INVESTIGATING GROUP PROBLEM POSING FOR SECONDARY STUDENTS IN A LINEAR FUNCTIONS INTERVENTION

by

Robert Anthony Mixell

A dissertation submitted to the Faculty of the University of Delaware in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Education

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ABSTRACT

Mathematics classrooms have reflected little change over the past century, consisting of the learning of inert, disjoint procedures with few connections between mathematics concepts and students’ everyday experiences. However, problem posing, that is, the reformatting of existing problems and the generation of new ones, has been promoted by the National Council of Teachers of Mathematics (NCTM, 1989) as a valuable practice for deepening students’ mathematical thinking. It has also been shown to be associated with problem-solving ability, conceptual understanding, and mathematical habits of mind such as divergent thinking. Furthermore, educational policy and research have promoted collaborative student interactions in mathematics.

In this study, I investigated the process of group problem posing for beginning problem posers. That is, I explored characteristics of an individual student’s engagement with the activity, as well as students’ interactions with one another, so as to understand the successes and struggles of student participation and collaboration. In addition, I explored the products of group problem posing, that is, what kinds of problem scenarios beginning problem posers would create. Because the linear function is a foundational topic for high-school Algebra students, and because mathematical connections and multiple representations have been promoted by the NCTM (2000), I analyzed how students incorporated visual representations, as well as the rate of change and the y-intercept.

I led a five-day intervention in an Integrated Mathematics 2 classroom with five groups of students (3-4 students per group) in an ethnically-diverse high-school in a small, mid-Atlantic town. Students were heterogeneously grouped according to
gender, mathematics disposition, and mathematics performance. As Activity Theory served as the basis for the intervention, students were daily provided with mediating artifacts (i.e., criteria, cultural artifacts, “I Notice, I Wonder” chart) to support their daily creation of problem scenarios. Students were also given the opportunity to periodically define, enact, and negotiate roles within their groups to promote student collaboration. In addition, to encourage their incorporation of representations and mathematics concepts, students participated in daily lessons regarding the rate of change and y-intercept, which fostered connections between concepts and between representations. Problem scenarios from all five groups were coded and analyzed with respect to how student groups incorporated representations and mathematics concepts into their scenarios. Videotaped sessions of one group of three students were recorded, transcribed, and analyzed to investigate student interactions and engagement.

The results indicated that all student groups created problem scenarios that included both mathematical information and everyday items (e.g., food or beverages) from the cultural artifacts. However, student groups faced difficulties with incorporating visual representations into their scenarios that were mathematically meaningful, that had explicit purposes, and that were explicitly mathematically connected. Regarding mathematics topics, most student groups often found difficulties with incorporating the rate of change and the y-intercept in productive ways. Furthermore, each of the three students varied in the degree and ways in which they participated in group problem posing throughout the intervention. As two of the students took a more cooperative, rather than collaborative, approach to group problem posing, the third student struggled to participate in the activity. Further results and implications of the findings for practitioners and researchers are discussed.
Chapter 1

INTRODUCTION

Many mathematics classrooms within the United States have reflected little change over the past century, promoting inert learning and procedurally-oriented and unchallenging content situated within fragmented lessons (Hiebert et al., 2005). What Papert (1993) calls instructionism may be knowledge-centered, but it is not learner-centered. Thus, little engagement or understanding is fostered, and few connections to authentic, everyday situations are made (Bransford et al., 2000). Recent efforts that have been made to effect change have included adopting a more conceptually-based set of standards (i.e., the Common Core State Standards for Mathematics, 2010), and developing more conceptually-based curricula (i.e., NSF-supported curricula such as Everyday Math, Core Plus, Interactive Mathematics Project, or other curricula such as the LieCal Project, Cai et al., 2011; Moyer et al., 2011). Yet classroom instruction still often lacks an emphasis on conceptual understanding, rigor, applications, connections, and authentically mathematical practices (Hiebert et al., 2005; Pianta et al., 2007), and expectations of students are often lowered on cognitively demanding tasks (Stein et al., 1996). Next, I expand on the idea of problem posing. Then, I propose collaboration within group problem posing as a productive approach and Activity Theory as a useful lens for studying group problem posing.
Problem Posing

One idea for how to effect instructional change in the classroom that both researchers and educational policy-makers have suggested, but that has not yet gained much traction, is to engage students in problem posing, that is, the generation of new problems and the reformatting of existing problems (Silver, 1994). The idea itself is not new: For over 25 years, the National Council of Teachers of Mathematics (NCTM, 1991) has promoted the use of problem posing as a means to increase students’ conceptual understanding, mathematical reasoning, communication, curiosity, and interest in mathematics. Also, Moses, Bjork, and Goldenberg (1993) stated that problem posing should be a part of the mathematical experiences of all students. More recently, the NCTM (2000) has advocated for the “formulation of interesting problems based on a wide variety of situations, within and outside of mathematics” (p. 258).

There is also empirical support for the use of problem posing. For example, research studies have shown that problem-posing ability is positively associated with greater problem-solving ability (English, 1998; Silver & Cai, 1996). The practice of problem posing has also been used as a measure of curricular effectiveness for student learning (Cai et al., 2013). In other studies, students across the K-12 spectrum have shown an adeptness at creating problems, whether in elementary school (Lampert, 1986; Etheridge et al., 1992), in middle school (Cai & Hwang, 2002; Cai, 1998; Silver & Cai, 1996), or in high school (Kimball, 1991). Additionally, the creation of mathematical problems has served to foster perceived ownership in one’s learning (Barlow & Cates, 2006), as well as increase students’ mathematical self-efficacy and their positive attitudes toward mathematics (Akay & Boz, 2010).

Despite the numerous calls for students to pose problems and despite the research that demonstrates its benefits, problem posing remains a relatively rare
activity in American classrooms (Van Harpen & Presmeg, 2013). Furthermore, few studies have examined what kinds of learning environments are conducive to helping students pose solvable, complex, and real-world problems. In other words, mathematical problem posing represents a gap in instructional practice and in the mathematics education research. This study attempts to address the gap in research so as to inform instructional practice.

**Collaboration**

As stated above, because of a lack of research, how best to engage students in problem posing is an open question. However, research suggests that collaboration with peers is an important component of meaningful problem solving. For example, researchers like Langer-Osuna (2017), Wood (2013), and others who have focused on understanding meaningful and non-meaningful intellectual mathematical collaboration suggest that students have more opportunities to meaningfully collaborate when they share the intellectual authority in a mathematical activity. Thus, as students benefit from collaborating in group problem solving, it may be that group problem posing could also be a productive approach. However, thus far group problem posing where students collaborate has not been well studied and research is needed to determine what collaborative problem posing looks like, especially at the beginning stages.

**Activity Theory**

One way to view collaboration is with an Activity Theory lens (Engestrom, 1987 and Leont’ev, 1978, as cited in Anthony, 2012; Engestrom, 2001; Vygotsky, 1978). According to Activity Theory, when subjects engage in an activity, two components which mediate the activity are the roles the subjects play and the artifacts
with which they work. In the context of collective problem posing, little research exists regarding what roles students should assume and what artifacts should mediate problem posing to support students’ ability to create authentic and original problems.

In summary, I discussed the benefits of problem posing, the potential importance of collaboration, and the possibility of Activity Theory as a useful lens to understand group problem posing. Furthermore, I showed the lack of research regarding group problem posing. For these reasons, I embarked on a qualitative study to examine group problem posing. Now, I will provide a review of prior research about these topics, as well others pertaining to the study.
Chapter 2

LITERATURE REVIEW

This literature review addresses four related topics. The first topic is problem posing. I will present prior research on the activity of problem posing and its benefits for students. Second, I will discuss the importance of collaboration with respect to problem posing. Third, I will present the Activity Theory framework and research that shows how it relates to the activity of problem posing. I also present literature on how Activity Theory can be used as a lens to analyze classroom learning environments. The fourth topic of the literature review is linear functions. I present research on the importance of linear functions, because it is the specific topic for which students in my study created problems.

Problem Posing

Problem posing is a mathematical activity of which mathematics education researchers have long advocated (NCTM, 1989). In this section, I will first present ways that problem posing is defined. Next, I will present several benefits of problem posing as cited in the literature.

Defining Problem Posing

Silver (1994) broadly views problem posing as an activity or process and defines it as the generation of new problems and the re-formatting of given problems. Three kinds of problem posing are discussed in the literature based on when a problem is solved and its instructional use. One kind of problem posing involves recreating an original problem for the purpose of solving it. This kind of problem posing aligns with Silver’s view that problem posing is a thinking process, or a method of planning used
for solving a more complex problem. Similarly, Polya (1954) described problem posing as a heuristic device of thinking of a “related, more accessible problem,” and Davis (1985) described it as further reducing or enhancing a problem to have better insights into the original problem (Silver, 1994).

A second kind of problem posing is the generation of alternative problems that are related to a previously solved problem. This is similar to Polya’s (1957) fourth stage of problem solving called “looking back” (Silver, 1994). This type of problem posing also corresponds to Brown and Walter’s (1983) thinking process of “What if?” and “What if not?” which emphasizes the addition of various affordances and constraints to an existing problem, and then solving the newly-generated problem (Silver, 1994).

A third kind of problem posing is also referred to as problem formulation. The main goal of problem formulation is not to solve an original problem, or to create a new problem from a previously-solved problem. Instead, the creation of a new problem is the end goal itself. Such problems can serve as a means of instruction (Kilpatrick, 1987), or an instructional tool to reveal levels of expertise regarding mathematical knowledge and understanding (Rash, 1997). This kind of problem posing may also serve as an instructional approach for deepening conceptual understanding (Cai et al., 2013). Problem posing can support the use, development, and application of procedural knowledge or the understanding of how concepts and principles can be applied to a problem context (Mestre, 2002; Rash, 1997). It should be noted that this kind of posed problem may eventually be solved by others. Therefore, like Kimball’s (1991) student creation of problems, the goal of this kind of problem posing is primarily to create a new problem as an end product for oneself and
perhaps secondarily for others to eventually solve. It was this third view that was the focus of my study.

**Benefits of the Activity of Problem Posing**

In this section, I will present findings regarding the benefits of the activity of problem posing for student learning. These benefits include mathematics achievement and problem-solving ability. I will also discuss problem-posing ability and ways to measure this construct.

**Mathematics Achievement**

Studies have shown that student participation in *problem posing* benefits their *mathematics achievement* in terms of skill fluency and conceptual understanding. For example, Cai et al. (2013) used student problem posing in a linear equations unit to deepen conceptual understanding. Also, Bernardo (2001) showed that the creation of analogous probability problems was connected to greater knowledge retrieval and rule application. This evidence provided a rationale for my study because it suggested that problem posing would benefit students in developing their understanding of linear functions.

**Problem Solving**

Another way in which problem posing is beneficial to students is that it can support students’ problem solving. Problem-posing activities can lessen anxiety and foster positive mathematical dispositions. They can also improve students’ understanding and problem-solving abilities (NCTM, 2000; Silver, 1994; cited in Cai & Hwang 2002). For example, studies have shown that students who engaged in problem-posing tasks in a problem-based curriculum have grown in their problem-
solving abilities (Cai & Nie, 2007; Lu & Wang, 2006; Clarke, Breed, & Fraser, 2004). Additionally, Kapur (2015) showed that there are benefits to problem posing and problem solving, as students who posed and solved their own problem had greater conceptual understanding than those who did not do both activities.

Problem-Posing Ability

A third benefit of problem posing is that it can provide insights into students’ cognition and may predict their mathematical performance. For example, problem posing ability can be a tool to measure mathematical creativity (Van Harpen & Sriraman, 2013; Yuan & Sriraman, 2011; Leung & Silver 1997). Second, one’s problem posing ability is predictive of students’ overall success in mathematics (Nicolaou & Philippou, 2007; English, 1998; Silver et al., 1996; Brown & Walter, 1993). Third, problem posing is also predictive of problem-solving ability (English, 1998; Silver & Cai, 1996, 1993). For example, Silver and Cai (1996) showed that the complexity of students’ posed problems is related to their problem-solving abilities.

If problem posing is to be beneficial to understanding students’ cognition and mathematical performance, then it must be measured in some way. Researchers have taken at least two different approaches to measuring problem-posing. One approach is measuring the solvability of the problems (Leung & Silver, 1997; Silver & Cai, 1996). Furthermore, Van Harpen & Sriraman (2013) measured solvability in terms of the number of solvable problems a student creates. Another approach to measuring problem-posing ability is looking at a problem’s complexity, that is, the number of necessary steps to solve it (Leung & Silver, 1997). Both approaches informed my understanding of students’ posed problems in my study.
Collaboration and Problem Posing

A position long-held by mathematics education policymakers and researchers is that collaboration is important for student learning in general (NCTM, 1989; National Research Council, 1989). Therefore, it stands to reason that collaboration would be important for problem posing in particular. I will now discuss collaboration in general and its specific importance within mathematics education. Then, I will define group problem posing based on my view of problem posing and collaboration.

Defining Collaboration

There are different perspectives on collaboration across disciplines, yet with some commonalities. Merriam Webster’s 1828 Dictionary defines collaboration as “work[ing] jointly with others or together especially in an intellectual endeavor,” such as a team of scientists working collaboratively on a study (Collaboration, 2019). As an example, from a business perspective, the Association for Information and Image Management (AIIM, 2019) defines collaboration as a “working practice whereby individuals work together to a common purpose to achieve a business benefit.” Moreover, according to this definition, working together entails having a common purpose, consensus in problem solving, an expectation for individual and pro-active engagement, collective negotiation to find the middle ground, individual sharing, and the consideration of alternatives (AIIM, 2019).

Researchers in the field of mathematics education also have their own understandings of collaboration with respect to participation in mathematical activities. One researcher, Langer-Osuna (2017), views collaboration as the activity where students position one another so that they share in the intellectual authority of a mathematical activity. According to this definition, examples of collaboration would
involve student sharing and explanation of ideas and building on one another’s ideas. Researchers agree that collaboration is important for student learning (NCTM, 1989; National Research Council, 1989). However, research has shown that collaboration is not always easy for students. For example, Wood (2013) showed that students may position themselves or others in ways that promote little collaboration in an activity.

Research on collaboration in mathematics education often involves the study of how student interactions can lead to their learning (O’Donnell, 2006; Webb & Palincsar, 1996). In mathematics, student interactions, such as expressing and defending one’s thoughts and questioning other’s thoughts can help students restructure, clarify, and notice discrepancies in their own thinking (Ball, 1993; Cobb, Yackel, & Wood, 1992; Hatano, 1988). Collaboration can also help a student develop problem-solving strategies and metacognitive awareness about their own understandings (Cooper, 1999).

Based on the view of Langer-Osuna (2017) and the benefits of particular student interaction in other studies, I define *collaboration* as students sharing in the intellectual work of a mathematical activity. This would include students sharing and explaining mathematical ideas with one another, as well as building onto and questioning one another’s ideas for a mathematical end-goal, such as problem solving, problem posing, or understanding an idea. It should be noted that this definition contrasts with a cooperative view of group work where group members play asynchronous, individualized roles (e.g., recorder, facilitator) (UNC Center for Faculty Excellence, 2017). This distinction between true collaboration, as I am defining it, and this asynchronous group activity became important in my study. As collaboration has
shown to have benefits to student learning in mathematics, I decided to incorporate group work into my problem-posing study, which I discuss next.

Defining Group Problem Posing

As stated above, a goal of this study was to explore what group work looks like in a problem-posing context. Very few studies have focused on the activity of group problem posing. As I view group work as ideally collaborative in nature, my view of collaboration informs my perspective on group problem posing. For this study, I define group problem posing as the process by which students work together to think about and create problem scenarios, as well as the product of problem scenarios they create.

In all, I only identified two studies that focused on group problem posing. One study was by Ellerton (2015), who analyzed group problem posing by pre-service and practicing middle-school teachers. Results indicated that teachers were often unsure about the nature of how group interactions were to function, and teachers commented on not receiving adequate feedback with regard to revising their posed problems. Ellerton’s analysis of group interactions for teachers corresponded to my desire to look at group interactions for beginning student problem posers.

The second study by Kontorovich, Koichu, Leikin, and Berman (2012) did look at high-school students’ group problem posing. The researchers extended Schoenfeld’s (1992) four-facet framework for problem solving (task organization, knowledge base, problem-posing heuristics and schemes, group dynamics and interactions) by adding a fifth facet (individual considerations and aptness) and by extending the framework to problem posing. For my study, I was mostly interested in
the second and fourth facets of the framework, namely the knowledge base and group dynamics and interactions.

For the second facet, Kontorovich et al. (2012) define the knowledge base as the mathematics facts and relevant competencies needed for composing a new problem. For my study, these included representations and concepts related to linear functions, including the rate of change and the y-intercept. For the fourth facet, Kontorovich et al. define group dynamics and interactions as the different functional roles that group members can play (e.g., generating ideas, mediating, recording). This corresponded to my study in that I wanted to investigate what roles beginning problem posers would identify and enact.

Beyond just problem posing in a group setting I wanted students to truly collaborate in problem posing. Therefore, regarding the process of group problem posing, I use the term high-quality group problem posing to refer to a truly collaborative activity consisting of students regularly sharing in the suggestion and explanation of information and the mathematics, building upon and critiquing/questioning one another’s ideas, asking for explanations of ideas and mathematics, and verifying the quality of their own problems. In summary, what was important for my study was a focus on group problem posing with the goal of true collaboration. Also recall that of Silver’s (1994) three views of problem posing, my study was focused on his third view, that is, the generation of problems as an end goal in and of itself rather than for the purposes of problem solving.

Group Problem Solving

Consistent with Kontorovich et al. (2012), I also considered research on group problem solving because more research exists in group problem solving than group
problem posing (e.g., Goos, Galbraith, & Renshaw, 2002; Lotan, 2003; Yackel & Cobb, 1991). One study that was informative to my own was Lotan (2003), who provided five design features for group-worthy tasks: (a) open-ended tasks, (b) multiple ways to show competence, (c) significant content, (d) interdependence and individual accountability, and (e), clear evaluation criteria. All of these elements for group problem solving connect with my study on group problem posing. In particular, problem posing is an open-ended task where students can show competency with multiple visual representations and mathematical understandings. I also tried to make the content significant to their lives by providing artifacts based on everyday experiences. My study also sought to have students become interdependent through collaboration. However, individual accountability was one feature that did not play a large role in my study. Finally, clear evaluation criteria were important for my study, because, as I will show later, I provided students with daily criteria as a support for their problem posing.

In addition to Lotan’s (2003) features of group-worthy tasks, other research in group problem solving also highlighted the value of collaboration in group work. First, Goos, Galbraith, and Renshaw (2002) explored how student collaboration in group problem solving can be used to mediate metacognitive awareness, as expressed through the problem-solving process. Successful problem solving was characterized by an active student approach of challenging others’ ideas, actively retaining strategies that were useful, and removing ideas that were not helpful to problem solving. In contrast, unsuccessful problem solving was characterized by a lack of critical engagement with other students’ thinking, thereby reflecting poor metacognitive awareness. These findings aligned to my study as they further informed my view of
high-quality group problem posing as active engagement of group members analyzing and evaluating one’s own and other’s thinking.

Second, Yackel, Cobb, and Wood (1991) analyzed the types of productive collaboration that occurred in a second-grade classroom for students working in small groups completing mathematical problem-solving tasks. One positive form of collaboration was that students successfully took turns using the solution work of others to develop their own solution methods. Second, problem reconceptualization occurred for the purpose of a student analyzing an incorrect solution method. Third, students sought to make sense of other students’ solution methods for the purpose of reaching group consensus about the solution. Similar to the Goos et al. (2002) study, this study informs my view of collaboration because students share in the intellectual work of solving problems. This occurred through the development of individual ideas, building off one another’s ideas (solution development), critique of other’s ideas (analyzing an incorrect solution method), and understanding other’s ideas (making sense of a solution method). For my study, I viewed all of these components as valuable to high-quality group problem posing.

Because collaboration was a focus of my study, a socio-cultural lens was warranted for understanding group problem posing. The three aforementioned studies informed my view of group problem posing. Kontorovich et al. (2012) informed my view of group dynamics and interactions in group problem posing. Lotan’s (2003) informed my view of group-worthy tasks. Goos et al. (2002) and Yackel et al. (1991) informed my view of collaboration in group work. Activity Theory further informed my understanding of group problem posing, which I will discuss next.
Activity Theory

In addition to analyzing group problem posing and describing what high-quality problem posing looks like, I sought to know what conditions were associated with maximizing student opportunities for collaboratively interacting in group problem posing and for creating mathematical, original, and authentic problems. Activity Theory served as the foundation on which to develop such conditions for the study. For this section, I will provide a short history of Activity Theory. I will then discuss the second-generation, its, components, and how they connect to the goals of my study.

