SOCIAL SECURITY POLICIES
IN AN OVERLAPPING GENERATIONS MODEL:
APPLICATIONS TO THE U.S. AND CHINA

by

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ABSTRACT

In this paper, I find a social security policy with intergenerational transfer may outperform in terms of welfare several balanced-budget alternatives as the economy undergoes a temporary demographic change. With the help of an overlapping generations model I quantify the effects of a demographic change on the economy in the steady state and along the transition path through numerical simulation. I calibrate the model to the U.S. to compare the various social security policy alternatives against fund accumulation in accommodating a population bulge. In addition, I calibrate the model to China to compare the various social security policy alternatives against intergenerational borrowing for the case of a reduced population growth.

Chapter 1

INTRODUCTION AND LITERATURE REVIEW

1.1 Introduction

In the past several decades, fertility has been declining and longevity has been increasing in many countries (Lee 2003). Population aging exerts great financial pressure on defined-benefit (DB), pay-as-you-go (PAYG) social security systems around the world. In a typical PAYG system, such as Social Security in the U.S., an individual is promised a stream of benefits upon retirement that depends on the worker's earning history, years of services, and age, and payments to the retirees in each year are financed through payroll taxes on workers in the same year. Solvency of a PAYG social security system depends critically on the ratio of the number of contributing members, which is usually the number of workers, to the number of beneficiaries, which is usually the number of retirees.[1] The declining fertility together with increasing longevity reduces the worker–retiree ratio and thus requires higher tax rates on the workers or lower benefits to the retirees to keep the budget balanced, or government borrowing to finance the promised benefit payments. The main goal of this study is to assess various social security policy options in terms of welfare performance in the face of a demographic event.

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[1] A closely related statistic is the dependency ratio, often defined as the ratio of the population aged 65 and over to the population aged 15–64. The Board of Trustees of the Federal Old-Age and Survivors Insurance and Federal Disability Insurance Trust Funds define the dependency ratio as the ratio of the population aged 65 and over to the population aged 20–64 in their annual reports to the Congress.

The U.S. experienced a baby boom after World War II. The total fertility rate (TFR) started to drop in the 1960s and continued to drop in the early 1970s. Although the TFR slightly increased from the late 1980s through the late 1990s, recent years saw a renewal of declining (Figure 1.1). According to Board of Trustees (2016), the number of covered workers per Old-Age, Survivors, and Disability Insurance (OASDI) beneficiary has declined since 2008 and is projected to continue to decline rapidly in the coming decade as the baby-boom generation begins to retire. In addition, the cost of the OASDI program has exceeded the cash (non-interest) income since 2010 and the OASDI Trust Fund is projected to be depleted in 2033 under the Trustees' intermediate assumptions. Many researchers and policymakers have proposed various reforms on Social Security to address its long-term solvency (see Section 1.2.2 for a brief discussion).

China's demographic trend and social security system are very different from those of the U.S. The TFR in China was high (at around 6) in 1950–1970, followed by a sharp decline by more than 50% in the 1970s, and has remained below the replacement level since the early 1990s (Peng 2011; Zeng and Hesketh 2016; Cai 2013; Figure 1.1). The TFR by the time of the 2010 population census is estimated to be about 1.5 (Cai 2013). The end of 2015 marked the end of China's one-child policy and the start of a universal two-child policy (Wang et al. 2016; Zeng and Hesketh 2016). National Bureau of Statistics of China estimated that the number of newborns in 2016 was 17.86 million, which was 1.31 million more than the number of newborns in 2015 and was the highest since the year of 2000.[2] With the universal two-child

[2] See http://www.stats.gov.cn/tjsj/zxfb/201701/t20170120_1455942.html (in Chinese) and http://news.xinhuanet.com/2017-01/23/c_1120364891.htm (in Chinese).

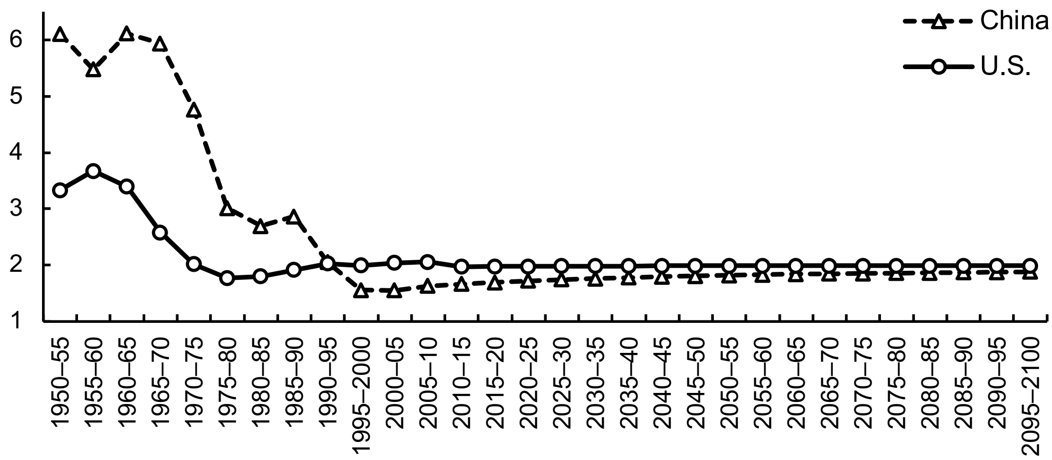

Figure 1.1: Historical and projected total fertility rates in the U.S. and China, 1950–2100.

policy having been effective for less than two years, it is difficult to assess the amount by which the TFR in China will be raised in the future due to the new policy. Zeng and Hesketh (2016) project that the TFR will be 1.81 in 2030. The United Nations' projection of China's TFR also converges to about 1.8 in the long run (Figure 1.1).

China has a multi-pillar social security system with a first pillar consisting of a defined-benefit, PAYG basic pension plan and a second pillar consisting of individual accounts (Cai and Cheng 2014; Liu and Sun 2016).[3] China's social security system faces considerable financial difficulties that are of historical origins, that are administrative, and that are caused by the demographic transition (Cai and Cheng 2014). In some provinces, the basic pension fund has been running deficits for some

[3] Urban workers and civil servants have additional coverage for medical care, disability, unemployment, and maternity.

years and funds in individual accounts have been diverted to finance the defined-benefit pillar, creating many "empty" individual accounts (Liu and Sun 2016). Such diversion of funds seriously jeopardizes the function of individual accounts and the Chinese government has been working on fully funding the individual accounts.

1.2 Literature Review

In this section, I review multiple strands of the vast literature on social security related to this study. The first subsection focuses on how the demographic transition, in particular changes in fertility, affects the social security system. The second subsection concerns a brief discussion of various reform proposals in the debate on Social Security reform in the U.S. The history of the social security system in China is reviewed in the last subsection.

1.2.1 Fertility and Social Security

Auerbach and Kotlikoff (1987) conduct a general-equilibrium analysis of the effects of the demographic transition on the economy and on social security in an overlapping generations (OLG) model. The authors consider two demographic scenarios: one "bust" scenario in which the fertility rate has a permanent decline and one "bust-boom-bust" scenario in which the fertility rate starts with a decline, followed by a temporary rise, and then followed by a permanent decline. They find via simulating the transition path that the payroll tax rate required to balance an unfunded, PAYG social security system has to rise substantially eventually in both scenarios, except that in the second scenario the boomers first slow down the increase in the payroll tax rate and then causes the rate to overshoot when the boomers retire. Despite the substantial increase in the payroll tax rate to finance social security, Auerbach and

Kotlikoff find considerable long-run welfare gains in both scenarios due to increased capital intensity and a decreased ratio of children to workers.

De Nardi et al. (1999) employ an OLG model to analyze the general-equilibrium implications of the demographic transition on Social Security. Compared to Auerbach and Kotlikoff's OLG model, De Nardi et al.'s model includes additional features, such as labor-augmenting technology, stochastic lifetimes and endowments, and an active bequest motive. The authors find that the projected demographic trends in the U.S. will result in a sharp rise in the labor income tax rate as well as a sizeable fall in the labor supply, the capital stock, and output, if Social Security is to be financed through the labor income tax.

Galasso and Profeta (2002) review several studies on how the demographic transition may affect the social security system from the perspective of the political economy. Bohn (1999, 2003) show for the U.S. that the median voter would gain from participating in Social Security in the face of the retirement of the baby boomers and thus Social Security is politically viable under majority voting. In the baseline case, population aging (resulted from either lower fertility or higher life expectancy) reduces the net present value of Social Security as the payroll tax rate has to be increased to maintain the legislated level of benefits and balance the budget, while it also raises the median voter's age and keeps the median voter's age above the critical age. Breyer and Stolte (2001) and Galasso and Profeta (2004) also find that population aging leads to an expanded PAYG social security program under majority voting. Boldrin and Rustichini (2000), however, show that under majority voting a PAYG social security system may be supported by rational voters but will be eventually be abandoned in the presence of a stochastic, asymptotically decreasing population growth rate.

While the above studies examine the economic and political consequences of the demographic transition on the social security system and assume exogenous fertility, many authors seek to understand the effects in the opposite direction, i.e., how expansion of social security programs in the U.S. and around the world affects fertility. Numerous theoretical and empirical studies usually identify a negative effect of social security expansion on fertility. See, e.g., Lapan and Enders (1990), Ehrlich and Kim (2005, 2007), Cigno and Rosati (1992, 1996), Cigno et al. (2003), Zhang and Zhang (2004), Boldrin et al. (2015).

1.2.2 Social Security Debate in the United States

The debate on how to reform Social Security in the United States to maintain financial sustainability of the program has persisted for over two decades. One direction of reform is a "parametric" reform, which is to change certain parameters of the existing Social Security program such as the retirement age, the tax rates, and the benefit levels. These actions may alleviate the financial burden on the Social Security system if the demographic trend continues as currently projected; however, they are not adaptive to unexpected future demographic events and they alone cannot financially secure the program once and for all (Wise and Woodbury 2009).

Another direction is a structural reform, which is to fundamentally change how the Social Security program is funded or how the Social Security benefits are distributed. Various proposals have emerged. Some propose partial or complete privatization of the current PAYG Social Security system (e.g., Feldstein 2005b). Privatization requires setting up personal retirement accounts (PRAs) and diverting some or all of the contributions to the PRAs. Individuals can invest their funds in the PRAs in government bonds as well as equities and have control over the portfolios. Upon retirement, instead of receiving benefits according to some legislated formula under the current defined-benefit Social Security system, individuals with PRAs receive whatever accumulated in their PRAs and can purchase an annuity if they would like to. A privatized (defined-contribution) Social Security system has multiple advantages over a PAYG (defined-benefit) system. First, privatization promotes national saving and a higher capital intensity, which enables individuals to enjoy higher consumption in the long run (Seidman 1986; Auerbach and Kotlikoff 1987). Second, because individuals have control over how funds in their PRAs are invested, a defined-contribution Social Security system allow individuals heterogenous in preferences to better tailor their retirement plans (Feldstein 2005a). Third, compared to the current PAYG Social Security system, which provides a rate of return equal to the growth rate of the tax base, a privatized system can offer a much higher rate of return by allowing investment of funds in equities (Feldstein 2005b; Seidman 1999). Fourth, a defined-contribution Social Security system provides a clear link between the taxes paid by individuals when they work and the benefits they receive when they retire, which helps reduce the labor-market distortion caused by the taxes on the wage and improve long-run efficiency of the system (Feldstein 2005a).

One of the disadvantages of privatization of Social Security is that a defined-contribution system reduces risk sharing by shifting various risks in wage income, investment income, and longevity to beneficiaries of the system instead of pooling the risking as a defined-benefit system does. Nishiyama and Smetters (2007) find that privatization of the PAYG Social Security system could induce efficiency loss in the presence of certain risks that are hard to insure with private markets. Other criticisms of Social Security privatization include that its administrative cost is potentially high, that individuals in a privatized Social Security system are left with a private annuity market which may be plagued by asymmetric information, that the survivors may be inadequately protected, and that not all individuals possess the ability to manage their PRAs well (Seidman 1999).

Some researchers propose to maintain the defined-benefit feature of the current Social Security system and finance the system by accumulating a large fund and investing the funds diversely in government bonds and equities (Seidman 1999; Lewis and Seidman 2002). This proposal offers individuals an inflation-protected annuity upon retirement as the current Social Security system does while addresses the system's solvency by paying for a considerable portion of benefits with investment returns on the accumulated fund and reducing reliance on concurrent taxes. The defined-benefit feature would insure the beneficiaries against various risks, the accumulated fund would increase national saving, and the lowered tax rate would reduce labor-market distortion and the induced deadweight loss. On the other hand, the fund accumulation policy has been criticized because a large fund may permit government to invest in corporate stocks of one firm but not another (Feldstein 2005b).

Lewis and Seidman (2004) show that the fund accumulation policy appeals to a social planner in the face of a temporary demographic bulge. The authors consider four policy alternatives in an overlapping generations model. These four policies include three balanced-budget PAYG policies under which the tax rate, the benefit level, or both have to be adjusted accordingly as the bulge progresses and a fund accumulation policy under which a fund is first gradually accumulated when the bulgers enter the economy and then gradually drawn down as the bulgers stay in and finally exit the economy. They compute the welfare gains and losses of various age cohorts and find that the fund accumulation policy is the only one among the four alternatives that does not impose a nontrivial welfare loss on any age cohort. In addition, the fund accumulation policy offers substantially more welfare gains to future generations compared to the other alternative policies. Yet, Lewis and Seidman note that the fund accumulation policy is not politically appealing because most current voters are better off under some PAYG policy.

Whether privatization or accumulating a large fund is adopted, transition from the current unfunded PAYG Social Security system to a funded system is not costless. However, it is possible to gradually phase in the new policy and spread the cost over multiple generations. Seidman (1986) finds that in a balanced-budget PAYG system an abrupt cut in the Social Security tax rate can considerably decrease the rest-of-life utility (i.e., utility from announcement of the policy change till death) of the retirees and the older workers, but delayed execution of a phase down of the tax rate can protect the retirees and the old workers at the cost of reducing the welfare of the young and the future workers. In an overlapping generations model with empirically plausible parameter values, Lewis and Seidman (2002) assess quantitatively the conversion from a PAYG system to a funded system, in particular the welfare outcomes of different age cohorts along the transition path. In line with Seidman (1986), Lewis and Seidman (2002) find that 1) when the total span of the phase-out of the PAYG system is fixed, the longer the delay of the tax cut is, the smaller the maximum loss for any age cohort is and the smaller the gain for the newborn is; and that 2) when the delay of the tax cut is fixed, the longer the phase-out is, the smaller the maximum loss for any age cohort is and the smaller the gain for the newborn is. This finding is also robust to changes in various parameters of the economy.

Aside from privatization and accumulation of a central fund, other Social Security reform proposals include conversion to a notional defined-contribution system (Auerbach and Lee 2006, 2009) or conversion to a means-tested system (Feldstein 1987). In a notional defined-contribution system, individual accounts are established as is done in a defined-contribution system. However, balances in individual accounts are notional in the sense that they are not backed up by accumulation of real capital but evolve according to some pre-announced formulas. Although a notional defined-contribution system provides many incentives that a funded defined-contribution system has, its financing is essentially PAYG and thus it suffers the low-return problem a PAYG defined-benefit system has. Some countries, such as Sweden, Italy, and Poland, have adopted a notional defined-contribution system.

In a means-tested system, individuals with sufficient means (income or wealth) are offered little or no benefit. A means-tested Social Security program often targets on individuals with low income or assets, and as a result, the size of the program can be financially small. However, a means-tested program may suffer from adverse selection in a world with heterogenous individuals and asymmetric information. Feldstein (2005a) argues that some rational farsighted individuals may be tempted to act as the myopic individuals who save too little for retirement and intentionally reduce their savings before retirement to qualify the means-tested program. Since the government cannot tell individuals of different types apart, adverse selection may render the means-tested program inferior to a universal one.

1.2.3 Social Security in China

Shortly after its founding, the People's Republic of China established the first pension system in 1951 (Cai and Cheng 2014). The first pension system included defined-benefit programs for old age, illness, disability, maternity, and death, and covered mostly industrial workers (and their dependents) in state-owned enterprises (SOEs) with an enrollment rate of one-fifth to one-sixth of the national population in the first few years of operation (Liu and Sun 2016; Liu 2010).[4] The retirement age was 60 for males, 55 for female civil servants, and 50 for female workers (Liu and Sun 2016). The replacement rate was 50–70% and the benefits were entirely financed by enterprises with a contribution rate of 3% of the wages (Cai and Cheng 2014).

In the early stage of China's first pension system, a national pool of reserve funds was accumulated (with 30% of the contributions from the SOEs going into this pool) and managed by the All-China Federation of Trade Unions (ACFTU), which allowed redistribution of contributions at the national level (Liu and Sun 2016). After the outbreak of the Cultural Revolution, the ACFTU was dissolved in 1966, and the reserve funds in the national pool were diverted for other government uses (Williamson and Deitelbaum 2005). Each SOE was then responsible for paying pensioners at its own expense (Cai and Cheng 2014), which was not a big problem in

[4] Civil servants and certain other workers had retirement and medical benefits through multiple other programs (Liu 2010).

the planned economy at that time since the SOEs were backed up by the central government.

The year of 1978 marked the dawn of a new era in Chinese economy history when the state initiated the reform and open-up policy. The SOEs were gradually reformed and became increasingly financially independent of the central government. On the other hand, from 1978 to the mid-1980s, the number of pensioners more than quadrupled and the worker–pensioner ratio declined by more than 75%, which resulted from incentives for early retirement introduced in the course of economic reforms, a sharp decline in fertility in the 1970s owing to the institution of the one-child policy, and other factors (Williamson and Deitelbaum 2005). Consequently, the SOEs with a large number of elderly employees were heavily burdened with pension finance and some were unable to pay the retirees (Liu and Sun 2016). Because pension benefits were administrated by individual SOEs, changing the employer after being laid off meant losing the entitlement to accumulated benefits (Cai and Cheng 2014). As a result, the reform of the SOEs was met with strong resistance and a pension reform became extremely urgent for further successful economic reforms (Cai and Cheng 2014; Liu and Sun 2016).

The State Council initiated a major pension reform in 1997, removing the SOEs from administrating the pension funds and establishing a three-pillar system for all urban workers.[5] The first pillar is a defined-benefit system (the basic pension fund)

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[5] State Council Document No. 26 in 1997. "Decision on the Establishment of a Unified Basic Old-Age Pension System for Enterprise Employees." Civil servants have been covered under a different pension system (Liu 2010). The State Council decided to merge the system covering civil servants to the system covering general urban workers in 2015; see

financed on a PAYG basis with contributions from both the employers and the employees; the second pillar is a defined-contribution system consisted of individual accounts funded by mandatory contributions from both the employers and the employees; and the third pillar is voluntary pension plans such as enterprise annuity plans (Cai and Cheng 2014). Both the basic pension fund and the individual accounts are directly managed by municipal or provincial authorities, not the central government. Other social insurance systems for medical care, disability, maternity, and unemployment for urban workers have been re-established in subsequent years.

Rural residents are covered by the New Rural Pension System, which was initiated in 2009.[6] The rural system is a two-pillar system which is consisted of a basic, flat-rate first pillar financed by the state and local budgets and a second pillar of subsidized individual accounts (Liu and Sun 2016). In 2014, the State Council decided to merge the urban and the rural pension systems into a unified system for all urban and rural residents.[7]

Currently for urban workers, the employer contributes 20% of the wage paid to the employee to the basic pension fund,[8] and the mandatory contribution rate to individual accounts is 8% of the wage, withheld by the employer. In 2014, more than

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http://www.mohrss.gov.cn/SYrlzyhshbzb/dongtaixinwen/shizhengyaowen/201501/t20150114_148951.htm (in Chinese).

[6] State Council Document No. 32 in 2009. "The State Council's Guidelines on the New Rural Pension Pilot."

[7] http://www.gov.cn/guowuyuan/2014-02/07/content_2591063.htm (in Chinese).

[8] In certain provinces and municipalities, the employer's contribution rate could be temporarily lowered to 19%. See http://www.mohrss.gov.cn/gkml/xxgk/201604/t20160419_238366.html (in Chinese).

800 million people were enrolled in the basic pension plan with a coverage rate about 80% of eligible population, the worker–pensioner ratio was 2.97, and the replacement rate for enterprise workers was 67.5%.[9] In addition, urban employers and employees make various mandatory contributions for medical, disability, maternity, and unemployment insurance.

Although the basic pension fund is managed by municipal or provincial authorities, the central government would subsidize the fund if the municipal or provincial fund runs into deficit, and in some provinces the basic pension fund has been in deficit even after being subsidized by the central government (Freedman and Zhang 2016). At the national aggregate level, the total contribution to the basic pension fund for workers amounted to 2.1 trillion yuan in 2015, which was less than the total payment of 2.2 trillion yuan in the same year, according to a government report by the Ministry of Finance.[10] Diversion of funds from individual accounts had been used to reduce deficits in some provinces and made individual accounts in those provinces "empty" (Liu and Sun 2016; Cai and Cheng 2014). It was reported that the size of the "empty individual accounts" was about 3.6 trillion yuan by the end of 2014.[11] Multiple factors may have caused current financial difficulty of the pension system in China. First is the declining worker–pensioner ratio. In some provinces, the

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[9] See http://politics.people.com.cn/n/2015/0701/c1001-27233744.html (in Chinese).

[10] See http://sbs.mof.gov.cn/zhengwuxinxi/shujudongtai/201611/t20161109_2454300.html (in Chinese).

[11] See http://news.xinhuanet.com/fortune/2016-09/27/c_129301401.htm (in Chinese).

ratio has been well below 2 for some years.[12] Second, there are pervasive compliance problems (Cai and Cheng 2014). The proportion of active contributors in participating people has been declining in recent years,[13] and some employers contribute less than they are mandated by underreporting the wages paid to their employees (Williamson and Deitelbaum 2005; Freedman and Zhang 2016). Third, the investment returns on the pension funds are very low, since the investment options (even for funds in the individual accounts) are mostly limited to bank deposits and government bonds (Williamson and Deitelbaum 2005; Freedman and Zhang 2016).

There have been a few initiatives to combat current and future financial difficulty that faces the social security system in China. In 2000, the National Council for Social Security Fund (NCSSF) was established to manage a reserve fund financed from the state budget. This fund is set up as a fund of last resort in preparation for future social security needs (Freedman and Zhang 2016), and its investment options are not restricted to bank deposits and government bonds but also include equities. As of 2015, the national reserve fund amounted to 1.9 trillion yuan and the average annual percentage yield since establishment was 8.8%.[14] Since 2015, the investment options for the basic pension fund has been expanded to include equities as well,[15] and multiple provinces have diverted the surpluses to invest in the stock market through

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[12] For instance, the ratio in Heilongjiang was 1.33 in 2015. See http://news.xinhuanet.com/politics/2016-08/23/c_129248928.htm (in Chinese).

[13] See http://politics.people.com.cn/n/2015/0701/c1001-27233744.html (in Chinese).

[14] See http://www.ssf.gov.cn/cwsj/ndbg/201606/t20160602_7079.html (in Chinese).

[15] State Council Document No. 48 in 2015. "Notice of the State Council on Issuing the Measures for the Administration of Investment in Basic Pension Funds."

the NCSSF for higher returns. The state also plans to raise the statutory retirement age, which is currently 60 for male workers and 50 for female workers,[16] to 65 for everyone.[17]

[16] The retirement age is 60 for male civil servants and 55 for female civil servants. Workers and civil servants in certain circumstances can retire at earlier ages.

