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Highlights

- Hydroacoustic waves resulting from tsunamigenic seabed movement are simulated by LBM.
- LBM can properly handle both complex generation and propagation of hydroacoustic waves.
- The effects of seabed unevenness and porosity are confirmed by LBM results.
- LBM efficiency makes it appealing for planetary scale applications.
Lattice Boltzmann approach for hydro-acoustic waves generated by tsunamigenic sea bottom displacement

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\section*{Abstract}

Tsunami waves are generated by sea bottom failures, landslides and faults. The concurrent generation of hydro-acoustic waves (HAW), which travel much faster than the tsunami, has received much attention, motivated by their possible exploitation as precursors of tsunamis. This feature makes the detection of HAW particularly well-suited for building an early-warning system. Accuracy and efficiency of the modelling approaches for HAW thus play a pivotal role in the design of such systems. Here, we present a Lattice Boltzmann Method (LBM) for the generation and propagation of HAW resulting from tsunamigenic ground motions and verify it against commonly employed modelling solutions. LBM is well known for providing fast and accurate solutions to both hydrodynamics and acoustics problems, thus it naturally becomes a candidate as a comprehensive computational tool for modelling generation and propagation of HAW.

\textit{Keywords:} tsunami, hydro-acoustics, Lattice Boltzmann, early warning

\section*{1. Introduction}

Submarine earthquakes and submarine mass failures (SMFs), such as landslides and slumps, can generate long gravitational free surface waves (or tsunamis), and pressure waves or hydro-acoustic waves, HAW. The latter are emitted by the column of water above the generation zone, which, due to its compressibility, acts as an oscillatory generator for quasi-horizontally traveling HAW. Tsunami waves can travel for long distances and are known for their dramatic effects on coastal areas. HAW travel roughly 10 times faster than the tsunami, effectively at the speed of sound in water. The presence of HAW in a pressure record therefore can anticipate the arrival of the tsunami, thus serving as a potential means for tsunami warning.

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Researchers have studied two frequency ranges for acoustic waves associated with tsunamigenesis: (1) low frequency HAW with characteristic frequencies $\sim 0.1$ Hz, and (2) T-waves, in frequencies in the range between 2 and 100 Hz. Low frequency HAW generated by seabed motion were measured during the Tokachi-Oki 2003 and Tohoku-Oki 2011 tsunami events by the Japan Agency for Marine-earth Sciences and TEChnology observatory and later have been used to estimate amplitude, duration, and velocity of bottom displacements, and as benchmarks for 3D and 2D numerical models (Nosov and Kolesov, 2007; Bolshakova et al., 2011; Abdolali et al., 2015d). During the 2012 Haida Gwaii event in Canada, bottom pressure signals were used to reveal the frequency ranges associated with gravity waves and HAW (Abdolali et al., 2015a). Higher frequency HAW records, or T waves, have also been applied to a number of tsunami characterization problems, including determination of the timing of the 1998 Papua New Guinea SMF event (Okal, 2003), estimation of the rupture length and velocity of bottom deformation of the 2004 Great Sumatra earthquake (de Groot-Hedlin, 2005), and estimation of the submarine landslide speed during the West Mata eruption in 2010 (Caplan-Auerbach et al., 2014). The variation of sound speed within the water column, multipathing of sound waves, and summation of direct and surface-reflected arrivals causes interference patterns in the hydro-acoustic spectrum, giving useful insights into the recognition of the type of bottom motion (i.e. earthquake, landslide or eruption), depth of occurrence and duration (Chierici et al., 2010).