History of Activity Theory

Activity Theory is used as a lens for understanding everyday human activities. Activity Theory began as a theory about individuals (subjects) accomplishing some motive, or goal (object). As part of this first generation of the theory, Vygotsky (1978) added the idea of mediating artifacts (signs, tools, and language), which support conceptual development and task accomplishment. Engestrom (1987) and Leont’ev (1978) conceptualized a second generation of the theory (see Figure 1). Their elaboration of the theory included not only individuals interacting with artifacts, but also complex social interactions, rules, and division of tasks, which according to Cole and Engestrom (1993), influence learning.¹

¹ Note that Engestrom (2001) developed a third generation of Activity Theory, which was not used to inform my study.
Now, I will discuss particular components of Second-Generation Activity Theory and how they applied to my study.

Components of Activity Theory that Guided the Study

For the purposes of my study I focused on two of the six components of Activity Theory, namely mediating artifacts and roles. Mediating artifacts may be used to support task accomplishment (Anthony, 2012; Vygotsky, 1978). Such artifacts can support problem creation (Cai et al., 2013; Van Harpen & Presmeg, 2013). The two artifacts I focused on were (a) criteria and (b) cultural artifacts.

Mediating Artifacts

For my study, I define criteria as the set of recommended problem features established by the instructor. The criteria focused on three categories: context, mathematical concepts, and representations. I chose criteria as a particular kind of mediating artifact to study with regard to problem posing because a student’s criteria and an instructor’s criteria may differ. Additionally, students’ perceptions of what constitutes a mathematics problem have been shown to impact their problem posing.
success (Lowrie, 2002). Therefore, my hypothesis was that if the instructor provides the criteria to the students as an artifact, this may standardize and solidify students’ understanding of what is being asked of them to support quality problem creation.

In addition to criteria, cultural artifacts, such as an everyday object or tool, was a second focus of the study. I define a cultural artifact as any tangible object or tool related to students’ experiences or interests for the goal of creating mathematical and original problems and to promote high-quality (collaborative) group problem posing. I focused on cultural artifacts because such artifacts may foster the posing of real-world problems.

While little research exists about cultural artifacts with problem posing, research has shown that cultural artifacts support students’ problem solving. For example, in Tomaz and David’s (2015) study, students were given an everyday artifact (i.e., a water bill), which helped them apply elements of their everyday experiences when trying to solve problems regarding pricing and daily water consumption. In other words, the everyday artifact helped learners identify with school-based knowledge at a personal level (Kafai, 2006). For this reason, I hypothesized that if students participate in problem posing with a cultural artifact, then they may similarly make original and authentic problems by introducing their own variables and reasonings based on personal experiences with the artifact. Also, I hypothesized that if the cultural artifact represents a shared experience among the group members, students would collaborate by sharing and building on one another’s ideas.

**Roles**

I define roles as the ways in which tasks and responsibilities are shared (Cole & Engestrom, 1993). This is regarding both the horizontal division of tasks among
group members, as well as the vertical division of individual power or status among the group (Engestrom, 1999, as cited in Bourke, Mentis & O’Neil, 2013). To my knowledge, no studies exist where students are assigned different roles to improve the posing of problems. However, Brown et al. (1989) discussed how novice apprentices can become acculturated to a community of practice through adopting roles that are more central to participation in an authentic mathematics task. I decided to examine students’ roles because how tasks are distributed and/or shared may have a profound effect on how students engage in a mathematical activity. Tomaz & David (2015) illustrated how the re-negotiation of roles resulted in shared power among the teacher and students in a mathematical activity.²

Another conjecture I had was that as group members re-negotiate the effectiveness of their roles over time, they will begin to share in the intellectual work in a way that is more authentic to the practice of doing mathematics (e.g., stating, critiquing, and building on one another’s ideas). One thing to keep in mind is that while roles may be viewed from an individual perspective, for my study, I also wanted to consider when (if at all) students shared their roles when engaging in group problem posing. This is because the degree to which students share roles or maintain individualized roles may affect the quality of group problem posing. Now, I will discuss another important element regarding students’ creation of problems, namely, the mathematical context.

² Another approach of analyzing student interactions would be to look at mathematical micro-identities, as did Wood (2013). However, in this study, I chose not to focus on this psychological perspective, but rather on the sociological component of roles, which is more consistent with Activity Theory.
**Linear Functions**

The mathematical context is an important feature of the problem scenarios students pose. What I mean by the mathematical context is both the mathematical instruction students receive before problem posing as well as the mathematical topic for which students are to pose problems. In this study, the mathematical context I focused on was linear functions. Next, I will present how the field of mathematics education views linear functions as it relates to high-school students.

**Importance of Linear Functions**

Functions are an important concept in mathematics (CCSS, 2010; NCTM, 2000). In particular, linear functions are a foundation for elementary algebra (Pierce, Stacey & Bardini, 2010) and is crucial to all mathematics beyond Algebra 1, up to calculus (Stewart, 2015). Linear functions are typically taught before students encounter quadratic and exponential functions (Larson & Boswell, 2014; Larson et al., 2006; Schoen & Hirsch, 2003). Another aspect of linear functions is the addition and subtraction of functions to produce new ones (CCSS, 2010). The foundational importance of linear functions in secondary education served as the rationale for using linear functions as the mathematical context for group problem posing for my study of high-school students. Two important concepts related to linear functions are the rate of change and the y-intercept.

**Constant Rate of Change and Y-Intercept**

Two key features of linear functions are the constant rate of change and the y-intercept (CCSS, 2010). According to the Common Core State Standards, a rate of change is a multiplicative comparison between the change in one quantity and a unit interval change in another quantity. Second, for a linear function, the rate of change is
constant (CCSS, 2010). Finally, the y-intercept is defined as the “constant term of a linear model” (CCSS, 2010, p. 81).

Not only should students explore the rate of change and the y-intercept numerically, the real-world context also important (Leinhardt et al., 1990). In particular, students are to “interpret parameters of a linear function” such as the rate of change and y-intercept “in terms of a context” (CCSS, 2010, B.5). Thus, it was important for students in my study to not only to focus on the rate of change and the y-intercept, but to interpret them in real-world contexts.

Research has shown a few challenges that exist for supporting student development of their conception of linear functions. First, Paz and Leron (2009) revealed that an “embodied scheme,” or an action on objects may be foundational to their early conception of linear functions. However, this way of thinking about functions may conflict with their understanding of a composition of functions where the initial function itself now serves as an input variable rather than an action on an object. Second, for students’ conceptions of the rate of change, Stump (2001) showed that high-school Pre-calculus students may struggle with interpreting the slope as a measure of a rate of change. Finally, in their examination of students’ conceptions of the rate of change and y-intercept, Pierce, Stacey, and Bardini (2010) showed that when students develop conceptions about the rate of change and the y-intercept for a linear equation \((y = mx + c)\), they may not provide a verbal or symbolic description about the y-intercept \((c)\) as if it were not an important part of the function.

Additionally, research has shown a variety of conceptions and misconceptions about the rate of change. For example, Thompson (1994) showed that students may have misconceptions regarding the concepts of speed, average speed, and rate.
Zaslavsky et al. (2002) showed that students, teachers, mathematicians, etc. can have different perspectives regarding the slope, including either an analytic one (slope as an unchanging property of a linear function like the derivative), a visual one (slope as a changing property dependent upon the graph of a linear function), or a combination of the two. Thus, the rate of change is not a trivial concept for high-school students.

Nonetheless, researchers have identified several supports for helping students explore linear functions. For example, Moschkovich (1998) found that students may draw on various supports for helping themselves to develop their conceptions about certain linear functions concepts like the x-intercept, such as using the x-intercept within a specific case, using other concepts (e.g., rate of change) to help understand the x-intercept, and using descriptive language to refine how it is used. Regarding the rate of change and the y-intercept, Pierce, Stacey, and Bardini (2010) showed that contextual information supported students’ understanding of the variables within a linear function and the key features, such as the rate of change and the y-intercept. For my study on problem posing, I wanted to investigate the use of an instructional intervention that supported students in incorporating these two concepts into their problem scenarios.

**Visual Representations**

Representations, in particular, visual representations (e.g., graphs, equations, tables, and pictorial representations) play a prominent role in mathematics education literature and policy. The National Council of Teachers of Mathematics (2000) has promoted students’ use of multiple representations and connections between representations. Blanton and Kaput (2011) stress how a variety of visual representational forms, including symbols, function tables, and pictorial
representations, can support students’ functional thinking. The Common Core State Standards (2010) has also called for students to create various representations and interpret aspects of functions from such representations. For example, the Standards emphasize graphing as a key visual tool and its connection to other types of representations. Particular standards include “interpret[ing] key features of graphs and tables in terms of their quantities, and sketch[ing] graphs showing key features given a verbal description of the relationship,” “graph[ing] linear functions and show[ing] intercepts,” “calculat[ing] an interpret[ing] the average rate of change of a function (presented symbolically or as a table),” “estimat[ing] the rate of change from a graph,” and “graph[ing] functions expressed symbolically and show[ing] key features of the graph” (CCSS, 2010, p. 69).

Benefits of Student Use of Visual Representations

Mathematics education literature has shown a relationship between the use of representations, the opportunities students have to learn, and students’ eventual learning and performance outcomes. For one, Brenner et al. (1997) found that students’ use of multiple representations in Algebra in cooperative learning groups was related to students’ increased use of various representations when solving problems. Second, Huntley et al. (2000) also showed that curricula such as the Core-Plus Mathematics Project (CPMP) promoted the use of multiple representations in Algebra. In a comparison of students taught with CPMP with students taught with a traditional curriculum, CPMP users reflected greater growth in not only the ability to move among various kinds of representations, but also growth in student understanding, skill, and problem-solving ability. Third, Lloyd & Wilson (1998) found that when a teacher emphasized the use graphs, tables, and rules for understanding
functions and promoted both a co-variation and correspondence conception of functions, students were provided with greater opportunities to develop a deeper understanding of functions.

As such studies illustrate the benefits of using multiple representations for student learning and performance, I wanted to develop an instructional intervention that supported students in including representations in their problem scenarios and making connections between them (in addition to their incorporation of linear function concepts) to see what kinds of difficulties they would experience (Note that while there are other representational forms besides visual ones, I chose to focus on visual representations. Therefore, any time I talk about representations in this study, I am referring to those that are visual, versus those that are verbal in nature). As both representations and linear functions are important in high-school mathematics, I wanted students to have maximal opportunities to achieve high-quality group problem posing with regard to the products of group problem posing, not just the process. I define the products of high-quality group problem posing as the successful incorporation of representations into students’ problem scenarios in mathematically meaningful and explicitly connected ways with explicit purposes, as well as the productive incorporation of linear functions concepts, such as the rate of change and the y-intercept. At times when student groups were not able to incorporate such mathematical information, my goal was to investigate what difficulties students may have with doing so.

**Purpose of the Study**

As previously discussed, problem posing is a mathematical activity that has been shown to be beneficial to student learning and mathematics performance (e.g.,
Cai et al., 2013; Mestre, 2002; Bernardo, 2001; Rash, 1997). Additionally, collaboration has also been highlighted by policy makers and researchers as important practice for mathematics students (NCTM, 1989). However, little research has investigated the combination of the two. That is, few studies have explored the process, that is, what student interactions and engagement look like for group problem posing. This study aims to fill a void in the problem posing literature regarding group problem posing.

The mathematical context in which the study was situated was linear functions. As explained earlier, my reason for choosing linear functions and its related concepts (e.g., the rate of change and y-intercept) is because such concepts are foundational for learning Algebra (CCSS, 2010). Additionally, I argued the use of representations is beneficial for student learning (Huntley et al., 2000; Brenner et al., 1997). The goal of my study was to reveal the kinds of difficulties students experience in the products of group problem posing, that is, with the incorporation of the rate of change, the y-intercept, and representations into their posed problems scenarios. Therefore, I hoped that this study would help practitioners with identifying what difficulties may arise when student groups attempt to pose problem scenarios, and how to support students with making mathematical and viable problems involving concepts of linear functions.

Finally, certain elements of Activity Theory, such as the use of roles and mediating artifacts, have been associated with productive group work. Such elements were included in the development of an intervention for maximizing students’ opportunities to collaborate and support their group problem posing. Thus, I hoped that this study would help mathematics educators understand whether roles and mediating artifacts would support or hinder collaboration and the posing of contextual
and mathematically valid problems with representations. I sought to answer the following three questions:

**Research Questions**

1. What difficulties do secondary students reveal about incorporating representations into their problem scenarios?

2. Based on students’ productive and unproductive uses of the rate of change and the y-intercept, what difficulties do secondary students reveal about incorporating those concepts into their problem scenarios?

3. What does engagement and interactions look like for beginner group problem posers?
Chapter 3

METHODS

Setting and Participants

I led a six-day intervention at a public high school in a small, mid-Atlantic town. The intervention took place in an Integrated Mathematics 2 classroom consisting of 30 students who were mostly tenth-graders. As the study was exploratory, the sample size was suitable for investigating what the activity of group problem posing looked like. Having students for five consecutive school days enabled me to have enough time to determine any changes in the characteristics about the problem scenarios they posed, as well as characteristics of their interactions. As I was only interested in problem posing with respect to linear functions, Integrated Mathematics 2 students were ideal for this study. Such students would have had experiences with linear function topics the previous year. In addition, as the school had a relatively high population of African-American and Hispanic students, my hope was that the study would serve to benefit students from groups historically underrepresented in the field of mathematics.

For student recruitment, I visited the school one week in advance to discuss the nature and aims of the study with the students. I asked them if they would like to take part in the study. I also informed them that they would receive a $5 gift card to a local restaurant (Chick-Fil-A) if they participated. If students decided not to be an official participant in the study, they were reminded that they would still take part in the lesson and problem-posing experiences, as it would serve to reinforce their knowledge of linear functions. However, I also told them that their created problems would not be analyzed. Furthermore, the classroom would also be arranged so that these students
will not be captured on any video-recordings. Each student was then provided with an assent from, on which they notified me whether or not he or she would participate in the study. Consent forms were then given to students who were willing to participate. Of the 30 students in the classroom, 17 served as official participants, as they returned both consent and assent forms.

As the high school had block scheduling, Integrated Mathematics 2 was a one-semester course and was taught for approximately 80 minutes per day.

**Student Groups**

Two days before the intervention, I purposely grouped the students in preparation for group problem posing. The students were given a short survey asking for the following self-reported data: (1) gender, (2) prior performance in mathematics (I am great at/good at/okay at/lousy at math), (3) mathematics disposition (I love/like/dislike/detest math), and (4) previous high-school mathematics courses taken, along with grades for the course(s). These measures were used to heterogeneously assign students to groups as a means to maximize student diversity of mathematical understanding and experiences for the effective creation of problems. Performance in mathematics as teacher-reported grades was also consulted to further validate the heterogeneity of the groups. All personal information was kept in confidence by the researcher.

In total, five groups were created from the 17 official participants (see Table 1 for student groups). Students were placed into groups of 3 or 4 so as to encourage a variety of viewpoints during the problem-posing activity. Due to issues with social group interactions and absences, some students were moved to different groups, either on the first or last day of the study.
Table 1  Groups of Study Participants

<table>
<thead>
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</thead>
<tbody>
<tr>
<td>1*</td>
<td>S1</td>
<td>Male</td>
<td>love math</td>
<td>“good at”</td>
<td>100 (A)</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>Male</td>
<td>like math</td>
<td>“okay at”</td>
<td>88 (B+)</td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>Female</td>
<td>like math</td>
<td>“okay at”</td>
<td>82 (B-)</td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>Female</td>
<td>detest math</td>
<td>“lousy at”</td>
<td>70 (C-)</td>
</tr>
<tr>
<td>2**</td>
<td>S1</td>
<td>Female</td>
<td>like math</td>
<td>“good at”</td>
<td>94 (A)</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>Female</td>
<td>dislike math</td>
<td>“okay at”</td>
<td>98 (A)</td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>Female</td>
<td>like math</td>
<td>“okay at”</td>
<td>20 (F)</td>
</tr>
<tr>
<td>3</td>
<td>S1</td>
<td>Female</td>
<td>dislike math</td>
<td>“okay at”</td>
<td>92 (A-)</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>Male</td>
<td>like math</td>
<td>“good at”</td>
<td>86 (B)</td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>Female</td>
<td>dislike math</td>
<td>“lousy at”</td>
<td>33 (F)</td>
</tr>
<tr>
<td>4</td>
<td>S1</td>
<td>Male</td>
<td>like math</td>
<td>“okay at”</td>
<td>82 (B-)</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>Male</td>
<td>dislike math</td>
<td>“lousy at”</td>
<td>47 (F)</td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>Female</td>
<td>like math</td>
<td>“okay at”</td>
<td>82 (B-)</td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>Male</td>
<td>like math</td>
<td>“good at”</td>
<td>19 (F)</td>
</tr>
<tr>
<td>5***</td>
<td>S1</td>
<td>Female</td>
<td>love math</td>
<td>“good at”</td>
<td>100 (A)</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>Female</td>
<td>like math</td>
<td>“okay at”</td>
<td>32 (F)</td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>Male</td>
<td>dislike math</td>
<td>“okay at”</td>
<td>84 (B)</td>
</tr>
</tbody>
</table>

*S4 moved to Group 3 on day 1 for social/group issues.
**S3 stopped attending class.
***S3 moved to Group 2 on day 5 as S1 and S2 were both absent.

The primary goal for the creation of heterogeneous groups was for students to have the opportunity to interact with others from various backgrounds and mathematical dispositions. It was my conjecture that by having students grouped in such ways, they would have a greater likelihood of creating problem scenarios that were not only mathematically complex, but also interesting and original as students with different life experiences could come together to make something unique.

The 13 non-participating students were also seated in groups. However, these students were kept out of the view of the camera so as to avoid being video or audio-recorded. These students were still asked to take part in the lessons and afterwards to take part in problem posing sessions. On the final two days when they chose not to
work on problems, students were given an alternative assignment (i.e., worksheets) that reviewed the ideas from the daily lesson.

**Primary Intervention Features**

Besides heterogeneous groups, other main features of the intervention were informed by Activity Theory. Specifically, the components of Activity Theory used were mediating artifacts (criteria, cultural artifacts, and organizer) and roles.

**Problem-Posing Criteria**

For the mediating artifact of problem-posing criteria, I hypothesized that as students received daily criteria that clearly identified what elements were to be incorporated into their posed problems, they would be more likely to incorporate such elements into their scenarios. Consistent with Lotan’s (2003) fifth feature for group-worthy tasks, that is, *clear evaluation criteria*, I presented daily problem-posing criteria to the students at the beginning of each problem-posing experience. Furthermore, to scaffold their problem posing, I gradually increased the number of criteria from one day to the next (see Appendix A for daily criteria).

**Cultural Artifacts**

For the mediating artifact of cultural artifacts, I hypothesized that as group members have opportunities to pose problems based on artifacts connected to their cultural experiences, students would have more ideas about how the artifacts could be incorporated into their posed problems. Consistent with Lotan’s (2003) third feature for group-worthy tasks, that is *significant content*, my goal was to provide students with a cultural artifact that was significant to their everyday experiences. Therefore, I surveyed students about their favorite supermarket and their favorite fast-food
restaurant. Based on a questionnaire of 26 students at the school, Wendy’s (a fast-food restaurant) and ShopRite (a supermarket) were two popular places for which students regularly visited. Thus, a ShopRite advertisement of food items and a Wendy’s menu were utilized in my study as the cultural artifacts to mediate the activity of problem posing (see Appendices B and C).

The artifacts also provided unit rates and prices which provided contexts in which to pose problems involving the rate of change and the y-intercept. Thus, I hypothesized that the mathematical features of the cultural artifacts would also support students’ problem posing.

Organizer

An additional artifact that I hypothesized would be helpful with group problem posing was an “I Notice, I Wonder” chart (see Appendix D). Such a chart has been used as part of the Notice and Wonder Protocol by Daniel Venables (ASCD, 2014). During brainstorming, students in my study were asked to store numerical information from the cultural artifacts onto this organizer, as well as any other ideas and wonderments they had about the artifact items and mathematics to be developed for problem creation. My hypothesis was that if students had an opportunity to record their thoughts during brainstorming activities, this would provide students with a greater opportunity to draw from their ideas when developing their problems.