[17] See http://www.xinhuanet.com/health/2015-12/03/c_128492954.htm (in Chinese).

Chapter 2

THE MODEL AND STEADY-STATE RESULTS

2.1 The Model

This section lays out the model used in this study, which is based on the models used in Seidman (1983, 1986) and Lewis and Seidman (2002, 2004). First, I describe the basic population dynamics and the behaviors of the participants in the model economy, including competitive firms, households, and a government. Then, I proceed to solve the model in the steady state and along the transition path. Finally, I discuss welfare evaluation in the steady state and along the transition path. Full model derivation is included in the Appendix.

2.1.1 The population

Consider an OLG model populated by *J* generations. An individual is born at age 0, works from age 1 to age *R*, retires at age *R* + 1, and lives through age *J*. The population growth rate is assumed to be *n*, i.e. the number of individuals born in year *v* + 1 is (1 + *n*) times the number of individuals born in year *v*. Thus, the number of age-*t* individuals in year *v* is $(1+n)^{v-t}$. Let $M_L^U(X)$ be defined as follows:

$$M_L^U(X) = \sum_{j=L}^{U} X^j = \begin{cases} \frac{X^{U+1}-X^L}{X-1}, & if\ X \neq 1, \\ U-L+1, & if\ X = 1. \end{cases} \tag{1}$$

Then the total number of workers in year *v* is

$$L_v = \sum_{t=1}^{R}(1+n)^{v-t} = (1+n)^v M_1^R\left(\frac{1}{1+n}\right). \tag{2}$$

The total number of retirees in year *v* is

$$Q_v = \sum_{t=R+1}^{J}(1+n)^{v-t} = (1+n)^v M_{R+1}^J\left(\frac{1}{1+n}\right). \tag{3}$$

Dividing (3) by (2) gives the worker–retiree ratio:

$$\omega = L_v/Q_v \ = M_1^R\left(\frac{1}{1+n}\right)/M_{R+1}^J\left(\frac{1}{1+n}\right). \tag{4}$$

2.1.2 Production

The aggregate production function is a labor-augmented Cobb–Douglas production function:

$$Y_v = K_v^{\alpha}[(1+g)^v L_v]^{1-\alpha}, \tag{5}$$

Where Y_v is the aggregate output in year *v*, K_v is the aggregate capital stock in year *v*, *g* is the growth rate of productivity, and α is the share of capital in the production function. Profit maximization gives the year-*v* wage rate w_v and the capital rental rate r_v:

$$w_v = \mathrm{MPL}_v = \partial Y_v/\partial L_v \ = K_v^{\alpha}(1+g)^v(1-\alpha)[(1+g)^v L_v]^{1-\alpha}, \tag{6}$$

$$r_v = \mathrm{MPK}_v = \partial Y_v/\partial K_v = \alpha K_v^{\alpha-1}[(1+g)^v L_v]^{1-\alpha}. \tag{7}$$

The capital–labor ratio is then

$$\frac{K_v}{L_v} = \frac{\alpha}{1-\alpha}\frac{w_v}{r_v}. \tag{8}$$

Define effective labor

$$E_v = (1+g)^v L_v \tag{9}$$

and per effective labor terms such as

$$k_v = \frac{K_v}{E_v},\ \ y_v = \frac{Y_v}{E_v},\ \text{etc.}$$

Then the expressions for the production function, the wage rate, and the capital rental rate simplify to, respectively,

$$y_v = k_v^\alpha, \quad w_v = (1-\alpha)k_v^\alpha(1+g)^v, \quad r_v = \alpha k_v^{\alpha-1}. \tag{10}$$

Define the per-effective-labor wage rate

$$w_v^e = \frac{w_v}{(1+g)^v L_v} = (1-\alpha)k_v^\alpha. \tag{11}$$

2.1.3 The government

In a PAYG social security system, the government levies the labor income tax rate $t_{w,v}$ on all workers and pays lump-sum social security benefits $b_v w_v$ to retirees, where b_v is the replacement rate. The government maintains a yearly balanced budget:

$$t_{w,v}\omega = b_v. \tag{12}$$

Consider a fund accumulation policy under which the government accumulates a fund *F* whenever there is a surplus and depletes the fund whenever there is a deficit. The accumulated funds are invested with a return *r*, which is the market interest rate (and equals the marginal product of capital). Let

$$H_v = t_{w,v} w_v L_v + rF_v - b_v w_v Q_v. \tag{13}$$

A surplus (deficit) occurs when receipts of labor income taxes, $t_{w,v} w_v L_v$, plus investment returns, rF_v, are more (less) than social security payments, $b_v w_v Q_v$, i.e., when $H_v > 0 (< 0)$. The law of motion of the fund is

$$F_{v+1} = F_v + H_v. \tag{14}$$

The government is also allowed to borrow funds at the market interest rate to finance the social security payments. $F_v < 0$ indicates that the government is borrowing; otherwise $F_v > 0$. When $F_v < 0, rF_v$ is the interest payment. The equations (13) and (14) both apply when the government is borrowing.

Fund accumulation affects accumulation of capital in the economy. The law of motion of the aggregate capital stock becomes

$$K_{v+1} = K_v + S_v + H_v^*, \tag{15}$$

where S_v is private savings and H_v^* is public savings given by

$$H_v^* = \begin{cases} H_v, & \text{if } F_v \geq 0 \text{ and } F_{v+1} \geq 0, \\ F_{v+1}, & \text{if } F_v < 0 \text{ and } F_{v+1} \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

2.1.4 Households

Consider an age-*t* individual in year *v*. Denote her consumption in year *v* by $C_v(t)$. She chooses her consumption path to maximize her lifetime utility subject to her lifetime budget constraint:

$$\max \textstyle\sum_{j=1}^{J} (1+\rho)^{-j} (C_{v-t+j}^{1-\gamma}(j) - 1)/(1-\gamma)\,, \tag{16}$$

$$\text{subject to } \textstyle\sum_{j=1}^{J} \frac{C_{v-t+j}(j)}{(1+r)^j} = \sum_{j=1}^{R} \frac{(1-t_w)w_{v-t+j}}{(1+r)^j} + \sum_{j=R+1}^{J} \frac{B_{v-t+j}}{(1+r)^j}, \tag{17}$$

where ρ is the subjective discount rate, γ is the elasticity of intertemporal substitution of consumption, and *r* is the interest rate. When $\gamma = 1$, the period utility function breaks down to logarithm, i.e., $\ln C_{v-t+j}(j)$. It is shown in the Appendix that the solution is given by

$$C_{v-t+j}(j) = \left(\frac{1+r}{1+\rho}\right)^{\sigma j} \Gamma \cdot w_{v-t},$$

where

$$\Gamma = \left[(1-t_w) M_1^R \left(\frac{1+g}{1+r}\right) + t_w \omega M_{R+1}^J \left(\frac{1+g}{1+r}\right)\right] / M_1^J \left(\left(\frac{1+r}{1+\rho}\right)^{\sigma} \frac{1}{1+r}\right) \tag{18}$$

and $\sigma = 1/\gamma$

The rate of return r^{ss} is that a worker receives on social security is defined implicitly by

$$\sum_1^R t_w w_t/(1+r^{ss})^t = \sum_{R+1}^J B_t/(1+r^{ss})^t. \tag{19}$$

2.1.5 The steady state

Suppose the government adopts a balanced-budget policy. The year-*v* resource constraint in the economy is

$$Y_v = C_v + S_v, \tag{20}$$

The law of motion of capital stock, assuming no depreciation, is

$$K_{v+1} = K_v + S_v. \tag{21}$$

In effective labor terms, (18) and (20) become

$$y_v = c_v + s_v \tag{22}$$

and

$$s_v = (1+g)(1+n)k_{v+1} - k_v. \tag{23}$$

In the steady state, I have

$$y_v = \bar{y}, \quad c_v = \bar{c}, \quad s_v = \bar{s}, \quad k_v = \bar{k}. \tag{24}$$

From (22), I have

$$\bar{y} = \bar{c} + \bar{s}. \tag{25}$$

From (23), I have

$$\bar{s} = (1+g)(1+n)\bar{k} - \bar{k} = (n+g+ng)\bar{k}. \tag{26}$$

From (10), I have

$$\bar{y} = \bar{k}^\alpha, \quad \bar{w}^e = (1-\alpha)\bar{k}^\alpha, \quad \bar{r} = \alpha\bar{k}^{\alpha-1}. \tag{27}$$

Equations (24–27) lead to a nonlinear equation of the steady-state interest rate $\bar{r}$:

$$r = \frac{\alpha}{1-\alpha}(r - n - g - ng)/[M_1^J\left(\left(\frac{1+r}{1+\rho}\right)^{\sigma}\frac{1}{(1+n)(1+g)}\right)/M_1^R\left(\frac{1}{1+n}\right)\cdot\Gamma - 1], \quad (28)$$

where Γ is given by (18). (28) is to be solved numerically with MATLAB's fzero function. After obtaining $\bar{r}$, the other steady-state variables can be recovered through Equations (24-27).

2.1.6 The transition path

When a demographic event occurs, an individual's choices change from what they are in the steady state. An individual who is age *t* in year *v* when the event occurs chooses a rest-of-life consumption path to maximize the rest-of-life utility:

$$\max \sum_{j=t}^{J}(1+\rho)^{t-j}(C_{v-t+j}^{1-\gamma}(j) - 1)/(1-\gamma), \quad (29)$$

subject to $\sum_{j=t}^{J}\frac{C_{v-t+j}(j)}{(1+r)^{j-t}} = A_v(t)(1+r) + Z\sum_{j=t}^{R}\frac{(1-t_w)w_{v-t+j}}{(1+r)^{j-t}} + \sum_{j=X}^{J}\frac{B_{v-t+j}}{(1+r)^{j-t}}$, (30)

where $A_v(t)$ is steady-state private wealth of an age-*t* individual in the beginning of year *v*, $Z = 1$ if $t \leq R$, $Z = 0$ if $t \geq R+1$, $X = R+1$ if $t \leq R$, $X = t$ if $t \geq R+1$. It is shown in the Appendix that

$$\begin{aligned} &A_{v+1}(t+1) \\ &= (1+r)^t\overline{w}^e(1+g)^{v-t}\{\left[(1-t_w)M_1^{X'}\left(\frac{1+g}{1+r}\right) + Z'\omega t_w M_{R+1}^t\left(\frac{1+g}{1+r}\right)\right] \\ &-\left[(1-t_w)M_1^R\left(\frac{1+g}{1+r}\right) + \omega t_w M_{R+1}^J\left(\frac{1+g}{1+r}\right)\right]M_1^t\left(\frac{(1+r)^{\sigma-1}}{(1+\rho)^{\sigma}}\right)/M_1^J\left(\frac{(1+r)^{\sigma-1}}{(1+\rho)^{\sigma}}\right)\}, \end{aligned} \quad (31)$$

where $X' = t, Z' = 0$ if $t \leq R$ and $X' = R, Z' = 1$ if $t \geq R+1$, and that

$$\begin{aligned} &C_v(t) = \\ &\left\{A_v(t)(1+r) + \left(\frac{1+r}{1+g}\right)^t\left[Z(1-t_w)w_v M_t^R\left(\frac{1+g}{1+r}\right) + B_v M_X^J\left(\frac{1+g}{1+r}\right)\right]\right\}\cdot \\ &\left[\frac{(1+r)^{\sigma-1}}{(1+\rho)^{\sigma}}\right]^t/M_t^J\left(\frac{(1+r)^{\sigma-1}}{(1+\rho)^{\sigma}}\right). \end{aligned} \quad (32)$$

2.1.7 Welfare evaluation

To facilitate welfare evaluation of different social security policy alternatives, I compute the individual's utilities along the balanced growth path and along the transition path. Consider an individual born in year *v*. Her lifetime consumption stream is $\{C_{v+j}(j)\}_{j=1}^{J}$ and thus her lifetime utility is

$$U_v = \sum_{j=1}^{J} \frac{1}{(1+\rho)^j} \frac{[C_{v+j}(j)]^{1-\gamma}-1}{1-\gamma}. \tag{33}$$

Denote the individual's consumption stream along the balanced growth path with a certain baseline social security policy by $\{C_{v+j}^{*}(j)\}_{j=1}^{J}$ and the corresponding lifetime utility by U_v^*. If $U_v > U_v^*$ ($U_v < U_v^*$), this individual is better (worse) off from the baseline.

Utilities computed here are ordinal rather than cardinal. However, the relative magnitude of U_v and U_v^* does not tell one by how much the individual is better or worse off from the baseline. One the other hand, consumption is cardinal. Relative difference in the utility may be translated into relative difference in consumption. Consider increasing the consumption stream (i.e., consumption in each period) by δ^L. The general lifetime utility with the new consumption stream would be

$$U_v(\delta^L) = \sum_{j=1}^{J} \frac{1}{(1+\rho)^j} \frac{[(1+\delta^L)C_{v+j}(j)]^{1-\gamma}-1}{1-\gamma}. \tag{34}$$

If $U_v(\delta^L) = U_v^*$, then the individual achieves the same utility as in the baseline by increasing her consumption stream by δ^L. If $U_v > U_v^*$ ($U_v < U_v^*$), then $\delta^L < 0$ ($\delta^L > 0$), which means that the gains or losses in lifetime utility can be translated into the proportion of lifetime consumption stream that has to be forgone or compensated in order to achieve the same utility level. Equate $U_v(\delta^L)$ and U_v^*, solve for δ^L, and I have

$$\delta^L = \left[\frac{U_v^* + \frac{1}{1-\gamma}\sum_{j=1}^{J}\frac{1}{(1+\rho)^j}}{U_v + \frac{1}{1-\gamma}\sum_{j=1}^{J}\frac{1}{(1+\rho)^j}}\right]^{\frac{1}{1-\gamma}} - 1, \tag{35}$$

where U_v^* is the baseline lifetime utility and U_v is the lifetime utility under an alternative social security policy to be compared against.

2.2 Calibration and Steady-state Results

In this section I explore the long-term impact of a permanent change in the population growth rate or the labor productivity growth rate on the economy and welfare by examining the impact on the steady state of the detrended model economy. First, I calibrate the model to the U.S. economy and the Chinese economy. Table 2.1 lists the calibrated model parameters. For the case of the U.S. and the case of China, following Lewis and Seidman (2004), I set the share of capital in the production function to $\alpha = 0.3$. For the case of the U.S., I set the population net growth rate to n = 0.01, which is the average net growth rate of the U.S. population rate over 1965–2015 (excluding the baby-boom years).[18] I set the labor productivity net growth rate to g = 0.019, which is the average annual net growth rate of the gross domestic product (GDP) per capita in the U.S. over 1965–2015[19]. The subjective discount factor $\rho = 0.0025$ and the elasticity of intertemporal substitution of consumption $\sigma = 1/\gamma = 2/3$ are taken from Smets and Wouters (2007).

[18] World Bank, "SP.POP.GROW: Population growth (annual %);" author's calculation.

[19] U.S. Bureau of Economic Analysis, "Table 7.1. Selected Per Capita Product and Income Series in Current and Chained Dollars;" author's calculation.

Table 2.1: Baseline Model Calibration.

Parameter	Description	Value
Case of the U.S.		
n	Population (net) growth rate	0.01
g	Labor productivity (net) growth rate	0.019
α	Share of capital in production function	0.3
ρ	Subjective discount factor	0.0025
σ	Elasticity of intertemporal substitution of consumption	0.67
R	Number of years in labor force	45
J	Number of years in labor force and retirement	66
t_w	Social security tax rate	0.15
Case of China		
n	Population (net) growth rate	0.013
g	Labor productivity (net) growth rate	0.049
α	Share of capital in production function	0.3
ρ	Subjective discount factor	0.002
σ	Elasticity of intertemporal substitution of consumption	0.5
R	Number of years in labor force	40
J	Number of years in labor force and retirement	60
t_w	Social security tax rate	0.22

I set $R = 45$ and $J = 66$ for both the case of the U.S. so that the individuals live 45 years working and 21 years in retirement.[20] Given the calibrated value of n, R, and J, the resulting worker–retiree ratio in the steady state of the detrended model for the U.S. is $\omega = 3.00$.[21] The social security tax rate is set to $t_w = 0.15$, matching the current

[20] If an individual enters the labor force at the age of 20, then $R = 45$ implies that the individual retires at the age of 65. As of 2017, the full retirement age in the U.S. is 66 to 67 (see Full Retirement Age). The value of $J - R = 21$ is related to life expectancy upon retirement. Life expectancy at 65 was 19.4 years in the U.S. in 2015 (National Center for Health Statistics 2017, Table 15).

[21] According to Board of Trustees (2016), the number of covered worker per OASDI beneficiary was about 2.8 in 2015, and is projected to decline to about 2 over the century.

rate in the U.S.[22] As a result, the steady-state replacement rate for the U.S. is calculated as $b = \omega t_w \approx 44.9\%$.

For the case of China, I set the population net growth rate to $n = 0.013$, which is the average net growth rate of the Chinese population over 1965–2015[18]. I set the labor productivity net growth rate to $g = 0.049$, basing on the projected long-term growth rate of the Chinese economy by Wang and Zhou (2017).[23] I take the subjective discount factor $\rho = 0.002$ and the elasticity of intertemporal substitution of consumption $\sigma = 1/\gamma = 1$ from Chang et al. (2012). I set $R = 40$ and $J = 60$ for the case of China.[24] Given the calibrated value of n, R, and J, the resulting worker–retiree ratio in the steady state of the detrended model would be $\omega = 2.97$.[25] I set the social

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[22] Office of Retirement and Disability Policy, "OASDI and SSI Program Rates & Limits, 2016."

[23] Wang and Zhou (2017) projected a GDP growth rate of 5.5% per year in 2021–2030 in the baseline for China. The average annual GDP growth rate in China over 1979–2014, reported by Wang and Zhou (2017), was 9.7%, and the average annual real GDP per capita growth rate in China over 1979–2014 was 8.7% (World Bank, "NY.GDP.PCAP.KN: GDP per capita (constant LCU);" author's calculation). Assuming a stable population growth rate and stable inflation, I extrapolate the projected long-term growth rate of the real GDP per capita in China to be 5.5% / 9.7% * 8.7% = 4.9%.

[24] If an individual enters the labor force at the age of 20, then $R = 40$ implies that the individual retires at the age of 60. China is planning to increase the statutory retirement age to 65 for both males and females by 2045 (see http://www.xinhuanet.com/health/2015-12/03/c_128492954.htm (in Chinese)). The value of $J - R = 20$ is related to life expectancy upon retirement. Life expectancy was 20.4 years for age 60–64 in China in 2015 (Cai et al. 2016).

[25] According to Department of Population and Employment Statistics et al. (2016), the worker–retiree ratio in China's Urban Employees Basic Pension Insurance was about 2.9 in 2015.

security tax rate in China to $t_w = 0.22$, leading to a social security replacement rate of $b = \omega t_w \approx 65.4\%$ that is consistent with the current replacement rate in China.[26]

Table 2.2 presents selected steady-state variables of the detrended model with baseline parameterization and some variants. The variants are to show how a permanent change to the population growth rate or the labor productivity growth rate affects the economy with a PAYGO social security system in the long run.

In the baseline steady state, the U.S. model economy has an interest rate of 5.3% and an average private saving rate of 16.4%. When the net population growth rate decreases to zero (variant 1), the worker–retiree ratio drops from the baseline value of 3.00 to 2.14 and the social security replacement rate drops from the baseline value of 44.9% to 32.1%. Due to decreased population growth, capital per effective labor, k, increases by 16.3% from the baseline. As a result, the interest rate drops to 4.8%. A lower interest rate induces a lower average private saving rate of 11.9%. Another way to explain the lowered private saving rate is that a lower population growth rate produces less young savers relative to old dis-savers and thus reduces the saving rate. Consumption per effective labor, c, increases by 10.3% from the baseline. However, due to distortionary wage tax, each individual's lifetime utility decreases from the baseline by 1.1% in equivalent consumption units.

[26] A news report (*in Chinese*) in 2015, citing the official 2014 Annual Report on China's Social Security Development compiled by the Ministry of Human Resources and Social Security of China, revealed that the replacement rates in 2009–2014 ranged from 65.9% to 67.6%. *Pensions at a Glance 2017* published by the Organization for Economic Co-operation and Development listed that the gross pension replacement rates in China was 76.0% for men and 65.1% for women.

Table 2.2: Steady State Results.

	n	g	t_w	ω	b	r	k	c	s/y	Δk	Δc	$\Delta(s/y)$	δ^L
Case of the U.S.: Baseline													
	1.0%	1.9%	15%	3.00	44.9%	5.3%	7.35	1.75	16.4%				
Case of the U.S.: Variants													
1	0%	1.9%	15%	2.14	32.1%	4.8%	13.67	1.93	11.9%	16.3%	10.3%	−4.5%	−1.1%
2	1.5%	1.9%	15%	3.55	53.3%	5.6%	10.92	1.67	18.3%	−7.1%	−4.4%	1.9%	0.8%
3	1.0%	1.0%	15%	3.00	44.9%	4.5%	15.05	1.95	13.4%	28.0%	11.5%	−3.0%	−14.9%
4	1.0%	2.5%	15%	3.00	44.9%	6.0%	10.05	1.64	17.7%	−14.5%	−6.1%	1.3%	11.4%
5	0%	1.9%	21.0%	2.14	44.9%	5.2%	12.12	1.88	10.9%	3.1%	7.5%	−5.5%	−6.3%
6	1.5%	1.9%	12.6%	3.55	44.9%	5.4%	11.49	1.69	18.9%	−2.2%	−3.7%	2.6%	2.4%
Case of China: Baseline													
	1.3%	4.9%	22%	2.97	65.4%	7.0%	7.92	1.36	26.7%				
Case of China: Variants													
7	0.975%	4.9%	22%	2.69	59.1%	6.9%	8.20	1.39	25.8%	3.5%	2.2%	−0.8%	−1.7%
8	2.0%	4.9%	22%	3.69	81%	7.4%	7.36	1.30	28.3%	−7.1%	−4.4%	1.6%	4.2%
9	1.0%	7.5%	22%	2.97	65.4%	9.2%	5.41	1.18	29.0%	−31.7%	−13.7%	2.3%	80.8%
10	1.0%	2.5%	22%	2.97	65.4%	5.2%	12.07	1.65	21.9%	52.4%	20.8%	−4.8%	−40.7%
11	0.975%	4.9%	24.3%	2.69	65.4%	7.1%	7.90	1.39	25.2%	−0.3%	2.0%	−1.5%	−2.4%
12	2.0%	4.9%	17.7%	3.69	65.4%	7.1%	7.91	1.31	29.8%	−0.1%	−4.3%	3.1%	4.5%

Note. Parameters remaining the same across the baseline and the variants include α, ρ, σ, R, and J.

Variant 5 tries to restore the original social security replacement rate under no population growth by raising the social security tax rate to 21.0%. The resulting capital per effective labor is only 3.1% higher than the baseline, and the interest rate is at 5.2%, close to the baseline value of 5.3%. Higher social security savings would displace private savings, so the average private saving rate is decreased by 5.5 percentage points from the baseline. With a higher distortionary tax rate, an individuals' lifetime utility decreases from the baseline by 6.3% in equivalent consumption units, despite the same social security replacement rate. The comparison of variant 1 and variant 5 with the baseline shows that individuals in an economy with a PAYGO social security system suffer from a lower population growth rate. An increased tax rate may be able to restore the social security replacement rate and largely capital per effective labor but may make individuals even worse off in welfare.

