SQUARE WAVE FORM TO INCREASE CYCLOTRON BEAM

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The present form of push-pull, tuned grid, tuned plate, quarter wave line oscillator used for cyclotrons gives a sine wave voltage-time curve. While this oscillator has the advantages of simplicity, its wave form is not the most efficient for beam production. For cyclotrons producing quantities of radioelements or neutrons for research, the largest beam possible is desirable. A substantially square wave form produced by superimposing frequencies of f, $3f$, $5f$, and $7f$ cycles per second offers a solution.

The sine wave has three disadvantages for maximum beam production: (1) the loss of ions from the beam due to increased path length, and consequent increase in probability of collision, during their acceleration on the low voltage portions of the curve, (2) the accumulated defocusing of ions on the increasing side of the curve, and (3) the spread of the path centers and consequent energy loss and ion loss to the dees, both due to low values of the radius.

With a sine wave, only the ions accelerated at the peak voltage reach the target by the shortest path. At any lower voltage the path is correspondingly longer. Thus, through the time the voltage is increasing or decreasing, the lengths of the ion paths from the ion source to the target are decreasing and increasing respectively.

The increase in beam due to the decreased path length for the square wave can be calculated as follows. In a beam of deuterons moving in its circular path through the gas in a cyclotron chamber we can expect a diminution of current as expressed by

$$i = i_0 \epsilon^{-as} \tag{1}$$

where i_0 is the initial current, i the current after a path distance s, and a is a coefficient of extinction. This relation depends only on the probability of collision. On collision, the deuteron is lost from the beam which decreases according to equation (1). This relation,

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however, does not represent the total change in beam current from the ion source to the exit slit. Some ions will be lost due to defocusing, path wandering, getting out of resonance, etc. For the increase in beam current due to the decrease in path length, however, it is sufficient.

If we establish the time-voltage relation as given in Fig. 1, we have the two limits of path length. When θ is 0° the voltage is at a maximum and the path s to the target is the shortest. As

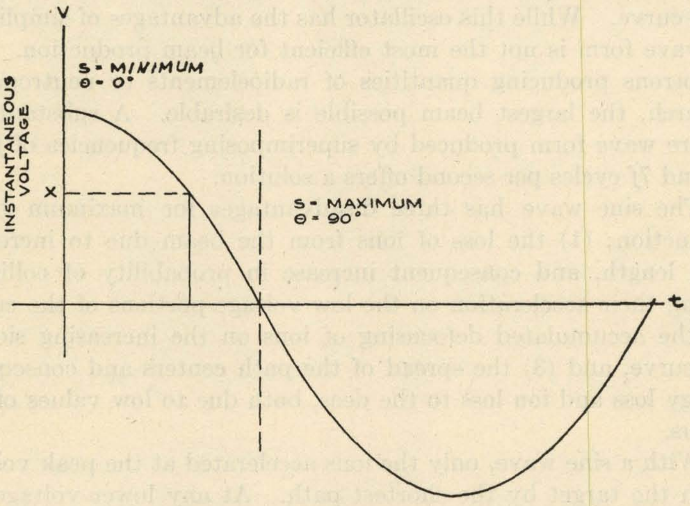

FIG. 1.
Plot of instantaneous voltage against time showing the variations and limits of path length s.

θ approaches 90°, the voltage approaches zero and hence the path s approaches ∞. Thus the path can be represented by

$$s = c \sec \theta \tag{2}$$

or

$$s = \frac{c}{\cos \theta} \tag{3}$$

where c is a constant. We can then rewrite (1) as

$$i = i_0 \epsilon^{-(k/\cos \theta)} \tag{4}$$

where k is a constant containing c and a.

It is obvious from (4) that when θ is 90°, i is zero. This is to be expected since the ions starting at the ion source would never reach the target if they must pass over an infinite path length which would provide a certainty of collision and loss from the beam.

Since $x = V \cos \theta$, (4) can be written as

$$i = i_0 \epsilon^{-\mathrm{k}(V/x)} \tag{5}$$

which gives us a picture of the beam current for a sine voltage wave where V/x is a variable quantity depending on value of θ. If we assume an ideal square wave, V/x would remain constant and (5) would be

$$i = i_0 \epsilon^{-\mathrm{k}_1} \tag{6}$$

for the entire time of the half cycle. In practice x would not be absolutely constant, of course, and i would vary accordingly. The target current would increase with the number of harmonics added to the square wave generator since we are approaching the ideal condition as represented by (6).