Roles

Finally, for roles, I hypothesized that as group members had opportunities to regularly try out particular roles during group problem posing over time, the roles they would choose for one another would gradually lead to higher quality group problem
posing (i.e., collaboration). Consistent with an aspect of Lotan’s (2003) fourth feature of group-worthy tasks, that is, *interdependence*, I was interested in what kinds of roles students would play during group problem posing. To facilitate role negotiation, students were asked to complete a role negotiation worksheet (see Appendix E) at the end of each problem-posing experience. On that worksheet, students were asked to analyze the usefulness of their roles, as well as what roles they thought were to be included, removed or shared for the next problem-posing experience.

**Intervention Procedures**

As the instructional design was geared toward providing students the optimal conditions for posing problems here, I will explain how the components related to such conditions were administered each day. As shown in Table 2 below, the general intervention plan consisted of an introduction to the topic of problem posing (Day 1), five problem-posing experiences (Days 1 through 5), and four lessons on linear functions (Days 2 through 5).

As can be seen in the table, the criteria, cultural artifacts, and roles were relevant components of the intervention on each of the five days. However, aspects of each of these elements changed from day to day. First, the goals of the criteria slightly changed from day to day. For example, the number and type of representations and the mathematical topics required to be incorporated into students’ problems changed as the lessons changed over time. Second, students were asked to implement items from one cultural artifact (i.e., ShopRite advertisement) from Days 1 to 3 into their problems. However, on Days 4 and 5, they were then introduced to a new artifact (i.e., Wendy’s menu) and were asked to incorporate an item or items from either one or both of the two artifacts. Third, group members were given the opportunity to
negotiate and identify their roles at the beginning of the problem-posing activity on Day 1, and were subsequently provided with more opportunities to do so on Days 2 through 5. Next, I will briefly outline what happened each day of the intervention.

Table 2    Timeline for Seven-Day Lesson and Problem-Posing Intervention

<table>
<thead>
<tr>
<th>Day</th>
<th>Lesson</th>
<th>Problem-Posing Session</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>None</td>
<td>Discussion of Problem-Posing (20 min)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Discussion of Roles</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Discussion of Representations</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Students Pose Problems (30 min)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-Criteria, ShopRite artifact, include one representation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Debriefing Session (10 min) -negotiate roles</td>
</tr>
<tr>
<td>2</td>
<td>Features of Linear Functions (slope, y-intercept) (30 min)</td>
<td>Students pose problems (30 min)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-Criteria, ShopRite artifact, include at least one representation, use rate of change and y-intercept</td>
</tr>
<tr>
<td>3</td>
<td>Features of Linear Functions (slope, y-intercept) (30 min)</td>
<td>Students pose problems (30 min)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-Criteria, ShopRite artifact, description of how to use “I Notice, I Wonder” chart, include two representations, use rate of change and y-intercept</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Debriefing session (10 min) -negotiate roles</td>
</tr>
<tr>
<td>4</td>
<td>Adding and Subtracting Functions (30 min)</td>
<td>Students pose problems (30 min)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-Criteria, ShopRite and Wendy’s artifacts, “I Notice, I Wonder” chart, include two representations, use rate of change, y-intercept, and/or adding/subtracting linear functions</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Debriefing session (10 min) -negotiate roles</td>
</tr>
<tr>
<td>5</td>
<td>Parallel and Intersecting Functions (30 min)</td>
<td>Students pose problems (30 min)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-Criteria, ShopRite and Wendy’s artifacts, “I Notice, I Wonder” chart, include two representations, use rate of change, y-intercept, and/or adding and subtracting linear functions and/or intersection of functions</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Debriefing session (10 min) -negotiate roles</td>
</tr>
</tbody>
</table>
Day 1

On Day 1, I, the principal investigator, assigned participants and non-participants to their groups. I first introduced the students to problem posing. To discourage students from creating problems that were structurally identical to those I would provide during instruction, I showed sample problems to help students see the difference between pairs of problems that were structurally similar, such as the cancer radiation problem (Duncker, 1945) and the military fortress problem (Gick & Holyoak, 1980), from those that were structurally different. This aligns with Silver’s (1994) goal of problem posing as creating original problems (see Appendix F on “What is Problem Posing?”). Then, I led a discussion about what are criteria of good problems, what a representation is, and what roles students can play in their groups. Afterwards, group members identified what roles they wanted to play, and began to pose their problem scenarios using the criteria and the ShopRite advertisement. At the end of the problem-posing activity, group members completed the role negotiation worksheet.

Day 2

Day 2 began with a lesson on the rate of change and the y-intercept (see Appendix G for all lesson materials from Days 2 through 5). Multiple representations were used to help students understand these concepts. Similar to Day 1, I then led a discussion about the criteria for high-quality problem posing, that is, the inclusion of at least one representation, the use of the rate of change and y-intercept, and the incorporation of information from the ShopRite advertisement. The criteria also stipulated that students should be clear whether they wanted the solver to use the
representation for a particular purpose. Note that because of time constraints, students did not evaluate and re-negotiate their roles at the end of this period.

Day 3

On Day 3, similar to Day 2, students received another mathematics lesson also on the rate of change and the y-intercept to further strengthen their understanding of the linear function concepts. Then, students were asked to create a problem scenario involving the rate of change and y-intercept, two representations instead of one, and information from the ShopRite advertisement. I also displayed a problem scenario created by one group (see Appendix H for Day 3 and Day 4 sample problem scenario) on the Smart Board to help students to understand how to incorporate representations into their problem scenarios. I also showed students how to use the “I Notice, I Wonder” chart to help them organize their thinking before posing their problem scenarios. Afterwards, groups were given the rest of the period to create their problem scenarios. Students took the last few minutes to fill in the role worksheet.

Day 4

On Day 4, I led a lesson on adding linear functions in addition to the rate of change and the y-intercept. Similar to the previous lessons, I emphasized connections between multiple representations. After the lesson, students were asked to create a problem scenario with either two representations given in the problem, or one given and one to be created by the solver. In addition to the rate of change and the y-intercept, students were asked to incorporate adding and/or subtracting linear functions into the problem scenario. Students were also asked to use the “I Notice, I Wonder” chart to brainstorm their ideas and to use Wendy’s menu along with the ShopRite
advertisement. Students then engaged in problem scenario creation. During this time, the sample problem from Day 3 was again shown to the students to support their incorporation of representations into their problem scenarios. For the last few minutes of class, groups completed the role negotiation worksheet as usual.

Day 5

On Day 5, students were taught the last lesson, which was on the intersection of linear functions. Again, I emphasized the use of multiple representations for understanding the intersection of linear functions, as well as the rate of change and y-intercept. Then, students were asked to create problem scenarios involving the intersection of linear functions, as well as all of the other criteria from the previous day. They were also again asked to use the “I Notice, I Wonder” chart. I also showed a problem scenario from one of the groups for how to incorporate more than one representation, how representations can mathematically connect to one another, and how the rate of change, y-intercept, and adding/subtracting functions can be incorporated into one problem scenario (see Appendix I for Day 5 sample problem scenario). After students had time to work on their problem scenarios, they completed the role negotiation worksheet.

Data Collection

There were three types of data that were collected: (a) students’ posed problem scenarios (including “I Notice, I Wonder” chart and solutions), (b) video-recordings of student engagement, and (c) group role negotiation worksheets. First, I collected groups’ problem scenarios each day to determine how students incorporated representations and mathematics into their posed problem scenarios. The “I Notice, I
Wonder” chart was also collected from the groups so as to understand the progression of students’ thinking and development of their problem scenarios. This evidence was used to answer the first and second research questions.

Second, I video-recorded student interactions for one group to examine what group problem-posing activity looks like. The videos were recorded by myself and a graduate student and were also transcribed. This evidence was used to answer the third research question.

Third, I collected the role negotiation worksheets at the end of the problem-posing sessions to further understand the group member interactions and participation during the problem-posing activity. This was also used to answer the third research question.

**Analysis**

Here, I will discuss how the collected data was measured and analyzed to answer each research question. For the first and second research questions, the unit of analysis was the problem scenario. My definition of a problem scenario first consisted of only the set of complete problems, that is, situations where an actual solvable problem was created. As groups at times wrote down the same problem on their brainstorming note sheet (e.g., “I Notice, I Wonder” chart) and then wrote the problem elsewhere on a sheet of paper, such duplications of a problem scenario were disregarded. I also initially excluded all incomplete problem scenarios. This resulted in a total of 22 problem scenarios. However, I eventually included the incomplete scenarios as long as mathematical representations and contextual elements existed because such situations helped to illustrate the progression of students’ problem posing when introduced to a new mathematical topic. This resulted in a total of 29
problem scenarios and a new operational definition of a problem scenario as any attempt made by an individual or group of students toward creating a problem as long as it included a contextual situation, representation, or any combination of these items.

Research Question #1

In order to answer the first research question about difficulties students had with incorporating representations into their posed problem scenarios, I had to analyze the 29 posed problem scenarios. Open coding (Strauss & Corbin, 1994) was used to develop a set of codes by which to analyze the problem scenarios. The scenarios were coded according to whether a representation was provided by the solver or asked to be created, as well as the number of times a representation was either provided or asked to be created (none, one, or more than one). From my analysis, the codes that emerged were: (a) no representations, (b) representation provided, (c) one representation asked to be created, (d) more than one representation with one provided and one asked to be created, and (e) more than one representation with two asked to be created. Note that the codes involve different combinations of two of the daily criteria given to students (i.e., stipulation of the number of representations to be provided and stipulation of either providing or asking for a representation in their problem scenarios).

Another code that emerged from this set of codes was that when students did incorporate a representation, they either did or did not provide an explicit purpose for the representation. It was common in many problem scenarios for students not to include an explicit purpose, even though they were asked to provide a purpose for their representation by their daily problem-posing criteria. Therefore, I decided to add this code to my code book.
At this point, nine codes existed (see Table 3). However, upon further analysis, other characteristics emerged. Three such characteristics were (a) representations being or not being mathematically meaningful, (b) representations being or not being mathematically connected to another representation, and (c) representations in their problem solutions rather than their problem scenarios. *Mathematically meaningful* was used to analyze one representation. The way I define a representation being mathematically meaningful is one that illustrates a mathematical idea in connection to the problem context. For example, if a student tries to illustrate an exponential relationship between two variables by creating a linear graph, this would not be mathematically meaningful, as the mathematical relationship would not make sense to the problem context.

*Mathematically connected* was used to analyze two representations in the same problems. The way I define representations as being mathematically connected is when the posers are explicit about how two representations relate to one another, even if they do not show the same relationship. The third characteristic, use of representations in students’ problem solutions rather than their problem scenarios, is self-explanatory.
Table 3  Representations Analytic Framework

<table>
<thead>
<tr>
<th>Number of Representations</th>
<th>Format: Given or to be created</th>
<th>Explicit Purpose: Yes/No</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>One</td>
<td>Given by poser</td>
<td>Yes/No</td>
</tr>
<tr>
<td></td>
<td>To be created by the solver</td>
<td>Yes/No</td>
</tr>
<tr>
<td></td>
<td>Mathematically meaningful</td>
<td>Yes/No</td>
</tr>
<tr>
<td></td>
<td>Problem solution rather than problem scenario</td>
<td>Yes/No</td>
</tr>
<tr>
<td>Multiple</td>
<td>Given by poser and to be created by solver</td>
<td>Yes/No</td>
</tr>
<tr>
<td></td>
<td>To be created by solver</td>
<td>Yes/No</td>
</tr>
<tr>
<td></td>
<td>Mathematically connected</td>
<td>Yes/No</td>
</tr>
</tbody>
</table>

Note: Examples for each code will be discussed in the results sections.

I used the Representations Analytic Framework to code the 29 problem scenarios. Then, to establish reliability, I had a colleague code the scenarios as well. Upon comparing our results, we obtained 90% reliability and we resolved any discrepancies in our interpretations. Note that mathematically meaningful, mathematically connected, and problem solution rather than problem scenario were not included in the reliability test because these characteristics emerged in later analyses and they occurred very infrequently. Based on how I coded the results, I was able to address the first research question, as I will show in the results section.

Research Question #2

The second research question addressed difficulties students had with incorporating mathematics concepts, namely the rate of change and the y-intercept, into their problem scenarios. In order to analyze students’ problem scenarios based on the mathematics used, I first identified scenarios where students actually incorporated the rate of change (27 of 29). Most problem scenarios either contained a specified unit rate or some type of equation or graph that either implicitly or explicitly included a slope and/or a y-intercept. However, problem scenarios varied based on the degree of mathematical information contained in the problem relating to the rate of change or y-
intercept, the accuracy of this information, and whether sufficient contextual information was tied to the mathematical information to illustrate a relationship between two variables.

As a result of this initial analysis of the 29 problem scenarios, what became most salient in the 27 remaining problems was that students incorporated the rate of change or the y-intercept in either productive or unproductive ways. More specifically, productive meant that the ways in which the rate of change or y-intercept were incorporated were (a) mathematically accurate, (b) non-trivial, (c) useful for accomplishing the desired task in the problem scenario, and (d) connected to the problem context. In contrast, unproductive meant that at least one of these four characteristics was not present. This became the primary lens that I used for my analysis of this research question.

In contrast to the representation analysis, I did not code each problem according to the four criteria that constituted what it meant for the concept to be incorporated productively. Instead, I first analyzed instances where the rate of change was used unproductively. Next, I analyzed the structure of the problem scenarios in which the students used the rate of change productively and compared them to the various ways in which a problem involving a constant rate of change and y-intercept can be structured to see if any of the ways were not represented. Finally, I analyzed instances where two rates of change were incorporated to see if they were used productively. From these three sub-analyses, I developed claims about the difficulties students encountered when posing problem scenarios involving the rate of change and the y-intercept. To establish validity and reliability, I presented these claims and
evidence to a Mathematics Educator. Based on our discussions, the claims were refined and agreed upon.

Research Question #3

The third research question addressed beginning problem posers’ interactions and engagement in group problem posing. As previously mentioned, video recordings of only one of the five groups was analyzed to obtain a rich description of how beginning problem posers participated in the activity. From the video recordings, I first transcribed the activity of the three group members, as well as the teacher.

Next, the transcriptions from each of the five days were then analyzed according to characteristics of engagement. The unit of analysis was the student. Analysis of the transcripts determined (a) what roles each student fulfilled each day, (b) what each student accomplished in their daily roles, and (c) how long on each day a student participated in a particular aspect of problem posing. In addition, I analyzed students’ responses from the role negotiation worksheets, which revealed the roles each student identified with on each day. These analyses provided an overall picture of the similarities and differences in participation for beginning problem posers, commonalities in what each student was able to accomplish, and each students’ general progression of participation in the problem-posing activity from Days 1 to 5.

To obtain a deeper understanding of how beginning problem posers may interact and engage in group problem posing, I addressed the research question by creating a rich description of the interactions. I initially contemplated focusing first on students’ interactions with one another and then on students’ interactions with the teacher. However, in my analysis of these interactions, I observed that two of the students interacted with one another in ways that highly contrasted the interactions and
engagement of the third student. Hence, I decided to structure my descriptions by first focusing on interactions between these two students. These descriptions of these two students’ interactions, in addition to responses from the role negotiation worksheets, illustrate what beginning problem posers may understand regarding what kinds of roles they were to play and what it means for them to share in the intellectual work of problem posing. Finally, as the interactions and engagement of the third student highly contrasted those of the first two, I focused the rest of my description on excerpts of this student’s interactions with the other two and the teacher, along with her responses on the role negotiation sheets. These descriptions of the third student, along with her role negotiation worksheet responses, illustrate the struggles beginning problem posers may have with initially participating in group problem posing, as well as what may support their participation.
Chapter 4

RESULTS FOR RESEARCH QUESTION 1

General Findings

First, I will provide several general observations about the 29 problem scenarios posed by the five groups over the five-day intervention. Only two of the five groups on average created at least one problem per day (Group 1 and Group 2). While 10 of the 29 scenarios (34%) contained more than one part, about two-thirds of the problem scenarios (19 or 66%) did not contain more than one part.

Besides the existence of multiple parts, scenarios varied according to the number of contextual features used. Such features consisted of actual food or beverage items. The cultural artifacts served as sources for the types of items used in each problem scenario. In conjunction with the ShopRite advertisement used on Day 1 through Day 3, all 19 scenarios created on these days included some aspect of this artifact. The Wendy’s menu was introduced on days four and five in addition to the ShopRite advertisement. However, groups seemed to prefer the Wendy’s menu over the ShopRite advertisement as none of the 10 scenarios created on Day 4 or Day 5 consisted of items from the ShopRite advertisement. This seems to make sense as teenagers would likely purchase fast-food items over supermarket items.

Another type of contextual feature included references to individuals or groups, such as an actual name or names (e.g., Alex, Natalia) or a pronoun referring to one of the posers (“I”) or the solver (“you”). Nineteen of the scenarios (66%) referred to at least one person, and 5 of the scenarios included more than one person. Yet, about one-third (10, or 34%) were “dehumanized” in the sense that they did not include any proper names or personal pronouns.
Next, I discuss the ways in which students incorporated representations into their posed problem scenarios. For these results, I used the Representations Analytic Framework to develop claims that address the first research question about what difficulties students have with incorporating representations during group problem posing.

**Research Question #1: What Difficulties do Secondary Students Reveal about Incorporating Representations into their Problem Scenarios?**

In general, from the analysis of the five student groups’ problem scenarios over the five-day intervention, students at times showed promise in their ability to incorporate representations and provide explicit and meaningful roles for the representations within their problem scenarios. However, there were often instances where students either struggled or failed to provide purposes for equations, graphs, tables, and pictorial representations. More specifically, I will answer this research question by providing evidence for three claims:

1. Students had difficulties with incorporating even a single representation into their problem scenarios.

2. Students who did incorporate a single representation in their posed problem scenarios had difficulty making the purpose explicit and the representation *mathematically meaningful* to the problem scenario.

3. Students who incorporated multiple representations had difficulty making the purposes of all the representations *explicit*, making all the representations *mathematically meaningful*, and explicitly *mathematically connecting* them to one another.

Each of these three claims focuses on none, one, and multiple representations, respectively. The reason for doing so is because the problem scenarios students created contained either none, one or multiple representations and each of these three problem scenario categories displays the students’ difficulties represented by the
claim. (Again, note that when I refer to a representation in this study, I am referring to any visual display of a mathematical relationship, including graphs, tables, equations, and pictorial representations).

In discussing each claim, I provide evidence for sub-claims from my categorization of the problem scenarios based on the aforementioned Representations Analytic Framework. My presentation of the claims and sub-claims is hierarchically organized to show the progression of student groups’ abilities to incorporate representations during the five-day intervention. Justifications for this hierarchy are provided throughout this results section. Where the order of presentation of information is not in accordance with the sequence of the hierarchy of claims, explanations are also provided to justify such occurrences.

**Difficulties with Incorporating Even a Single Representation into Their Problem Scenarios**

As stated above, the first claim is that some students had difficulties incorporating even a single representation into their problem scenarios. This claim emerged from my observations that groups often incorporated no representations into their problem scenarios. This claim is based on two sub-claims. The first sub-claim is that students had difficulties incorporating a representation in a problem scenario because they did not provide evidence of representations whatsoever. The second sub-claim is that students had difficulties incorporating representations in a problem scenario, because they used a representation only in the solution, not in the problem scenario. Student groups’ posed problem scenarios, solution strategies, frequency counts, and explanations of daily requirements (as expressed through daily criteria) will be used as evidence for these sub-claims.
No Evidence of Representations Whatsoever

At the lowest level, some student groups’ problem scenarios were devoid of representations. This was in spite of the following supports provided during the intervention to encourage them to use representations. On Day 1, I held a discussion about what a representation was, as well as the different kinds of representations (i.e., graphs, tables, equations, diagrams) that illustrate mathematical relationships. Additionally, every day during the intervention, student groups were asked to incorporate at least one representation into their problem scenarios, either by having them provide the representation in the scenario or having them ask for the solver to create it from the scenario. In spite of these supports, 10 of the 29 scenarios created (34% of all scenarios) contained no representations (see Table 4 for frequency counts for use of representations). Of particular note, Group 4 failed to include or mention the creation of any representations in their problem scenarios whatsoever.