When the net population growth rate increases by 50% to 1.5% per year (variant 2), the worker–retiree ratio rises from the baseline value of 3.00 to 3.55 and the social security replacement rate rises from the baseline value of 44.9% to 53.3%. Due to increased population growth, capital per effective labor, *k*, decreases by 7.1% from the baseline. As a result, the interest rate rises to 5.6%. A higher interest rate induces a higher average private saving rate of 18.3% Another way to explain the higher private saving rate is that a higher population growth rate produces more young savers relative to old dis-savers and thus raises the saving rate. Individuals' lifetime utility increases slightly by 0.8% in equivalent consumption units.

Variant 6 tries to keep the social security replacement at the original level under increased population growth by reducing the social security tax rate to 12.6%. The new values of capital per effective labor and the interest rate are close to the corresponding baseline values. In addition, individuals' lifetime utility is higher by 2.4% in equivalent consumption units from the baseline. The comparison of variant 2 and variant 6 with the baseline shows that individuals in an economy with a PAYGO social security system can benefit from a higher population growth rate and enjoy a lower social security tax rate.

Variant 3 and variant 4 keep baseline population growth but change the labor productivity growth rate. A lower (higher) labor productivity growth rate results in higher (lower) capital per effective labor and thus a lower (higher) interest rate. The private saving rate would be lower (higher) as well. Compared with a relative change in the population growth rate, a positive relative change in the labor productivity growth rate seems to cause a much larger positive impact to individuals' lifetime utility.

In the baseline steady state, the model economy of China has an interest rate of 7.0% and an average private saving rate of 26.7%. Because of population growth and higher labor productivity growth, the Chinese economy has a higher interest rate and a higher private saving rate than the U.S. economy does. Variants 7–12, compared with the Chinese baseline, tell a similar story as in the U.S. case. Note in particular that in variant 7, when the net population growth rate is reduced by 25% to 0.975% per year, an average individual suffers a loss of 1.7% in equivalent consumption units.

Chapter 3

MANAGING A DEMOGRAPHIC BULGE: THE CASE OF THE U.S.

This chapter applies the OLG model described in Chapter 2 to the U.S. and examines the welfare implications along the transition path of various social security policy options in the face of a demographic bulge.

3.1 A Demographic Bulge: The Bulge Effect

For the case of the U.S., I assume that individuals are born at age 0, enter the labor force at age 20, work for 45 years, retire at age 65, live another 21 years of retirement, and then die at age 86. The parameter values used are as calibrated in Table 2.1. In addition, I assume that a demographic bulge hits the economy in year −19 and lasts $T_B = 18$ years.[27] The demographic bulge temporarily raises the number of births in a year by 50 percent.[28] Individuals born between the year −19 and the year −2 (inclusive) are called the "bulgers."

I illustrate how a population bulge alters the worker–retiree ratio over time as the bulgers enter and then exit the labor force (Figure 3.1). From the beginning, when the first bulge cohort enters the labor force, to the end when the final bulge cohort

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[27] The U.S. experienced a baby boom from mid-1946 to mid-1964 (Hogan et al. 2008), which lasted about 18 years.

[28] I follow Lewis and Seidman (2004) and assume that the demographic bulge increases the number of people born in a year by 50%, which is an approximation to the situation experienced by the U.S. population during the post-war baby boom.

exits the economy, it takes 84 years. From year 1 to year 18, the bulgers enter the labor force and raise the labor population (workers plus retirees), so the worker–retiree ratio rises from 3 to 3.68. Starting in year 45, the bulgers retire and exit the labor force, significantly lowering the worker–retiree ratio. The worker–retiree ratio first decreases below 3 in year 50 and falls to the bottom in year 63 at about 2.09 when the last bulgers retire. Finally, from year 63 to year 84, the population composition and the worker–retiree ratio gradually return to their respective steady-state (pre-bulge) levels.

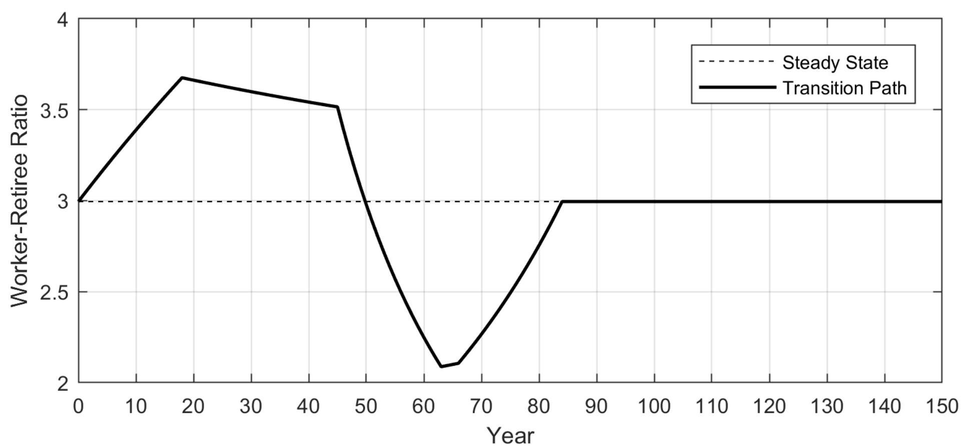

Figure 3.1: The worker–retiree ratio along the transition path in the face of an 18-year demographic bulge entering the labor force in year 1.

To capture the effects of the demographic bulge on the economy and welfare, I assume in this section that the economy has no social security program, or equivalently, that the social security tax rate is $t_w = 0$. When the bulgers enter the labor force, capital per effective labor falls initially. As a result, the interest rate, which equals the marginal product of capital, will increase initially, whereas the effective wage rate, which equals the marginal product of labor, will fall initially

(Figure 3.2). After all the bulgers are in the labor force, the worker–retiree ratio starts to fall, leading to a decrease in the interest rate and an increase in the effective wage rate. Both the interest rate and the wage rate will eventually return to the steady-state level after the bulgers exit the economy.

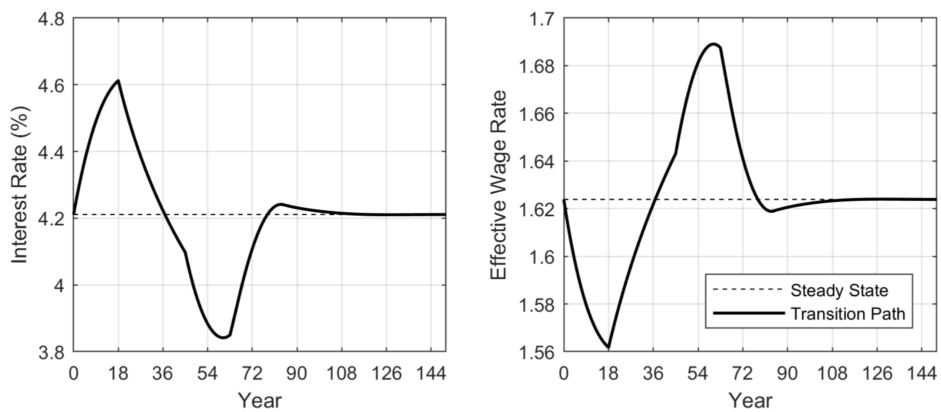

Figure 3.2: The interest rate and the effective wage rate along the transition path in the face of the demographic bulge.

I examine a selected age cohort in detail to uncover the reason for gains or losses due to the "bulge effect." I take a person who is born 10 years after the first bulgers are born, i.e., a person from the age −10 cohort, as an example (Figure 3.3). The left panel shows relative changes in the individual's consumption and income of various sources along the transition path due to the presence of the bulge. This individual enters the labor force in year 11. According to Figure 3.2, the effective wage rate in year 11 is lower than the steady-state (no-bulge) level, so in early working years this individual earns less wage income.

The interest rate, however, is higher than the steady-state (no-bulge) level initially. A higher interest rate has two competing effects on consumption: on the one hand, it boosts capital income and thus has a positive wealth effect; on the other hand, it provides an incentive to save more and consume less, i.e., an intertemporal substitution effect. The combined effect, as the left panel of Figure 3.3 shows, is that the individual chooses to decrease consumption. The right panel of Figure 3.3 plots the cumulative rest-of-life change in welfare in equivalent consumption units. This individual suffers a significant welfare loss at early ages because of lowered consumption.

As the interest rate drops later, the individual starts to increase consumption and reduce the loss in welfare. Yet, during this person's retirement starting in year 56, the interest rate is mostly less than its steady-state (no-bulge) level. Hence, during retirement, this person receives much less capital income, which leads to a lower consumption level as well as a lower utility level. Compared to the total rest-of-life utility in the absence of the bulge, the total rest-of-life utility in the presence of the bulge is 2.1% less in equivalent consumption units for this individual.

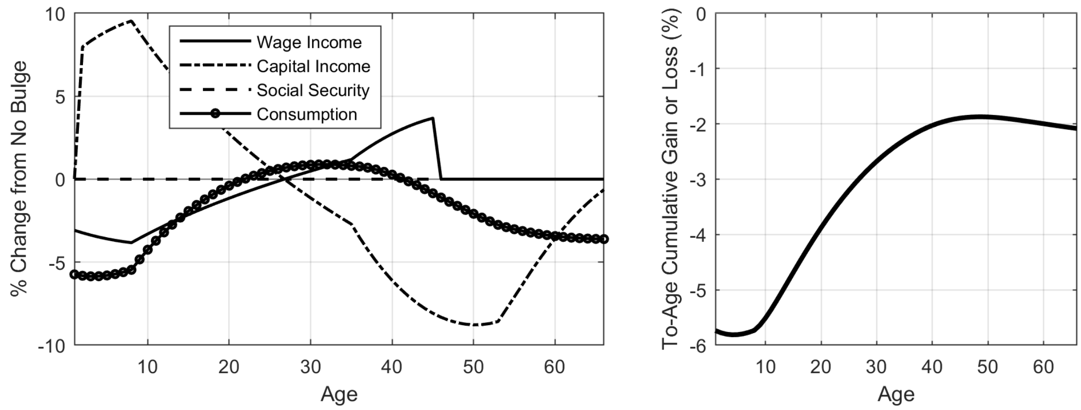

Figure 3.3: Relative changes in consumption, wage income, capital income and the social security benefit (left) and the cumulative gain or loss in welfare in equivalent consumption units due to the presence of the bulge (right) for the age −10 cohort along the transition path without a social security program in the face of the demographic bulge.

To assess how cohorts of different ages are affected by the demographic bulge, I calculate the difference in rest-of-life utility achieved with and without the bulge for each age cohort. Figure 3.4 shows the percentage gains or losses in rest-of-life welfare. An age cohort is numbered after the age of individuals in the age cohort when the first bulgers are born. For example, the age 0 cohort is the first generation of bulgers. The individuals in the age 45 cohort are age 65 in their first year of retirement when the first bulgers enter the labor force at age 20.

The cohorts aged less than or equal to 14 but more than or equal to −27 as well as those aged less than or equal to −73 experience losses, which are 36% of the 150 age cohorts examined. The remaining age cohorts experience gains. The age −8 cohort suffers the largest loss of 2.12% while the age 39 cohort enjoys the largest gains of 2.44%.

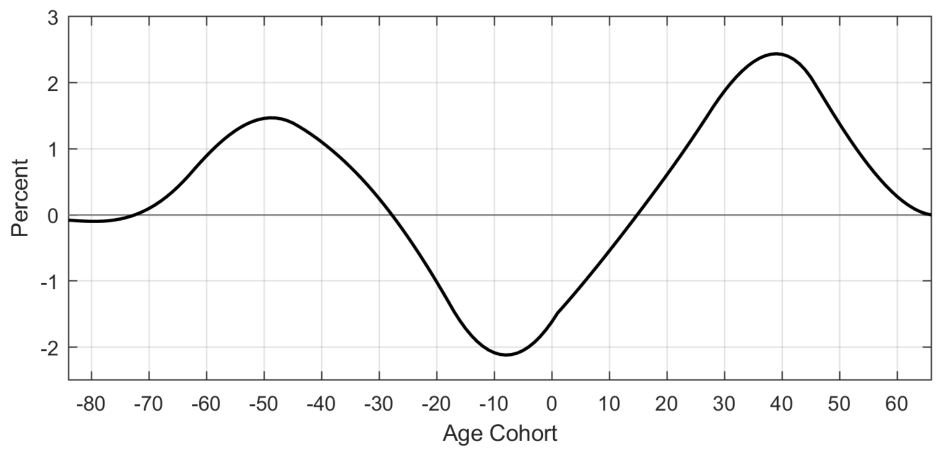

Figure 3.4: Gains or losses of different age cohorts in rest-of-life welfare in the face of the demographic bulge.

3.2 Policy 1: Fixed Tax Rate, Balanced Budget

The first social security policy to consider is a balanced-budget policy with a fixed social security tax rate. Under this policy, the social security manager sets benefit per retiree B each year to balance the budget with a fixed tax rate of 15% ($t_w = 15\%$). Balanced budget leads to the replacement rate $B/w = t_w\omega = 0.15\omega$, so B/w varies directly with ω (Figure 3.5). As the bulgers gradually enter the labor force over 18 years, the replacement rate increases to 55.1%. From year 18 through year 45, the bulgers age but keep working in the labor market, and as a result, the replacement rate remains constant at 52.5% for 30 years. After year 45, the bulgers begin to retire until the replacement rate drops to the lowest of 31.3% in year 63 when the last cohort of bulge exits the labor market. As the bulgers gradually finish the 21-year retirement period starting from year 63, the replacement rate gradually rises to the steady-state level.

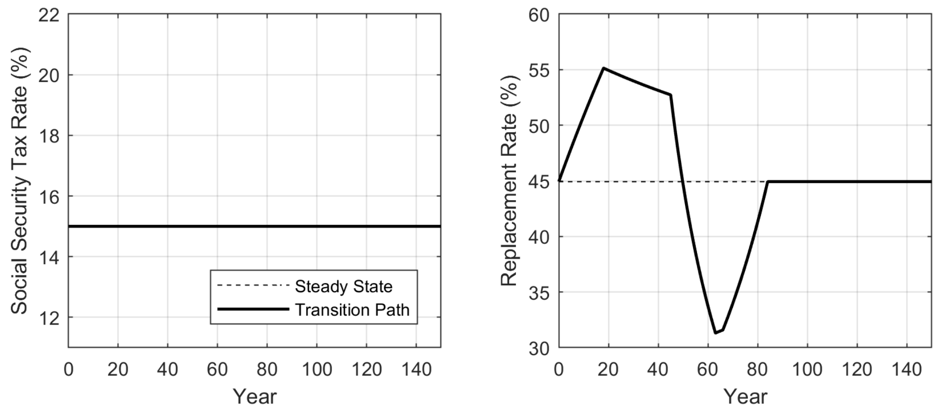

Figure 3.5: The social security tax rate and the replacement rate over time under policy 1 (fixed tax rate, balanced budget).

As the bulgers enter the labor force, the capital–labor ratio decreases, leading to an increase in the interest rate and a decrease in the effective wage rate (Figure 3.6). After all bulgers have entered the labor force in year 18, the working population gradually stabilizes but capital increases because of the bulge, so the capital–labor ratio begins to rise. This leads to a decrease in the interest rate and an increase in the effective wage rate. After year 45, the bulgers begin to retire, and growth of the working population slows down, resulting in an accelerated rise in the capital–labor ratio and the effective wage rate, and an accelerated decline in the interest rate. As the bulgers gradually exit the labor force from year 63, the capital–labor ratio, the effective wage rate, and the interest rate gradually return to their respective steady-state levels.

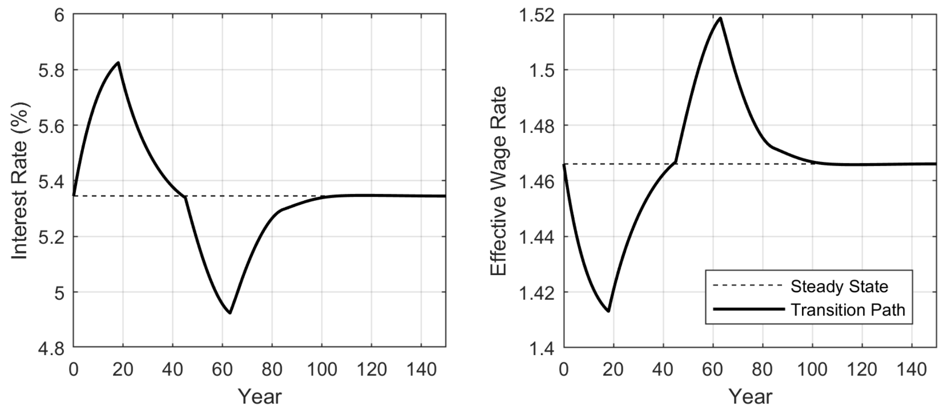

Figure 3.6: The interest rate and the effective wage rate over time in the face of the demographic bulge under policy 1 with a fixed tax rate and a balanced budget.

I examine a selected age cohort in detail to uncover the reason for gains or losses for the age cohort under policy 1. I take a person from the age –10 cohort, i.e., a person born 10 years after the first bulgers are born, as an example (Figure 3.7). This individual enters the labor force in year 11. According to Figure 3.6, in year 11, the effective wage rate is lower than its steady-state (no-bulge) level and the interest rate is higher than its steady-state (no-bulge) level. The lower wage rate decreases the individual's wage income initially, which exerts a negative wealth effect on consumption. In addition, the higher interest rate imposes a strong intertemporal substitution effect that favors more saving. The combined effect reduces consumption levels and significantly lowering welfare levels at early ages of this individual (Figure 3.7).

When this individual retires in year 56, the low interest rate (Figure 3.6) reduces the individual's capital income from pre-retirement savings. In addition, the replacement rate is low (Figure 3.5), which further lowers the individual's retirement income. The low retirement income level forces the individual to reduce consumption and thus suffer a welfare loss. The downturn of the curve upon retirement in the right panel of Figure 3.7 reflects this welfare loss in retirement. Compared to the total rest-of-life utility in the absence of the bulge, the total rest-of-life utility in the presence of the bulge is 3.36% less in equivalent consumption units under policy 1 for this individual.

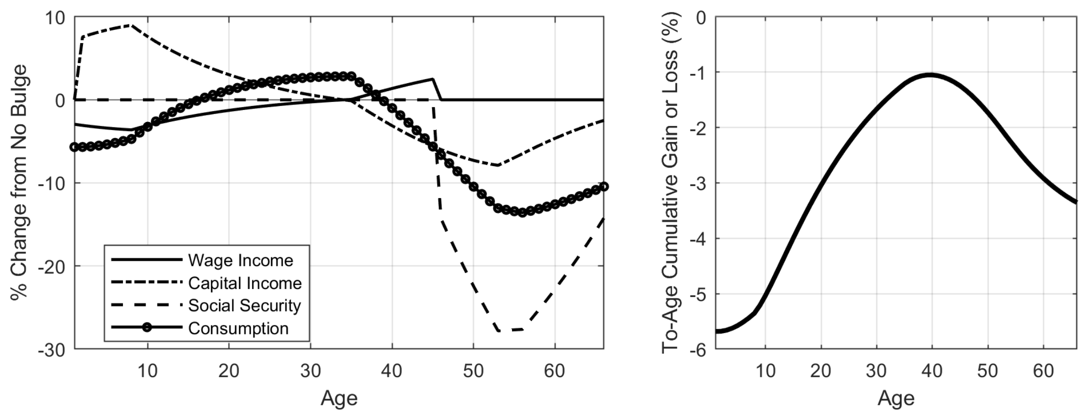

Figure 3.7: Relative changes in consumption, wage income, capital income, and the social security benefit (left) and cumulative gain or loss in rest-of-life welfare (right) for the age −10 cohort along the transition path in the face of the demographic bulge under policy 1 with a fixed tax rate and a balanced budget.

Figure 3.8 shows the percentage gains or losses of different age cohorts in rest-of-life under policy 1. The cohorts aged less than or equal to 7 but more than or equal to −37 suffer losses. The remaining age cohorts enjoy gains. Among the 150 age cohorts examined, 28.7% of them suffer losses. The age 39 cohort experiences the largest gain of 5.45%, while the age −11 cohort suffers the worst loss of 3.37%.

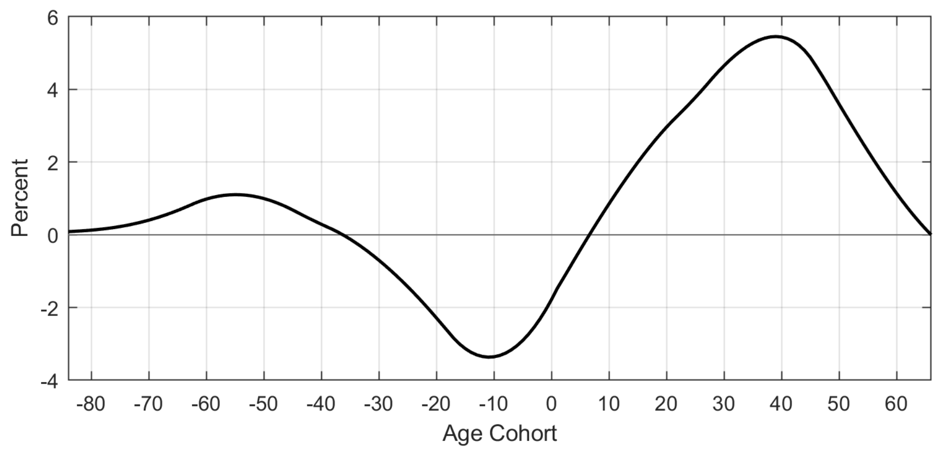

Figure 3.8: Gains or losses of different age cohorts in rest-of-life welfare in the face of the demographic bulge under policy 1 with a fixed tax rate and a balanced budget.

Figure 3.9 shows the implicit rates of return of social security for different age cohorts in the face of the bulge under policy 1. Figure 3.9 tries to explain why the social security program under policy 1 is attractive to cohorts of age 4–65, as a means of compulsory saving, when there is a demographic bulge. In the case of no bulge, the steady-state implicit rate of return of social security is $r^{ss} = (1 + n)(1 + g) - 1 = 2.92\%$. For cohorts of age 4–65, their implicit rates of return are no less than the steady-state value of 2.92%, and hence they prefer putting money into the social security program. For the other age cohorts, they would rather invest their money on their own. The implicit rate of return is the lowest for the age −12 cohort at 2.20%.

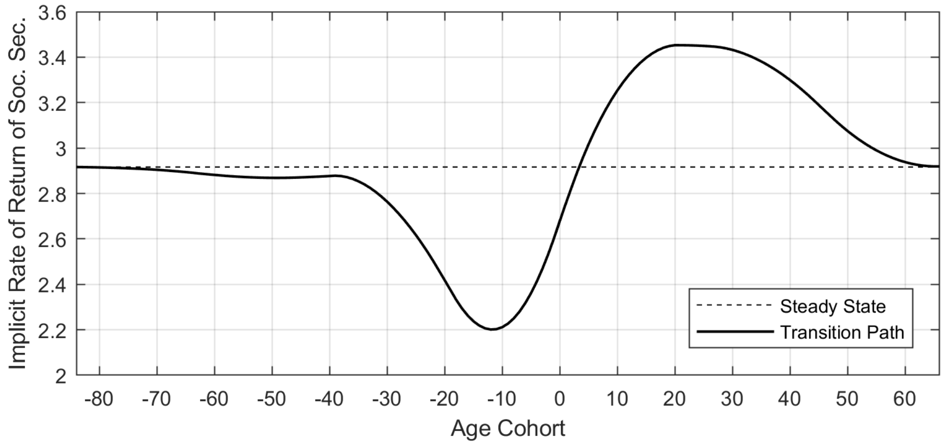

Figure 3.9: Implicit rates of return (percent) of social security for different age cohorts in the face of the demographic bulge under policy 1 with a fixed tax rate and a balannced budget.