To date, HAW as precursors of tsunamis have been numerically modeled by means of linear models (Nosov and Kolesov, 2007; Bolshakova et al., 2011) derived from the Compressible Euler (or inviscid Navier-Stokes) equations (hereinafter CEE). As these approaches are computationally demanding, resort has been made to vertically integrated formulations of the same set of equations, as in Sammarco et al. (2013) and Abdolali et al. (2015c). These models proved to be not only accurate enough to capture the main far-field features of HAW for idealized cases, but also efficient enough to be employed to reproduce real tsunamis (Abdolali et al., 2014, 2015a; Cecioni et al., 2015; Abdolali et al., 2015d). The main drawback of these depth-integrated models lies in leaving out both the horizontal component of bed deformation and vertical variations of the propagation medium. These omissions may be the limiting factor to accuracy in all cases when the multiple reflection between uneven seafloor and surface causes non-trivial propagations patterns. The interplay between the abovementioned aspects occurs when wave reflec-
tion, induced by uneven seafloor, gives rise to modification of the grazing angle of wave fronts, which in turn allows for high frequency bands to be trapped into the SOFAR channel (Johnson et al., 1963; Talandier and Okal, 1998; de Groot-Hedlin and Orcutt, 1999; Williams et al., 2006). In order to simulate these phenomena, resort has to be made to fully 3D (or 2D in the vertical x-z plane) models whose computational demand makes their employment over large areas, even for the reproduction of mid-frequencies bands, often prohibitive. The approach that we propose in this work is to solve the non-linear compressible Navier-Stokes equations by means of the Lattice Boltzmann Model (LBM), which guarantees enough computational efficiency to make the non-depth-integrated solution feasible also for real cases. Moreover, as further explained later, LBM, being a fully non linear fluid dynamics solver, does not rely on any assumption either on the magnitude or the type of bottom displacement, thus effectively opening the possibility to directly simulate HAW deriving by complex SMFs. Regarding computational efficiency, in the specific field of acoustics, LBM has been proven to require a lower number of operations to advance the solution for a fixed time interval, given a fixed absolute dispersion error, than is commonly required in conventional finite difference methods (Marié et al., 2009). Moreover, the ease of parallel implementation and the extreme scalability of the LBM lie at the heart of its huge worldwide spread as a computational fluid dynamics solver. Several works (Bernaschi et al. (2009) and Obrecht et al. (2013), for example) have reported quasi-ideal speed-up of LBM on thousands (16384) of cores or GPU. These impressive performances naturally suggest the LBM as an efficient computational framework very attractive for performing coupled hydro-acoustic/hydrodynamic simulations for real and deferred time tsunami simulation.

The LBM approach not only allows us to overcome the aforementioned limitations of the depth integrated models, but provides further intrinsic advantages also over the 3D CEE models: it opens the possibility to integrate an hydro-acoustic solver with an efficient and extremely flexible hydrodynamic solver, leading to what is known as the “direct approach” that is, modeling the hydrodynamic processes leading to the formation of HAW, just like their counterpart in aeroacoustics (de Jong et al., 2013). In perspective such LBM direct approach could be easily coupled with LBM wave models in order to perform efficient planetary-scale hydro-acoustic simulations (Metz et al., 2016).

The aim of this work is to present and validate a LBM model for the gener-
ation and short range propagation (∼10 times the fault length) of pressure waves in an idealized 2D (x−z) compressible ocean. Generation consists in a fast vertical displacement of a portion of the seafloor. No free surface is modeled at this stage, and thus the gravity mode representing the main tsunami gravity wave does not appear as part of the solution. Validation is carried out employing results from a previously validated model. Section 2 provides an overview of formation of HAW. Section 3 describes the Lattice Boltzmann Method for compressible fluids, scaling and forcing boundary condition. Verification of the LBM model is carried out for constant and varying geometries against both numerical and analytical solutions of CEE in Section 4. Conclusions and perspectives are given in Section 5.

2. Hydro-acoustic waves generated by seabed movement

HAW recorded during tsunami events are often characterized by highly energetic power spectra, which segregate into discrete peaks centered around cutoff mode frequencies. Their energy content is comparable to that of the tsunami gravity wave. The dominant frequency range in the wave spectrum can be expressed by a discrete set of normal frequencies \( f(n) \) given by

\[
f(n) = (2n - 1) \frac{c}{4h}, \quad n = 1, 2, 3, \ldots
\]

where \( h \) and \( c \) are respectively the depth and the speed of sound of water. It has previously been highlighted (Nosov and Kolesov, 2007) that the assumption of perfectly reflecting seabed could introduce inaccuracies in the modelling of the propagation of HAW. Indeed, introducing a single underlying sediment layer, acting together with the water column, lowers the spectral peak frequencies, \( \gamma(n) \), which are then determined from the following transcendental equation (Abdolali et al., 2015b):

\[
\tan \left[ \frac{2\pi \gamma(n) h}{c} \right] \tan \left[ \frac{2\pi \gamma(n) a}{c_s} \right] = \frac{\rho_s c_s}{\rho c}
\]