Table 4 Frequency Counts for Problem Scenarios Based on Representations

<table>
<thead>
<tr>
<th>Number of Representations</th>
<th>Format: Given or to be created</th>
<th>Frequency</th>
<th>Explicit Purpose: Yes/No</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>NA</td>
<td>10</td>
<td>NA</td>
</tr>
<tr>
<td>One</td>
<td>Given by poser</td>
<td>6</td>
<td>0/6</td>
</tr>
<tr>
<td></td>
<td>To be created by the solver</td>
<td>7</td>
<td>5/2</td>
</tr>
<tr>
<td></td>
<td>Mathematically meaningful</td>
<td>7</td>
<td>NA</td>
</tr>
<tr>
<td></td>
<td>Representation in problem solution rather than in problem scenario</td>
<td>4</td>
<td>NA</td>
</tr>
<tr>
<td>Multiple</td>
<td>Given by poser and to be created by solver</td>
<td>3</td>
<td>2/1</td>
</tr>
<tr>
<td></td>
<td>To be created by solver</td>
<td>3</td>
<td>2/1</td>
</tr>
<tr>
<td></td>
<td>Mathematically connected</td>
<td>5</td>
<td>NA</td>
</tr>
</tbody>
</table>
To illustrate problem scenarios that contained no representations, consider two problems from Group 4. On Day 2, Group 4 created a problem scenario that incorporated a lasagna coupon from the ShopRite advertisement, shown in Figure 2 below.

Figure 2  Group 4, Day 2 problem scenario.

On Day 3, groups again used the ShopRite advertisement to create problem scenarios. This time, Group 4 chose to focus their problem scenario on the purchase of DiGiorno pizza. Group 4 asked the solver to find out the total price for two pizzas (see Figure 3).

Figure 3  Group 4, Day 3 problem scenario.
Note that neither of the problems provided a representation for the solver to use, nor did it ask the solver to create a representation. Thus, it appears that these students may have had difficulty incorporating a representation in any way into their scenarios. An alternative explanation is that the students lacked the motivation to include a representation into their scenarios. However, the students included meaningful and realistic purposes in both problem scenarios. The effort needed to choose at least one item from the ShopRite advertisement (i.e., pizza or lasagna) and to incorporate real-world prices to find the total cost also illustrates at least some degree of student motivation for posing problems. Another alternative explanation is that students lacked the amount of time to practice incorporating representations into the problems presented above. Yet, the groups already had two days to practice posing problem scenarios.

A third alternative explanation is that the ShopRite advertisement hindered students’ abilities to incorporate a representation into their problem scenarios. However, students also failed to incorporate representations when asked to use the Wendy’s menu. For example, on Day 4, the group chose to write their scenario involving one of the fast-food sandwiches from the Wendy’s menu, the “Dave’s Single” (see Figure 4). However, as seen in the figure, they did not include a representation.

Figure 4   Group 4, Day 4 problem scenario.
Group 3 also used the Wendy’s menu on Day 4 to create a problem scenario without incorporating a representation. As shown in Figure 5, Group 3 created a scenario involving another sandwich, the “Baconator.” Yet again, no representation was provided or referred to in the problem scenario.

![Figure 5](image)

**Figure 5** Group 3, Day 4 problem scenario.

These four problem scenarios stand in contrast to problems some groups created that did include one or more representations. For example, consider the problem scenario Group 2 posed on Day 4 (see Figure 6). Notice that Group 2 asked for two kinds of representations (i.e., a graph and an equation). Recall that I counted asking for a representation was one way to incorporate a representation into one’s problem scenario.
This problem scenario is evidence that if students are given criteria that specifically asks them to incorporate representations into their problem scenarios, some beginning problem posers will be able to do so. However, the fact that other groups did not have any representation in their scenarios, as shown in Figures 2 through 5, suggests that some beginning problem posers may have difficulties with incorporating any evidence of a representation. I interpreted this group of problem scenarios as the lowest level on the hierarchy because they provided no evidence of a representation.

Next, I will discuss the second sub-claim. This sub-claim deals with a category of problems that is one level above the previous category because students at least made an attempt to incorporate a representation somewhere into their work.
Evidence of Representations Only in the Solution

At the next lowest level, some groups did use representations when finding the solutions to their problem scenarios, even though they still did not incorporate a representation into their problem scenarios. In particular, Groups 1 and 3 created problem scenarios with no representations provided or asked for, yet included representations when solving at least one of their problem scenarios. Note that during the intervention, I did not explicitly ask groups to solve their problem scenarios using representations. Thus, this was a spontaneous action on the part of these groups.

An example of a problem scenario that contained no representations but that was solved by the group with a representation was created by Group 3 (see Figure 7).

![Figure 7](image)

Figure 7   Group 3, Day 2 problem scenario.

In this example, Group 3 worked from a “2 for $5” deal from the ShopRite advertisement for boxes of crackers to show a relationship between cost and the quantity of items bought. In this problem scenario, the students did not provide, nor did they ask for the creation of a representation to illustrate the relationship between the two variables or to solve the problem. Yet, Group 3 used a representation to solve the problem, as shown in Figure 8 below.
Figure 8  Group 3 solution to Figure 7 scenario using a visual representation.

Here, Group 3 used a visual representation, that is, a series of rectangles, to depict the relationship between the number of items bought and the total cost at each stage of the iteration. Darkened squares represent the number of cracker boxes bought, and the total number of squares in each iteration represents the total cost in dollars. Figure 7 shows the problem posers’ ability to iterate the visual up to 10 boxes to find the answer of $25. However, the problem itself did not provide or ask the solver to create or use any representations. By not including or asking for the creation of a representation, these students may have had difficulty distinguishing between using a representation to solve a problem and including a representation in the problem scenario.

Similar to Group 3, Group 1 created at least one problem scenario in which they used a representation to solve the problem without providing or asking for a
representation in the problem scenario. Figure 9 below shows one of these problem scenarios.

\[ y = mx + b \]

Shoprite sells frozen fruit for $6.99. Danielle bought some frozen fruit for her friends. If she bought frozen fruit and had $22 from the start and ended up with $1, how many boxes of frozen fruit did she buy?

Figure 9  Group 1, Day 1 problem scenario #3.

Here, Group 1 incorporated the rate of change concept, introducing $6.99 as the unit rate at which frozen fruit is sold. Interestingly, in contrast to the Group 3 problem scenario, note that Group 1 included the generic formula for slope-intercept form, “\(y = mx + b\),” above their problem scenario. Yet, the problem scenario did not refer to the equation. Therefore, I did not interpret the equation as sufficient evidence to count as a representation in the problem scenario. Nonetheless, Group 1 included a symbol-based representation (i.e., an equation) when it presented the solution to their scenario, shown as Figure 10.
Here, like Group 3, Group 1 was able to develop a plausible equation and use it to solve for the correct answer of 3 fruit boxes. Therefore, they demonstrated the ability to use a representation to find the solution, even though they did not provide or ask for one in their problem scenario.

In these two examples, Groups 1 and 3 have shown how to meaningfully incorporate at least one representation into their solution. However, they failed to either provide or ask for the creation of a representation for the solver, even after receiving explicit, daily criteria directing them to do so. This suggests that even when students use representations to solve problems, they may experience difficulties with incorporating representations into their problem scenarios. However, solving problems with representations seems a level above posing problems with no representations whatsoever. This is because the latter examples at least showed that students understood something about representations and the roles they could play in problems.

Next, I will discuss claims that emerged from problems that incorporated a single representation into their problem scenarios. I interpreted such scenarios as a
level higher than the scenarios that either did not include any representations or only included a representation into the solution. As I will show, there were three sub-claims about ways that single representations were incorporated into problem scenarios.

**Difficulties When a Single Representation is Incorporated into a Problem Scenario**

When students did incorporate a representation into a problem scenario, my analysis suggested three additional difficulties that may occur for beginning problem posers. First, students may have difficulty with making a representation mathematically meaningful to the problem scenario. Second, students may have difficulty making the purpose for a representation explicit. Third, even when students successfully show how to incorporate graphs and tables in mathematically meaningful and purposefully-explicit ways, they may still have difficulty with doing so for equations.

**Evidence of a Representation that is Not Mathematically Meaningful**

As stated in the daily criteria, student groups were asked to either provide a representation in their problem scenario or to ask for the creation of at least one representation by the solver. The groups were also to be explicitly clear what the purpose of the representation was to serve in the scenario. Furthermore, the representation was to be *mathematically meaningful* to the problem scenario. Recall that I defined a mathematically meaningful scenario as one that illustrates a mathematical relationship aligned to the problem context. However, one interesting finding is that for single-representation problems when student groups *provided* one representation (as opposed to asking for its creation), the representation was almost *never* mathematically meaningful to the scenario context. Six of the 29 problem
scenarios incorporated one given representation into the problem (see Table 4). For two of these six situations, representations (i.e., graphs) were simply provided, but no problem scenario was written. However, for the four problem scenarios where a representation was provided by the posers and a problem scenario was written, three of the four problem scenarios were incomplete. That is, each one of the three scenarios simply lacked the written information necessary to serve as a plausible problem. This lack of information prevented the problem from being mathematically meaningful to the problem scenario.

To illustrate how student groups provided a representation that was not mathematically meaningful, consider the following two scenarios. On Day 2, Group 5 provided a scenario involving the price of a Snickers (a type of candy bar) and “something else” (see Figure 11). On Day 4, Group 1 created a scenario involving the Wendy’s menu focused on the number of sales for the Baconator sandwich in comparison to another fast-food sandwich not included in the Wendy’s menu, the Big-Mac (see Figure 12).

![Figure 11](image1.png)

Figure 11  Group 5, Day 2 problem scenario.
Figure 12  Group 1, Day 4 problem scenario #2.

Note that in addition to the two groups having failed to complete each of their problem scenarios, each scenario lacked information necessary to support the mathematical meaningfulness of the representation. From Figure 11, the equation “c = 2.50x + 1.25” is given, thereby serving to potentially represent a relationship between two variables. However, the mathematical meaningfulness of the equation to the problem scenario is problematic with the fact that the group did not identify the unknown item that costs $2.50, referring to it as “something else.” Furthermore, the group never clarified what the x and c variables represent with respect to the cost of the candy bar and the cost of the unknown item.

Similar to the Group 5 scenario in Figure 11, the non-existence of important contextual and mathematical detail by Group 1 in Figure 12 prevented their representation, in this case, a graph, from being mathematically meaningful to the problem scenario. As shown in Figure 12, the graph reflected the number of sales for the Baconator sandwich. However, the graph lacked a variable label on the x-axis and
numerical scale values on both the x- and y-axes, which prevented the graph from being meaningfully meaningful.

The scenarios shown above illustrate my claim that students may have difficulty with providing a mathematically-meaningful representation into their problem scenarios. One possible explanation for the lack of mathematical meaningfulness for their representations may be that the amount of practice students had with incorporating representations within their posed problem scenario was why these groups were not able to provide mathematically meaningful representation into their problem scenarios. However, I did not deem this possible explanation as compelling because one of the groups created such a problem scenario later on in the intervention after having had multiple opportunities to practice posing problem scenarios (e.g., Group 1 created their scenario on Day 4).

An alternative explanation is that students failed to make their provided representation meaningful because they wrote incomplete problem scenarios. While I agree that this explanation may be a contributing reason to the representation not being mathematically meaningful, there still existed in my data set some complete single-representation problem scenarios whose representations were not mathematically meaningful. To illustrate this, consider the problem scenario created by Group 5. In the problem shown in Figure 13, the group incorporated two types of sandwiches from the Wendy’s menu, that is, the Baconator sandwich and the Homestyle Sandwich.
Figure 13  Group 5, Day 4 problem scenario.

This problem scenario by Group 5 is different than the previous two in that the group did provide a written goal for solving the problem or showing a mathematical relationship, thereby making the problem scenario complete. At the beginning, as shown in Figure 13, the group provided a question, “Why is the ‘Baconator’ more expensive from the ‘Homestyle chicken sandwich’?” Here, the question gave the solver a sense of direction as to what the problem scenario was to address, namely the prices of two sandwiches. Toward the end of the problem scenario, the group included an even more explicit purpose, to “Figure out how much the home-style is, and what the price difference is.” Thus, the solver can understand that he or she is to find out not
only the price of the Homestyle sandwich, but also the difference in its price compared to that of the Baconator sandwich.

In spite of Group 5 being able to provide an explicitly written goal for the problem, it lacked information that would have made the bar graph *mathematically meaningful* to the problem scenario. For example, the number of orders (on the y-axis) and the total accumulated cost for a certain number of Baconator sandwich orders (written within each bar) both appear to represent the height of the bars, which is not mathematically meaningful. Perhaps the most notable issue is the non-existence of information pertaining to the Homestyle sandwich. Without this information, a student would not be able to find the difference between the prices of the two sandwiches. Thus, even though this problem scenario was complete, the lack of information within the graph prevented it from being meaningfully meaningful to the problem. Therefore, because neither of the aforementioned alternative explanations seem plausible in this case, I concluded that the skill of providing a representation that is mathematically meaningful is a difficult one for beginning problem posers.

To show a contrast to the prior three problem scenarios in Figures 1, 12, and 13, the only single-representation problem scenario where a representation was given and was able to be used meaningfully was by Group 1 on Day 5. In this scenario (see Figure 14), Group 1 graphed the daily purchase of a Baconator sandwich by one individual named John. Meanwhile, his friend Daniel buys a 10-piece McNugget (fried chicken pieces) and a large soft drink.
My interpretation of the graphs provided in this problem scenario was that they were mathematically meaningful for illustrating the mathematical relationships between the variables and for solving the problem. In addition to variable names and
labels, the group included consistent numerical scales for both variables that depicted relationships between time vs. total cost. Also, Group 1 showed that Daniel’s graph started at a total cost slightly higher than the starting point of John’s graph. This is because the price for the combined 10-piece McNugget and soft drink costs more than the Baconator sandwich. In order to answer the question, “When would they spend the same amount of money?” the larger cost of Daniel’s daily meal was represented by a steeper line than that of John’s graph, allowing for the lines to intersect and for a solution to be found. Thus, the graphs were mathematically meaningful to the scenario.

Overall, on the problem-posing hierarchy, the problem scenario in Figure 14 showed a higher degree of problem-posing ability than those from Figures 11 through 13, because the graphs in Figure 14 were mathematically meaningful. Moreover, according to my interpretation, all of these examples (Figures 11-14) would be at a level above the examples where students did not incorporate a representation into their problem scenarios at all (Figures 2-10). Next, I will present the second sub-claim about when student groups ask for one representation in their problem scenarios.

**Evidence of a Representation whose Purpose is Not Explicit**

When student groups asked for a single representation to be created in their problem scenarios, sufficient mathematical information was given for the representation to be mathematically meaningful if the representation was an *equation*. However, the purpose of the equation was never made explicit. I view these scenarios with equations as a level above the previous ones (Figures 11-13) where the representation was provided, but the mathematical information about the representation was insufficient. This is because even though the purpose of the single-
representation scenarios with equations was never made explicit, the equation was always mathematically meaningful.

Of all 29 problem scenarios, 2 of the 29 (see Table 4) fit this category. Specifically, these scenarios consisted of the posers asking the solver to create an equation, but without providing an explicit purpose for its creation. Group 5 created these two scenarios (see Figures 15 and 16).

![Figure 15](image1)

**Figure 15**  Group 5, Day 1, problem scenario #1.

![Figure 16](image2)

**Figure 16**  Group 5, Day 1, problem scenario #2.
These problem scenarios were similar in a number of ways. First, the overall structure of these problems was essentially the same. In particular, both problems presented information pertinent to the problem, asked for the creation of an equation, and then asked for a particular question. Second, both scenarios involved the solver finding the total cost for purchasing a certain number of items, whether “5 bags of artichokes” or “26 pizzas.” Third, the group included sufficient information in each scenario to create an equation, such as the unit rate (i.e., $3 per bag of artichokes and $2 per pizza) and in the case of the second problem, the y-intercept or starting point (i.e., the sales tax is $0.88).

Unlike many of the single-representation problems where the group did not ask a question, the equations in these scenarios were mathematically meaningful because (a) the group did ask a question, either “How much would it cost if you bought 5 bags of artichokes?” or “How much would it cost if you bought 26 pizzas?” and (b) both examples include enough information for the solver to use the equation to answer the question in each scenario.

Nonetheless, neither scenario provided an explicit purpose for asking the solver to create the equation. This is noteworthy because the daily criteria asked students to provide an explicit purpose for any type of incorporated representation. Hence, my claim in this section is that just because secondary students are provided daily criteria for including a purpose for representations, one cannot assume that students will not have difficulty doing so.

Next, I will discuss another set of problem scenarios where the purpose for a given representation is made explicit. My interpretation is that this new set of scenarios is one level above the previous ones because the posers explicitly ask for a
representation to be created. This new set revealed another difficulty students may experience.

Evidence of a Mathematically-Meaningful Representation with an Explicit Purpose Only When the Representation Is a Graph or a Table

This sub-claim emerged from my observation that single-representation problem scenarios that had an explicit purpose always asked for the creation of graphs and tables rather than equations. In particular, 5 of the 29 scenarios were single-representation scenarios whose purposes for the asked-for representation were explicitly given (see Table 4 for frequency counts). Additionally, 4 of these 5 scenarios also asked for representations that were mathematically meaningful to the problem scenario. However, surprisingly, none of these scenarios involved equations.

Two of the problem scenarios from Days 1 and 2 created by Group 2 are shown below in Figures 17 and 18 respectively. Numerical information and product names from the ShopRite advertisement were incorporated into both problem scenarios. In each of the scenarios, the group asked for the solver to create a graph.

The price of Vanilla ice cream increased 25¢ per month, from October to January. The price is now $1.99. Make a graph to show the increase of price.

Figure 17 Group 2, Day 1 problem scenario.
Note that for each scenario, Group 2 was able to provide an explicit purpose for the asked-for graph. In Figure 17, the group asked for a graph “to show the increase of price” for the ice cream over time. In Figure 18, the group asked for a graph to “show how much you’re saving” with the “buy one, get one half off and the third one free” deal for Herr’s food snacks.

In addition, not only were the purposes for the representations made explicit in both scenarios, both contained sufficient information for the creation of a graph that is mathematically meaningful. For the ice cream problem (Figure 17), two important pieces of information mentioned are the price for ice cream ($1.99) and the time months “October to January.” The solver was also told that January was the current month, and the price of vanilla ice cream “increased by 25 cents per month.” Thus, he or she would be able to repeatedly subtract $0.25 from $1.99 to obtain the prices for December, November, and October to “show the increase in price.” In Figure 19, Group 2 shows that a mathematically meaningful graph can be created from the information provided in this problem scenario.
Group 2 did not provide a solution for the Herr’s snacks problem (see Figure 18). Nonetheless, sufficient information was provided that would have allowed for the creation of a mathematically meaningful graph. Specifically, the group provided the original snack price of $1.99, stated the discounts for buying more than one, and stated that more than one snack item would be purchased. This information would allow for the solver to sketch a coordinate plane showing two graphs for the accumulated price (y-axis) and the number of snacks (x-axis).

Similar to the problem scenario in Figure 17, note that the group did not ask for the creation of an equation to represent the mathematics in the problem scenario. The use of equations to mathematically represent the scenario would have taken much more mathematical sophistication than with the process of creating a graph to illustrate the relationship between two variables. This is because the application of the different discounts after each successive purchase of a snack would result in the creation of a
piece-wise relationship, which is much easier to display on a graph and more difficult to represent in a multi-part equation.

Tables were also asked for by one group, and the purpose of the table was made explicit. In particular, Group 3 created a problem scenario involving the cost of bagel bites by providing the cost for two packages of bagel bites ($11), and then by asking for the total cost for ten boxes (see Figure 20). Then, the group provided an explicit purpose for the table, that is, “to solve the problem.”

Figure 20  Group 3, Day 1 problem scenario.

This scenario did have the necessary price for 2 packages of bagel bites ($11). Such information could have been easily used to create a table to depict the relationship between the number of bagel bite packages purchased and the total cost, which then could have been used to find the price of 10 boxes ($55). Thus, the table fulfilled the explicitly given purpose to “solve the problem above.” However, note that the problem scenario did not require the use or creation of an equation by the solver.

In summary, this evidence shows that groups provided explicit purposes when graphs and tables were asked to be created, but not when equations were asked to be created. I view these scenarios involving graphs and tables as being at a higher level
on the problem-posing hierarchy compared to the scenarios involving equations because the scenarios involving graphs and tables had an explicit purpose. My data suggests it may be more difficult for students who ask for the creation of an equation in their problem scenario to reach that level.