3.3 Policy 2: Fixed Replacement Rate, Balanced Budget

The second social security policy to consider is a balanced-budget policy with a fixed replacement rate. Under this policy, the social security manager sets B to fix the replacement rate at 45%, i.e., $B/w = 45\%$. Balanced budget leads to $t_w = (B/w)/\omega = 0.45/\omega$, so the tax rate t_w varies inversely with ω (Figure 3.10). As the bulgers gradually enter the labor force over 18 years, the tax rate drops from 15% in year 0 to 12.24% in year 18. After year 45, the bulgers begin to retire, and therefore the tax rate hits the highest level of 21.56% in year 63. As the bulgers gradually exit the labor force after year 63, t_w gradually falls to the steady-state level of 15%.

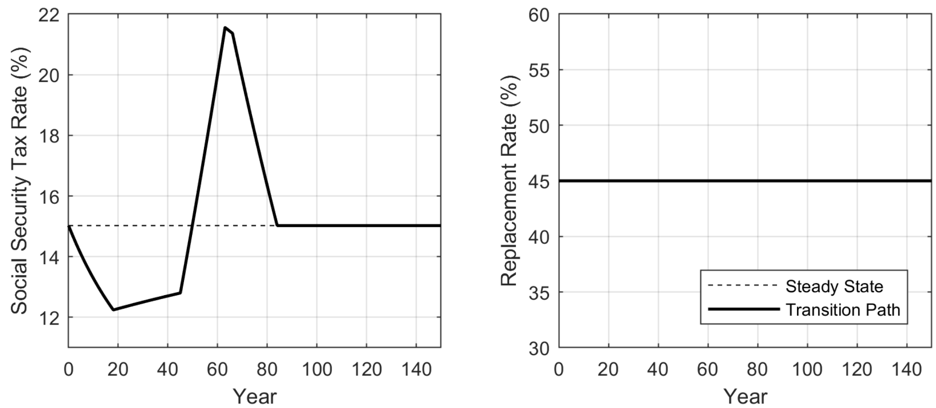

Figure 3.10:The social security tax rate and the replacement rate over time under policy 2 with a fixed replacement rate and a balanced budget.

As the bulgers enter the labor force, the capital–labor ratio decreases initially, leading to an increase in the interest rate and a decrease in the effective wage rate (Figure 3.11). After all bulgers have entered the labor force, the working population stabilizes but the capital–labor ratio begins to rise due to the bulge. This leads to a decrease in the interest rate and an increase in the effective wage. After year 45, the first bulgers begin to retire, growth of the working population slows down, and the capital–labor ratio rises, which further decreases the interest rate and increases the effective wage rate. The effective wage rate and the interest rate then fluctuate over time but will eventually return to their respective steady-state levels after the bulgers exit the economy.

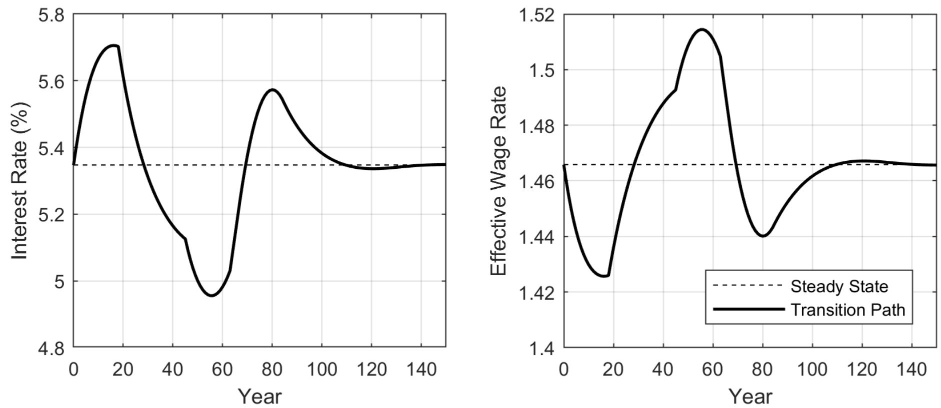

Figure 3.11: The interest rate and the effective wage rate over time in the face of the demographic bulge under policy 2 with a fixed replacement rate and a balanced budget.

I examine a selected age cohort in detail to uncover the reason for gains or losses for the age cohort under policy 2. I take a person from the age −10 cohort, i.e., a person born 10 years after the first bulgers are born, as an example (Figure 3.12). This individual enters the labor force in year 11. As can be seen in Figure 3.11, in year 11, the effective wage rate is lower than its steady-state (no-bulge) level and the interest rate is higher than its steady-state (no-bulge) level. The lower wage rate exerts a negative wealth effect on consumption by lowering the individual's wage income. On the other hand, the higher interest rate induces the individual to substitute saving for consumption intertemporally. The combined effect is reduced consumption and welfare levels at early ages of this individual (Figure 3.12).

As the individual ages, the interest rate soon starts to fall, and the effective wage rate starts to rise, both of which boost consumption. The right panel of Figure 3.12 shows that the cumulative to-age welfare flips from a loss to a gain at age 22. In retirement (starting in year 56), the individual's social security benefit is generally higher than the no-bulge level, thanks to the higher effective wage rate and a fixed replacement rate. Yet, the individual has mostly lower than no-bulge consumption levels, largely because of less cumulative pre-retirement savings. Compared to the total rest-of-life utility in the absence of the bulge, the total rest-of-life utility in the presence of the bulge is 1.07% more in equivalent consumption units under policy 2 for this individual.

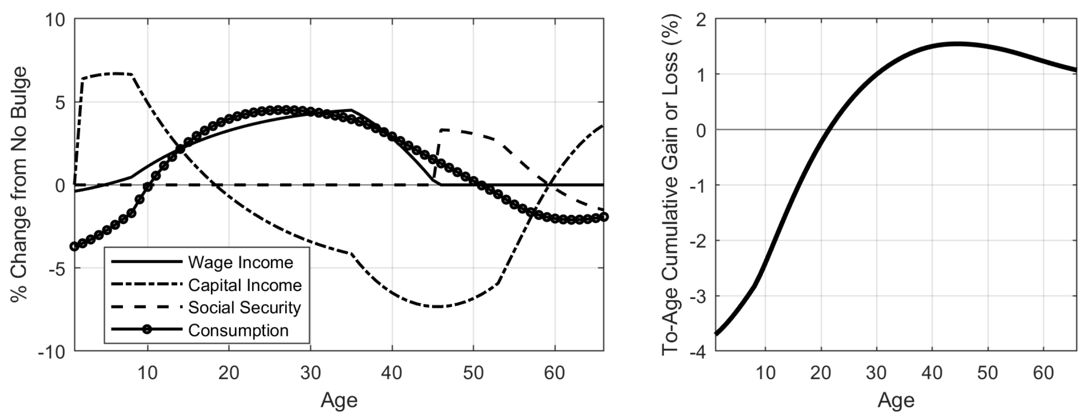

Figure 3.12: Relative changes in consumption, wage income, capital income, and the social security benefit (left) and cumulative gain or loss in rest-of-life welfare (right) for the age −10 cohort along the transition path in the face of the demographic bulge under policy 2 with a fixed replacement rate and a balanced budget.

Figure 3.13 shows the percentage gains or losses of different age cohorts in rest-of-life welfare under policy 2. The cohorts younger than age −33, i.e., those born at least 34 years after the first bulgers are born, experience losses. The cohorts aged 57–65 also experience trivial losses. The remaining age cohorts enjoy gains. 40% of the 150 age cohorts examined suffer welfare losses. The age 59 cohort suffers the largest loss of 2.95%, while the age 25 cohort enjoys the largest gains of 1.52%.

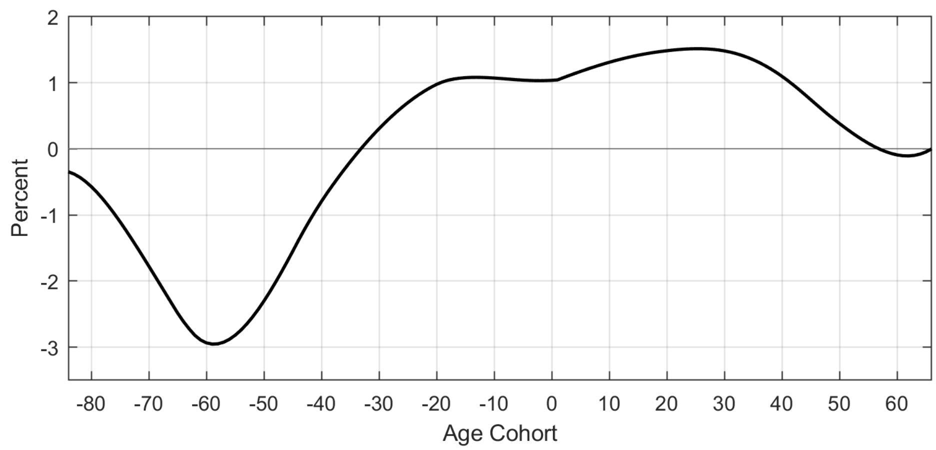

Figure 3.13: Gains or losses of different age cohorts in rest-of-life welfare in the face of the demographic bulge under policy 2 with a fixed replacement rate and a balanced budget.

Figure 3.14 shows the implicit rates of return of social security for different age cohorts in the face of the bulge under policy 2. Figure 3.14 tries to explain why the social security program under policy 2 is attractive to cohorts of age younger than 33 but older than –21, as a means of compulsory saving, when there is a demographic bulge. In the case of no bulge, the steady-state implicit rate of return of social security is $r^{ss} = (1+n)(1+g) - 1 = 2.92\%$. For cohorts younger than 33 but older than age –21, their implicit rates of return are no less than the steady-state value of 2.92%, and hence they would find it beneficial to put money into the social security program. For the other age cohorts, they would rather invest their money on their own. The implicit rate of return is the lowest for the age −50 cohort at 2.40%.

Figure 3.14: Implicit rates of return (percent) of social security for different age cohorts in the face of the demographic bulge under policy 2 with a fixed replacement rate and a balanced budget.

3.4 Policy 3: Tax Rate Ceiling, Balanced Budget

The third social security policy to consider is a balanced-budget policy with a ceiling on the social security tax rate. Under this policy, the social security manager maintains a balanced budget throughout. At first the benefit–income ratio is fixed at 45%, i.e., the replacement rate $B/w = 45\%$, and the tax rate t_w varies to achieve a balanced budget. However, a certain ceiling (e.g. of 15%) is set on the social security tax rate and the replacement rate has to be reduced when the tax ceiling is reached (Figure 3.15). As the bulgers gradually enter the labor force over 18 years, the tax rate drops from 15% in year 0 to 12.22% in year 18. After year 45, the bulgers begin to retire, and the tax rate starts to climb dramatically. When the tax rate reaches 15% in year 50, the replacement rate needs to be adjusted to guarantee the balanced budget and the 15% tax ceiling. The replacement rate is reduced and drops to the bottom of 31.33% in year 63 when the last bulgers retire. The replacement rate gradually rises to the initial level of 45% in year 84 when all bulgers exit the economy.

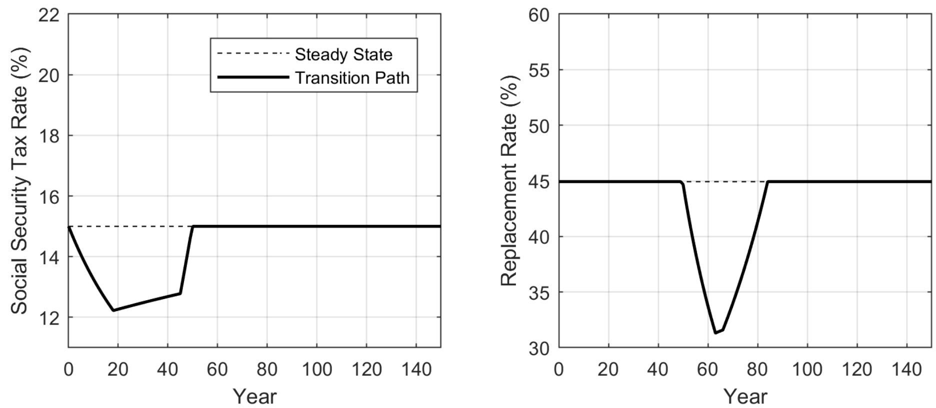

Figure 3.15: The social security tax and the replacement rate over time under policy 3 with a ceiling on the tax rate and a balanced budget.

As the bulgers enter the labor force, the capital–labor ratio decreases, leading to an increase in the interest rate and a decrease in the effective wage rate initially (Figure 3.16). After all bulgers have entered the labor force in year 18, the working population stabilizes but capital increases because of the bulge and the decrease in the tax rate, so the capital–labor ratio begins to rise. This leads to a decrease in the interest rate and an increase in the effective wage rate. After year 45, the bulgers begin to retire, and growth of the working population slows down, so the effective wage rate continues rising while the interest rate keeps falling. The effective wage rate and the interest rate will eventually return to the initial levels after the bulgers gradually exit the economy (Figure 3.16). The tax rate in policy 3 returns to steady state level faster than in policy 2 so the effective wage rate and the interest rate in policy 3 converges to steady state faster than policy 2.

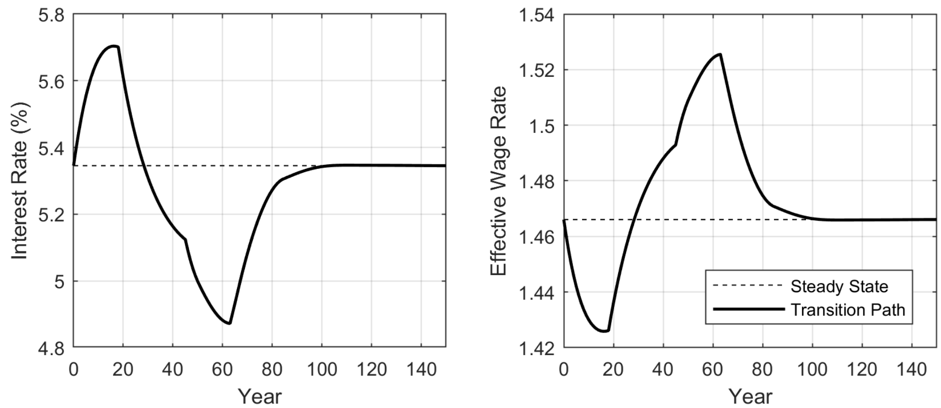

Figure 3.16: The interest rate and the effective wage rate over time in the face of the demographic bulge under policy 3 with a ceiling on the tax rate and a balanced budget.

I examine a selected age cohort in detail to uncover the reason for gains or losses for the age cohort under policy 3. I take a person from the age −10 cohort, i.e., a person born 10 years after the first bulgers are born, as an example (Figure 3.17). The individual enters the labor force in year 11. As can be seen in Figure 3.16, in year 11, the effective wage rate is lower than its steady-state (no-bulge) level and the interest rate is higher than its steady-state (no-bulge) level. The lower wage rate negatively pressures consumption due to the wealth effect from a lower wage income. In addition, the higher interest rate has a substitution effect which makes the individual postpone consumption and save more. The combined effect reduces consumption and welfare levels at early ages of this individual (Figure 3.17).

As the individual ages, the interest rate soon starts to drop while the effective wage rate starts to rise, both of which boost consumption. The right panel of Figure 3.17 shows that the cumulative to-age welfare flips from a loss to a gain at age 22. In retirement (starting in year 56), the individual's social security income is significantly less than that in the steady state, because now the replacement rate is lowered by a lot under policy 3 during the individual's retirement. In addition, this individual does not have sufficient pre-retirement saving. As a result, retirement consumption drops to levels considerably lower than the same-period steady-state levels. Upon retirement, the individual still has a to-age welfare gain; yet by the end of retirement the gain has been turned into a loss. Compared to the total rest-of-life utility in the absence of the bulge, the total rest-of-life utility in the presence of the bulge is 0.81% less in equivalent consumption units under policy 3 for this individual.

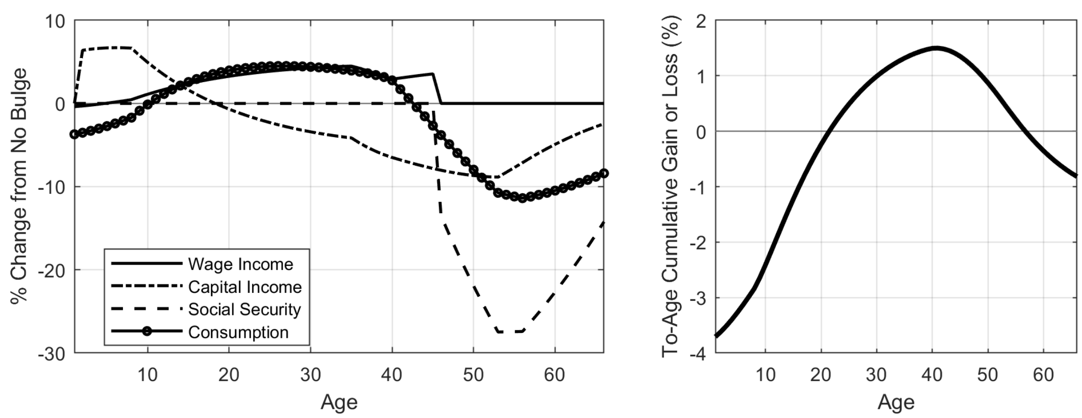

Figure 3.17: Relative changes in consumption, wage income, capital income, and the social security benefit (left) and cumulative gain or loss in rest-of-life welfare (right) for the age −10 cohort along the transition path in the face of the demographic bulge under policy 3 with a ceiling on the tax rate and a balanced budget.

Figure 3.18 shows the percentage gains or losses of different age cohorts in rest-of-life welfare under policy 3. The cohorts aged less than or equal to 0 but more than or equal to −18, which are mostly the bulgers, suffer losses. The cohorts aged 57–65 also suffer some losses in welfare due to the bulge. The remaining cohorts enjoy welfare gains. 18.7% of the 150 age cohorts examined experience some welfare losses.

The age −10 experiences the largest loss of 0.82%, while the age −37 cohort enjoys the highest gain of 1.61%.

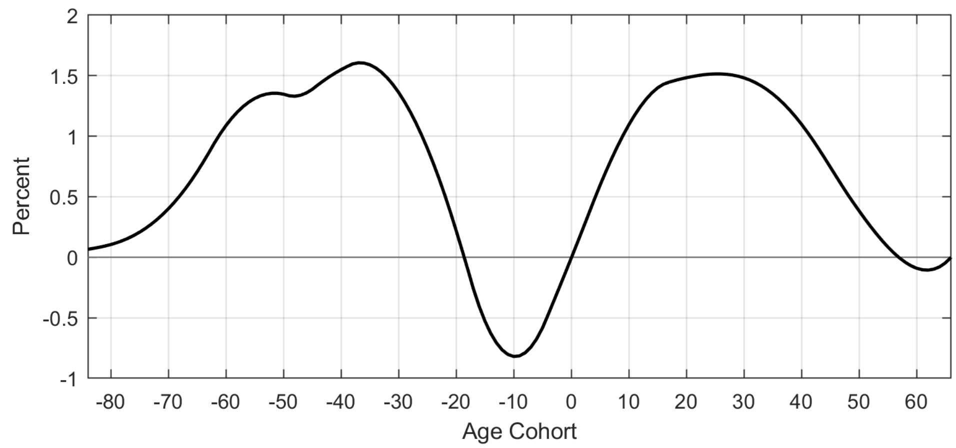

Figure 3.18: Gains or losses of different age cohorts in rest-of-life welfare in the fact of a demographic bulge under policy 3 with a ceiling on the tax rate and a balanced budget.

Figure 3.19 shows the implicit rates of return of social security for different age cohorts in the face of the bulge under policy 3. Figure 3.19 tries to explain why the social security program under policy 3 is attractive to cohorts of age younger than –29 but older than –42 and cohorts of age younger than 33 but older than –3, as a means of compulsory saving, when there is a demographic bulge. In the case of no bulge, the steady-state implicit rate of return of social security is $r^{ss} = (1+n)(1+g) - 1 = 2.92\%$. For cohorts younger than –29 but older than age –42 and cohorts younger than 33 but older than age –3, their implicit rates of return are no less than the steady-state value of 2.92%, and hence they would find it beneficial to put money into the social security program. For the other age cohorts, they would rather invest their money on their own. The implicit rate of return is the lowest for the age −14 cohort at 2.60%.

Figure 3.19: Implicit rate of return (percent) of social security for different age cohorts in the face of the demographic bulge under policy 3 with a ceiling on the tax rate and a balanced budget.

3.5 Policy 4: Fund Accumulation

Finally, I consider a fourth policy under which a special fund is established to gradually accumulate social security surplus as the bulgers enter the labor force and is then gradually depleted as the bulgers retire. When the bulgers enter the labor force and raise the worker–retiree ratio, keeping the social security tax rate and the replacement rate fixed at their respective steady-state levels generates a social security surplus. When the bulgers stay in the labor force, maintaining the steady-state replacement rate does not necessarily require the steady-state tax rate, so the tax rate could be lowered, or the replacement rate could be raised, to boost welfare. Figure 3.20 shows one possible design of the trajectories of the social security tax rate and the replacement rate under this policy, in which the social security tax rate is lowered from 15% to about 14.4% from year 19 through year 96 and the replacement rate is raised from about 44.9% to 53% from year 49 through year 66. With the funds accumulated when the bulgers stay in the labor force, the bulgers' benefits in retirement are guaranteed even if the tax rate remains unchanged. Figure 3.21 shows the size of the accumulated fund over time. The accumulated fund is depleted in year 96.

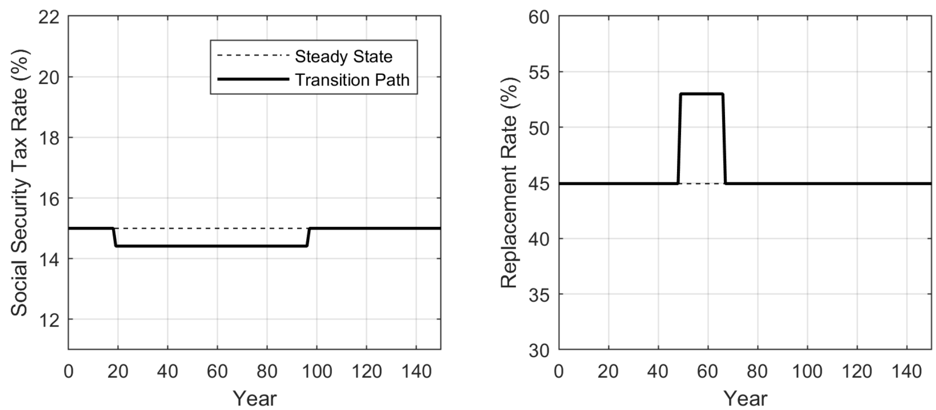

Figure 3.20: The social security tax rate and the replacement rate over time under policy 4 with fund accumulation.