where \( a \), \( c_s \) and \( \rho_s \) are respectively thickness, sound speed and density of the sediment layer; \( \rho \) is water density. Equation (2) relies on the treatment of the sediment layer as a “fluid like” layer (Chierici et al., 2010). More complex and physically grounded approaches have also been proposed (Balanche et al., 2009; Maeda and Furumura, 2013), but require the coupling between different
Another important aspect related to the propagation of HAW into complex geometries is the filtering effect induced by shallow regions. Indeed HAW generated at large depths, dominated by the first normal mode, undergo a cut off condition when propagating into shallower regions. This shift in the power spectrum should be taken into account when planning the arrangement of new measurement probes, or accounted for when analyzing arriving signals at shallow depths. A test case investigating the filtering effect of the shallow areas is included in the following.

Stiassnie (2010) provided an analytic and detailed calculation for wave radiation by a piston bottom displacement, in a compressible ocean of constant depth to calculate the nondimensional free surface and dynamic bottom pressure acoustic-gravity wave modes at a large distance from the source within the framework of a two-dimensional linear theory. This analytical solution has been employed as a reference solution in the following. In order to better explain the generation and propagation dynamics, the reader is referred to the provided supplemental video, showing the pressure field in the whole domain for a test case introduced later on.

3. LBM model for hydro-acoustic wave propagation

Our hydro-acoustic simulations are based on the well-known Lattice Boltzmann equation with the Bhatnagar-Gross-Krook collisional (LBGK), whose main aspects are briefly described in the following.
The LBGK equation reads as follows (Succi, 2001; Aidun and Clausen, 2010):

$$f_i(\vec{x} + \vec{c}_i \Delta t, t + \Delta t) = f_i(\vec{x}, t) + \frac{\Delta t}{\tau} (f_{i}^{eq} - f_i(\vec{x}, t)) \quad i = 0, \ldots, Q - 1 \quad (3)$$

where $f_i(\vec{x}, t)$ is a set of probability distribution functions (PDFs), representing the probability of finding a particle at position $\vec{x}$ and time $t$ with a lattice velocity $c_i$, $i$ spanning over all the $Q$ lattice directions (see Fig. 2). In the above equation, the left hand side is the lattice transcription of the free flight of the molecules streaming along the lattice directions, while the right hand side describes the collisional relaxation of the set of PDFs towards the set of truncated low-Mach number expansion of the Maxwell-Boltzmann distribution (Chen and Doolen, 1998):

$$f_{i}^{eq} = w_i \rho \left[ 1 + \frac{(\vec{c}_i \cdot \vec{u})}{c^2} + \frac{(\vec{c}_i \cdot \vec{u})^2}{2c^4} - \frac{\vec{u} \cdot \vec{u}}{2c^2} \right] \quad i = 0, \ldots, Q - 1 \quad (4)$$

where the $w_i$ are the weights of the discrete Maxwellian $\vec{u}$ the macroscopic flow velocity, and $c$ the lattice speed of sound given by

$$c^2 = \sum_i w_i c_i c_i \quad (5)$$

In the present work the two-dimensional nine speed (D2Q9) lattice is employed which supports discrete reconstruction of exact moments up to second order. This formulation allows for recovery of the Navier-Stokes equations for weakly compressible flows with speed of sound $c^2 = 1/3$ (He and Luo, 1997), in the form of eq.(6)

$$\begin{cases}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \\
\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = -\frac{1}{\rho} \nabla p + \nu \Delta \vec{u}
\end{cases} \quad (6)$$

with an ideal equation of state $p = \rho c^2$. It can also be shown (Succi, 2001) that the fluid kinematic viscosity can be controlled via the collisional relaxation time $\tau$ through the following relation:

$$\nu = c^2 \left( \tau - \frac{\Delta t}{2} \right) \quad (7)$$
It is worth noting that a first-order Mach expanded set of equilibria could have, in principle, been employed along with a five speed D2Q5 lattice thus reducing the computational burden considerably. This is due to the fact that, in the benchmark cases considered in this work, nonlinear terms in the hydrodynamic equations are negligible. Nevertheless, we chose to employ the "classical" D2Q9 lattice (see Fig. 2) supported by a 2nd-order expansion of the Maxwell-Boltzmann distribution functions, which permits to recover the weakly-compressible Navier-Stokes equations as hydrodynamic limit, since, in perspective, the aim will be to deploy a full set of efficient kinetic-based computational techniques able to provide accurate and fast solution for coupled hydrodynamic-hydro-acoustic problems. Moreover, in order to mimic the presence of a sediment layer, with a different speed of sound compared to sea water, we employ a variant of the LBM (Viggen, 2014) which allows for non-uniform spatial distribution of the speed of sound by modifying the set of equilibrium distribution functions accordingly. More precisely, in this model the set of equilibrium distribution functions is defined as follows:

\[
\begin{align*}
    f^0_{eq} &= w_0\rho \left[ \frac{c_k^2}{c^2} + \frac{1}{w_0}(1 - \frac{c_k^2}{c^2}) - \frac{\vec{u} \cdot \vec{u}}{2c^2} \right] \\
    f^i_{eq} &= w_i\rho \left[ \frac{c_k^2}{c^2} + \frac{(\vec{c}_i \cdot \vec{u})^2}{c^2} + \frac{(\vec{c}_i \cdot \vec{u})^2}{2c^4} - \frac{\vec{u} \cdot \vec{u}}{2c^2} \right] \quad i = 1, ..., Q - 1
\end{align*}
\]  

(8)

where \( c_k \) is the local speed of sound. For \( c_k = c \) the set of discrete equilibria reduces to that of eq. (4). It can be shown (Viggen, 2014) that this extended model allows for recovery of the quasi-compressible Navier-Stokes equations.
with an ideal equation of state \( p = \rho c^2 \). The LBM algorithm consists of a series of successive streaming and local collision steps, interleaved by imposition of boundary conditions and calculation of macroscopic variables as discrete statistical moments performed in the velocity space:

\[
\begin{align*}
\rho(\vec{x}, t) &= \sum_i f_i(\vec{x}, t) \\
p\vec{u}(\vec{x}, t) &= \sum_i f_i(\vec{x}, t)\vec{c}_i
\end{align*}
\]

(9)

### 3.1. Scaling and numerical simulation details

In order to set up the numerical simulations, a proper scaling of the problem from physical units (PU) to lattice units (LU) needs to be imposed. Lattice units for space and time are assumed to be integers, which is tantamount to assuming \( \Delta x_{LU} = \Delta t_{LU} = 1 \). Since we are dealing with propagation of pressure waves, Mach scaling (\( Ma_{PU} = Ma_{LU} \)) seems appropriate. The Mach number is defined as \( Ma = \frac{u_{ref}}{c} \) where \( u_{ref} \) is a suitable reference fluid velocity, while \( c \) is the speed of sound of the system. In this study the pressure waves are generated by a displacement of the sea floor, i.e. a fault of length \( L_f \), moving at velocity \( u_f \) for a time \( t_f \). The reference velocity was chosen as the speed of the displacement. Since such velocity is \( u_{PU}^f \approx 1 \text{ m/s} \) and the usual speed of sound of water is \( c_{PU} \approx 1500 \text{ m/s} \), the Mach values at hand are relatively low (\( Ma \approx 10^{-3} \)). Given that the speed of sound in a D2Q9 based LBM model is \( c_{LU} = 1/\sqrt{3} \), the value of the fault velocity in lattice units can be determined as \( u_{LU}^f = Ma \cdot c_{LU} \). The frequency of the generated pressure waves depends mainly on the travel time of pressure perturbations between the source at the sea floor and the surface. This mechanism is accounted for in the scaling of the physical domain: travel time, namely \( h/c \), was scaled by fault duration \( t_f \) leading to a numerical depth:

\[
h_{LU} = h_{PU} \frac{c_{LU}^f t_{LU}^f}{c_{PU}^f t_{PU}^f}
\]

(10)

The lattice spacing and time step in physical units are then determined to be:

\[
\Delta x_{PU} = \frac{h_{PU}}{h_{LU}}
\]

(11)

\[
\Delta t_{PU} = \frac{t_{PU}^f}{t_{LU}^f}
\]

(12)
When an additional layer of sediment is modelled, this portion of the domain is flagged with a different speed of sound according to the formulation reported in Viggen (2014). The Mach scaling is enforced there as well, and the thickness of the layer is determined by applying the same procedure as for the uniform speed of sound.