I will now discuss in what ways student groups attempted to incorporate more than one representation into their problem scenarios. Even though the incorporation of multiple representations into their problem scenarios is an advance in problem posing, I will show that there were still issues with providing mathematically meaningful representations with explicit purposes, as well as with a new issue of relating representations to each another.

Difficulties When Multiple Representations are Incorporated into a Problem Scenario

When students did incorporate multiple representations into a problem scenario, my analysis suggested three new difficulties that may occur for beginning problem posers. Specifically, students who incorporate multiple representations, may have difficulty (a) making the representations mathematically-meaningful, (b) making the representations mathematically-connected, and (c) giving each representation an explicit purpose. Recall that I define two representations as being mathematically connected when the posers are explicit about how two representations relate to one another, even if they do not show the same relationship. When I refer to two representations being explicitly mathematically connected, I mean that the posers include enough language in the problem scenario that alludes to this mathematical connection. However, even when groups were able to include representations that are mathematically connected, I will show that not every representation served an explicit
purpose, not every representation was mathematically meaningful, and not every representation was explicitly mathematically connected.

I will discuss this claim in the form of three sub-claims. First, I will discuss the sub-claim that students may have difficulties with making multiple representations in their problem scenarios explicitly mathematically connected. Next, I will discuss the second sub-claim that even when students can provide explicitly mathematically-connected representations, students may have difficulties with making each representation mathematically-meaningful and having an explicit purpose. Finally, I will discuss the last sub-claim that, although a rare occurrence in my study, it is possible for beginning problem posers to incorporate mathematically-connected and mathematically-meaningful representations, all with explicit purposes.

Evidence of Incorporating Two Representations That are Not Explicitly Mathematically Connected

One category of multiple-representation problem scenarios consisted of those where no explicit mathematical connection existed between any of the representations in the problem scenario. On Days 1 to 3 of the intervention, the criteria stated for student groups to incorporate at least one representation into their problem scenarios. Then, on Days 3 to 5, the criteria stated that groups were to include two representations into their problem scenarios. In addition, students were told to either “have [the] solver create a representation from another given representation” or to “have the solver use two given representations of the same data.”

In spite of these criteria directions, only 6 of the 29 problem scenarios contained more than one representation. Of these 6 multiple-representation scenarios, 2 did not contain any representations that had both the characteristics of being
mathematically meaningful and having an explicit purpose. In this sub-section, I will discuss these two problem scenarios created by Group 1. In particular, I will focus on the mathematical connectedness of the representations (or lack thereof) within each scenario.

On Day 3, Group 1 created a multiple-representations scenario involving the ShopRite advertisement for soda and chips (see Figure 21). Here, the group asked for two equations, one for Doritos and one for Coke.

![Figure 21](image)

Group 1, Day 3 problem scenario #2.

What stood out in this problem from other multiple-representation problem scenarios is the absence of an explicit connection between the two equations. The equations essentially served to represent different mathematical relationships, either the quantity of chip bags vs cost or the quantity of soda bottles versus cost. Yet, even though both of these equations represented the total cost of an item, the posers did not make an effort to explain how such equations were mathematically related to one another. I interpreted this situation as a possible difficulty students may have with making mathematical connections between the representations in their scenarios.
In addition to the lack of explicit mathematical connections between the two equations, an explicit purpose was missing as well. However, I considered the two equations being asked for as mathematically meaningful. My criteria for deciding whether any representations were mathematically meaningful was if sufficient information was included in the problem scenario for the solver to create a representation. In this case, there was sufficient information (i.e., unit rates) to display a bivariate relationship, either the number of Dorito chip bags versus cost or soda bottles purchased versus cost.

As shown, Group 1 failed to create a scenario with explicit connections between its representations. However, this was not always the case. Group 1 also created a problem scenario involving the ShopRite advertisement involving the cost of soda (see Figure 22). This scenario was different than the previous one in that there was one representation, a table, being given, and another one, an equation, being asked for as opposed to two of the same kind of representation being asked for.

The scenario in Figure 22 had one advantage over the one in Figure 21 in that the group did make an explicit connection between the two representations in the former problem. This is apparent as the group asked the question, “What would be a good equation for this table?” Such language suggested that an equation was to be created that displayed the same mathematical relationship that existed in the table. Because the table and equation are mathematically connected, I therefore considered the scenario in Figure 22 as on a higher level of the problem-posing hierarchy than the two-equation problem scenario.
Figure 22  Group 1, Day 2 problem scenario.

Despite the differences between Figures 21 and 22 being either explicitly or not explicitly mathematically connected, the two scenarios are similar in that they both contain mathematically-meaningful representations, yet without explicit purposes. Note that the Table in Figure 22 is mathematically meaningful because it displays the relationship between cost and the number of packs of soda. The corresponding equation asked to be created could also be mathematically meaningful as it could display this same relationship. Also similar to Figure 21, Figure 22 did not provide a verbally explicit purpose for the incorporation of either the table or the equation for answering the questions “What would be the total price of 11 sodas?” and “If you were to buy 10 sodas, how much would the 10th soda cost?”

These findings provide evidence that students at times may have difficulties with making explicit mathematical connections between their representations in their
problem scenarios. An alternative explanation for why some students did not include explicit connections between the representations was because they were not aware they were to do so. However, beginning on Day 3, the group was told to either “have [the] solver create a representation from another given representation” or to “have the solver use two given representations of the same data.” Based on this evidence, just because a group is able to include more than one representation in a problem scenario, this does not imply that students will not experience difficulty with explicitly expressing the mathematical connections between them.

In addition to these two problem scenarios provided by Group 1, groups sometimes created multiple-representation problem scenarios where exactly one representation was mathematically meaningful and had an explicit purpose. I consider such problem scenarios on a higher level than the previously discussed multiple-representation scenarios due to the fact that those student groups were able to successfully follow the problem-posing criteria by providing an explicit purpose for one representation as opposed to none.

**Evidence for Incorporating Multiple Representations where Only One Representation is Mathematically Meaningful and Has an Explicit Purpose**

A second category of multiple-representation problem scenarios consisted of those where only one representation was both mathematically meaningful and had an explicit purpose and the other did not. Over the five-day intervention, several groups created problem scenarios of this type. I will discuss problem scenarios posed by Groups 2 and 3 to illustrate the difficulty of creating a two-representation problem scenario with each representation having both characteristics.
On Day 3, recall that groups were asked to incorporate two representations into their problem scenarios. Groups had the option to either provide the representations or to ask for their creation by the solver. They also were asked to provide purposes for how each one was to explicitly pertain to the problem scenario.

By Day 3, Group 3 was able to create a multiple-representation problem scenario involving the cost of a bag of chips, namely the Doritos brand ($2.50). The group provided an equation “\(y = 2.50x + 0\)” and from the equation showed a mathematical understanding of $2.50 as the rate of change and 0 as the starting point of no initial price. Then, the group explicitly wrote a purpose for a representation by asking the solver to “make a table showing how much it would cost for 10 bags of chips.” (see Figure 23).

Note that in this problem scenario, not only is the purpose for the creation of the table made explicit. The table is also mathematically meaningful, as enough information, such as the number of bags of chips (10) and the unit rate ($2.50) can be used to display the relationship between total cost and the number of chip bags purchased. Furthermore, the posers state “use the table to make the graph.” The creation of a graph would also be mathematically meaningful for displaying the relationship between these two variables. However, after finding the cost of 10 bags of chips from using the table, no explicit purpose is provided for the creation of the graph.
This problem scenario illustrates that the incorporation of more than one mathematically meaningful representation with an explicit purpose pertaining to the problem scenario is not a trivial task for beginning problem posers. An alternative explanation for why Group 3 was not able to incorporate a second representation that was both meaningful and whose purpose was explicit could be that the group did not have sufficient time to practice doing so. However, Group 3 had already created
problem scenarios on Days 1 and 2 that were mathematically meaningful and had an explicit purpose. Having two more days to incorporate at least one more mathematically meaningful representation was still not sufficient for the group to incorporate a second representation that was both mathematically meaningful and whose purpose was explicit into their problem scenario.

Group 2 also provided a problem scenario incorporating multiple representations. However, in contrast to the Group 3 problem scenario, Group 2 did not provide but asked for both representations to be created. Nonetheless, one representation being asked for was mathematically meaningful with an explicit purpose and the other did not have an explicit purpose.

Here, Group 2 created a problem scenario involving the sale of juice (see Figure 24). The scenario involved the purchase of a certain number of juice containers for a certain amount of money, that is, “2 Dole pineapple juices for $15”, “6 pineapple juices for $15”, and “6 apple juices for $5.94.” Even though some mathematical issues existed with regard to unit rates, the numbers of sales, etc., there is sufficient information for the creation of a mathematically meaningful table to illustrate the difference of prices for the sales of at least two of the three juices. Furthermore, the problem scenario provided an explicit purpose for the table, that is, to “show the difference for both sales.”
Figure 24  Group 2, Day 3 problem scenario.

Despite asking for a mathematically meaningful table with an explicit purpose in the scenario, Group 2 failed to provide an explicit purpose for the second representation. This occurred when the group then asked the solver to “Find the slope and y-intercept” and then “make a graph.” Yet, the group did not mention any reason for its creation pertaining to the problem scenario.

Thus, in these two problem scenarios, Groups 2 and 3 incorporated a second representation, but appeared to have difficulty with providing a valid purpose for the second representation in their problem scenarios. Again, this is in spite of groups receiving daily criteria informing them to create at least one representation in their problem scenarios on Days 2 through 3 and to create two representations on Days 4 and 5, all with the stipulation of providing an explicit purpose for each incorporated representation. Hence, just because student groups are able to incorporate multiple representations into their problem scenarios, this does not imply that each representation will be mathematically meaningful and have an explicit purpose, even after having received daily criteria directing them to do so. I will now discuss the only scenario where a group was able to create a scenario where more than one representation was mathematically meaningful and had an explicit purpose. This will represent the highest level in the problem-posing hierarchy.
Evidence of a Multiple-Representation Scenario with More Than One Representation Being Mathematically Meaningful and with an Explicit Purpose

So far, we have seen many different ways in which groups have created problem scenarios with incomplete understandings of how to include representations. Now, I will show a multiple-representation scenario where more than one representation was incorporated and all representations were both mathematically meaningful and had an explicit purpose.

By Day 4, Group 2 was also able to incorporate more than one representation into its problem scenario involving the Wendy’s menu (see Figure 25). In this situation, the group compared the cost of the meals of two individuals, Jason and Susan. Jason purchased a Baconator sandwich meal every day from Monday to Friday, and Susan correspondingly purchased a Spicy Chicken sandwich meal over the same time period. Group 2 then provided an explicit purpose for the creation of two graphs in order to “show how much they would spend every week.” These graphs would be mathematically meaningful because they would have been able to display the bivariate relationship represented in the problem regarding cost (how much each person spent) and time (in a week). Furthermore, enough information existed in the problem (i.e., cost of sandwiches as unit rates) to sketch the graphs. In addition, Group 2 sketched the graphs to illustrate the trend of how much is spent in one week as part of the solution (see Figure 25).
Figure 25 Group 2, Day 4 problem scenario with problem solutions. (Figure 5 represented with graphical and algebraic solutions to chapter to facilitate the presentation of the claim.)

What makes this problem even more impressive is that Group 2 included not just one, but two additional representations that were both mathematically meaningful and had explicit purposes. After asking the solver to sketch the graphs, Group 2 then asked the solver to “Use equations from the graphs to find who spends more on the sixth day.” Even though the wording “create an equation” is not used, it is implied in the phrase “Use equations from the graphs.” The purpose of the equations is explicit in the phrase “find[ing] who spends more on the sixth day.” Furthermore, the equations
are mathematically meaningful due to their connectedness to the graphs which illustrate the same bivariate relationship.

Note that this scenario is the only one of the 29 problem scenarios that contained more than one representation having both the characteristics of being mathematically meaningful and having an explicit purpose. Thus, this problem scenario stands hierarchically above the previously discussed multiple-representation scenarios, in addition to all of the other scenarios involving or lacking representations. This is surprising because all five groups had five total days to create problem scenarios that contained at least one representation and received daily criteria directing them to include a certain number of representations every day, as well as to make it explicit how such representations were to be used.

In summary, these findings regarding multiple-representation problem scenarios further emphasize the notion that the students in the intervention had difficulties with incorporating more than one representation into their problem scenarios. One cannot assume that just by providing daily directives for students to incorporate multiple representations in explicit and meaningful ways that students will do so. Yet, because Group 2 was able to create a multiple-representation problem scenario with explicitly written statements asking for the creation and meaningful use of graphs and equations (see Figure 25), this is proof of concept that adherence to the criterion is doable for beginning problem posers.

In this chapter, I have provided evidence that students may have difficulties with incorporating both single and multiple representations into their problem scenarios in ways that are purposefully explicit, mathematically meaningful, and explicitly mathematically connected. In the next chapter, I will discuss how students
may also have difficulties with incorporating mathematics concepts, namely the rate of change and the y-intercept, in productive ways.
Chapter 5

RESULTS FOR RESEARCH QUESTION 2

In this chapter, I will discuss the ways in which students incorporated mathematics concepts essential to linear functions (i.e., the rate of change and the y-intercept) into their posed problem scenarios.

Research Question #2: Based on Students’ Productive and Unproductive Uses of the Rate of Change and the Y-Intercept, What Difficulties Do Secondary Students Reveal about Incorporating those Concepts into their Problem Scenarios?

To address this research question, I analyzed how student groups incorporated two mathematical concepts associated with linear functions into their problem scenarios: rate of change and the y-intercept. This analysis showed that students incorporated these concepts in both a productive and an unproductive manner.

As mentioned in the methods section, when I discuss using the rate of change or y-intercept in a productive manner, I am referring to the use of the concept in ways that are mathematically accurate, non-trivial, useful for accomplishing the desired task in the problem scenario, and connected to the problem context in ways that sufficiently promotes a mathematical idea or fosters connections between mathematical values and ideas. In contrast, when I discuss using these mathematical concepts in an unproductive manner, I mean that the concept is used in ways that are not mathematically accurate, trivial, not useful for accomplishing the task within the problem scenario, and/or are not connected to the problem context in ways that either insufficiently promotes a mathematical idea or prevents connections between mathematical values and ideas in the scenario.
Similar to the three claims presented in the last chapter about single-and multiple-representation problem scenarios, three claims will be discussed in this chapter as well. The first claim focuses on no use of a rate of change or students’ *unproductive* uses of a single rate of change. The second claim highlights students’ *productive* uses of one rate of change, in addition to the use of either an implicit or explicit y-intercept. The final claim focuses on how students use two rates of change in both *unproductive* and *productive* ways.

The three specific claims that will be addressed are the following:

1. Students had difficulties with incorporating a rate of change into a problem scenario in a *productive* manner.

2. Students who did incorporate a rate of change in a productive manner had difficulties with incorporating the rate of change concept into a problem scenario productively to find the independent variable, as well as had difficulties with incorporating a non-zero y-intercept in a productive manner.

3. Students who did incorporate one rate of change into a problem scenario in a productive manner had difficulties with using two distinct rates of change in a productive manner.

Similar to the first research question, I will also provide evidence for sub-claims to better illustrate nuanced aspects of each of the three primary claims. Student groups’ posed problem scenarios, frequency counts, and explanations of daily requirements as expressed through criteria will be used as evidence for the sub-claims.

**Difficulties with Incorporating a Rate of Change into a Problem Scenario in a Productive Manner**

As previously stated, the first claim is that students had difficulties with including a rate of change in a linear function problem scenario in a productive manner. This claim is based on four sub-claims. First, students may have difficulties
with incorporating a rate of change in a problem scenario. Second, students may have
difficulties with including sufficient context so that the rate of change is useful. Third,
students may have difficulties making a problem scenario in which the rate of change
is constant. Fourth, students may have difficulties with making a problem scenario that
asks the solver to find the rate of change. The first sub-claim involves no rate of
change. Sub-claims two through four fit into my understanding of using the rate of
change in an unproductive manner.

**No Evidence of a Rate of Change in a Problem Scenario**

At the most basic unproductive level, student groups at times did not
incorporate a rate of change into their linear function problem scenarios. This was in
spite of the students being given the ShopRite advertisement (and later the Wendy’s
menu) containing rate of change values and daily criteria explicitly asking them to
incorporate the rate of change concept into their problem scenarios. Moreover, on
Days 2 and 3, groups received 30-minute instruction that highlighted the mathematical
concepts of the rate of change and the y-intercept.

Two problem scenarios in particular were devoid of any kind of information
about a rate of change, whether in the form of numerical value, in a table, in a graph,
or even in the solution to a problem. The first problem scenario, as shown in Figure
26, was created by Group 4 on Day 2.
The second such type of problem scenario was created on Day 4. On that day, groups also received 30-minute instruction involving the understanding of a rate of change, but also involving the addition and subtraction of functions, both algebraically and graphically. Similar to Days 2 and 3, students were provided with explicit criteria asking them to incorporate information from a cultural artifact (this time, the Wendy’s menu and/or the ShopRite ad) into their problem scenarios. In addition, they were asked to incorporate the idea of adding and subtracting functions, along with the rate of change and y-intercept, into their scenarios. On that day, Group 4 created the following scenario (see Figure 27):

It is clearly evident that no rate of change exists in either of the problem scenarios. In Figure 26, the group asked for the cost of three lasagnas with the use of a coupon. However, neither the original price of a Stouffer’s lasagna, nor the price using
the coupon is provided. This makes it impossible to know the rate of change. Similarly, in Figure 27, the group asked, “How much could $20 buy on the Wendy’s menu?” However, without any prices as rates included in the problem scenario, no rates of change could be determined, let alone accomplishing the task of finding the maximum amount that can be purchased. From these unproductive problem scenarios, I interpreted that Group 4 had difficulties with incorporating a rate of change into their scenarios.

Some alternative explanations, besides that these students were experiencing difficulties with incorporating a rate of change into their problem scenarios, were entertained. First, perhaps Group 4 did not realize early on in the intervention they were supposed to include a rate of change into their problem scenarios. However, Day 2 was already the second time the group was given criteria asking them to include information from the ShopRite advertisement containing a myriad of product unit rates. In addition, they were explicitly asked on Day 2 to include rates. Furthermore, by Day 4, they still created a problem without a rate.

Second, perhaps groups were not successful at including rates of change because it was the first time they were being asked to do so. However, from Days 2 through 4, the group was given opportunities to practice incorporating rates of change into its scenarios. This suggests that is not trivial for students to include rates of change in problem scenarios.

**Evidence of a Rate of Change in a Problem Scenario without Sufficient Context**

At a somewhat more advanced, but still unproductive level, some groups did include a rate of change value in their problem scenarios, but without incorporating sufficient contextual information to make the rate of change useful. First, consider one
of the five problem scenarios created by Group 4. On Day 1, Group 4 created the following mathematical sentence (see Figure 28):

![Figure 28](image)

Figure 28  Group 4, Day 1 problem scenario.

Note that although the 10 from “10x” and the 5 from “5x” could be rates of change, no contextual information is provided for the solver to understand what numbers or variables represent what quantities in the problem scenario.3 Because the equation is not directly connected to any type of contextual elements from the advertisement, one cannot assume that the coefficients 10 and 5 actually serve as rates of change, such as the price of items. On the one hand, it is possible that the students intended x to represent the number of items purchased, 5 and 10 to represent the unit prices of two different items, and the statement 10x+5x to represent the cost of purchasing the same number of each item. Also, it is possible they intended x to represent the unit rate and 5 and 10 to each represent the quantity of the same item.

On the other hand, it is possible the students intended an item to have two different prices, which would not make sense, unless they meant a “buy one, get one half-off” type of problem scenario, with 10 serving as the original price and 5 serving as the halved price. However, without the contextual information, it is not possible to

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3 In the problem in Figure 28, other mathematical issues exist regarding this equation (e.g., arithmetic mistakes). However, for the purposes of this study, I have ignored the discussion of other mathematical issues not directly pertaining to rates or y-intercepts.
determine if these numbers or variables reflect any of these cases. Such a problem scenario provides evidence that students may have difficulty with including sufficient contextual information in their problem scenarios. Only by adding such information could the rate of change be used productively.