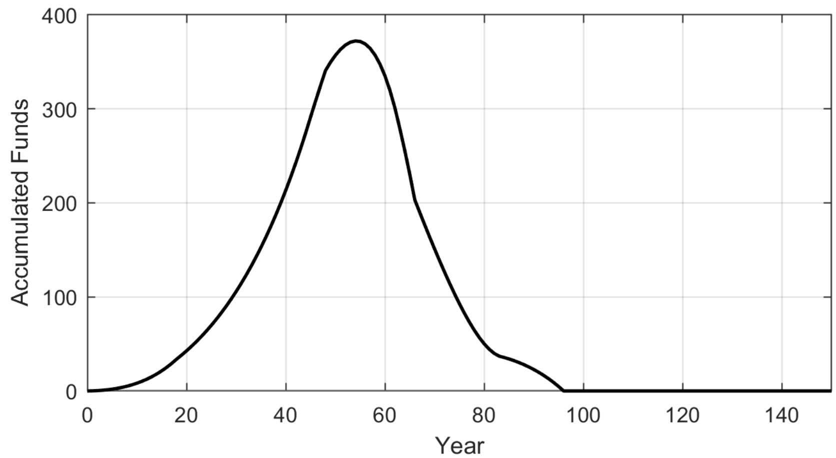

Figure 3.21: Funds accumulated in the face of the demographic bulge over time under policy 4.

As the bulgers enter the labor force, the capital–labor ratio decreases, leading to an increase in the interest rate and a decrease in the effective wage rate initially (Figure 3.22). The social security program runs a surplus and the fund starts to accumulate. After all bulgers enter the labor force, the working population stabilizes but capital increases because of the bulge and a decrease in the tax rate, so the capital–labor ratio begins to rise. This leads to a decrease in the interest rate and an increase in the effective wage rate. After year 45, the bulgers begin to retire, and growth of the working population slows down, resulting in an accelerated rise in the capital–labor ratio and the effective wage rate, and an accelerated decline in the interest rate. From year 54, the social security manager starts to use the accumulated funds to pay benefits and capital falls, so the effective wage rate decreases while the interest rate increases. However, the effective wage rate and the interest rate will eventually return to the initial levels after the bulgers gradually exit the economy (Figure 3.22).

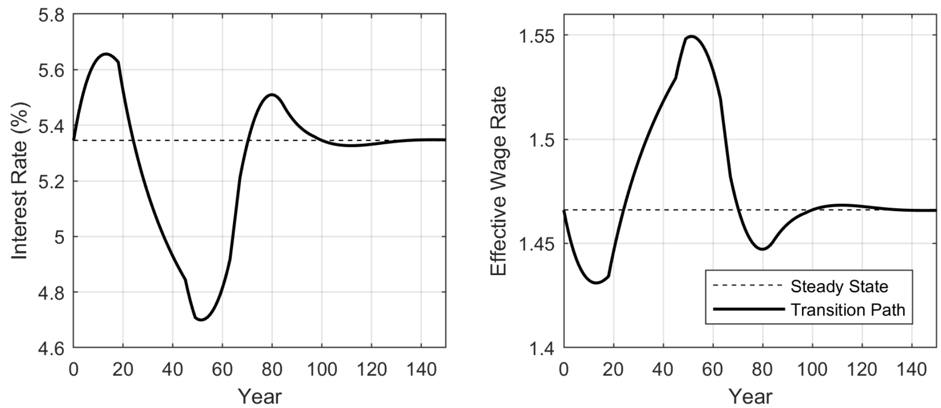

Figure 3.22: The interest rate and the effective wage rate over time in the face of the demographic bulge under policy 4 with fund accumulation.

I examine a selected age cohort in detail to uncover the reason for gains or losses for the age cohort under policy 4. I take a person from the age −10 cohort, i.e., a person born 10 years after the first bulgers are born, as an example (Figure 3.23). This individual enters the labor force in year 11. As can be seen in Figure 3.22, in year 11, the effective wage rate is lower than its steady-state (no-bulge) level and the interest rate is higher than its steady-state (no-bulge) level. The lower wage rate exerts a negative wealth effect on consumption while the higher interest rate induces an intertemporal substitution effect on consumption. The combined effect reduces consumption and welfare at early ages of this individual (Figure 3.23).

As the individual ages, the interest rate soon starts to decline while the effective wage rate starts to rise, both boosting consumption. The right panel of Figure 3.23 shows that the cumulative to-age welfare flips from a loss to a gain for the first time at age 35. In the first half of retirement (starting in year 56), the individual's social security income is much higher than in the steady state, thanks to an increased replacement rate at that time. The higher social security income helps keep consumption at higher-than-steady-state levels in the early years of the individual's retirement. In the second half of the individual's retirement, social security income drops as the replacement rate drops, and consumption and welfare follow the drop. Policy 4 makes the total rest-of-life utility for this individual in the presence of the bulge almost no different from the rest-of-life utility for this individual without the bulge.

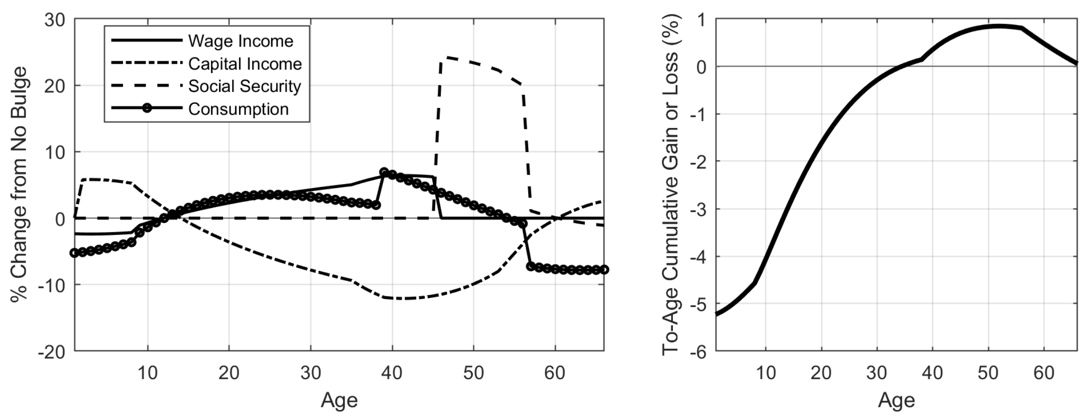

Figure 3.23: Relative changes in consumption, wage income, capital income, and the social security benefit for the age −10 cohort along the transition path in the face of the demographic bulge under policy 4 with fund accumulation.

Figure 3.24 shows the percentage gains or losses of different age cohorts in rest-of-life welfare under policy 4. The vast majority of the age cohorts experience gains. Only cohorts of age −7, 18, and 57–65, which are only 7.3% of the 150 age cohorts examined, suffer minor losses, and the worst loss is merely 0.1%. The age −23 cohort enjoys the largest gain of 2.57%.

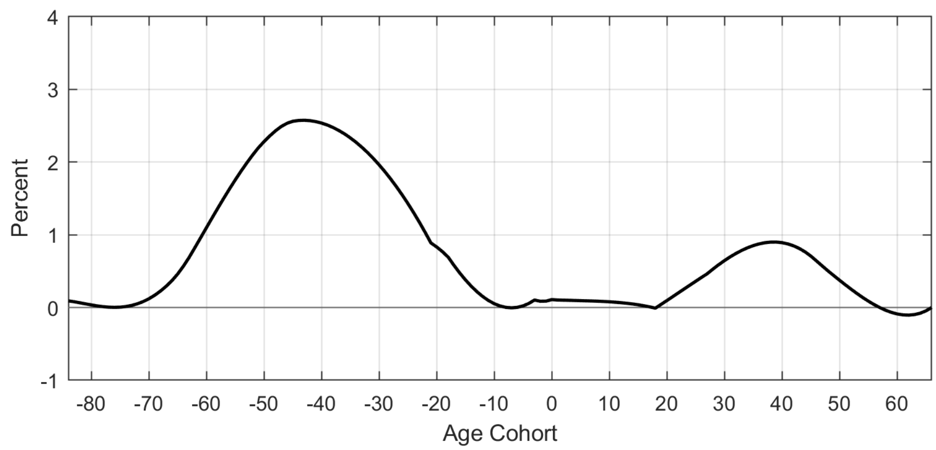

Figure 3.24: Gains or losses of different age cohorts in rest-of-life welfare in the face of the demographic bulge under policy 4.

Figure 3.25 shows the implicit rates of return of social security for different age cohorts in the face of the bulge under policy 4. Figure 3.25 tries to explain why the social security program under fund accumulation is attractive to cohorts of age younger than –33 and cohorts of age younger than 31 but older than –23, as a means of compulsory saving, when there is a demographic bulge. In the case of no bulge, the steady-state implicit rate of return of social security is $r^{ss} = (1 + n)(1 + g) - 1 = 2.92\%$. For cohorts of age younger than –33 and cohorts of age younger than 31 but older than age –23, their implicit rates of return are no less than the steady-state value of 2.92%, and hence they prefer putting money into the social security program. For the other age cohorts, they would rather invest their money on their own. The implicit rate of return is the lowest for the age 43 cohort at 2.86%.

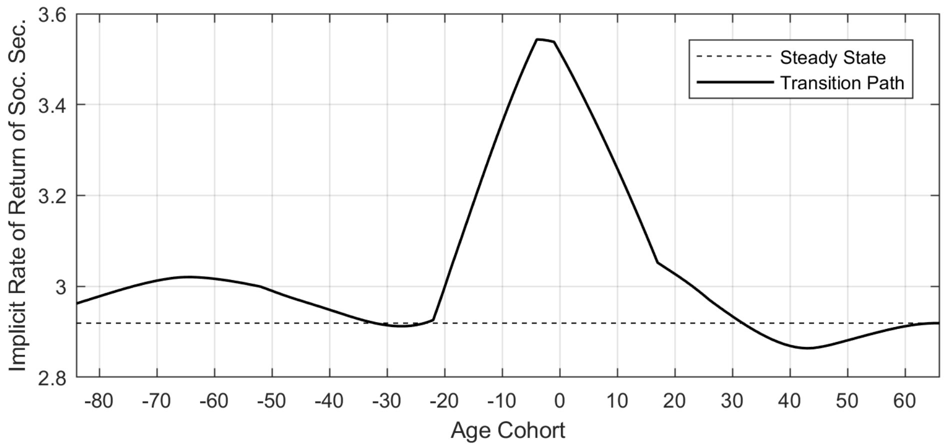

Figure 3.25: Implicit rates of return (percent) of social security for different age cohorts in the face of the demographic bulge under policy 4 with fund accumulation.

3.6 Comparison of Policy Alternatives

This section compares the four social security policy alternatives examined in the previous sections in terms of welfare in the face of the demographic bulge. Table 3.1 presents some welfare metrics of the four policy alternatives and Figure 3.26 collects the gains and losses of different age cohorts under the policy alternatives. Policy 2 with a fixed replacement rate and a balanced budget induces the most losing age cohorts and the second largest maximum loss and the smallest maximum gain among all age cohorts. Under policy 2, the losing cohorts are mostly the post-bulge generation, who have to pay higher social security taxes to sustain the fixed replacement rate when the bulgers retire. The bulgers and the pre-bulge generation generally enjoy welfare gains. Policy 2 also has the second lowest minimum implicit rate of return of social security.

Table 3.1: Welfare evaluation of various social security policy options in the face of the demographic bulge in the U.S.

	Proportion	Maximum Loss[a]		Maximum Gain[a]		Minimum r^{ss} [b]	
	Losing	Amount	Age Cohort	Amount	Age Cohort	Rate	Age Cohort
No Social Security							
Bulge Effect	36.0%	2.1%	−8	2.4%	39	—	—
Balanced-Budget Policies							
#1: Fixed Tax Rate	28.7%	3.4%	−11	5.5%	39	2.20%	−12
#2: Fixed Replacement Rate	40.0%	3.0%	−59	1.5%	25	2.40%	−50
#3: Tax Rate Ceiling	18.7%	0.8%	−10	1.6%	−37	2.60%	−14
Non-Balanced-Budget Policy							
#4: Fund Accumulation	7.3%	0.1%	62	2.6%	−43	2.86%	43

[a] The amount of gain or loss in utility is measured in terms of increase or decrease in equivalent consumption units.

[b] r^{ss} is the implicit rate of return of social security. $r^{ss} = (1 + n)(1 + g) - 1 = 2.92\%$ in the steady state.

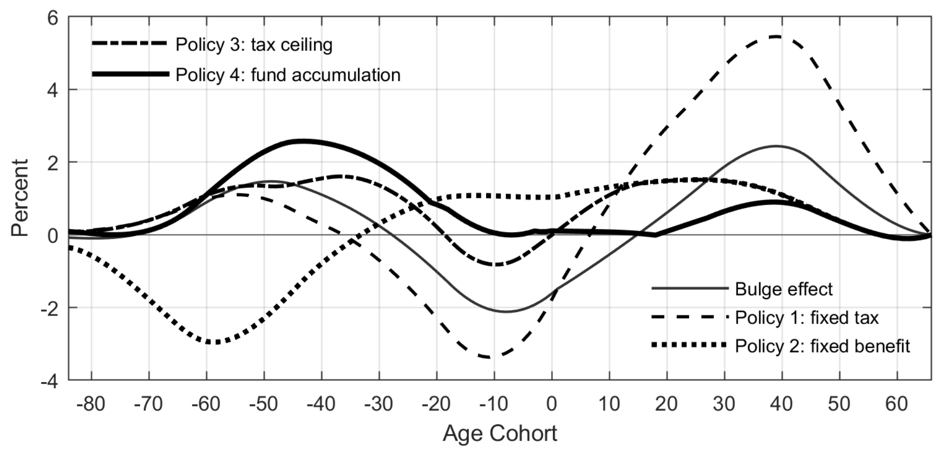

Figure 3.26: Gains or losses (percent) of different age cohorts in the face of the demographic bulge under various policy options.

Policy 1 with a fixed tax rate and a balanced budget ranks second in the resulting proportion of losing cohorts and first in the maximum loss incurred by any age cohort. Under policy 1, the pre-bulge generation enjoys elevated replacement rates when bulgers enter the labor force and also rest-of-life welfare gains in general. The bulgers and some age cohorts following the bulgers suffer welfare losses. In addition, policy 1 has the lowest minimum implicit rate of return of social security.

Policy 3 with a ceiling on the social security tax rate has the second least proportion of losing age cohorts. Moreover, the maximum loss of any age cohort is considerably less than that under policy 1 or policy 2. The social security tax rate and the replacement rate trajectories are exactly the same for the first 49 years under policy 3 and policy 2. As a result, the curve of the rest-of-life welfare gains or losses under policy 3 coincide with that under policy 2 for cohorts aged more than or equal to 17. Despite of the same beginning in policy rates, policy 3 saves the post-bulge generation from suffering a non-trivial welfare loss as under policy 2. On the other hand, the bulgers inflict some welfare losses, partly because of the lowered replacement rate during their retirement years due to the ceiling on the tax rate.

The social security tax rate and the replacement rate trajectories are identical after year 84 under policy 3 and policy 1. The curve of rest-of-life welfare gains or losses under policy 3 almost coincide with that under policy 2 for cohorts aged less than −59. Despite of the same ending in policy rates, policy 3 is significantly better at "smoothing" the welfare curve than policy 1. Overall, policy 3 seems to be a well-rounded policy option that avoids imposing extreme losses on any age cohorts. In terms of the minimum implicit rate of return of social security, policy 3 ranks the second highest.

Policy 4 with fund accumulation produces the least proportion of losing age cohorts and at the same time a negligible maximum rest-of-life loss of 0.1% among all age cohorts. It also achieves a maximum rest-of-life gain higher than that under policy 2 or 3, as well as the highest minimum implicit rate of return of social security. Compared with policy 3, policy 4 is able to avoid imposing non-trivial losses on some of the bulge age cohorts and some of the pre-bulge age cohorts because of a temporarily elevated replacement rate from year 49 to year 66. Furthermore, the social security manager has the budget to raise the replacement rate at that time since the manager has accumulated a large fund prior to that. The social security program performs intergenerational transfer of wealth from the current workers to the current retirees without accumulation of a fund. However, fund accumulation enables the social security program to achieve wealth transfer to future generations and helps better smooth over time the impact on welfare of a demographic bulge.

Chapter 4

MANAGING A DEMOGRAPHIC SHRINK: THE CASE OF CHINA

This chapter applies the OLG model described in Chapter 2 to the case of China and examines the welfare implications along the transition path of various social security policy options in the face of a demographic shrink.

4.1 A Demographic Shrink: Parameters

For the case of China, I assume that individuals are born at age 0, enter the labor force at age 20, work for 40 years, retire at age 60, live another 20 years of retirement, and then die at age 80. The parameter values used are as calibrated in Table 2.1. In addition, I assume that a demographic shrink hits the economy in year -19 and lasts $T_S = 36$ years, which means the shrink hits the labor force in year 1.[29] The demographic shrink temporarily decreases the number of births in a year by 25 percent.[30]

[29] The exact year in which China's one-child policy was started is debated, but it is generally thought to be in the late 1970s or the early 1980s (Zhang 2017, Wang et al. 2016). The universal two-child policy, started in 2016, marked the end of the one-child policy (Wang et al. 2016). Assuming the one-child policy started in 1980 and ended in 2015, I set the length of China's demographic shock to $T_S = 36$ years in the simulation.

[30] I calculated from World Bank Data (SP.DYN.CBRT.IN: Birth rate, crude and SP.POP.TOTL: Population, total) that the number of births in China in 1979–2009 was about 609 million. Wang and Cai (2010) estimated that at most 200 million births in the period of 1979–2009 had been prevented by the one-child policy in China. Hence, I approximate the effect of China's one-child policy on population growth in the simulation as a 25% reduction in the number of births.

I illustrate how a demographic shrink may alter the worker–retiree ratio over time as the shrinking generations enter and then exit the labor force (Figure 4.1). A shrink lasting for 36 years hits the economy in the year −19, which results in the birth rate falling from year −19 through year 16. The first shrinking generation enters the labor force in year 1 and the last shrinking generation exits the economy in year 96. From year 1 to year 36, the shrinking generations gradually enter the labor force, leading to a decrease in the growth rate of the working population and at the same time a decrease in the worker–retiree ratio from 2.97 to 2.29. Starting at year 40, the shrinking generations retire and exit the labor force, which significantly increases the worker–retiree ratio. The worker–retiree ratio first rises above the steady-state level in year 51 and hits the ceiling in year 76 at about 3.96 when the last shrinking generation retires. Finally, from year 76 to year 96, the population composition and the worker–retiree ratio gradually return to their steady-state (pre-shrink) levels.

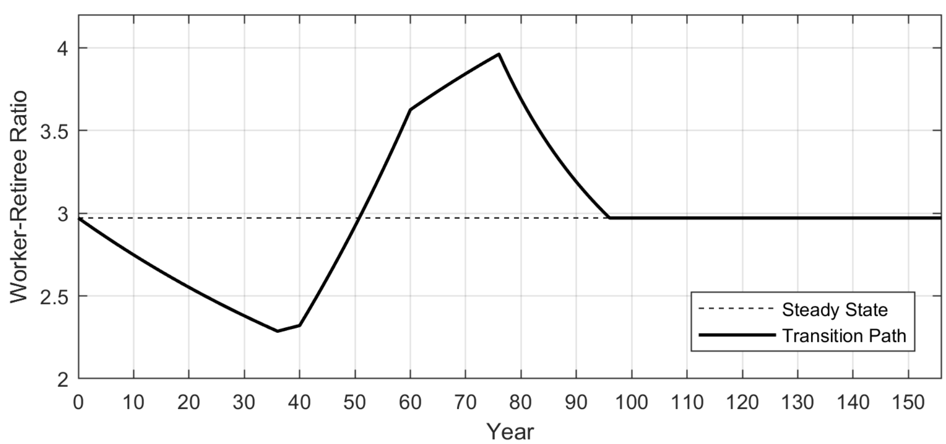

Figure 4.1: The worker–retiree ratio along the transition path in the face of a 36-year demographic shrink entering the labor force in year 1.

To capture the effects of a demographic shrink on the economy and on welfare, I assume in this section that the economy has no social security program, or equivalently, that the social security tax rate is $t_w = 0$. When the shrinking generations enter the labor force, capital per effective labor rises initially. As a result, the interest rate, which equals the marginal product of capital, will decrease initially, whereas the effective wage rate, which equals the marginal product of labor, will increase initially (Figure 4.2). After all the shrinking generations are in the labor force, the working population stabilizes but capital decreases due to the demographic shrink so the capital–labor ratio begins to fall. This leads to an increase in the interest rate and a decrease in the effective wage rate. Both the interest rate and the wage rate will eventually return to the steady-state levels after the shrinking generations exit the economy.

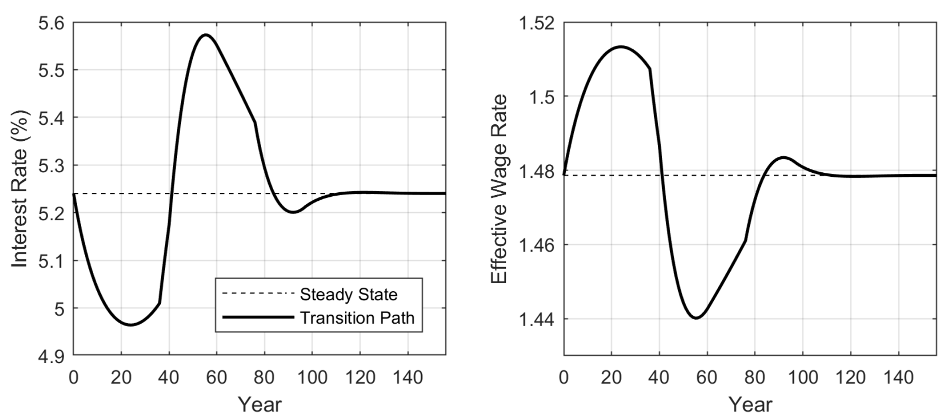

Figure 4.2: The interest rate and the effective wage rate along the transition path in the face of the demographic shrink.

I examine selected age cohorts in detail to explain the underlying reasons for gains or losses under "shrink effect." I take a person who is born 20 years after the first shrinking generation is born, i.e., a person from the age −20 cohort, as an example (Figure 4.3). The left panel shows relative changes in the individual's consumption and income of various sources along the transition path due to the presence of the shrink. This individual enters the labor force in year 21 when the individual is 20 years old. According to Figure 4.2, the effective wage rate in year 21 is much higher than the steady-state (no-shrink) level, so initially this individual earns more wage income.

The interest rate, however, is lower than the steady-state (no-shrink) level initially. A lower interest rate has two competing effects on consumption: on the one hand, it lowers capital income and thus has a negative wealth effect; on the other hand, it lowers the incentive to save and hence increases the incentive to consume, i.e., an intertemporal substitution effect. The combined effect, as the left panel of Figure 4.3 shows, is that the individual chooses to boost consumption. The right panel of Figure 4.3 plots the cumulative rest-of-life change in welfare in equivalent consumption units. This individual enjoys a significant welfare gain at early ages because of high consumption.