The bottom displacement was modelled by adding the external momentum to the standard second order bounce back, as described in Bouzidi et al. (2001). No body force such as gravity was modeled. The condition at the boundary opposite to the one containing the fault is of specified pressure type, through equilibria 4 as discussed in Mohamad and Succi (2009). No surface movement is modelled, assuming its effect on HAW generation to be negligible (Smith, 2015). Viscosity was set as low as numerical stability allowed, in order to match as closely as possible the inviscid reference solution; the minimum relaxation time still providing stable results was found to be \( \tau_{LU} = 1/2 + 10^{-4} \).

For the reference numerical solution, the governing linear equation and boundary conditions are formulated in terms of the velocity potential and solved by means of a finite-element solver. Description of linear models for weakly compressible fluids for the cases of rigid and permeable bottoms is presented in Sammarco et al. (2013) and Abdolali et al. (2015c). Given the symmetry of the problem about the mid-point of the earthquake, computations are undertaken only for half of the physical domain. In order to reduce the computational costs, the domain was kept as small as possible by resorting to a Sommerfeld radiation condition along a boundary placed at a finite distance from the wave source. In order to correctly reproduce the wave field, the maximum mesh size and time step are chosen to be 100m and 0.05s respectively.

4. Results

In this section the validation of the model is presented. Three series of synthetic simulations have been carried out. They encompass generation and propagation of HAW in a 2D ocean, the former consisting in a fast vertical displacement of a portion of the bottom. All test cases have been run with the duration of the bottom movement set to \( t_f = 2s \). Numerical and physical
details of each test case are reported in Table 1. Reference solutions are yielded by both linear analytical and numerical (CEE) formulations.

<table>
<thead>
<tr>
<th>Test case</th>
<th>$\Delta x$ (m)</th>
<th>$\Delta t$ (s)</th>
<th>$N_z$</th>
<th>$N_x$</th>
<th>$Ma$</th>
<th>$u_f$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat seabed</td>
<td>37</td>
<td>$1.4 \times 10^{-2}$</td>
<td>40</td>
<td>$10^4$</td>
<td>$6.67 \times 10^{-4}$</td>
<td>1.0</td>
</tr>
<tr>
<td>Variable depth</td>
<td>52</td>
<td>$2.0 \times 10^{-4}$</td>
<td>77</td>
<td>$10^4$</td>
<td>$1.34 \times 10^{-3}$</td>
<td>2.0</td>
</tr>
<tr>
<td>Sediment layer</td>
<td>37</td>
<td>$1.4 \times 10^{-2}$</td>
<td>40 $+ 18^*$</td>
<td>$10^4$</td>
<td>$6.67 \times 10^{-4}$</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 1: Numerical details of the validation test cases (LBM). *thickness of sedimentary layer

4.1. Flat seabed

In this benchmark the depth of the ocean is uniform and the bottom rigid. The model is tested by comparing the signal at a probe placed on the bottom at 50 km distance from the epicenter of the fault against potential flow solution. Moreover a frequency analysis is carried out in order to compare the frequency of the amplitude peaks with the cutoff frequencies given by eq.(1). Such comparisons are depicted in Fig. 3, panels (c) and (d). The black and light gray lines represent potential flow solver output and the analytical solution of Stiassnie (2010) respectively while the gray line corresponds to nonlinear LBM solver. The frequency peaks coincide with the cut off frequencies for ideal impermeable bottom ($f^{(1)} = 0.25$ and $f^{(2)} = 0.75$ Hz). The correct reproduction of the signal modulation is an important characteristic of such models, since several relationships have been put forward (Chierici et al., 2010) between the features of the signal and those of the bed movement.

In Fig. 4 the vertical distribution of pressure is plotted in time at two different distances from the epicenter. The first location, above the fault (10km from the epicenter), shows the superimposition of several high frequency modes onto low frequency once. These modes do not propagate away from the fault location and are thus referred to as “evanescent”; as such, they are barely visible in a record measured at a further location (see lower panel in the same figure), where LBM recovers the reference solution, confirming its capability to correctly interpret technically interesting HAW.

Since the LBM model is here proposed as an integrated hydrodynamic-acoustic simulation tool, it is important to assess its capabilities to provide accurate acoustic results even when viscosity needs to be set according to a
dynamic scaling (Reynolds scaling) in addition to a sonic one (Mach scaling) as done in this work. In order to show this property, a comparison was carried out between two LBM simulations with viscosities 10 times higher, and the respective spectra compared. As shown in Fig. 5, no relevant differences can be seen between the two simulations, proving the abovementioned property.