Group 5 also created a scenario with insufficient contextual information. Such a lack of information made it difficult to understand the relationship between the mathematical values in the problem. As shown in Figure 29, Group 5 created the following scenario on Day 2:

\[
\begin{align*}
\text{Start with 1 \text{ Snickers}} \\
\text{Buy second case for } 2.50 \\
\text{C = 2.50x + 1.25}
\end{align*}
\]

Figure 29 Group 5 Day 2 problem scenario (Figure 11 re-presented from the previous chapter to facilitate the presentation of the claim).

Unlike the Group 4 problem scenario in Figure 28, some context was provided in Figure 29 to show a connection between elements of the ShopRite advertisement. For example, Group 5 gave the price for Snickers as $1.25 and included it as the y-intercept in the equation. This makes this scenario stand apart from Figure 28 where no context was provided whatsoever. Also, unlike Group 4’s scenario, Group 5 does appear to include an explicit rate of change, namely the $2.50.
In spite of the inclusion of this rate of change, two issues exist with this problem scenario. For one, the rate of change of $2.50 is not in reference to any particular item from the ShopRite advertisement. As the reference to the $2.50 is “something else,” the rate of change is being used in a less contextually meaningful (and unproductive) manner than if this price was connected to an actual item from the ShopRite advertisement. Furthermore, as previously mentioned, this problem scenario was incomplete because there was no specific task involving the rate of change which the solver was asked to accomplish. Thus, even if an explicit reference of a food item was made to the rate of change of $2.50, the rate could not be used to solve a problem because no problem existed.

These findings suggest that students may have difficulties with incorporating sufficient contextual information about the rate of change into their problem scenarios. An alternative explanation for why Groups 4 and 5 failed to do so, besides that they were having difficulties with doing so, is that, as proposed for the previous claim, this was their first time they included rates of change into their problem scenarios. However, on Day 1, Group 5 was able to complete two problem scenarios where each one incorporated the rate of change in connection with the price of an actual item from the ShopRite advertisement. Furthermore, as the scenario was created on Day 2, they already had two consecutive days where they were asked to incorporate information from the ShopRite advertisement into their problem scenarios. Therefore, my interpretation was that including sufficient contextual information about a rate of change so that it can be productively used for solving a problem is non-trivial.

Next, I will discuss the third sub-claim. Not only may students have difficulties with incorporating a rate of change and with including sufficient context into a
completed problem scenario so the rate of change is useful, they also may have difficulties making a problem scenario in which the rate of change is constant.

Evidence of a Rate of Change in a Problem Scenario that is Not Constant

As previously shown, student groups did not always use the rate of change in productive ways. However, there also existed problem scenarios where a single rate of change was included and was explicitly connected to the context of the cultural artifact. Yet, the rate of change was used unproductively because it was used in a mathematically inaccurate way. That is, the rate of change was not constant (i.e., the rate of change did not represent a linear relationship between two variables). Despite the instruction students received about linear relationships and constant rates of change on Days 2 through 5 of the intervention, groups at times failed to include a constant rate of change.

As described in research question 1, Group 5 created a problem scenario on Day 4 involving the cost of two sandwich meals from the Wendy’s menu: the Baconator sandwich meal and the Homestyle sandwich meal (see Figure 30). The goal of the problem scenario was to find the price of the Homestyle sandwich when given the price of the Baconator meal. The bar graph displays the order number on the vertical axis and the price of a number of Baconator meals within each bar.
From this scenario, note the mathematical issues that exist that reflect a lack of understanding about rates of change for linear functions. First, an issue exists with the price of the Baconator sandwich meal. Based on the graph, when the order number increases by 1, the price per order for the Baconator meal doubles, thereby increasing the rate of change rather than keeping it constant at $8.59. In other words, the price increases exponentially from one order number to the next, rather than linearly. This exponential relationship is also reflected by the key underneath the bar graph that states, “Price per order for Baconator x 2 each time.”

Second, while the price increases exponentially, the visual representation looks linear in nature. Specifically, the relative heights of the bars on the graph appear to
increase at a constant rate. Thus, the prices written on each bar that represent the price of the Baconator meal do not correspond to the bar heights.

This exponential increase in the price of a Baconator meal from order to order is clearly non-realistic, non-linear, and thereby unproductive for illustrating how these two variables represent a linear relationship. This non-linear relationship was surprising. Not only did the Wendy’s menu not contain prices that depicted exponential relationships between two variables, the students only received instruction on linear functions from Days 2 through 5, and received no instruction based on non-linear relationships during the intervention.

These findings suggest that students may have difficulties with including a constant rate of change into a linear functions problem scenario. One possible explanation for the inclusion of a non-linear relationship may be that because the group used a bar graph, this inhibited them from accurately representing a constant rate of change in their problem scenario (i.e., the issues represented in the bar graph may have been more due to the representation type than students’ understanding of linear rates of change). However, other groups also created problem scenarios containing non-linear mathematical relationships that did not involve bar graphs.

For example, Group 2 created a problem scenario involving the purchase of Herr’s snacks (see Figure 31) where the customer used the following deal: “buy one, get one half off, and the third one free.”
Figure 31  Group 2, Day 2 problem scenario #2 (Figure 18 re-presented from the previous chapter to facilitate the presentation of the claim).

This problem scenario asked for the creation of a representation to depict the relationship and to find out how much is saved. However, if a graph or table is to be created from the “buy one, get one half off, and the third one free” situation reflecting price versus quantity purchased, such a representation would display a non-linear relationship (i.e., a piecewise function). This is contradicted when the posers also asked the solver to find the slope, which implies a constant rate of change. By asking for a singular rate of change for a mathematical relationship depicted by a piecewise function, the students indicate they may have had difficulties with including a rate of change that is constant into a linear function problem.

Group 1 showed a similar struggle when they incorporated a deal on sodas in the following problem scenario in Figure 32. Here, five packs of sodas can be bought for $10, as opposed to purchasing a single pack of soda for $3.69.
Figure 32  Group 1, Day 2 problem scenario (Figure 22 re-presented from the previous chapter to facilitate the presentation of the claim).

This problem scenario is somewhat more sophisticated than those created by Groups 2 and 5. For one, a table is provided so that the solver can clearly see the mathematical relationship between the number of packs and the total cost. Also, additional questions are asked regarding total cost and the unit cost for a soda.

Nonetheless, similar to the previous two scenarios, this scenario too reflects a non-linear relationship between two variables. Toward the end of the scenario, Group 1 asked the solver, “what would be a good equation for this table.” I suspect that this group of posers did not realize that a multi-part equation with each part representing a different part of the domain of this function would be necessary. Thus, by asking for a single equation for the table, this illustrated an unproductive use of the rate of change concept.
In summary, analysis of the data suggests that it is non-trivial for students to incorporate a single, constant rate of change into their problem scenarios by writing their scenarios in ways that make it possible for the solver to use the rate of change in a productive manner, that is, to accomplish tasks that correspond to a linear relationship. Otherwise, if the intent of the problem scenario is to reflect a non-linear relationship, the mathematical items which they ask the solver to provide should be worded in ways that avoid confusion with features of linear functions.

**No Evidence of Asking for a Single Unit Rate in a Problem Scenario**

The fourth sub-claim regarding students’ unproductive incorporation of the rate of change concept is with regard to students requiring an understanding of how to ask for the unit rate. From the 29 problem scenarios, none of the scenarios asked the solver to use the independent and dependent variable to find the rate of change. Eleven of the 29 involved a productive use of a single rate of change. However, the creation of a problem asking for the dependent variable (such as the total cost or amount saved) was the most prevalent kind of productive, single rate of change problem created during the intervention. In fact, this occurred in 10 of these 11 scenarios. Examples of such problem scenarios are the following created by Groups 3 and 4 (see Figures 33 and 34 respectively):

![Figure 33](https://example.com/figure33.png)

**Figure 33** Group 3, Day 3 problem scenario (Figure 7 re-presented from the previous chapter to facilitate the presentation of the claim).
In Figure 33, Group 3 provided a rate of change (i.e., 2-for-5 deal) and the independent variable (i.e., 10 cracker boxes) when asking for the total cost. Similarly, in Figure 34, Group 4 provided a rate of change (i.e., $4.39 Dave’s single sandwich) and the independent variable of time (i.e., one month) to find out the total amount spent for one month. Thus, the rate of change is used in each scenario to productively fulfill the task of finding the total cost (dependent variable). Again, 10 out of 11 of the productive, single rate of change scenarios were of this type. However, none of the 11 scenarios asked for the solver to find a single unit rate.

This evidence suggests that students may have difficulties with how to ask for the solver to find the rate of change in their problem scenarios. An alternative explanation for why students did not ask the solver to find the rate of change in a single rate of change problem scenario may be that the students did not have enough experience with how to find the rate of change from a bivariate data set. However, during the lessons on Days 2 and 3, students used an inductive method of exploring a bivariate relationship by creating visual patterns, creating tables from such patterns, creating graphs from the tables, and finally creating equations. In spite of students having opportunities to find the rate of change in the lessons, students failed to ask for the solver to find the rate of change in any of their problem scenarios involving a
single rate of change. This evidence suggests that students may have difficulties with structuring a problem scenario in a way that requires the solver to find the rate of change.

From the previous discussion, it has been shown that there are various ways in which students have *unproductively* incorporated a single rate of change into their scenarios. Next, I will discuss two sub-claims that emerged about student groups that incorporated a single rate of change, and in some cases, the concept of the y-intercept, in a *productive* manner.

**Difficulties When the Rate of Change is Productively Incorporated into a Problem Scenario**

Even when students incorporate the rate of change concept productively, some experienced difficulty with asking the solver to find the independent variable, as well as with incorporating a non-zero y-intercept in a productive manner. As stated above, this claim emerged from analysis of problem scenarios that productively incorporated rates of change. I will begin by discussing and providing evidence for the first sub-claim that students may have difficulties with incorporating the rate of change productively into their problem scenarios for finding the value of the independent variable. Then, I will discuss the second sub-claim about students’ difficulties with incorporating a contextual, non-zero y-intercept productively. Explanation of student groups’ posed problem scenarios, frequency counts, daily lesson content, and explanations of daily requirements (as expressed through daily criteria) will be used as evidence for these sub-claims.
Evidence of Only One Problem Scenario Asking for the Independent Variable

In general, groups at times were successful in creating problem scenarios involving a single rate of change in a productive manner. From the 29 total problem scenarios, 11 of them involved a productive use of a single rate of change. However, all but one of these scenarios involved the solver finding the dependent variable, that is, a monetary value involving the total cost, price, or amount saved, rather than the corresponding independent variable (number of items). I interpreted the overrepresentation of problem scenarios involving a single rate of change that asked solvers to find amounts of the dependent variable (10 of 11) and the underrepresentation of problem scenarios that asked solvers to find amounts of the independent variable (1 of 11), as indication that posing the latter kind of problem scenarios would be non-trivial for beginning problem posers.

The only example of a problem scenario that incorporated a single rate of change in a productive manner and that asked the solver to find a quantity of the independent variable was a problem scenario created by Group 1. In this scenario in Figure 35, the posers stated that the price of a single box of frozen fruit was $6.99 (i.e., the rate of change) and the amount spent on fruit was $21 (i.e., an amount of the dependent variable). Then, they asked for the total amount of boxes of frozen fruit that were purchased (i.e., the corresponding amount of the independent variable). Note that, as stated above, this problem scenario was the only productive single rate of change scenario asking for the solver to find the independent variable (i.e., number of items) when given the rate of change and the corresponding dependent variable (i.e., total cost or total amount spent).
It is understandable that students productively created problem scenarios involving finding the total cost or amount spent (i.e., amounts of the dependent variable) because such problem scenarios were consistent with the daily lesson scenarios that were geared primarily toward finding amounts of the dependent variable. However, students were also given instruction on Day 1 to not create problem scenarios that were slight contextual modifications of those provided, such as in daily lessons. Again, Group 1 was the only group to create a problem scenario that asked for a solution that was not the dependent variable. Therefore, the results suggest students may have difficulties with creating problem scenarios requiring the solver to find the independent variable. Next, I will discuss students’ difficulties with incorporating the initial value, or y-intercept of a linear function into their problem scenarios.

**Evidence of a Problem Scenario Lacking a Productive Non-Zero Y-Intercept**

Evidence also suggested that students had difficulties with incorporating a non-zero y-intercept in a productive manner. This was in spite of students having been asked to incorporate the y-intercept concept on all five days of the intervention.
Students also received daily lessons where both zero and non-zero contextual y-intercepts could be incorporated into problem contexts. Of the 12 scenarios in which a single rate of change was productively used, there were three categories of problem scenarios related to the y-intercept: 7 problem scenarios did not explicitly address the y-intercept, 2 explicitly included a y-intercept of zero, and only 3 explicitly included a non-zero y-intercept.

In the first category of problem scenarios, 7 out of 12 problem scenarios where a single rate of change was incorporated in a productive manner, excluded the explicit mentioning of a y-intercept, or starting point, for a productive purpose. Five of these 7 scenarios (i.e., Figures 3, 4, 5, 7, and 20) illustrated that often, groups asked for the dependent variable by providing the rate of change (e.g., a unit rate or the price for two items) and the independent variable, that is, the number of items (or the time, which translates into the number of items), without any acknowledgement of or reference to a y-intercept. One example is shown below (see Figure 36):

![Figure 36](image)

Figure 36  Group 3, Day 4 problem scenario (Figure 5 re-presented from the previous chapter to facilitate the presentation of the claim).

In this problem scenario and in the other four, the y-intercept would have been zero. Ignoring the y-intercept in the problem scenario would not affect a solver’s ability to obtain a solution. However, recall that students learned about y-intercepts
during lessons on Days 2 through 5 and were repeatedly encouraged to address a $y$-intercept in their problem scenarios. This suggested to me that students may have had difficulties with incorporating the $y$-intercept concept in their problem scenarios.

In the second category of problem scenarios, there were 2 cases in which students did, to some degree, reference a $y$-intercept that was zero. For example, on Day 1, Group 5 created a problem scenario involving the sale of artichokes. As shown in Figure 37, the problem informed the solver that one bag of artichokes costs $\$3$, and two bags cost $\$6$. Then, the solver is asked to find the equation for the situation if $P$ is the price and $c$ is the number of artichoke bags. In this example, even though the $y$-intercept $b$ in the equation would have been 0, the students referenced it in their equation.

![Figure 37](image)

**Figure 37**  Group 5, day 1 problem scenario #1 (Figure 15 re-presented from the previous chapter to facilitate the presentation of the claim).

The second case occurred on Day 3. On this day, Group 3 created a problem scenario with an explicit $y$-intercept of zero. This is shown in Figure 38:
Figure 38  Group 3, Day 3 problem scenario (Figure 23 re-presented from the previous chapter to facilitate the presentation of the claim).

For the two problem scenarios in Figures 37 and 38, the inclusion of the zero y-intercept is productive because it is mathematically accurate. If someone is, for example, buying only chips and begins with no bags of chips, then it makes sense for the initial cost, or y-intercept, to be zero. The explicit reference to the y-intercept suggests that students reveal their awareness of this important feature of linear functions. Despite this explicit reference, there may have been aspects of the y-intercept concept that were still causing some difficulties for Group 3. One reason was
because in Figure 38, Group 3 states that the y-intercept is zero because “that’s how many you start with.” Based on this language, I was not certain whether the posers meant that the number of chip bags was the reason for the total cost beginning at zero, or if they meant that the number of chip bags being zero was the y-intercept. This evidence suggests that students may have difficulty with the contextual quantities that can and cannot serve as the y-intercept.

In the third category of problem scenarios, only 3 existed that included a non-zero y-intercept. However, some of the problem scenarios still used the y-intercept concept in an unproductive manner. Again, this was in spite of their receiving lessons involving the y-intercept concept on Days 2 through 5 of the intervention.

One example from this category occurred on Day 4, when Group 1 created a problem scenario involving the sales for the Baconator sandwich, with time (i.e., years) as the independent variable (see Figure 39). In this scenario, the graph depicts the relationship between the years and the number of sales. From the graph, a non-zero y-intercept is shown along the y-axis, in addition to a constant rate of change. Here, the y-intercept served a productive purpose by representing the starting point for a linear function about a real-world situation of sales versus time.
Even though a y-intercept served a productive purpose, two unproductive aspects existed regarding how the y-intercept was to be used. First, the posers asked the solver to answer the question, “what equation would fit this graph?” However, the graph lacked the precision for determining what the coordinates of the y-intercept actually are. This would prevent the solver from determining the precise value of the y-intercept, because the solver must incorporate this value into a linear equation.

Another unproductive issue for how the y-intercept was used by Group 1 is that the y-intercept is not required for solving an aspect of the problem scenario. In particular, the posers ask in #2, “by mid 2020, how many sales would the Baconator have?” Because the graph is already given, the solver can simply look at it to determine what number of sales (in millions) matches up with the time between 2020 and 2021.
Therefore, as the y-intercept is not required for solving this problem, it is not being used in a productive manner.

A second example from this category of problem scenarios, where a non-zero y-intercept is included in both productive and unproductive ways, was one created by Group 5. In this scenario, the posers ask for the cost of 26 pizzas when given the rate of change of $2 per pizza (see Figure 40). However, unlike the artichoke problem scenario the group created on the same day (see Figure 37), here, the group included a y-intercept, namely a sales tax of $0.88.

This problem scenario differs from the Baconator sandwich sales scenario in Figure 39. In Figure 40, the solver has a precise y-intercept value (i.e., $0.88) to use for creating the linear equation. Additionally, in the solution of this problem (see Figure 41), the equation can be productively used to find the total cost of 26 pizzas via substitution. Furthermore, the value of $0.88, as it is non-zero, would actually be used in the calculation of the total cost. Therefore, in one sense, the y-intercept is being used in a productive manner. However, in another sense, the y-intercept is not productive in that sales tax is used in a non-normative way. Specifically, the posers
identify $0.88 as a fixed quantity, even though, as the number of pizzas purchased changes, so also should the sales tax. I interpreted this incorrect use of the sales tax as an indication of students having difficulty with incorporating the y-intercept concept into their problem scenarios.

Figure 41  Solution to Group 5, Day 1 problem scenario #2 seen in Figure 40.

In sum, the underrepresentation of problem scenarios that explicitly and productively addressed the y-intercept concept suggests that students may struggle with productively including the y-intercept concept into a problem scenario. Additionally, students may also have difficulties with including contextual quantities that can serve as the y-intercept, so as to enable it to be mathematically accurate and contextually meaningful to the scenario. Next, I present findings from instances where students attempted to incorporate two rates of change into their problem scenarios.

Difficulties When Two Rates of Change are Incorporated into a Problem Scenario

On the last two days of the intervention (i.e., Days 4 and 5), students participated in lessons pertaining to the use of two rates of change. On Day 4, groups received instruction with regard to adding and subtracting linear functions. On Day 5, groups received instruction pertaining to intersecting linear functions. On both days,
groups were provided with criteria asking for each one to incorporate the rate of change, y-intercept, and the concept of adding and subtracting functions into their problems scenarios. However, on Day 5, they were also asked to include the concept of intersecting functions. A total of 8 out of 29 problem scenarios contained more than one rate of change. In many cases, this was done both productively and unproductively. In particular, analysis of these 8 problem scenarios suggested that when involving two distinct rates of change, students may have difficulties with (a) providing sufficient contextual information for understanding and relating the two rates to one another and other elements of the problem and with (b) incorporating two rates of change in mathematically non-trivial ways.

Evidence of a Problem Scenario with Two Rates of Change Lacking Sufficient Contextual Information for Relating Two Rates of Change to One Another

In this sub-section, I provide evidence that when students include two rates of change in their problem scenarios, they may have difficulties with including sufficient contextual information for relating the two rates to one another. This evidence comes from 2 of the 8 problem scenarios. The first problem scenario came on Day 5. Group 3 created a problem scenario in which two linear functions were represented on the same graph. Each line represented a different rate of change (see Figure 42). This group identified the price of two items, the Baconator sandwich ($6.39) and a soft drink ($1.59) (i.e., two rates of change, dollars per sandwich and dollars per drink, respectively). As seen in the figure, the group scaled the horizontal and vertical axes for the graph, but did not include labels indicating the quantities that the axes represented.
A productive aspect of this problem scenario is that Group 3 appeared to use the rates of change correctly to generate the two lines with slopes that appear to match the rates of change. However, an unproductive aspect is that there is little contextual information that ties together the two rates of change. Specifically, the solver is not provided with information about how these prices related to one another in a story situation, or what quantities are represented on each axis. It may be inferred from the price of the Baconator sandwich and the drinks that perhaps the x-axis represents the number of orders of each (or the number of days or weeks when an order is made) and the y-axis the total cost. However, the two variables are not clearly indicated on the graph. Additionally, no problem is provided to give clues about how the rates are related. Thus, the solver is left to make sense of how the two graphs, as well as the two values from the Wendy’s menu were to relate to one another for a productive purpose.