As the interest rate rises, the individual starts decreasing consumption, which reduces the gain in welfare. Yet, during this person's retirement starting at year 61, the interest rate is mostly above its steady-state (no-shrink) level. Hence, during retirement, this person receives much more capital income, which leads to an increased consumption level as well as a higher utility level. Compared to the total rest-of-life utility in the absence of the shrink, the total rest-of-life utility in the presence of the shrink is 1.71% more in equivalent consumption units for this individual.

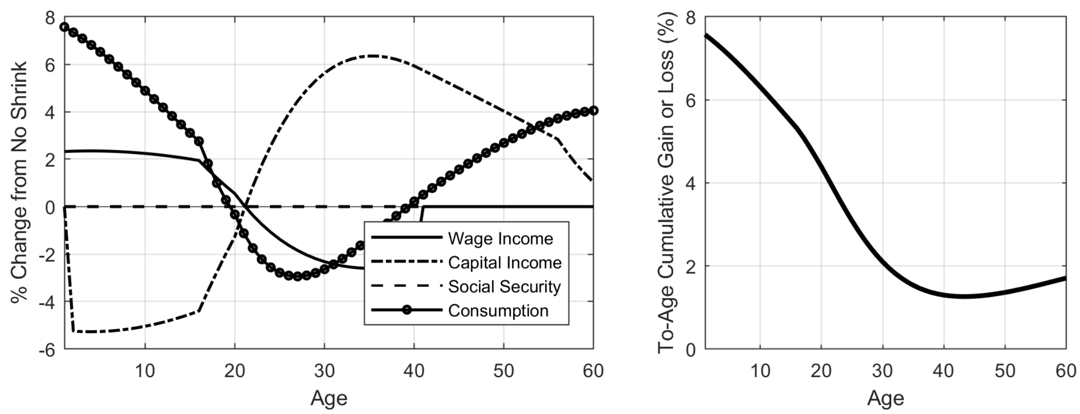

Figure 4.3: Relative changes in consumption, wage income, capital income, and the social security benefit (left) and the cumulative gain or loss in welfare in equivalent consumption units due to the presence of the shrink (right) for the age −20 cohort along the transition path without a social security program in the face of the demographic shrink.

To assess how cohorts of different ages are affected by the demographic shrink, I calculate the difference in rest-of-life utility achieved with and without the shrink for each age cohort. Figure 4.4 shows the percentage gains or losses in rest-of-life welfare. An age cohort is numbered after the age of individuals in the age cohort when the first shrinking generation is born. For example, the age 0 cohort is the first shrinking generation. The individuals in the age 40 cohort are 60 years old in their first year of retirement when the first shrinking generation enters the labor force at age 20.

The cohorts aged less than or equal to −37 but more than or equal to −77 as well as those aged 7−60 experience losses, which are 59.87% of the 150 age cohorts examined. The remaining age cohorts experience gains. The age 30 cohort suffers the largest loss of 1.67% while the age −13 cohort enjoys the largest gains of 1.93%.

Figure 4.4: Gains or losses of different age cohorts in rest-of-life welfare in the face of the demographic shrink.

4.2 Policy 1: Fixed Tax Rate, Balanced Budget

The first social security policy to consider is a balanced-budget policy with a fixed social security tax rate. Under this policy, the social security manager sets benefit per retiree B each year to balance the budget with a fixed tax rate of 22% and sets a replacement rate $B/w = t_w \omega = 0.22\omega$, so that B/w varies directly with ω. As the shrinking generations gradually enter the labor force over 36 years, the replacement rate drops from 65.4% in year 0 to 50% in year 36. After year 40, the first shrinking generation begins to retire, and the replacement rate climbs to a peak of 87% in year 76. As the shrinking generations gradually exit the labor force after year 76, the replacement rate gradually declines to the steady-state level. Figure 4.5 shows the path of the replacement rate under policy 1 over time.

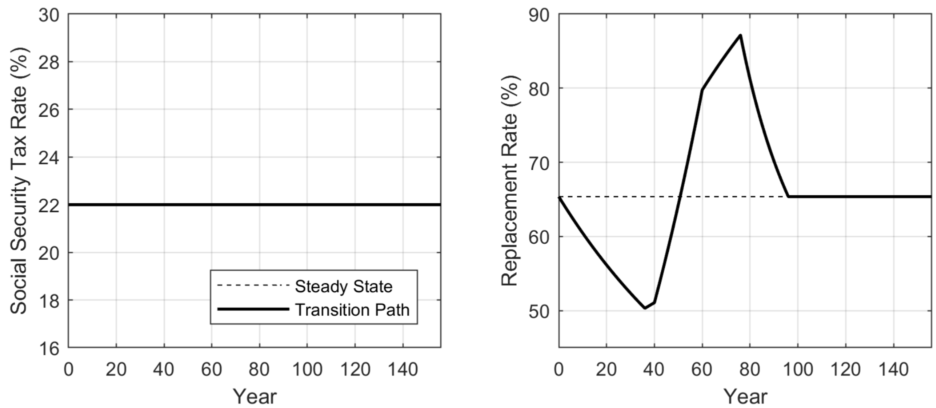

Figure 4.5: The social security tax and the replacement rate over time under policy 1 with a fixed tax rate and a balanced budget.

As the shrinking generations enter the labor force, the capital–labor ratio increases, leading to an increase in the effective wage rate and a decrease in the interest rate (Figure 4.6). After all shrinking generations have entered the labor force, the working population stabilizes but capital decreases because of the demographic shrink, so the capital–labor ratio begins to fall after year 36. This leads to a decrease in the effective wage rate and an increase in the interest rate. After year 40, the shrinking generations begin to retire so growth of the working population speeds up, resulting in an accelerated decline in the capital–labor ratio and the effective wage rate, yet an accelerated rise in the interest rate. The effective wage rate and the interest rate gradually return to the steady-state levels when the shrinking generations gradually exit the economy (Figure 4.6).

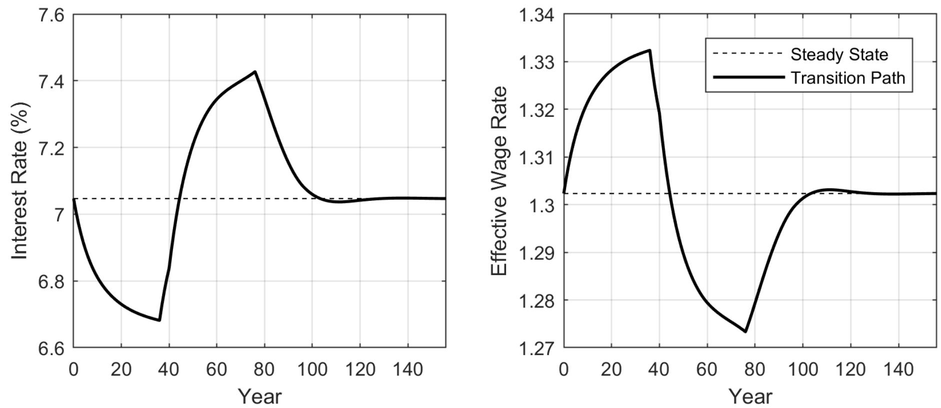

Figure 4.6: The interest rate and the effective wage rate over time in the face of the demographic shrink under policy 1 with a fixed tax rate and a balanced budget.

I examine a selected age cohort in detail to uncover the reasons for gains or losses for the age cohort under policy 1. I take a person from the age −20 cohort, i.e., a person born 20 years after the first shrinking generation is born, as an example (Figure 4.7). According to Figure 4.6, in year 21, the effective wage rate is higher than its steady-state (no-shrink) level and the interest rate is lower than its steady-state (no-shrink) level. The higher wage rate increases the individual's wage income initially, which imposes a positive wealth effect on consumption. In addition, the lower interest rate exerts a strong intertemporal substitution effect that favors more consumption. The combined effect results in high consumption levels and significantly high welfare levels at early ages of this individual (Figure 4.7). However, the decreased effective wage rate and the increased interest rate lower the consumption as well as welfare after year 36.

When this individual retires in year 61, the high interest rate (Figure 4.6) raises the individual's capital income from pre-retirement savings. In addition, the replacement rate is higher than its steady-state (no-shrink) level (Figure 4.5), which gives a further boost to the individual’s social security income. The high retirement income level provides the individual an incentive to consume more and thus enables welfare gains. The upturn of the curve upon retirement in the right panel of Figure 4.7 reflects welfare gains in this individual’s retirement. Compared to the total rest-of-life utility in the absence of the shrink, the total rest-of-life utility in the presence of the shrink is 5.55% more in equivalent consumption units under policy 1 for this individual.

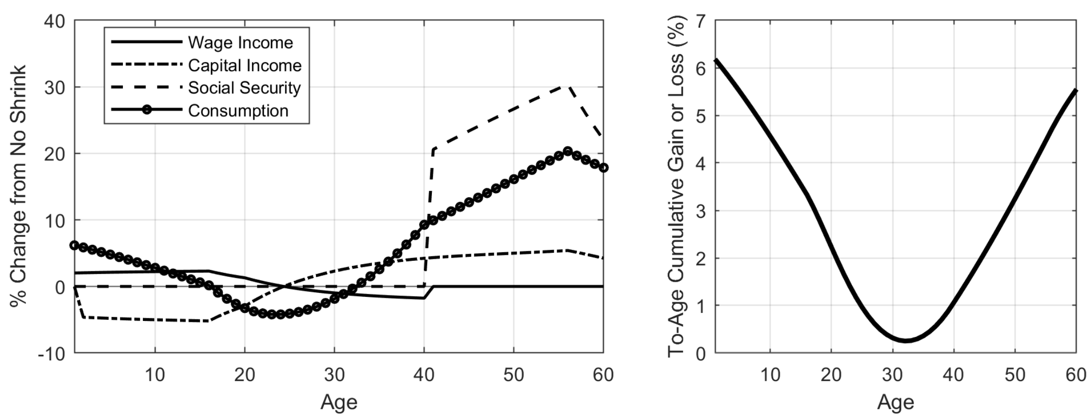

Figure 4.7: Relative changes in consumption, wage income, capital income, and the social security benefit (left) and the cumulative gain or loss in welfare in equivalent consumption units due to the presence of the shrink (right) for the age −20 cohort along the transition path in the face of the demographic shrink.

Figure 4.8 shows the percentage gains or losses of different age cohorts in rest-of-life welfare under policy 1. The cohorts aged less than or equal to 60 but more than or equal to 0 as well as those aged less than or equal to −51 experience losses, which are 67.52% of the 150 age cohorts examined. The remaining age cohorts experience gains. The age 21 cohort suffers the largest loss of 5.98% while the age −22 cohort enjoys the largest gains of 5.62%.

Figure 4.8: Gains or losses of different age cohorts in rest-of-life welfare in the face of the demographic shrink under policy 1 with a fixed tax rate and a balanced budget.

Figure 4.9 shows the implicit rates of return of social security for different age cohorts in the face of the shrink under policy 1. Figure 4.9 tries to explain why the social security program under policy 1 is attractive to cohorts of age younger than –3, as a means of compulsory saving, when there is a demographic shrink. In the case of no shrink, the steady-state implicit rate of return of social security is $r^{ss} = (1 + n)(1 + g) - 1 = 6.26\%$. For cohorts of age younger than –3, their implicit rates of return are no less than the steady-state value of 6.26%, and hence they would find it beneficial to put money into the social security program. For the other age cohorts, they would rather invest their money on their own. The implicit rate of return is the lowest for the age 15 cohort at 5.53%.

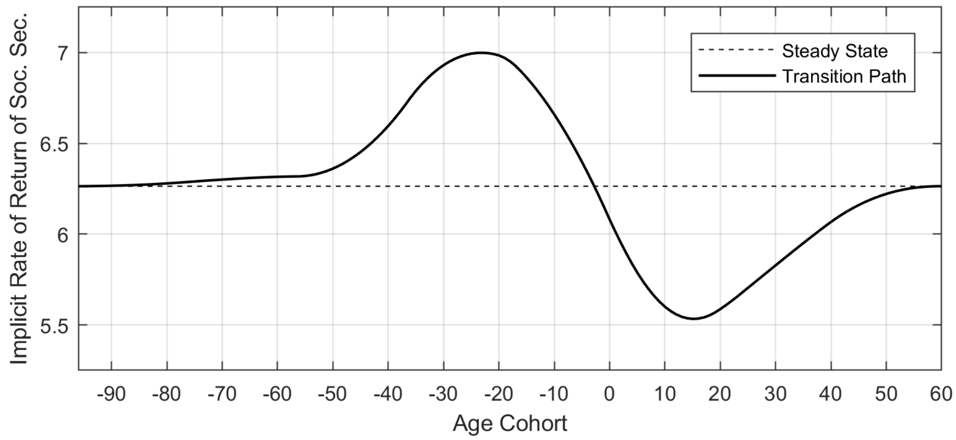

Figure 4.9: Implicit rates of return (percent) of social security for different age cohorts in the face of the demographic shrink under policy 1 with a fixed tax rate and a balanced budget.

4.3 Policy 2: Fixed Replacement Rate, Balanced Budget

The second social security policy to consider is a balanced-budget policy with a fixed replacement rate. Under this policy, the social security manager sets benefit per retiree B to maintain the replacement rate at 65.4%, i.e., $B/w = 65.4\%$, and sets the social security tax rate $t_w = (B/w)/\omega = 0.65/\omega$, so that t_w varies inversely with ω. As the shrinking generations gradually enter the labor force over 36 years, the tax rate rises from 22% in year 0 to 28.86% in year 36. After year 40, the shrinking generations begin to retire and the tax rate declines significantly to the lowest level of 16.66% in year 76. As the shrinking generations gradually exit the labor force after year 76, the tax rate gradually rises to the steady-state level of 22%. Figure 4.10 shows the path of the tax rate under policy 2 over time.

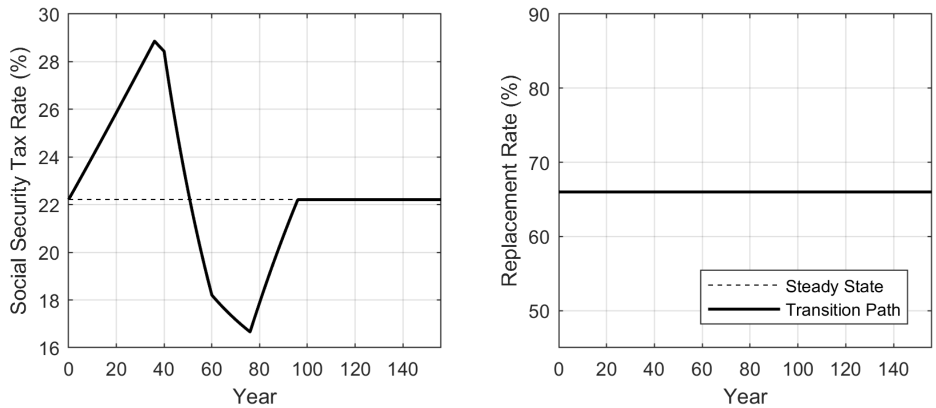

Figure 4.10: The social security tax rate and the replacement rate over time under policy 2 with a fixed replacement rate and a balanced budget.

As the shrinking generations enter the labor force, the capital–labor ratio increases, leading to a decrease in the interest rate and an increase in the effective wage rate initially. The rising tax rate and the shrink result in a quick drop of capital, which leads to a decrease in the effective wage rate. As the shrinking generations gradually exit the economy, the effective wage rate and the interest rate gradually return to the steady-state levels (Figure 4.11).

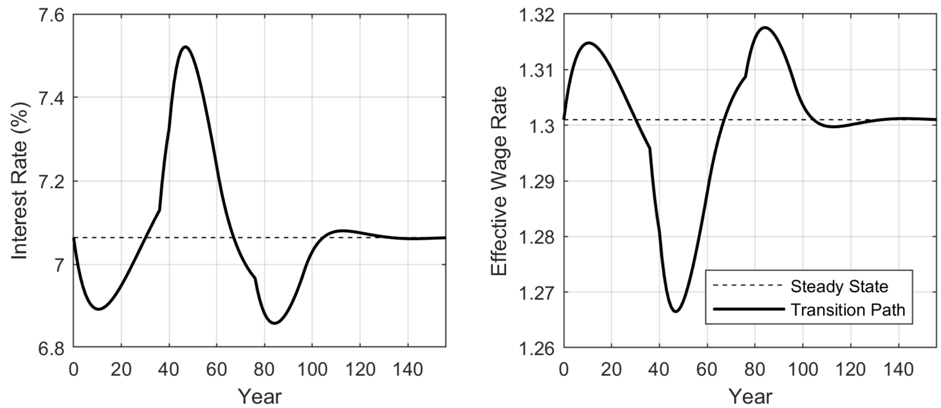

Figure 4.11: The interest rate and the effective wage rate over time in the face of the demographic shrink under policy 2 with a fixed replacement rate and a balanced budget.

I examine a selected age cohort in detail to uncover the reason for gains or losses for the age cohort under policy 2. I take a person from the age −20 cohort, i.e., a person born 20 years after the first shrinking generation is born, as an example (Figure 4.12). As can be seen in Figure 4.11, in year 21, the effective wage rate is higher than its steady-state (no-shrink) level and the interest rate is lower than its steady-state (no-shrink) level. The higher wage rate and lower interest rate should have provided an incentive to consume more. However, the high tax rate (Figure 4.10) offsets such incentives, which reduces consumption and welfare levels at early ages of this individual (Figure 4.12).

As the individual ages, the wage rate begins to drop while the interest rate begins to rise from year 30. The lower wage rate negatively pressures consumption due to the wealth effect from a lower wage income. In addition, the higher interest rate has a substitution effect causing the individual to postpone consumption and save more. Therefore, consumption continues to drop. Because of the high interest rate and low consumption at early ages, this individual accumulates more savings to consume in middle ages, which leads to an increase in consumption and welfare gains during retirement. Compared to the total rest-of-life utility in the absence of the shrink, the total rest-of-life utility in the presence of the shrink is 3.32% less in equivalent consumption units under policy 2 for this individual.

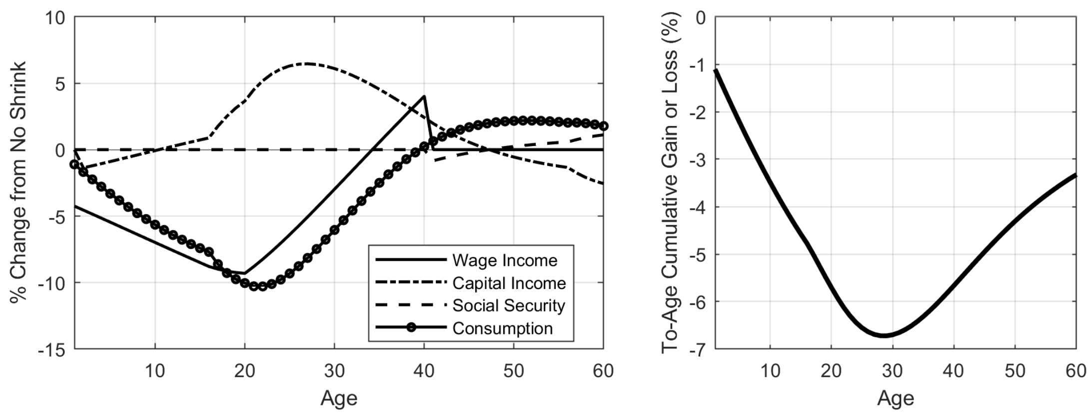

Figure 4.12: Relative changes in consumption, wage income, capital income, and the social security benefit (left) and the cumulative gain or loss in welfare in equivalent consumption units due to the presence of the shrink (right) for the age −20 cohort along the transition path in the face of the demographic shrink.

Figure 4.13 shows the percentage gains or losses of different age cohorts in rest-of-life welfare under policy 2. The cohorts aged less than or equal to 44 but more than or equal to −38 experience losses, which are 52.23% of the 150 age cohorts examined. The remaining age cohorts experience gains. The age −12 cohort suffers the largest loss of 3.78% while the age −58 cohort enjoys the largest gains of 3.61%.

Figure 4.13: Gains or losses of different age cohorts in rest-of-life welfare in the face of the demographic shrink under policy 2 with a fixed tax rate and a balanced budget.

Figure 4.14 shows the implicit rates of return of social security for different age cohorts in the face of the shrink under policy 2. Figure 4.14 tries to explain why the social security program under policy 2 is attractive to cohorts of age less than or equal to −34 as well as those aged more than or equal to 31, as a means of compulsory saving, when there is a demographic shrink. In the case of no shrink, the steady-state implicit rate of return of social security is $r^{ss} = (1 + n)(1 + g) - 1 = 6.26\%$. For cohorts aged less than or equal to −34 as well as those aged more than or equal to 31, their implicit rates of return are no less than the steady-state value of 6.26%, and hence they would find it beneficial to put money into the social security program. For the other age cohorts, they would rather invest their money on their own. The implicit rate of return is the lowest for the age −6 cohort at 5.63%.

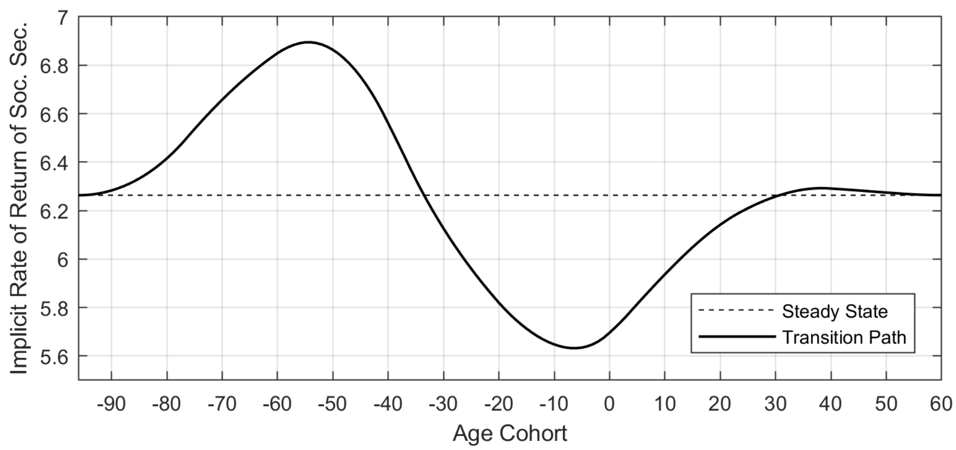

Figure 4.14: Implicit rates of return (percent) of social security for different age cohorts in the face of the demographic shrink under policy 2 with a fixed replacement rate and a balanced budget.

4.4 Policy 3: Replacement Rate Ceiling, Balanced Budget

The third social security policy to consider is a balanced-budget policy with a ceiling on the replacement rate. Under this policy, at first the tax rate t_w is fixed at 22%, and the replacement rate B/w varies to achieve a balanced budget. However, a ceiling of 65.4% is set on the replacement rate and the social security tax rate has to be reduced when the replacement rate ceiling is reached (Figure 4.15). As the shrinking generations enter the labor force in year 1, the benefit has to be reduced to keep the tax rate of 22%. The replacement rate drops to the bottom of 50.3% in year 36 when the last shrinking generation enters the labor force. After year 40, the shrinking generations begin to retire, and the replacement rate starts to rise dramatically while the tax rate is fixed. When the replacement rate reaches 65.4% in year 50, the tax rate needs to be adjusted to guarantee the balanced budget. The tax rate decreases and hits the bottom of 16.5% in year 76 when the last shrinking generation retires. The tax rate gradually rises to the steady-state level of 22% in year 96 when all shrinking generations exit the economy.