The second advantage of the square wave form is the reduction of the number of ions lost by defocusing on a portion of the sine curve. Rose [1] derives the equation for the deflection of an ion due to the field variation of the single electric lens of a cyclotron. A system of rectangular coordinates with the origin at the center of the cyclotron chamber is chosen. The z axis is taken in the vertical direction and the x axis in the direction of the electric field. The equation is:

$$(\Delta\alpha) \text{ field variation} = -\frac{eV_0 \sin \theta}{E} \frac{\omega}{v} z \tag{7}$$

where $\Delta\alpha$ = deflection in the lens at any instant along the z axis
e = charge of the ions
$2V_0$ = peak potential across the dees
v = velocity of the ion
ω = circular frequency of the oscillating electric field
E = energy at the center of the accelerating region
θ = phase of the electric field relative to the position of the ion, i.e., the time t is measured from the instant the ion passes the center ($x = 0$) and θ is the phase of the electric field at $t = 0$.
z = displacement of the plane of the ion path as it enters the lens.

[1] M. E. Rose, "Focusing and Maximum Energy of Ions in the Cyclotron," *Phys. Rev.*, 53, 392–408 (1938).

This equation shows that the deflection is towards the median plane for positive θ and away from it for negative θ. When θ is positive, the electric field is decreasing during acceleration and the result is a focusing of the ions toward the median plane. When θ is negative the electric field is increasing during acceleration and the deflection is defocusing. However, from the equation for the deflection due to energy change

$$(\Delta\alpha)\text{ energy change} = -bz\frac{e^2}{2E^2}\cos^2\theta \qquad (8)$$

it is seen that a focusing action always results. Practically, the focusing due to energy change is relatively unimportant. However, it serves to give a net focusing for small negative values of θ. Therefore, as pointed out by Rose, only slightly more than half of the radio frequency cycle is effective in producing focused ions. It is desirable, then, to produce focused ions for the beam over a greater portion of the radio frequency cycle.

In equation (7), $(\Delta\alpha)_{\text{f.v.}}$ is zero peak dee potential since $\theta = 0$. For a square wave the dee potential is not a function of θ and will remain the same peak value for the condition of $\theta = 0$. Hence, $(\Delta\alpha)_{\text{f.v.}}$ for a square wave form will always be zero and there will be neither focusing nor defocusing. By the same reasoning, $\cos^2\theta$ of (8) will always be constant for a square wave and $(\Delta\alpha)$ en. ch. will always have a value which, as pointed out above, is always toward the median plane. Thus for a square wave, the net result for the field variation and energy change is a focusing action toward the median plane.

The sine wave produces a displacement of the ion path centers.[2] This displacement will vary with the initial phase of the ion relative to the voltage on the dees so that the centers will be spread out over a region near the center instead of being concentrated at a point. This spread will result in a variation of path radii from R to $R - \Delta R$ where R is the radius from the center of the region of centers to the deflector exit slit or the internal target and ΔR is the radius of the region.

This variation in R produces a beam nonhomogeneous in energy. More serious, however, is the loss of ions to the dees for a large number of the ions with path centers within a certain portion of the path center region. It can be seen from Fig. 2 that ions with path

[2] R. R. Wilson, "Theory of the Cyclotron," *J. App. Phys.*, 11, 781–796 (1940).

centers over a portion of the path center region will be lost by hitting the dees. Of course, this number can be diminished by placing the internal target at a smaller radius but this means sacrificing energy. The deflector exit slit can be made larger but this will give a more diffuse beam and a spread of energy.

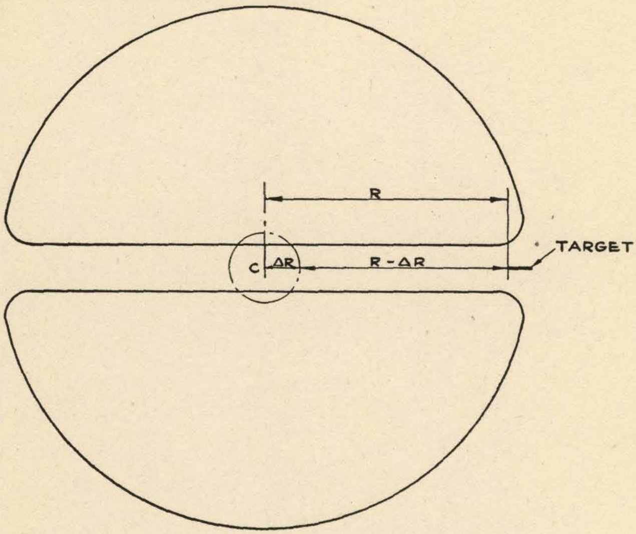

FIG. 2.
Illustrating the spreading of path centers over a region ΔR due to sinusoidal voltage applied to the dees.

With a square wave, the instantaneous accelerating voltages will be nearer the same values over the entire half cycle than for a sine wave. Consequently the ions accelerated during this half cycle will have more uniform paths with centers bunched instead of spread over a region as for the sine wave. The position of the ion source can then be adjusted so as to locate the path centers as near to the magnetic center as possible. This would give the maximum beam current and maximum homogeneous energy.