The second problem scenario that informed this finding also came on Day 5. Group 2 created a problem scenario also involving the Wendy’s menu (See Figure 43).
For this problem scenario, the group provided the price of a Baconator sandwich (i.e., $8.59) and a Dave’s Double sandwich (i.e., $7.59).

A productive aspect of this problem scenario is that two rates of change are provided. Another productive aspect is that the posers do state the problem to be solved, namely to “make a problem to show where the two prices intersect.” However, an unproductive aspect is that insufficient contextual information is provided that would enable the graphs representing the two rates would intersect. For example, if Kim also buys a shake on the first day in addition to the Dave’s Double, then she would initially have a greater total cost than Willard on Day 1. However, over time, her total accumulated cost will be less than Willard’s total accumulated cost, thereby allowing the graphs to intersect. The lack of such information in Figure 43 obscures any potential connection between the two rates.

Based on these two problem scenarios, I concluded that it may be challenging for beginning problem posers to include contextual information into two-rate problem scenarios to show how the rates are related. Two alternative explanations for why Groups 2 and 3 did not include such contextual information may be that the groups did not have enough time to complete their problem scenarios, or perhaps this was just the
first time they tried to incorporate two rates. However, groups had two days to practice creating two-rate problems (Days 4 and 5). Thus, it appears that students may need additional supports to include sufficient contextual information into their problem scenarios involving two rates of change that would enable the solver to ascertain how the rates of change relate to one another.

Evidence of a Problem Scenario with Two Mathematically-Trivial Rates of Change

In addition to incorporating appropriate contextual information, students had difficulties with making the rates in two-rate problem scenarios mathematically non-trivial. This sub-section emerged from analysis of three problem scenarios in which students did include sufficient context into their two-rate-of-change scenarios. Yet, the rates in these problem scenarios were mathematically trivial.

The first of these problem scenarios occurred on Day 1, when Group 1 created a problem scenario involving two ShopRite items. In Figure 45, the price of a Stouffer’s party lasagna (i.e., $13.99) and Hot Pockets (i.e., $9.99) were incorporated into the problem scenario. The task was to determine, when beginning with $15, the purchase of which item would then allow for an additional purchase of a drink and leftover change.
Figure 44  Group 1, Day 1 problem scenario #1.

Group 1 is to be commended for incorporating sufficient context to connect the two rates of change in their problem scenario (i.e., Ben buys lasagna or Hot Pockets). Also, in contrast to the problem scenarios presented in the previous category, this scenario contains an actual task to be solved, that is, to determine which item will enable more leftover money to make a second purchase. However, the rates of change are mathematically trivial. This is because only one lasagna meal or only one Hot Pocket (a type of turnover or sandwich) is to be purchased. Without a number greater than one representing the number of lasagnas or Hot Pockets purchased, the rate of change for each item, and the total cost of an amount of that item are the same amount. Therefore, by fixing the independent variable at one, the total cost never increases, which trivializes the use of the rate of change.

The second and third of these problem scenarios were created by Group 2. For both of these problems, there was something mathematically trivial. On Day 3, Group 2 created a problem scenario involving the price of different kinds of juices from the ShopRite advertisement (see Figure 45). In this scenario, the group indirectly provided

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4 Note that I am ignoring the additional issue that the problem posers neglected to include the price of a drink.
the rate change for purchasing pineapple juice, that is, “2 Dole pineapple juice[s] for $5,” and directly provided the rate of change for purchasing apple juice, that is, “each juice is $0.99.” The problem scenario asked solvers to find the slope and y-intercept and make a graph. On Day 4, Group 2 created another problem scenario, again involving two rates of change (see Figure 46). This time, the group incorporated two items from the Wendy’s menu. The group provided the rate of change for purchasing the Baconator sandwich ($8.59) and the Spicy Chicken sandwich ($6.99) in their scenario. This problem too asked solvers to find the slope and y-intercept, and then asked solvers to use the graphs to answer a question.

Figure 45 Group 2, Day 3 problem scenario (Figure 24 re-presented from the previous chapter to facilitate the presentation of the claim).

Group 2 can be commended for incorporating two rates of change into each problem scenario, and for asking additional questions in Figure 46 that would employ the rates of change productively, that is, for solving a problem. Nonetheless, one way the rates of change were not used in mathematically productive ways in either problem scenario was that the solver is asked to “Find the slope.” This problem statement is unproductive because the slopes are already given to the solver. Specifically, in Figure 45, the slope is given when solvers are told “Each juice is $0.99.” Similarly, in the
second problem, solvers were provided the slope in the form of the prices for the Baconator and the Spicy Chicken Sandwich. Thus, I interpreted asking solvers to find the slopes for each item as mathematically trivial and thus unproductive.

Figure 46  Group 2, Day 4 problem scenario (Figure 5 re-presented from the previous chapter to facilitate the presentation of the claim).

It should also be noted that in addition to the aforementioned trivial aspect of using two rates of change, another unproductive issue was that in both problem scenarios, the solver is asked to find a singular slope when two rates of change were represented (i.e., find the slope). From my analysis of this issue, I concluded that students may have difficulty with how to ask mathematically non-trivial questions about two rates of change. For example, when students provide the rate of change, they should not ask for the slope, which is a trivial question. Also, when they provide two rates of change, they should not ask for a single slope, because it is not certain to
which rate of change the poser is referring. In general, problem scenarios with two
rates of change may include the same difficulties as problem scenarios with one rate of
change, and can include additional difficulties with using them in productive ways.

In this chapter, I have provided evidence that students can often struggle with
incorporating mathematics concepts, such as the rate of change and the y-intercept, in
ways that are unproductive to the problem scenario. That is, the mathematical concept
is used in an inaccurate or non-trivial way, is not useful for accomplishing the task
within the problem scenario, and/or is not connected to the problem context. These
difficulties occur even when students are provided with daily criteria, lessons on such
mathematics concepts, and the use of cultural artifacts.

The findings from this chapter and the previous one examined the products of
group problem posing and have shown that there is more for students to understand
regarding the incorporation of mathematical concepts (Chapter 5), and also with
representations in ways that are explicitly purposeful, mathematically meaningful, and
explicitly mathematically connected (Chapter 4). In addition to the quality of the
products, I will now discuss the process of student interactions and engagement during
group problem posing.
Chapter 6
RESULTS FOR RESEARCH QUESTION 3

Research Question #3: What Does Engagement and Interactions Look Like for Beginner Group Problem Posers?

In the previous two results sections, I have provided a detailed analysis of the products of student groups’ posed problem scenarios with respect to their incorporation of representations and mathematics topics. In this section, I will now provide a detailed description of what the process, or the activity of collective problem posing looks like for beginning problem posers. The purpose is to paint a picture of how a small group of secondary students went about engaging in the activity of problem posing for the first time during the five-day intervention. In order to show what group problem posing looked like, I will describe what occurred based on three foci:

1. General characteristics of engagement for each individual student
2. Interactions between two students engaged in aspects of problem posing
3. Interactions between a student who is struggling to collaborate, her peers, and the teacher

These three foci were chosen to together provide a comprehensive picture of what elements of what the group dynamics of problem posing looks like. Characteristics of engagement (#1) can reveal what each student does, what roles they fulfill, how much and for how long each group member participated in a certain kind of activity, and what the progression of engagement looks like for the individuals and group over the five-day intervention. As two of the three students were the primary contributors to the activity, their interactions (#2) can provide insights as to whether
such interactions reflect an intellectually shared atmosphere of critiquing and building on one another’s ideas, or one where students merely cooperate and take turns creating and solving a problem scenario. Such interactions can also reveal what students need to know about the activity of group problem posing in order to successfully engage in the activity. Finally, the third student struggled with getting involved in the activity. Her interactions between group members and the teacher (#3) revealed challenges involved with how to engage and direct students in the general problem-posing process. In addition to these foci, teacher whole-group directives and student answers from the roles negotiation worksheets provide further evidence for what can be learned about student understandings and the potential challenges of enacting group problem posing in a secondary student classroom for the first time.

Characteristics of Engagement

In this sub-section, I will provide a description of how three students engaged in the activity of problem posing during the five-day intervention. The pseudonyms of the three students I videotaped over the five days and will be reporting on are Peter, Zeke, and Jocelyn. Again, these three students were grouped heterogeneously according to race/ethnicity, gender, mathematical performance, and mathematical disposition. For each day of the intervention, I will first explain what the instructor’s instructions were. Then, I will describe each students’ engagement, in turn, in terms of four characteristics: (a) what roles they identified with, (b) what roles they actually fulfilled, (c) what they did or did not accomplished in their roles, and (d) how long they participated in a particular aspect of problem posing, and eventually. Then, I will summarize each student’s overall progression of engagement over the entire five days.
Day 1

On Day 1, the Teacher discussed the questions “What is problem posing?” and “What is a representation?” with the class. Group members were given the opportunity to identify what roles they thought were to be parts of the problem-posing process and which ones they would like to fulfill. Then, they were given the criteria directing them to create one scenario that incorporated the rate of change and y-intercept topics, one representation, and the ShopRite advertisement flyer (More details about the intervention can be found in the Methods chapter).

Peter was the most engaged student of the three during Day 1. Entering the problem-posing activity, Peter identified himself as the group leader and spent approximately 7 minutes creating two problem scenarios. Not only did Peter work on his own problem; he also checked over a problem Zeke created. Additionally, he spent several minutes throughout the period looking through the ShopRite advertisement. Peter also somewhat facilitated group activity by giving his problem to Zeke to check over his work and by giving the roles worksheet to Jocelyn for her to answer part of it. Finally, he filled in a role re-negotiation worksheet.

Zeke was nearly as engaged as Peter but spent more time waiting for Peter to finish his work. Before the problem-posing activity, Zeke identified his role as the “solver.” Zeke worked at solving Peter’s problems each time Peter completed a problem. Although Zeke identified himself as only the “solver” at the beginning, he was engaged in other tasks as well. In particular, Zeke checked over Peter’s first and second problem scenarios for errors or inconsistencies before solving them. He also created and solved his own problem scenario.

In contrast to Peter and Zeke who were similarly engaged in the problem-posing activity from start to finish, Jocelyn showed little evidence of engagement on
Day 1. At the start of the activity, Jocelyn identified her role as the note-taker. Yet, she kept her head down for most of the period. Only in the last minutes of the period did she become engaged when answering questions on the role re-negotiation worksheet.

**Day 2**

On Day 2, students were taught a lesson involving tiled patios involving the rate of change and y-intercept, and that incorporated various pictorial, tabular, graphical, and symbolic representations for linear functions. Then, for the problem-posing portion of the class, they were given criteria directing them to create one scenario involving the rate of change and y-intercept, one ShopRite item, and at least one representation.

Even though Zeke continued to identify himself as the solver, he played a greater part in the problem-creation process on Day 2. In fact, on this day, Zeke was the only student to create a problem scenario. He spent approximately 11 minutes working on his scenario and about 1 minute solving it. Zeke was also the only group member to fully engage in creating, checking, solving, and discussing scenarios during the entire period. During the time Zeke worked on a problem scenario, he also perused the ShopRite flyer and used his calculator. In addition to giving his problem to Peter to solve, Zeke also appeared to solve his own scenario (“I’m gonna do it again on a separate sheet and then you try to solve it.”). Zeke also answered Peter’s clarifying questions, helped Peter answer the parts of the scenario about his problem and checked over Peter’s answers to his problem. Clearly, Zeke assumed a much larger role than the solver role.

Peter continued to identify himself as the leader. However, on Day 2, Peter did not assume a leadership role. Instead, he was disengaged during the first part five-and
a-half minutes, seeming somewhat nervous or fidgety (tapping water bottle, biting fingernails). In fact, Peter seemed to imply that Zeke was now taking the lead when he asked, “Are you solving it too?” and Zeke responded, “No, I am just…I am making the problem.” During the next few minutes, Peter did not talk to anyone or write anything down. At best, he may have been trying to think of a problem, since when asked by the Teacher to explain the goal of working together to make a problem, Peter answered, “I can’t think of any problem.” Even though Peter did not create a problem on this day, he did check and solve Zeke’s scenario as stated above. Specifically, once Zeke was done, he read Zeke’s scenario, he asked clarifying questions and he solved the scenario.

Again, in contrast to Zeke and Peter, Jocelyn continued to struggle with engagement. At the end of Day 1, Jocelyn had indicated that she wanted to assume the role of writer and solver of problem scenarios. However, on Day 2, she kept her head down during almost all of the period. She provided no evidence of looking at the ShopRite advertisement, reading Zeke’s scenario, writing anything, or interacting with Peter or Zeke.

**Day 3**

On Day 3, students were taught a lesson on taxi fares that involved the rate of change and y-intercept and that made connections between tabular, graphical, and symbolic representations of linear functions. Then, each group was provided with criteria for creating a scenario that incorporated the rate of change and y-intercept, one ShopRite item, and two representations. Students were also told to use an “I Notice, I Wonder” chart to help structure the brainstorming of ideas and the writing down of artifact information before creating a problem.
On Day 3, Zeke was the only student who appeared to be fully engaged in creating a problem for the entire period. Contrary to Day 2, Zeke worked almost the entire class and without collaboration. Overall, Zeke spent about 16 total minutes creating his scenario. Zeke alternated between writing his problem scenario, looking at the Shoprite advertisement, and consulting his calculator. The teacher dropped by the Group 3 times to discuss the task and to encourage the group to collaborate. However, Zeke continued to work independently for the entire period (he did have a side conversation with Janice for about half a minute). On the roles sheet, Zeke now identified his role as a creator, rather than merely a solver.

In contrast to both of her Day 1 and Day 2 experiences, Jocelyn did actively engage in the problem-creation process. This only occurred during about half the thirty-minute period. For the early part of the period, Jocelyn disengaged, having her head on the desk and only occasionally raising her pen. However, she then began looking through the ShopRite ad, chose an item (Doritos) and wrote down information onto her “I Notice, I Wonder” chart. After the Teacher came by, Jocelyn finally began working on creating a problem on the chart. After a short side conversation with Zeke, Jocelyn went back to creating her problem. By the end of the activity, Zeke wrote that Jocelyn was a “Thinker”, as she “chooses what to make the problem about.”

Peter was even less engaged on Day 3 than Day 2. This was surprising given his self-identification as the leader on Day 1. A main reason Peter was so disengaged was because he spent significant time away in the restroom. However, even when he was present, he appeared disengaged (Peter: “Are we doing…What are we doing?”). At one point, he spent about one minute writing something down. Yet, soon after this, Peter disengaged again, spending about two minutes on his phone. Perhaps the only
engagement Peter exhibited on Day 3 was creating a representation associated with Jocelyn’s created problem scenario. Specifically, Peter spent about two minutes creating a table that reflected the mathematical relationship between total cost and the number of items purchased for Jocelyn’s scenario. Yet afterwards, Peter re-disengaged until he spent one minute filling out some information on the roles sheet. By the end of the period, Zeke identified Peter on the roles sheet as the “solver.”

Day 4

On Day 4, students were taught a lesson that revisited the tiled patios scenario from day 2 (the rate of change and y-intercept). As usual, the lesson involved graphical and symbolic representations of linear functions, but also introduced the idea of adding and subtracting linear functions. Afterwards, students were given criteria directing them to create one problem that incorporated not only the rate of change and y-intercept and two representations, but also the topic of adding and subtracting functions. This time, students were allowed to use the Wendy’s menu as well as the ShopRite advertisement. At the end of the period, students analyzed and re-negotiated their roles.

At the end of Day 3, Zeke identified himself as a “creator.” During Day 4, Zeke did fulfill this role, being the only student who created any scenarios (one completed scenario with two parts and one incomplete scenario). Between such events as class discussion, discussion with group, and looking at his lesson notes, Zeke worked on scenarios for the entire period. This work involved Zeke writing down the problem, examining the Wendy’s menu, and/or working on the calculator. He then spent one minute completing the roles sheet.
On Day 4, Peter was far from enacting his prior roles of leader, creator, or solver. After the whole-class discussion, Peter had a side-conversation with Jocelyn, and spent about two-and-a-half minutes filling in his missed lesson notes and discussing them with Zeke. He did spend almost one minute helping his group identify an item from the Wendy’s menu for the problem scenario. However, by this point, Peter re-disengaged, and began using his phone, or sat at his table doing nothing. He also asked, “What time do we leave this class?” and looked through a group member’s sketchpad, packed his belongings early, initiated off-topic conversations with Zeke and then with Jocelyn, left the table twice, and remained out of his seat for the last five minutes of the period.

On this day, Jocelyn had little engagement in the activity, which stood in contrast to her active engagement in creation of a problem on the previous day. Much of her participation on Day 4 was mostly passive. Specifically, Jocelyn spent most of the period looking through the Wendy’s and Shoprite artifacts. Jocelyn, like Peter, did help identify what item from the Wendy’s menu to include in the problem scenario. She also read through Zeke’s problem, commenting on what she liked and what the group could contextually consider when creating the scenario. However, for the last eighteen minutes of the period, no evidence existed of her participation.

**Day 5**

On Day 5, students were taught a lesson involving the intersection of linear functions, and the lesson again made connections between incorporated graphical and symbolic representations. Afterwards, students were given criteria directing them to create at least one problem that involving the rate of change and y-intercept, adding and subtracting functions and/or the intersection of functions. Groups were again to
include two representations and an item from the Wendy’s menu and/or the ShopRite advertisement. Again, students analyzed and re-negotiated their roles.

Zeke continued his “creator” role on Day 5 and was actively engaged throughout the period. Specifically, he alternated between consulting the Wendy’s and Shoprite artifacts, writing his problem scenario and using his calculator. Something noteworthy is that, in contrast to the previous four days, another group member participated in the development of the problem scenario. In particular, Zeke asked Peter to create the graphs for the scenario, and for almost one minute, they discussed the creation of the graphs. Zeke and Peter also spent about one minute discussing the mathematics of the problem and checking Peter’s graphs and solution. After they created the problem, Zeke continued to appear as actively engaged for most of the final ten minutes, either working on the calculator, looking through the lesson packet, exclusively writing, or working on his problem using the Wendy’s menu and a calculator. By the end of the activity, Zeke identified himself as the “problem creator” as he did on Day 3.

In contrast to the previous two days, Peter appeared to be actively engaged for part of the period (approximately fourteen total minutes). This length of time was greater than the degree of engagement from Days 3 and 4. He was not in the classroom for the first seven minutes of class and looked around for about five-and-a-half minutes while Zeke was creating the problem scenario. Specifically, Peter was actively engaged during several discussions with about creating the graph. He was also involved in developing the problem scenario, as he spent about ten minutes using the calculator to generate values and plot those values to create graphs. Peter also paid attention and answered a question during a Teacher-led whole-class discussion.
However, after Peter did his part with creating the graphs useful for solving Zeke’s problem scenario, he was no longer engaged in creating a problem. By the end of Day 5, Peter was still identified as the “solver.”

Again, Jocelyn was completely disengaged on Day 5. She kept her head down for almost six minutes into the period, and raised her head to tell Zeke, “I don’t know whether I should do work because I did not eat and because I am kind of mad.” For approximately the final half-hour no evidence existed of Jocelyn being engaged in any aspect of writing, solving, checking, or reading a problem scenario or looking at an artifact.

**Five-Day Summary**

The overall summary of the three students’ engagement is first, that Zeke was the only student who was consistently actively engaged in the problem-posing process. On Day 1 and 2, he stated he would be the solver. However, he not only solved two of Peter’s problems, he created his own scenarios. On Day 3-5, he identified himself as the creator, and continued to create at least one scenario for each day of the intervention. This involved consulting the two artifacts, writing down his scenarios and using the calculator to work out the mathematics of the problems. In addition to the creator role, he also appeared to solve his own problems and check over Peter’s work.

Peter began the intervention actively engaged, having identified himself as the “leader,” and having created two of his own problem scenarios. During Day 2, Peter assumed the role of “solver,” as he listened to Zeke’s mathematical explanations of the problem and worked to solve Zeke’s problem. By Day 3 however, Peter was mostly disengaged. He did create a tabular representation of Jocelyn’s problem. However, he
spent the first and last part of that day out of his seat, and was inactive during much of the time at his table. This inactivity was even more evident during Day 4 and 5. On Day 4, his only productive activity was helping his group decide what Wendy’s item to use for the scenario. On Day 5, his only active participation was listening to the teacher during whole-group instruction and creating the graphs for Peter’s problem scenario.