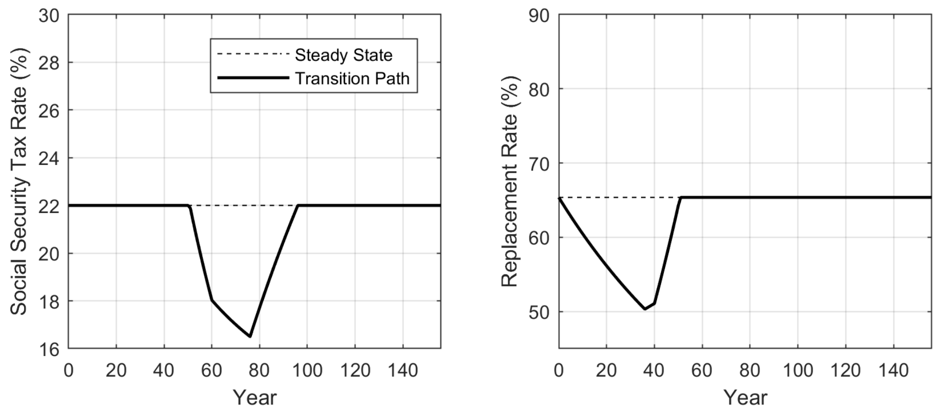

Figure 4.15: The social security tax rate and the replacement rate over time under policy 3 with a ceiling on the replacement rate and a balanced budget.

As the shrinking generations enter the labor force, the capital–labor ratio increases, leading to a decrease in the interest rate and an increase in the effective wage rate initially. After all shrinking generations have entered the labor force, the working population stabilizes but the shrinking generations reduce the capital. A fall in the capital–labor ratio prompts an increase in the interest rate and a decrease in the effective wage rate. When the shrinking generations exit the labor force, it causes a fluctuation of the wage rate and the interest rate over time. As the shrinking generations gradually exit the economy, the effective wage rate and the interest rate gradually return to the steady-state levels (Figure 4.16).

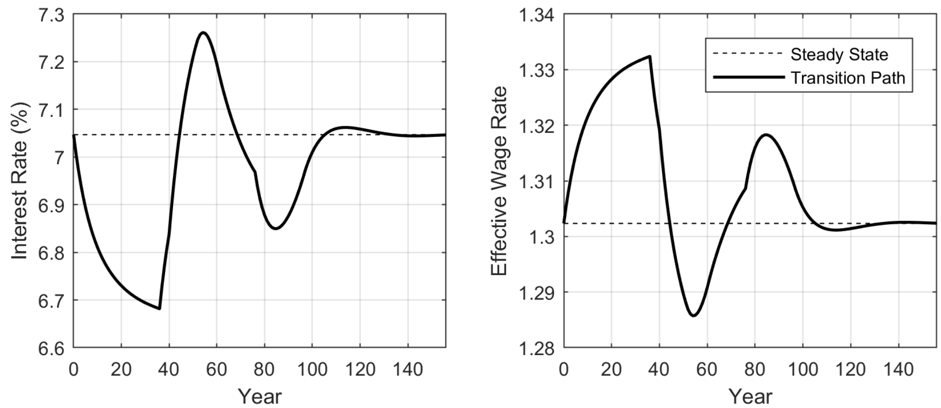

Figure 4.16: The interest rate and the effective wage rate over time in the face of the demographic shrink under policy 3 with a ceiling on the replacement rate and a balanced budget.

I examine a selected age cohort in detail to uncover the reason for gains or losses for the age cohort under policy 3. I take a person from the age −20 cohort, i.e., a person born 20 years after the first shrinking generations are born, as an example (Figure 4.17). The individual enters the labor force in year 21. As can be seen in Figure 4.16, in year 21, the effective wage rate is higher than its steady-state (no-shrink) level and the interest rate is lower than its steady-state (no-shrink) level. The higher wage rate increases the individual's wage income initially, also causing a positive wealth effect on consumption. In addition, the lower interest rate exerts a strong intertemporal substitution effect that favors more consumption. The combined effect results in high consumption levels and significantly high welfare levels at early ages of this individual (Figure 4.17).

When this individual retires in year 61, a higher interest rate raises the individual's capital income from pre-retirement savings. In addition, the higher effective wage rate with a fixed replacement rate further boosts the individual's retirement income. The high retirement income level provides the individual an incentive to consume more and thus get welfare gains. Compared to the total rest-of-life utility in the absence of the shrink, the total rest-of-life utility in the presence of the shrink is 1.05% more in equivalent consumption units under policy 3 for this individual.

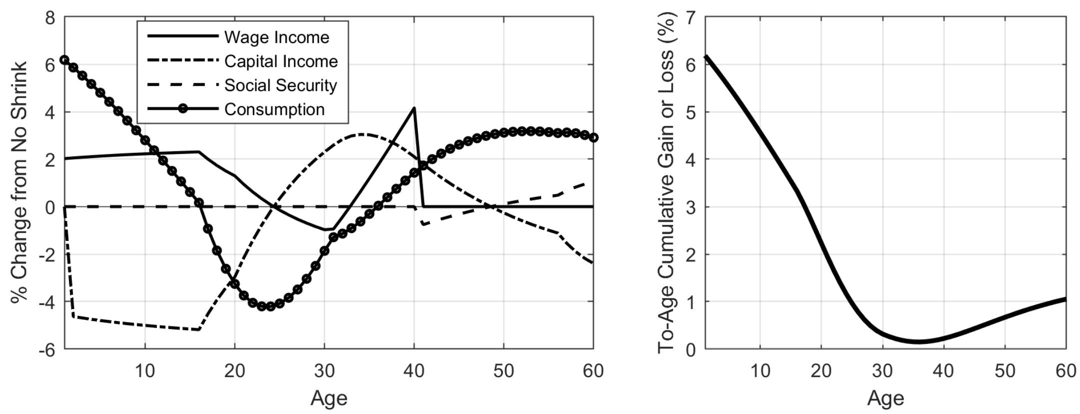

Figure 4.17: Relative changes in consumption, wage income, capital income, and the social security benefit (left) and the cumulative gain or loss in welfare in equivalent consumption units due to the presence of the shrink (right) for the age −20 cohort along the transition path in the face of the demographic shrink.

Figure 4.18 shows the percentage gains or losses of different age cohorts in rest-of-life welfare under policy 3. The cohorts aged less than or equal to 60 but more than or equal to −5, suffer losses. The remaining cohorts enjoy welfare gains. 41.4% of the 150 age cohorts examined experience some welfare losses. The age 20 cohort experiences the largest loss of −5.75%, while the age −57 cohort enjoys the highest gain of 3.57%.

Figure 4.18: Gains or losses of different age cohorts in rest-of-life welfare in the face of a demographic shrink under policy 3 with a ceiling on the replacement rate and a balanced budget.

Figure 4.19 shows the implicit rates of return of social security for different age cohorts in the face of the shrink under policy 3. Figure 4.19 tries to explain why the social security program under policy 3 is attractive to cohorts of age less than or equal to −19, as a means of compulsory saving, when there is a demographic shrink. In the case of no shrink, the steady-state implicit rate of return of social security is $r^{ss} = (1 + n)(1 + g) - 1 = 6.26\%$. For cohorts aged less than or equal to –19, their implicit rates of return are no less than the steady-state value of 6.26%, and hence they prefer putting money into the social security program. For the other age cohorts, they would rather invest their money on their own. The implicit rate of return is the lowest for the age 15 cohort at 5.53%.

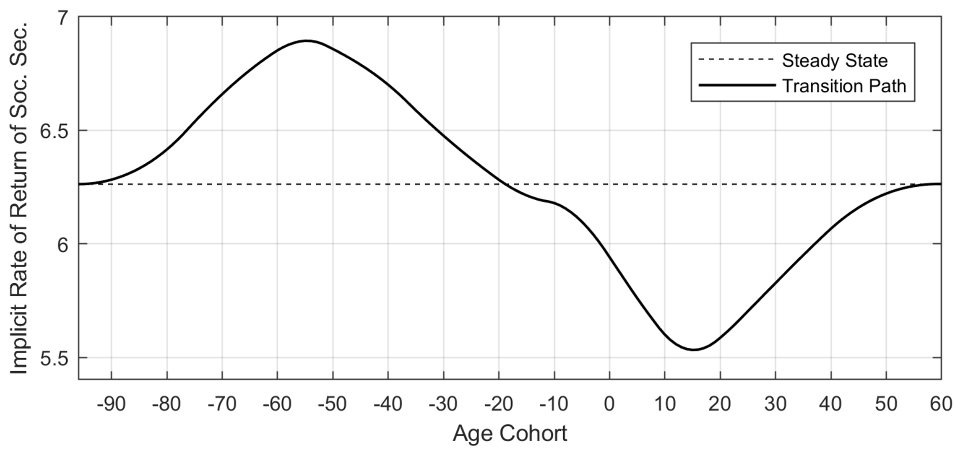

Figure 4.19: Implicit rates of return (percent) of social security for different age cohorts in the face of the demographic shrink under policy 3 with a ceiling on the replacement rate and a balanced budget.

4.5 Policy 4: Intergenerational Borrowing

Under policy 4, the social security manager sets a fixed tax of 22% after the first shrinking generations enter the labor force. The tax rate is raised from 22% to 38.79% from year 13 to year 21. From year 0 to year 36, the replacement rate is raised by 3% from the initial point of 65.37%. From year 36 to year 56, the replacement rate maintains at 65.37%. From year 56 to year 126, the replacement rate is raised to 69.94%. After year 126, the replacement rate drops to the initial value of 65.37%. The funds accumulated in early periods pay for the raised benefits until the funds are depleted in year 126 (Figure 4.21). The raised tax rate increases social security receipts to compensate for the deficit caused by the increased replacement rate and slow growth of working population. When the first shrinking generation starts to retire, the government has to borrow from external sources or issue debts to finance social security payments. After all shrinking generations exit the labor force, tax receipts exceed social security payments, and funds begin to accumulate. With the accumulated funds, the tax rate is kept low, and the replacement rate is kept high. Finally, in year 126 the funds are depleted.

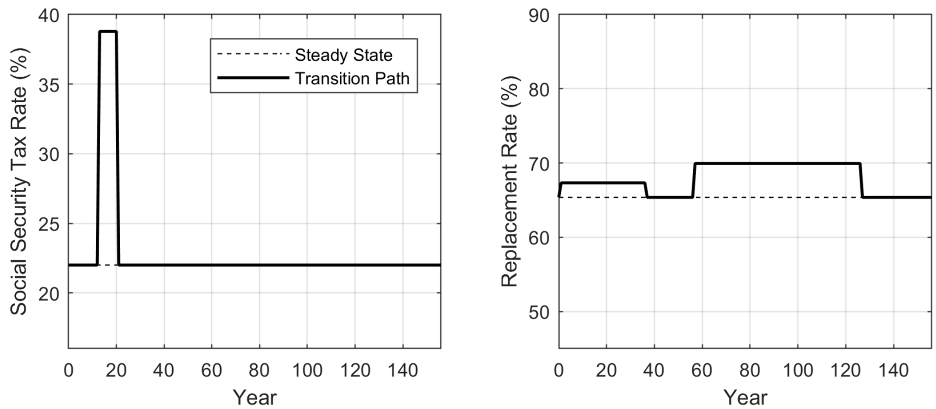

Figure 4.20: The social security tax rate and the replacement rate over time under policy 4 with intergenerational borrowing.

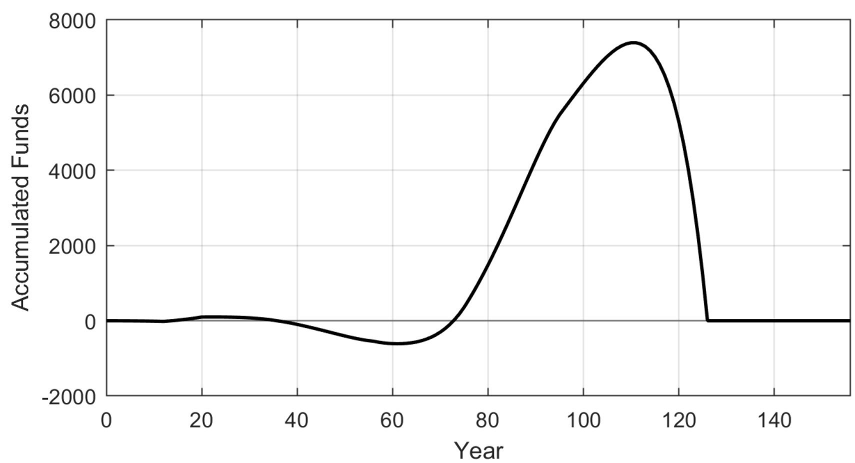

Figure 4.21: Funds accumulated in the face of the demographic shrink over time under policy 4 with intergenerational borrowing.

As the shrinking generations enter the labor force, the capital–labor ratio increases, leading to a decrease in the interest rate and an increase in the effective wage rate, initially. The wage rate and the interest rate fluctuate over time. As the shrinking generations gradually exit the economy and the accumulated funds get depleted, the capital–labor ratio, the effective wage rate, and the interest rate gradually return to the initial levels.

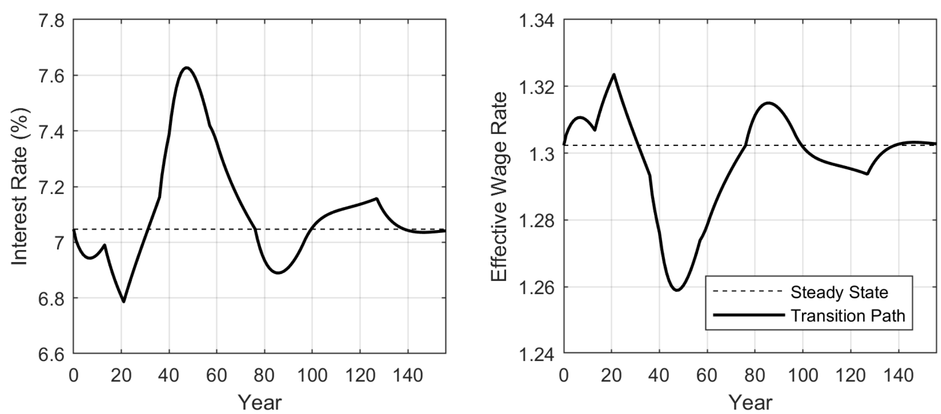

Figure 4.22: The interest rate and the effective wage rate over time under policy 4 with intergenerational borrowing.

I examine a selected age cohort in detail to uncover the reason for gains or losses for the age cohort under policy 4. I take a person from the age −20 cohort, i.e., a person born 20 years after the first shrinking generations are born, as an example (Figure 4.23). This individual enters the labor force in year 21. As can be seen in Figure 4.22, in year 21, the effective wage rate is higher than its steady-state (no-shrink) level and the interest rate is lower than its steady-state (no-shrink) level. The higher wage rate increases the individual's wage income initially, also causing a positive wealth effect on consumption. On the other hand, the lower interest rate exerts a strong intertemporal substitution effect that favors more consumption. The combined effect is high consumption levels and significantly high welfare levels at early ages of this individual (Figure 4.23).

As the individual ages, the effective wage rate soon starts to decline while the interest rate starts to rise, both of which decrease consumption. The right panel of Figure 4.23 shows that the cumulative to-age welfare flips from a gain to a loss for the first time at age 43. In retirement (starting at year 61), the individual's social security income is mostly much higher than in the steady state, thanks to the increased replacement rate at that time. The higher social security income helps keep consumption at higher-than-steady-state levels in the years of the individual's retirement. Compared to the total rest-of-life utility in the absence of the shrink, the total rest-of-life utility in the presence of the shrink is 1.95% more in equivalent consumption units under policy 4 for this individual.

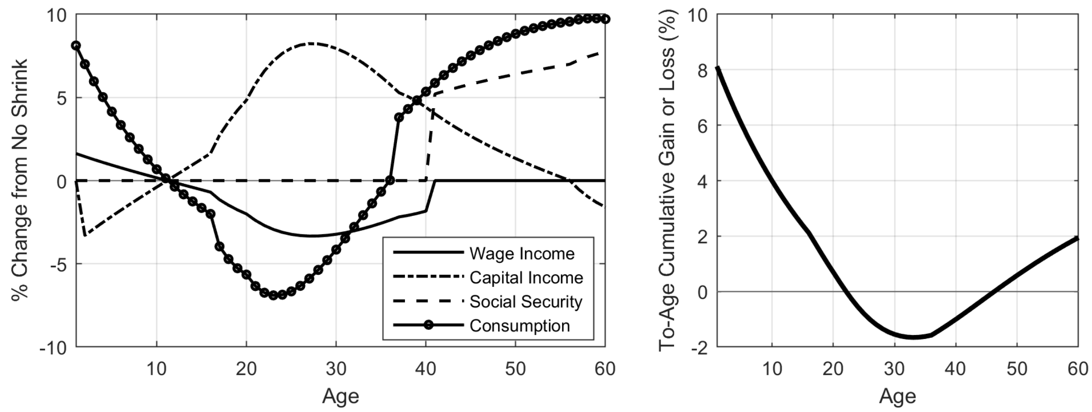

Figure 4.23: Relative changes in consumption, wage income, capital income, and the social security benefit (left) and the cumulative gain or loss in welfare in equivalent consumption units due to the presence of the shrink (right) for the age −20 cohort along the transition path in the face of the demographic shrink under policy 4 with intergenerational borrowing.

Figure 4.24 shows the percentage gains or losses of different age cohorts in rest-of-life welfare under policy 4. The cohorts aged less than or equal to 26 but more than or equal to −16, suffer losses. The remaining cohorts enjoy welfare gains. 27.4% of the 150 age cohorts examined experience some welfare losses. The age 7 experiences the largest loss of −3.59%, while the age −20 cohort enjoys the highest gain of 1.95%.

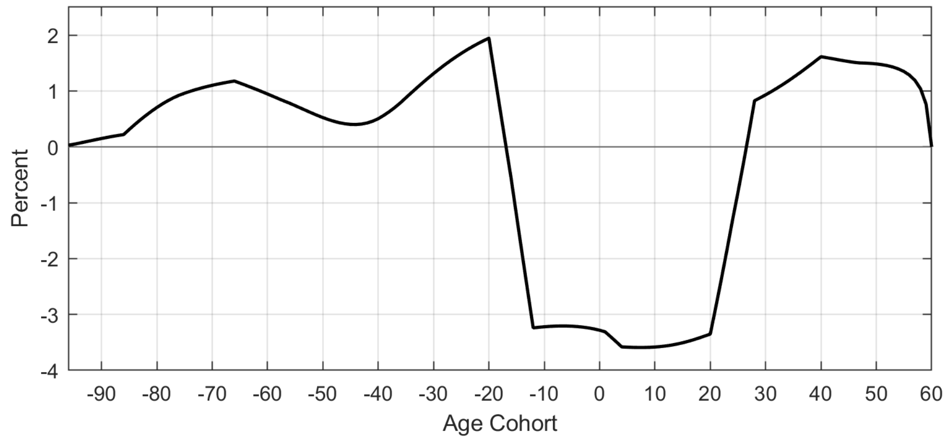

Figure 4.24: Gains or losses of different age cohorts in rest-of-life welfare in the face of the demographic shrink under policy 4.

Figure 4.25 shows the implicit rates of return of social security for different age cohorts in the face of the shrink under policy 4. Figure 4.25 tries to explain why the social security program under policy 4 is attractive to cohorts of age less than or equal to −18 as well as those aged more than or equal to 26, as a means of compulsory saving, when there is a demographic shrink. In the case of no shrink, the steady-state implicit rate of return of social security is $r^{ss} = (1+n)(1+g) - 1 = 6.26\%$. For cohorts aged less than or equal to −18 as well as those aged more than or equal to 26, their implicit rates of return are no less than the steady-state value of 6.26%, and hence they prefer putting money into the social security program. For the other age cohorts, they would rather invest their money on their own. The implicit rate of return is the lowest for the age 4 cohort at 5.67%.

Figure 4.25: Implicit rates of return (percent) of social security for different age cohorts in the face of the demographic shrink under policy 4 with intergenrational borrowing.

4.6 Comparison of Policy Alternatives

This section compares the four social security policy alternatives examined in the previous sections in terms of welfare in the face of the demographic shrink. Table 4.1 presents some welfare metrics of the four policy alternatives and Figure 4.26 collects the gains and losses of different age cohorts under the policy alternatives.

Table 4.1: Welfare evaluation of various social security policy options in the face of the demographic shrink in China

	Proportion	Maximum Loss[a]		Maximum Gain[a]		Minimum r^{ss} [b]	
	Losing	Amount	Age Cohort	Amount	Age Cohort	Rate	Age Cohort
No Social Security							
Shrink Effect	60.3%	1.7%	30	1.9%	−13	—	—
Balanced-Budget Policies							
#1: Fixed Tax Rate	67.9%	6.0%	21	5.6%	−22	5.53%	15
#2: Fixed Replacement Rate	52.6%	3.7%	−12	3.6%	−58	5.63%	−6
#3: Replacement Rate Ceiling	41.7%	6.0%	21	3.6%	−57	5.53%	15
Non-Balanced-Budget Policy							
#4: Intergenerational Borrowing	27.6%	3.6%	7	2.0%	−20	5.67%	4

[a] The amount of gain or loss in utility is measured in terms of increase or decrease in equivalent consumption units.

[b] r^{ss} is the implicit rate of return of social security. $r^{ss} = (1 + n)(1 + g) - 1 = 6.26\%$ in the steady state.

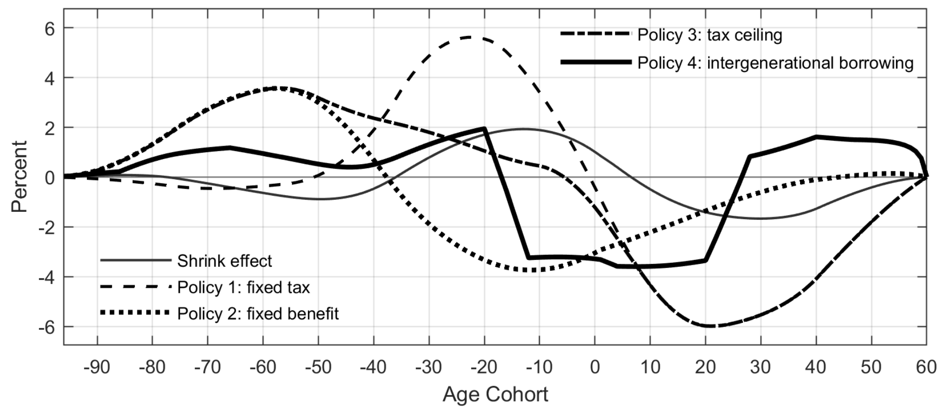

Figure 4.26: Gains or losses (percent) of different age cohorts in the face of the demographic shrink under various policy options.

Policy 1 with a fixed tax rate and a balanced budget induces the most losing age cohorts and the largest maximum loss among all age cohorts. Under policy 1, the losing cohorts are mostly the pre-shrink generation, who have to accept lower social security benefits to sustain the fixed replacement rate when the shrinking generations enter the labor force. The shrinking generations and the post-shrink generations generally enjoy welfare gains. Policy 1 and policy 3 both have the lowest minimum implicit rate of return of social security.

