (p. 4). Barnes (2008) describes the talk as “hesitant and incomplete” (p. 4) as opposed to presentational talk where the focus is on conveying finished ideas to an audience.

Cognitive: The Zone of Potential Construction (ZPC)

I made a video talking about the importance of the ZPC (here) based on the research of Norton and D’Ambrosio (2008).

The cognitive aspect of productive struggle is what differentiates productive struggle from non-productive struggle. An exaggerated example of non-productive struggle is having my eight year old spend an hour on a Calculus problem, but he is probably not going to learn any Calculus from his efforts as the concepts are well beyond his Zone of Potential Construction or zone “determined by the modifications of a concept a student might make in, or as a result of, interactive communication in a mathematical environment" (Steffe, 1991, p. 193).



So it is imperative for teachers to assess a student's current level of understanding (case history) in order to place him/her in an environment (offer him/her a prompt) that will help him/her refine their schema about a particular concept.

Emotional: Agency

> "Only when people have confidence in their ability to understand and when they experience cognitive incongruity about a target they value (because it is relevant to their lives) are they likely to engage in comprehension activity. Otherwise, they will be reluctant to engage in comprehension activity (which requires much mental effort), and they may suppress the motivation to comprehend." (Hatano, 1988, p. 59)

Here Hatano is explicit that learners need both a cognitive impasse as well as the confidence in themselves to grapple with their impasse. I have already discussed the importance of a cognitive impasse (or "cognitive incongruity" as Hatano referred to it), but a student who reaches an impasse will only take advantage of this learning opportunity if he or she perceives themselves as capable of resolving the impasse (Gresalfi et al., 2009; Warshauer, 2014).

When discussing how to get students to engage in Exploratory Talk, Solomon and Black (2008) noted, "In order to 'try out' new ways of thinking, we need to perceive ourselves as having some agency in or control over what we are doing" (p. 75). Encouragingly, when a student's participation in Exploratory Talk shifts the direction of the discussion (in an assumed meaningful way) that student's agency can be improved (Cobb et al., 1997).

One method teachers can use to promote student agency is offering challenging, ill-structured problems (Henningsen and Stein, 1997; Kapur, 2009; Kapur, 2014). A second method: "One of the ways that students have agency in their learning of mathematics is in developing their own symbols for mathematical relations" (Greeno et al., 1996, p. 36).

Key Mathematical Ideas Made Explicit

The second finding from Hiebert and Grouws (2007) concerning key features of classrooms that promote conceptual understanding is the advantage of making key mathematical ideas explicit—"treating mathematical connections in an explicit and public way" (p. 383). Hiebert and Grouws (2007) cite several studies where students taught mathematics meaningfully—where key ideas and connections were made explicit—demonstrated better retention of the mathematics on both conceptual as well as procedural assessments (Fuson and Briars, 1990; Brownell and Moser, 1949; Hiebert and Wearne, 1993; Good et al., 1983; Good and Grouws, 1979, Fawcett, 1938; Boaler, 1998). If we want students to make sense of a mathematical topic we need to be clear that the connections are being made clear to our students whether by the teacher or by their peers.

Student Accommodation Made

I'm using the "student accommodation made" to essentially mean—the student learned something meaningful about the mathematics. More specifically I am getting to the point that there was some change or reorganization of a student's cognitive structure about a specific topic (this distinction is opposed to a student *assimilating* some knowledge where a student's neuropathways were possibly reinforced or strengthened, and one could argue that is also learning).

I recognize that there are lots of good and necessary ideas about learning that I left out (such as Maslowe's Hierarchy of Needs) and that each of my three branches of productive struggle—social, cognitive, emotional—each have more than one major input developing them. I chose to only address one idea per branch as I believe attending to these three concepts—Exploratory Talk, ZPC, student agency—will have a profoundly positive impact on any teacher's classroom who is attempting to teach for conceptual understanding. I believe adding more ideas per branch would have detracted from the overall from the core of the framework.

10. Classroom Footage from my Integrated Math II Inclusion Class

The video can be found at the following URL: https://youtu.be/IrO9mx9cGlM

The video shows one of my classes working on a linear programming problem. The video demonstrates students working on whiteboards, both horizontal and vertical, and eventually shows groups working together to try to resolve an issue a single group member is having—can you have zero of one of the variables or do you need at least one of each? The video is an attempt to showcase my implementation of what I have learned about creating an environment where students productively struggle with the mathematics. I believe it accurately portrays a student-centered environment where students are working to make sense of mathematics not immediately apparent to them.

Appendix H

THOUGHTS ON TEACHER LIBERATION

I do recognize that the word "liberate" might not be the ideal word in my goal: "More than anything, I want teachers to feel liberated as they shift mathematical authority to the students and their students come to feel empowered in their classes."

However, in my own experience—moving away from a teacher-directed form of instruction towards a classroom where students built math knowledge from one another's ideas and legitimately struggled together to find resolution of these ideas—felt liberating. The only experience from my life I can relate it to is the experience of my Christian salvation. When I associate the feelings I now have towards teaching in a way that empowers rather than oppresses students, I cannot help but recall the following verse from the hymn *And Can It Be* (Charles Wesley, 1738):

> Long my imprisoned spirit lay,
> Fast bound in sin and nature's night;
> Thine eye diffused a quick'ning ray—
> I woke, the dungeon flamed with light;
> My chains fell off, my heart was free,
> I rose, went forth, and followed Thee.
> (URL of hymn being sung: https://youtu.be/nO43CHWBbbw?t=133)