Figure 3: The case of constant depth, \( h = 1500 \) m, \( a = 681 \) m, at point 50 km from the tsunamigenic source, with fault semi-length \( b = 15 \) km moving with (a) velocity \( u_f \) and (b) displacement \( \zeta \). (c, e) Time series of bottom pressure; and (d, f) corresponding spectra; (c, d) for single layer of water, (e, f) for water overlying sedimentary layer, \( c_s = 2000 \) m/s; obtained from CEE solver (Black line), theoretical solution (Light gray line) and LBM solver (Gray line). The vertical dashed line in panels (d, f) represent cut off frequencies yielded by eqs. (1) and (2).
4.2. Sediment layer

In this case the bottom of the sea is not considered to be purely reflective but mimics the behavior of a sediment layer with higher speed of sound and damping properties than water. The effect of the sediment layer on HAW is assumed to be the same as the one yielded by a “fluid like” layer (Chierici et al., 2010). In this case the effect of density variation is neglected in order to highlight the influence of the difference in celerity. Results are depicted in Fig. 3, panel (e) and (f). The down shift in frequency is captured with remarkable accuracy by the LBM model. For the case of water column interacting with sedimentary layer, the spectrum is peaked at $\gamma^{(1)} = 0.19$ and $\gamma^{(2)} = 0.56$ Hz, representing cut off frequencies evaluated by Eq. (2). The LBM model with non uniform speed of sound does not require any additional calculation compared to the standard uniform formulation, and thus the agility of the
Figure 5: Spectrum of the pressure signal above the fault for the flat seabed case, as calculated by the LBM model with two different viscosity values (given in lattice units). Vertical lines locate cutoff modes according to eq.(1). Viscosity does not induce either damping or shifting of the cutoff peaks.

LBM scheme is not spoiled.

4.3 Variable depth

This case consists of an ocean with non-uniform depth, with deep and shallow portions connected by a gentle slope of the seafloor as depicted in panel (a) of Fig. 6. The motivation of this test case is to check for the model capability of allowing only high frequencies to propagate from the deep part, where the generation occurs, to the shallow one. The results are presented in Fig. 6 in terms of time series of bottom pressure ($P$) and corresponding spectra ($\tilde{P}$) at point $P_1$, 190 km from the epicenter at 4 km water depth.
dominated by the first normal mode, $f_{P_1}^{(1)} = 0.09$ Hz (panels b, c) and point $P_2$, 340 km from tsunamigenic source at 2 km water depth (d, e). It can be seen from model results that HAWs with frequencies lower than $f_{P_2}^{(1)} = 0.19$ Hz cannot propagate in the waveguide in shallower depth and, as a result, are filtered. The reflected waves from slope are superimposed in the arriving wave trains and change the wave pack modulation.

Figure 6: The case of varying sea bottom with a tsunamigenic source at the deeper part. (a) The computational domain. (b, d) Time series of bottom pressure and (c, e) corresponding spectra at point $P_1$, 190 km from the tsunamigenic source at 4 km water depth (b, c), and point $P_2$, 340 km from the tsunamigenic source at 2 km water depth (d, e), obtained from CEE solver (Black line) and LBM solver (Gray line). The vertical dashed lines in panels (c, e) represent first cut off frequency for deep and shallow depth manifesting depth effect on frequency spectrum.
5. Conclusions and perspectives

In this paper, a Lattice Boltzmann Model (LBM) model is presented for the combined generation and propagation of hydro-acoustic waves (HAW) triggered by tsunamigenic bottom movement. The model is successfully validated against several benchmark cases obtained using conventional finite element techniques. The generation of HAW often involves complex hydrodynamics (e.g. landslides) and thus requires a fully non linear model to properly handle it; propagation, on the other hand, has been so far successfully carried out employing linear models, both 3D or depth integrated, the first being still too computational demanding for large scale/high frequencies applications, the latter neglecting important variations of quantities along the water column. The resort to LBM is in this sense twofold: it allows for efficient investigation of the effect of vertical propagating features, while still encompassing enough physics to simulate complex nonlinear hydrodynamics. Future work will be aimed at assessing the generation and propagation of high frequency HAW, which is beyond the capabilities of currently modeling approaches. In this regard, LBM is the natural candidate for playing a central role in the development of an integrated numerical framework for both real and deferred time investigation of tsunami waves.

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