Policy 2 with a fixed replacement rate and a balanced budget ranks second in the resulting proportion of losing cohorts. Under policy 2, the post-shrink generations enjoy a lower tax rate when the shrinking generations retire and also enjoy rest-of-life welfare gains in general. Most of the shrinking generations and pre-shrink generations suffer welfare losses.

Policy 3 with a ceiling on the replacement rate has the second smallest proportion of losing age cohorts. The social security tax rate and the replacement rate trajectories are exactly the same for the first 50 years under policy 3 and policy 1. As a result, the curve of rest-of-life welfare gains or losses under policy 3 coincide with that under policy 1 for cohorts aged more than or equal to 10. Despite of the same beginning in policy rates, policy 3 saves the post-shrink generation from suffering a non-trivial welfare loss, unlike policy 1. On the other hand, the pre-shrink generations inflict some welfare losses, partly because of the lowered replacement rate during their retirement years.

The social security tax rate and the replacement rate trajectories are identical after year 50 under policy 3 and policy 2. The curve of rest-of-life welfare gains or losses under policy 3 almost coincide with that under policy 2 for cohorts aged less than −50. Despite of the same ending in policy rates, policy 3 is significantly better at "smoothing" the welfare curve than policy 2 for the segment towards the left end.

Policy 4 with intergenerational borrowing produces the least proportion of losing age cohorts and at the same time the least rest-of-life loss among all age cohorts. It also achieves the highest minimum implicit rate of return of social security. Compared with policy 3, policy 4 is able to avoid imposing non-trivial losses through the temporarily elevated replacement rate on some of the shrink age cohorts and as well as some of the post-shrink age cohorts. Furthermore, the social security manager has the budget to raise the replacement rate at that time since the manager has accumulated a large fund prior to that. The social security program performs intergenerational transfer of wealth from the current workers to the current retirees without accumulation of a fund. However, fund accumulation enables the social security program to achieve wealth transfer to future generations and helps better smooth over time the impact on welfare of a demographic shrink.

Chapter 5

CONCLUSION

This paper examines four policy options for a pay-as-you-go social security program that can be adopted in response to a temporary demographic shrink or bulge. Policy 1 is keeping the tax fixed and adjusting the replacement rate for a balanced budget. Policy 2 is keeping the replacement rate fixed while varying the tax rate for a balanced budget. Policy 3 is setting either a tax ceiling or a replacement ceiling under balanced budget. Policy 4 is fund accumulation, in which a reserve fund is accumulated when there is a social security surplus and decumulated when there is a social security deficit. In the case of a demographic shrink, intergenerational borrowing at the beginning is essential for building funds. Through numerical simulation of an overlapping generations model, I conclude that the fund accumulation policy works better than the other three balanced budget policies in the face of either a bulge or shrink. With a bulge, the issue is how to spread the benefits of population growth so that every cohort enjoys gains. Compared with the bulge, the shrinking generations cannot generate extra benefits due to the reduced population growth. Therefore, what I focus on is how to distribute the loss through different age cohorts to improve welfare. The case of a population bulge is an application to the U.S. baby boom, while the case of a population shrink is an application to China's one-child policy. However, the model, the results, and the policy implications in my dissertation can be applied to other countries as well.

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Appendix

MODEL DERIVATION

A.1 Individual's Consumption Decisions

Consider an individual who is age t in year v. At age j (j = 1, 2, …, J) in year $v - t + j$, the individual consumes $C_{v-t+j}(j)$ and saves $S_{v-t+j}(j)$. Consumption and savings are funded by a wage income w_{v-t+j} less a social security tax at the rate of t_w when the individual is at a working age j (j = 1, 2, …, R) or a social security benefit $B_{v-t+j}(j)$ after the individual retires ($j = R + 1$, …, J). In addition, at age j (j = 1, 2, …, J) the individual receives investment income from her accumulated private wealth $A_{v-t+j}(j)$ at a rate of r. Thus, the individual's intertemporal budget constraint at age j is given by

$$\begin{aligned} &C_{v-t+j}(j) + S_{v-t+j}(j) \\ &= rA_{v-t+j}(j) + \mathbf{1}_{j \le R}(j)(1 - t_w)w_{v-t+j} + \mathbf{1}_{j>R}(j)B_{v-t+j}(j), \end{aligned} \tag{A.1}$$

where $\mathbf{1}_{j \le R}(j) = 1$ when $j \le R$ and 0 when $j > R$ and similarly $\mathbf{1}_{j>R}(j) = 1$ when $j > R$ and 0 when $j \le R$. At the end of each year, the saved funds go to the individual's private wealth. Thus,

$$A_{v-t+j+1}(j + 1) = A_{v-t+j}(j) + S_{v-t+j}(j). \tag{A.2}$$

The individual is born with no private wealth so $\mathrm{A}_{\mathrm{v-t+1}}(1) = 0$.

Substitute (A.2) into (A.1) to eliminate $S_{v-t+j}(j)$ and I obtain

$$\begin{aligned} &C_{v-t+j}(j) + A_{v-t+j+1}(j + 1) \\ &= (1 + r)A_{v-t+j}(j) + \mathbf{1}_{j \le R}(j)(1 - t_w)w_{v-t+j} + \mathbf{1}_{j>R}(j)B_{v-t+j}(j), \end{aligned} \tag{A.3}$$

Divide both sides of (A.3) by $(1+r)^j$ and sum over 1, …, J to obtain

$$\sum_{j=1}^{J}\frac{C_{v-t+j}(j)}{(1+r)^j}+\sum_{j=1}^{J}\frac{A_{v-t+j+1}(j+1)}{(1+r)^j}$$
$$=\sum_{j=1}^{J}\frac{A_{v-t+j}(j)}{(1+r)^{j-1}}+\sum_{j=1}^{R}\frac{(1-t_w)w_{v-t+j}}{(1+r)^j}+\sum_{j=R+1}^{J}\frac{B_{v-t+j}(j)}{(1+r)^j}.$$

Adjust the index in the second term on the left-hand side and in the first term on the right-hand side of the above equation and it is reduced to

$$\sum_{j=1}^{J}\frac{C_{v-t+j}(j)}{(1+r)^j}+\frac{A_{v-t+J+1}(J+1)}{(1+r)^J}$$
$$=A_{v-t+1}(1)+\sum_{j=1}^{R}\frac{(1-t_w)w_{v-t+j}}{(1+r)^j}+\sum_{j=R+1}^{J}\frac{B_{v-t+j}(j)}{(1+r)^j}. \tag{A.4}$$

Recall $A_{v-t+1}(1)=0$. Additionally, note that at age J, the individual is about to die so that he or she would consume all the private wealth and save nothing to maximize the utility in the terminal period, i.e., $A_{v-t+J+1}(J+1)=0$. Then, (A.4) is further reduced to (17), which is the individual's lifetime budget constraint.

Consider the individual's optimization problem. Construct the Lagrangian

$$L=\sum_{j=1}^{J}(1+\rho)^{-j}\frac{C_{v-t+j}^{1-\gamma}(j)-1}{1-\gamma}$$
$$-\lambda\left[\sum_{j=1}^{J}\frac{C_{v-t+j}(j)}{(1+r)^j}-\sum_{j=1}^{R}\frac{(1-t_w)w_{v-t+j}}{(1+r)^j}-\sum_{j=R+1}^{J}\frac{B_{v-t+j}(j)}{(1+r)^j}\right].$$

The first-order condition is

$$\frac{\partial L}{\partial C_{v-t+j}(j)}=(1+\rho)^{-j}C_{v-t+j}^{-\gamma}(j)-\lambda\frac{1}{(1+r)^j}=0,$$

which gives

$$C_{v-t+j}(j)=\lambda^{-\sigma}\left(\frac{1+r}{1+\rho}\right)^{\sigma j}, \tag{A.5}$$

where $\sigma={}^{1}/_{\gamma}$. Substitute (A.5) into the individual's lifetime budget constraint and I have

$$\sum_{j=1}^{J}\lambda^{-\sigma}\left(\frac{1+r}{1+\rho}\right)^{\sigma j}\frac{1}{(1+r)^j}=\sum_{j=1}^{R}\frac{(1-t_w)w_{v-t+j}}{(1+r)^j}+\sum_{j=R+1}^{J}\frac{B_{v-t+j}(j)}{(1+r)^j},$$

from which I obtain

$$\lambda^{-\sigma} = \frac{\left[\sum_{j=1}^{R} \frac{(1-t_w)w_{v-t+j}}{(1+r)^j} + \sum_{j=R+1}^{J} \frac{B_{v-t+j}(j)}{(1+r)^j}\right]}{\sum_{j=1}^{J} \left[\left(\frac{1+r}{1+\rho}\right)^{\sigma} \frac{1}{(1+r)}\right]^j},$$

Substitute the above equation back into (A.5) to obtain

$$C_{v-t+j}(j) = \frac{\left(\frac{1+r}{1+\rho}\right)^{\sigma j} \left[\sum_{j=1}^{R} \frac{(1-t_w)w_{v-t+j}}{(1+r)^j} + \sum_{j=R+1}^{J} \frac{B_{v-t+j}(j)}{(1+r)^j}\right]}{\sum_{j=1}^{J} \left[\left(\frac{1+r}{1+\rho}\right)^{\sigma} \frac{1}{(1+r)}\right]^j} \tag{A.6}$$

A.2 The Balanced Growth Path

Before discussing the balanced growth path of the model, a brief discussion of the aggregate variable is necessary. Consider in particular the aggregate consumption in year *v*, C_v. The population in entire year *v* consists of individuals of age 1 through *J*. Each individual at age *j* consumes $C_{v-j}(j)$ and the number of individuals at age *j* is $(1+n)^{v-j}$. Therefore, the aggregate consumption in year *v* is

$$C_v = \sum_{j=1}^{J} (1+n)^{v-j} \cdot C_{v-j}(j). \tag{A.7}$$

Along the balance growth path, different aggregate variables may have different growth rates. The labor force is growing at a (gross) rate of 1+*n*. The effective labor force grows at a rate of (1+*n*) (1+*g*). As a result, the aggregate capital, the aggregate output, and the aggregate consumption would also grow at a rate of (1+*n*) (1+*g*).

The per effective labor terms, such as the per effective labor consumption C_v, therefore have well-defined steady states along the balanced growth path. The interest rate, which is also the return of investment, has a well-defined steady state. Along the balanced growth path, the wage rate for effective labor w_v^e has a well-defined steady state while the market wage rate w_v growth at a (gross) rate of (*1*+*g*), i.e.,

$$w_v = w_{v-t}(1+g)^t. \tag{A.8}$$

Assume all current retirees receive the same social security benefits and assume the social security program runs a balanced budget. Then,

$$B_v = \sum_{j=R+1}^{J}(1+n)^{v-j}B_v(j) = t_w \sum_{j=1}^{R}(1+n)^{v-j}\, w_v,$$

$$B_v(j) = \omega t_w w_v, \tag{A.9}$$

where $\omega = {}^{L_v}/_{Q_v}$ is the worker–retiree ratio. Note that *w* is constant over time. With (A.8) and (A.9), along the balanced growth path, (A.6) becomes

$$C_{v-t+j}(j)$$

$$= \frac{\left(\frac{1+r}{1+\rho}\right)^{\sigma j}\left[\sum_{j=1}^{R}\frac{(1-t_w)(1+g)^j w_{v-t}}{(1+r)^j} + \sum_{j=R+1}^{J}\frac{\omega t_w (1+g)^j w_{v-t}}{(1+r)^j}\right]}{\sum_{j=1}^{J}\left[\left(\frac{1+r}{1+\rho}\right)^{\sigma}\frac{1}{(1+r)}\right]^j}$$

$$= \frac{\left(\frac{1+r}{1+\rho}\right)^{\sigma j}\left[\sum_{j=1}^{R}\left(\frac{1+g}{1+r}\right)^j (1-t_w)w_{v-t} + \sum_{j=R+1}^{J}\left(\frac{1+g}{1+r}\right)^j \omega t_w w_{v-t}\right]}{\sum_{j=1}^{J}\left[\left(\frac{1+r}{1+\rho}\right)^{\sigma}\frac{1}{(1+r)}\right]^j}$$

Using the definition in (18), the above equation is further reduced to

$$C_{v-t+j}(j) = \left(\frac{1+r}{1+\rho}\right)^{\sigma j}\Gamma w_{v-t}. \tag{A.10}$$

Substitute (A.10) into (A.7) and I obtain

$$C_v = \sum_{j=1}^{J}(1+n)^{v-j}\, C_v(j) = \sum_{j=1}^{J}(1+n)^{v-j}\left(\frac{1+r}{1+\rho}\right)^{\sigma j}\Gamma w_{v-j}$$

$$= \sum_{j=1}^{J}\left[\left(\frac{1+r}{1+\rho}\right)^{\sigma}\frac{1}{(1+n)(1+g)}\right]^j (1+n)^v \Gamma w_v,$$

and

$$c_v = \frac{C_v}{(1+g)^v L_v} = \frac{M_1^J\left(\left(\frac{1+r}{1+\rho}\right)^{\sigma}\frac{1}{(1+n)(1+g)}\right)}{M_1^R\left(\frac{1}{1+n}\right)}\Gamma w_v^e. \tag{A.11}$$

From (25), (26), (27) and (A.11), I have

$$\bar{y} = \bar{k}^{\alpha} = \frac{M_1^J\left(\left(\frac{1+r}{1+\rho}\right)^{\sigma}\frac{1}{(1+n)(1+g)}\right)}{M_1^R\left(\frac{1}{1+n}\right)}\Gamma(1-\alpha)\bar{k}^{\alpha} + (n+g+ng)\bar{k}.$$

Multiply both sides by $\alpha\bar{k}^{-1}$ and apply (27) to obtain

$$\bar{r} = \frac{M_1^J\left(\left(\frac{1+r}{1+\rho}\right)^{\sigma}\frac{1}{(1+n)(1+g)}\right)}{M_1^R\left(\frac{1}{1+n}\right)}\Gamma(1-\alpha)\bar{r} + (n+g+ng)\alpha.$$

Subtract from both sides of the above equation $(1-\alpha)\bar{r}$ and I obtain

$$\alpha\bar{r} = \left[\frac{M_1^J\left(\left(\frac{1+\bar{r}}{1+\rho}\right)^{\sigma}\frac{1}{(1+n)(1+g)}\right)}{M_1^R\left(\frac{1}{1+n}\right)}\Gamma - 1\right](r\alpha)\bar{r} + (n+g+ng)\alpha.$$

Move the second term on the right-hand side to the left-hand side and the above equation can be further reduced to (28), which is the equation determining the steady-state interest rate $\bar{r}$. Once the steady-state interest rate is pinned down, the other steady-state variables may be recovered from (25-27).

To facilitate calculation of consumptions in the transition path, I derive in what follows an expression for the accumulated private wealth of an individual in a given year along the balanced growth path. Consider $A_{v-t+j+1}(j+1)$, which is the private wealth of an individual born in year *v-t* at age *j*+1. From (A.3) I have

$$\begin{aligned} &A_{v-t+j+1}(j+1) \\ &= (1+r)A_{v-t+j}(j) + 1_{j\leq R}(j)(1-t_w)w_{v-t+j} + 1_{j>R}(j)B_{v-t+j}(j) \\ &\qquad - C_{v-t+j}(j). \end{aligned}$$

Along the balanced growth path, $w_{v-t+j} = w_{v-t}(1+g)^j$, $B_{v-t+j}(j) = \omega t_w w_{v-t+j} = \omega t_w w_{v-t}(1+g)^j$ given by (A.9), and $C_{v-t+j}(j) = \left(\frac{1+r}{1+\rho}\right)^{\sigma j} \Gamma w_{v-t}$ given by (A.10).

Hence

$$
\begin{aligned}
&A_{v-t+j+1}(j+1) \\
&\quad = (1+r)A_{v-t+j}(j) + 1_{j\le R}(j)(1-t_w)w_{v-t}(1+g)^j \\
&\quad + 1_{j>R}(j)\omega t_w w_{v-t}(1+g)^j - \left(\frac{1+r}{1+\rho}\right)^{\sigma j} \Gamma w_{v-t}.
\end{aligned}
$$

Recursively apply the above equation and recall $A_{v-t+1}(1) = 0$. I obtain

$$
\begin{aligned}
&A_{v-t+j+1}(j+1) \\
&\quad = (1+r)^j A_{v-t+1}(1) \\
&\quad + \sum_{k=1}^{j} (1+r)^{j-k} \Bigg[1_{k\le R}(k)(1-t_w)w_{v-t}(1+g)^k \\
&\quad + 1_{k>R}(k)\omega t_w w_{v-t}(1+g)^k - \left(\frac{1+r}{1+\rho}\right)^{\sigma k} \Gamma w_{v-t} \Bigg] \\
&\quad = (1+r)^j \overline{w}^e (1+g)^{v-t} \Bigg\{ \sum_{k=1}^{j} 1_{k\le R}(k)(1-t_w)\left(\frac{1+g}{1+r}\right)^k \\
&\quad + \sum_{k=1}^{j} 1_{k>R}(k)\omega t_w \left(\frac{1+g}{1+r}\right)^k - \sum_{k=1}^{j} \left[\left(\frac{1+r}{1+\rho}\right)^{\sigma} \frac{1}{1+r}\right]^k \Gamma \Bigg\} \\
&\quad = (1+r)^j \overline{w}^e (1+g)^{v-t} \Bigg\{ \sum_{k=1}^{min(j,R)} (1-t_w)\left(\frac{1+g}{1+r}\right)^k \\
&\quad + \sum_{k=min\{j,R+1\}}^{j} \omega t_w \left(\frac{1+g}{1+r}\right)^k - \sum_{k=1}^{j} \left[\left(\frac{1+r}{1+\rho}\right)^{\sigma} \frac{1}{1+r}\right]^k \Gamma \Bigg\} \\
&\quad = (1+r)^j \overline{w}^e (1+g)^{v-t} \Bigg[(1-t_w) M_1^{min\{j,R\}} \left(\frac{1+g}{1+r}\right) \\
&\quad + 1_{j>R}(j) M_{R+1}^{j} \left(\frac{1+g}{1+r}\right) - M_1^j \left(\left(\frac{1+r}{1+\rho}\right)^{\sigma} \frac{1}{1+r}\right)\Gamma \Bigg].
\end{aligned}
$$

Let j=t and the above equation reduces to (31).

A.3 The Transition Path

Suppose a temporary demographic shock hits the economy in year *v*. The time during which the economy deviates from its balanced growth path is the transition periods. In each year in the transition periods, individuals re-optimize their consumption taking as given the current interest rate, the tax rate, and social security benefits, and assuming those conditions will remain unchanged indefinitely. Consider an age-*t* individual in year *v*. This individual will maximize his or her rest-of-life utility subject to the intertemporal budget constraint (A.1) (for $j \geq t$). I can consolidate the intertemporal budget constraints in different periods into a rest-of-life budget constraint as done in section A.1. Hence, the individual's optimization problem can be characterized by (29) - (30). Construct the Lagrangian

$$\begin{aligned} L^{(t)} \\ &= \sum_{j=t}^{J} (1+\rho)^{t-j} \frac{C_{v-t+j}^{1-\gamma}(j) - 1}{1-\gamma} \\ &\qquad - \mu_t \left[\sum_{j=t}^{J} \frac{C_{v-t+j}(j)}{(1-r)^{j-t}} - A_v(t)(1+r) \right. \\ &\qquad - 1_{t \leq R}(t) \sum_{j=t}^{R} \frac{(1-t_w) w_{v-t+j}}{(1+r)^{j-t}} \\ &\qquad \left. - \sum_{j=max\{t,R+1\}}^{J} \frac{B_{v-t+j}(j)}{(1+r)^{j-t}} \right]. \end{aligned}$$

The first-order condition is

$$\frac{\partial L^{(t)}}{\partial C_{v-t+j}(j)} = (1+\rho)^{t-j} C_{v-t+j}^{-\gamma}(j) - \mu_t \frac{1}{(1+r)^{j-t}} = 0,$$

from which I have

$$C_{v-t+j}(j) = \left(\frac{1+r}{1+\rho}\right)^{\sigma(j-t)} \mu^{-\sigma}. \tag{A.12}$$

Substitute (A.12) into the rest-of-life budget constraint (30) and solve for $\mu^{-\sigma}$. I obtain

$$\mu^{-\sigma} = \frac{\left[\frac{\left(\frac{1+r}{1+\rho}\right)^{\sigma}}{1+r}\right]^{t} \left[A_v(t)(1+r) + 1_{t\leq R}(t)\sum_{j=t}^{R}\frac{(1-t_w)w_{v-t+j}}{(1+r)^{j-t}} + \sum_{j=max\{t,R+1\}}^{J}\frac{B_{v-t+j}(j)}{(1+r)^{j-t}}\right]}{\sum_{j=t}^{J}\left[\frac{\left(\frac{1+r}{1+\rho}\right)^{\sigma}}{1+r}\right]^{j}}.$$

Substitute the above equation of $\mu^{-\sigma}$ back into (A.12) and I obtain

$$C_{v-t+j}(j) = \frac{\left(\frac{1+r}{1+\rho}\right)^{\sigma(j-t)}\left[\frac{\left(\frac{1+r}{1+\rho}\right)^{\sigma}}{1+r}\right]^{t} \left[A_v(t)(1+r) + 1_{t\leq R}(t)\sum_{j=t}^{R}\frac{(1-t_w)w_{v-t+j}}{(1+r)^{j-t}} + \sum_{j=max\{t,R+1\}}^{J}\frac{B_{v-t+j}(j)}{(1+r)^{j-t}}\right]}{\sum_{j=t}^{J}\left[\frac{\left(\frac{1+r}{1+\rho}\right)^{\sigma}}{1+r}\right]^{j}} \tag{A.13}$$

Since along the transition path individuals assume current economic conditions (such as the interest rate) will persist, I have

$$w_{v-t+j} = (1+g)^{j-t}w_v$$

and

$$B_{v-t+j}(j) = (1+g)^{j-t}B_v,$$

Then (A.13) becomes

$C_{v-t+j}(j)$

$$=\frac{\left(\frac{1+r}{1+\rho}\right)^{\sigma(j-t)}\left[\frac{\left(\frac{1+r}{1+\rho}\right)^{\sigma}}{1+r}\right]^{t}\left[A_v(t)(1+r)+1_{t\le R}(t)\sum_{j=t}^{R}\left(\frac{1+g}{1+r}\right)^{j-t}(1-t_w)w_v+\sum_{j=max\{t,R+1\}}^{J}\left(\frac{1+g}{1+r}\right)^{j-t}B_v\right]}{\sum_{j=t}^{J}\left[\frac{\left(\frac{1+r}{1+\rho}\right)^{\sigma}}{1+r}\right]^{j}}$$

Let j=t and the above equation is reduced to (32).

Along the transition path, the interest rate in year v, is determined as follows. First, the aggregate capital stock in year v is given by

$$K_v=\sum_{j=1}^{J}N_v(j)A_v(j)+F_v$$

where $N_v(j)$ is the number of individuals at age j in year v, $A_v(j)$ is the private wealth of individuals at age j in year v, and F_v is the public savings in year j. Then, the per-effective labor capital stock is given by

$$k_v=\frac{K_v}{(1+g)^v L_v}$$

where $L_v=\sum_{j=1}^{R}N_v(j)$ is the size of the labor force in year v. Finally, the interest rate in year v is determined as

$$r_v=\alpha k_v^{\alpha-1},$$

and the wage rate in year v is

$$w_v=(1-\alpha)k_v^{\alpha}(1+g)^v.$$