Jocelyn was mostly disengaged throughout the five-day intervention, except for Day 3. On Days 1 and 2, Jocelyn did not actively participate in any aspect of problem creation or group or class discussions. On Day 3, Jocelyn did actively participate in a discussion with the teacher and group mates about the creation of a problem. She also successfully chose an item from the ShopRite advertisement and wrote down her own scenario on the “I Notice, I Wonder” chart. On Days 4 and 5, she mostly returned to disengagement. However, there were exceptional moments on Day 4 when she did look through the ShopRite and Wendy’s menu and identified what item she liked to use for a problem, as well as commented on Zeke’s problem scenario.

This analysis provides a picture of the kinds of engagement patterns, including the kinds of roles students might assume, that teachers might expect from the first sustained experiences students have to engage in problem posing. My descriptions of these five days of problem posing suggests that assuming active roles will be challenging for some students and that working collaboratively will be another challenge.

However, all three students did successfully pose at least one problem scenario. This suggests that problem-posing is an activity that is within students’ zone
of proximal development (Vygotsky, 1987) and may just take more time and additional supports to foster meaningful engagement. In the next section, I will zoom in on how Peter and Zeke interacted with one another, to highlight the ways in which beginning problem posers collaborate to create and solve problems.

Interactions between Two Engaged Students during Group Problem Posing

In this second section, I will zoom in on the interactions between Peter and Zeke. I chose to focus on these specific interactions because they offer insights into the kinds of interactions between beginning problem posers that teachers might expect.

Beginning Problem Posers May Interact by Taking Turns with Problem Creation and Problem Solving

Throughout the five-day intervention, the goal was for groups to create problems by sharing the intellectual work. This would have entailed group members providing and explaining their own ideas, as well as building and critiquing one another’s ideas for a given problem scenario. All three students did participate in creating at least one problem scenario. However, even though Jocelyn successfully created a problem on Day 3 and provided some insights on other days, the primary weight of creating problems was carried by Zeke and Peter. Yet, rather than sharing in the intellectual work of creating problems, Zeke and Peter instead enacted a turn-taking approach. This consisted of the students taking turns creating scenarios and solving/checking over each other’s created problems. This approach was reflected in Zeke and Peter’s interactions as the creator took sole responsibility for creating the problem while the solver waited, after which the solver asked clarification questions and the creator provided answers or mathematical explanations about the creator’s
problem. To illustrate this interaction pattern, I will present evidence from Day 1 and 2. On Day 1, Peter created the problem and Zeke asked questions about the problem and tried to solve the problem. On Day 2, the roles reversed with Zeke creating the problem and Peter asking the questions and solving the problem.

During problem posing on Day 1, Peter began the activity identifying himself as the “leader” and Zeke identified his role as the “solver.” As the leader, Peter created a problem by himself while Zeke waited. He then gave his completed problem to Zeke. Zeke then read Peter’s problem while Peter waited. The following conversation ensued:

Peter: Does that make sense?

Zeke: Um, I am confused about the other part, ‘Afford a drink and change.’

Peter: My fault. (Peter reaches over and erases something) …And change for his mom, he is going to give the change back to his mom.”

Zeke then set about to solve Peter’s problem while Peter waited. About two minutes later, Zeke finished solving Peter’s scenario and passed the scenario back to Peter, who checked over Zeke’s work.

Peter: The drinks should be 3 dollars or less.

Zeke: Oh yeah, 3.

Peter: Yeah. At least 3 dollars and change.

In these episodes on Day 1, the turn-taking approach to collaboration is evident. Peter took the first turn by playing the role of creator. This meant that he was entirely responsible for creating the problem, during which time Zeke was entirely passive and provided no input. Upon creating his problems and giving them to Zeke, there was some interaction as he answered Zeke’s clarification questions. Then, it was Zeke’s
time to take a turn and solve the problem. During Zeke’s turn, Peter was entirely passive and provided no input. Once Zeke had an answer, there again was some interaction as Peter checked Zeke’s answers to ensure the solvability of the problems. Note that except for the times in between taking turns, Peter and Zeke did not work at the same time on the scenario.

The turn-taking roles reversed on Day 2. Zeke became the primary problem creator, and he provided the mathematical explanations to Peter’s clarifying questions. In contrast, Peter became the problem reader, the question asker and the problem solver. As with Day 1, while Zeke was creating the problem, Peter waited passively. Then, Zeke passed the problem to Peter. While Peter took time to read and solve Zeke’s problem, Zeke waited passively. Then, they had the following conversation:

Peter: How many [sodas] are in a bag?

Zeke: Um, a six pack but (shows Peter the artifact) It’s just, it is the actual deal right here. Every five packs you get, and it will start making every pack cost two dollars. But if it is less or more, every extra less will cost $3.69.

In this excerpt, Peter asked a clarification question as he desired to know from Zeke’s created problem how many sodas were in one bag. Zeke clarified Peter’s questions by saying that were six sodas were in one pack rather than in one bag and referenced the deal when five packs are purchased. After this clarification, Zeke worked alone to solve his own problem on the calculator while Peter waited passively. Then, Zeke passed the problem back to Peter and the conversation continued:

Peter: I’m confused.

Zeke: All right, So, if you had like (both Zeke and Peter looking at ShopRite advertisement), if you had four, if you bought four kegs of soda, it wouldn’t cost that (pointing to ShopRite advertisement) times
four because you wouldn’t have five. So, it would cost that times four (pointing to the ShopRite ad). But if you had five packs,

Peter: Five sodas?

Zeke: Packs, packs. So, basically, (pauses) every pack costs that much until you reach five. So (shows Peter the calculator) five, five packs would be five times two dollars would be ten dollars. But, if it was like 1 less than five, so if you had 4 packs, it would be that, because the deal is you have to get at least five.

Peter: Okay.

In this excerpt, Peter expressed confusion as he apparently did not understand the five-pack for $2 deal and Zeke clarified the mathematics by explaining the situation of having less than five packs (four packs) and the associated cost and by re-explaining the price of purchasing four packs. The final part of the conversation was as follows:

Zeke: But if you had like 6, it would be that (pointing to the ShopRite ad) and then that extra 1 besides the, the extra 1 above 5 is how much that (pointing to ShopRite ad). So, five (typing into calculator) would cost 10, or no, 5 (typing into calculator) would cost 10. Five times 2 dollars.” (pointing to ShopRite ad) “And then, because you have a sixth one, you need to get another 5 for the deal to count again, so that one would just be (typing into calculator) 3 dollars and 70 cents.

Zeke: It’s hard to explain (shaking his head)

Peter: I add these two (pointing to the calculator) and then

Zeke: That would be six, that would be six, um, that’s the cost of six packs. Um, let me graph it first. (pressing buttons on calculator)

In this final part of the episode, Zeke continued to provide a detailed explanation of the mathematics for how to obtain the total cost of a number of packs that were more than five, namely six packs. Meanwhile, Peter played his role as solver as he attempted to enact Zeke’s explanation of adding the cost for five packs to the
cost of one pack, stating, “I add these two…and then”. Then, Zeke verified Peter’s final answer, stating “that’s the cost of six packs.”

This episode was somewhat different from the episode on Day 1 because Zeke was more active while Peter solved the problem, than Peter had been when Zeke solved his problem on Day 1. Even though the two students conversed about the mathematics and the solutions, Peter primarily asked questions and Zeke primarily provided explanations and answers. Most of the interaction was around clarification of the problem scenario. When Zeke was creating the problem, Peter was passive. When Peter was solving the problem, Zeke answered Peter’s questions, but Peter was still expected to solve the problem scenario. Thus, their approach to collaboration continued to be a turn-taking approach.

Note that the turn-taking approach occurred despite the group having three opportunities (Days 1, 3, and 4) to reanalyze their roles, in terms of what roles contributed to or hindered the members sharing in the intellectual work of problem posing. Additionally, the Teacher the entire class in a discussion at the beginning of Day 2 about the goal of collaboratively creating problem scenarios. Next, I will discuss how Peter and Zeke were able to reach the point of both participating in the creation of the same problem scenario. However, this occurred with little collaboration between them.

**Moving from Turn-Taking, Beginning Problem Posers may Collaborate on Creating a Problem Scenario, but in Limited Ways**

As mentioned, the Teacher encouraged group members to share in the intellectual work of problem posing. Furthermore, on Day 2, the Teacher stated to the class:
Teacher: Okay, so what I am trying to get you guys to do is to work together to make the problem. Okay? So, make sure you guys, as best you can, two people, maybe three people, teaming up, running ideas off of each other, trying to work and build on making the problem together. Okay? ...So, try to be engaged in working with each other, trying to make the problem.

However, as stated above, Peter and Zeke interacted primarily by turn taking, one person creating the problem and the other person solving the problem. It was not until Day 5 that the teacher was able to move Peter and Zeke’s interactions closer toward sharing the intellectual work of creating a problem scenario. On this day, Zeke began creating a problem scenario (See Figure 47 below). The Teacher noticed this and stated said, “Maybe if you get Peter to graph, graph this problem here. Since you did most of the work so far.” This suggestion seemed to help because what followed was Zeke and Peter engaging in closer collaboration.
First, Zeke gave his scenario and the Wendy’s menu to Peter and provided an explanation of the problem scenario):

Zeke: So, there’s a, basically for each problem, there’s an event that you have to spend 50 dollars in two weeks at Wendy’s. John gets the, goes the first day and gets a Baconator for six dollars, 40 cents. Dan goes the second day and gets a ten-piece Chicken and a large soft drink. And then they get the same thing for two weeks straight. And then you can graph that one.
Next, Peter read the problem. After reading the problem, Peter asked a clarification question, “You said two weeks?” to which Zeke replied, “Yeah, for two weeks, so fourteen days, but Daniel doesn’t buy anything the first day.” Two minutes later, Peter asked another clarification question, “It’s the Baconator and nothing else?” to which Zeke responded, “Ah yeah. The six-forty-one.” For the next nine minutes, Peter used his calculator to generate values to plot points for the graph (See graphical representations in Figure 13). During this time period, Zeke worked on his calculator, apparently to find the solution for the problem scenario. Peter then gave the problem to Zeke, who looked over Peter’s work and stated, “That’s right.”

This episode illustrates a modest improvement in Peter and Zeke’s interactions away from turn taking and toward more sharing of the intellectual work of creating the problem. With Zeke writing out the scenario and Peter providing a graph for the scenario, Peter and Zeke provided the first instance of sharing in the role of creator for the same problem scenario, rather than each student developing his or her own scenario or one student creating a scenario and the other student solving it.

While Peter and Zeke’s interactions were not entirely collaborative on Day 5, compared to Days 1 and 2, I interpreted their interactions on Day 5 as a step in the right direction. Nevertheless, the fact that Peter and Zeke did not move entirely toward sharing in the intellectual work of creating the problem scenario and only began to move toward greater collaboration on the last day of the intervention, suggests the students may have had a different conception of shared intellectual work from the kind of collaborative effort the Teacher described. This episode suggests that having secondary students working in a collaborative way to create problems is a nontrivial instructional goal that may require significant supports. In the final sub-section, I will
now describe Jocelyn’s interactions with Peter, Zeke, and the Teacher during group problem posing and what they can reveal regarding how beginning problem posers engage in such a group activity.

Interactions Involving a Student who Struggled to Participate

In this third sub-section, I will talk about the interactions of the third group member, Jocelyn. Jocelyn appeared to be the least motivated student in the group, as expressed by having her head down at some point every day of the intervention. Jocelyn also appeared to express a lack of faith in her own mathematical ability. Before the intervention, Jocelyn identified herself as “okay at math.” However, on Day 3, in response to her not successfully enacting her role as note-taker, she responded that she was “not that smart with math or with making math work.”

As described previously, Jocelyn participated in the problem-posing activity least of the three students, and was almost completely disengaged during Days 1, 2, and 5. Yet, evidence suggests that she still did desire to participate in problem posing, but did not fully understand how to do so. Also, on Days 3 and 4, she did show signs of participation and it seemed to be because the problem her group was working connected with her personal interest. Next, I describe instances that suggest Jocelyn did not understand how to participate as well as the episode on Day 4 when Joselyn did participate.

Beginning Problem Posers May Not Understand How to Participate in Problem Posing

At times, Jocelyn appeared to lack the understanding about how to participate in collective posing problems. This was evident because she sometimes questioned what she was supposed to do, she relied on her classmates to direct her actions and she
consulted the teacher about the role she should play. This was also evident when she asked the teacher about her roles.

**Asking group what to do.** Jocelyn often sought direction from her group members. At the end of Day 1, when Peter and Zeke were finished with creating each of their problems, Jocelyn asked, “Do you want me to answer the question?” to which Peter nodded. Similarly, on Day 3, after having chosen the ShopRite item for the group to create their problem (Doritos) and having written down information on the “I Notice, I Wonder” chart, Jocelyn asked her group members, “So what else did you want me to do?” Zeke responded, “Just, ah, think of a problem for, ah, the Doritos. I am trying to think of something right now.”

On a third occasion on Day 4, after the Teacher told the class to begin their work, Jocelyn looked at the Wendy’s men and had a short conversation with Peter and Zeke:

Jocelyn: Is this from Wendy’s?
Peter: Yeah.
Jocelyn: What did you want me to write my name for?
Zeke: You can write down one of those two (points to paper) or you can make your own.
Jocelyn: (looks through the Wendy’s menu) Want me to just choose one randomly?
Zeke: That’s what the thinker does if you want to do that.

By Jocelyn asking whether or not the artifact was from Wendy’s, why she needed to write down her name, how to go about choosing the Wendy’s item, Jocelyn seemed unsure about how to participate, and exhibited a reliance on her group
members for direction. Next, I explain how Jocelyn consulted the Teacher about the role she should play.

**Asking the teacher about her role.** Even though Jocelyn did not greatly contribute to the problem-posing activity on some days, as mentioned in the summary section, she participated the most on Day 3. This was the only day when she actually created a problem scenario, albeit without group collaboration. Yet, this occurred in conjunction with teacher guidance for helping her to understand her role.

Jocelyn’s lack of understanding about her role began immediately on the first day of the intervention. On Day 1, Jocelyn chose the role of “note-taker.” In response to the question, “In what ways did any individual roles not help today with having group members equally participate in the problem-posing process?” Jocelyn stated that she could not do play her role “because they [she] couldn’t figure out what notes they were [she was] supposed to write.” Thus, it was evident that she lacked the knowledge of how to enact her role that day.

On Day 3, the Teacher presented a sample problem created by one group to the entire class and attempted to engage the class in a conversation about the incorporation of representations and mathematics. Then, the Teacher talked about the use of the “I Notice, I Wonder” chart and asked for the person who did the least amount of work to look through the ShopRite ad and to be the one to make the problem. A few minutes later, Jocelyn expressed her lack of participation, having mentioned to her group members, “I’m sorry if I’m not doing anything for note-taking.” The Teacher walked over to Jocelyn’s group.

Teacher: Okay, so go ahead and write down the numbers for that for Doritos, so like two for five dollars.
Jocelyn: So, this is note-taking?
Teacher: Yeah, you can use this sheet here, so anything you notice from here, that’s what you want to do, you just write down two for five dollars right here…Okay, so when you [Zeke] say something, she’s gonna write it down. If Peter says something, she’s gonna write it down. Okay, so think about any ideas, like we talked about slope-intercept, like, what would the slope be? You can write that down over here. What kind of problem can I make? Anything like that. You can jot that stuff down.

In this scenario, we see that in continuation from Day 1, it appeared that Jocelyn was unsure how to execute her note-taker role and that the teacher helped Jocelyn define the role.

**Beginning Problem Posers May Participate More When They Find Something of Personal Interest**

As already shown, Jocelyn appeared to struggle to participate in the group, and this was expressed throughout the five-day intervention. However, in contrast to her typical lack of participation, two episodes stood out that revealed her interest in an aspect of the problem posing activity. The first episode occurred at the beginning of Day 4. On that day, the teacher engaged the group in a conversation about the Wendy’s restaurant menu:

Teacher: You guys find anything interesting on here? That you like?

Jocelyn: Food-wise, I think all of them.

Teacher: All of them? Yeah. Why don’t you guys think of something you all like?

Peter: Number four. [Baconator Sandwich]
Jocelyn: Number four. [Baconator Sandwich]

Peter: You don’t?

Jocelyn: No, I want number four.

Peter: Do you like it? [Zeke nods]

Jocelyn: They throw a whole bunch of bacon in that Baconator.

This episode contrasted with Jocelyn’s previous participation patterns, as she was willing to at least participate in conversation about something related to the problem scenario. Furthermore, the participation here was collaborative in that at least each group member was able to state their opinions about an item from the artifact (i.e., the Baconator sandwich) and come to a consensus about using it for their problem. Later that day, after Zeke created a problem scenario involving Baconator sales, he teacher then returned to the group:

Teacher: Okay. So, what do you guys think of this? [Teacher hands problem scenario to Jocelyn who reads the problem]

Jocelyn: I like this.

Teacher: You like the idea?

Jocelyn: Um-hmm.

Teacher: What do you like about it?

Jocelyn: That it’s continuing, keep going off.

Teacher: It’s going up every year, okay. Can you think of any question you can maybe ask from that?

Jocelyn: Maybe there’s competition. There’s other marketing, something like that. Burger King, Hardee’s, whatever.

In this interaction, Jocelyn was clearly participating more than on the previous days. She critiqued the problem scenario and provided the idea of a competing
company, such as Burger King (Zeke ended up incorporating the idea into the next problem scenario that day as shown in Figure 11 below). It appeared that the combination of Jocelyn’s personal interest with the food item from the Wendy’s menu, and the Teacher’s personal questioning about what Jocelyn liked about Zeke’s problem corresponded to moments of greater participation.

![Figure 11](image)

Figure 11  Group 1, day 4 problem scenario #2. (Figure 12 re-presented from the representations chapter to facilitate the presentation of the claim.)

In spite of Jocelyn’s statements throughout the intervention suggesting her lack of motivation to participate, these two episodes revealed that Jocelyn was able to get to the point of exhibiting some interest in the activity. This was shown by her choosing an item of her liking for the group to create a problem, as well as her offering ideas about Zeke’s problem. Of course, this level of participation was still fairly low compared to Zeke and Peter, but it suggests that personal interest in
something like a food item that a student likes may increase participation in problem posing.

In general, it was evident that Jocelyn began the intervention not understanding how to participate in the activity of group problem posing. This was evident on Days 1 through 3 by her dependence upon her group members what she should do in particular instances of the activity. However, after having expressed her lack of understanding about the role of note-taker, she was able to successfully contribute, at least to some degree, in collaborative discussion with the teacher and group members. Furthermore, Jocelyn’s participation correlated with her interest not only with a food item (i.e. Baconator) from the Wendy’s menu, but with her creating a problem on Day 3 based on her preferred choice of an item from the ShopRite advertisement (i.e., Doritos). The Day 3 occurrence of successfully creating a problem scenario was in response to the Teacher asking the class to determine the person who did the least amount of work to “look through the ad and pick out something that is interesting to them.” Thus, the evidence suggests that students like Jocelyn who lack an understanding of how to participate in group problem posing may benefit from having teacher direction regarding understanding their role in participation, as well as finding something of personal interest in the activity.
Chapter 7

DISCUSSION AND CONCLUSION

This study was about problem posing, a mathematical activity for which many in the mathematics education community have advocated for almost two decades. In particular, I explored group problem posing, an activity which few studies exist, especially within specific areas of mathematics content. In particular, this study focused on linear functions, a content area most students are familiar with and content necessary to understand at the secondary level. Activity Theory served as the theoretical lenses to guide the development of an intervention that afforded students opportunities to collectively pose problems. With my five-day intervention, I set out to examine how students incorporate representations and mathematics concepts into problem scenarios and the difficulties they may have with doing so. I also wanted to explore the process by which students interact and engage with one another and the teacher around group problem posing.

For this discussion, I will first provide an overview of the findings for Research Question #1 and my interpretation of such findings. Then, I will discuss the significance of these findings and implications for future practice. Then, I will follow the same format for Research Question 2 and 3. Finally, I will talk about limitations of the study and steps for future research.

Research Question 1

The first research question was, “What difficulties do secondary students reveal about incorporating representations into their problem scenarios?” I will provide a short overview of the findings that answered this question, conclusions that
were drawn from the findings, as well as significance for the field and implications for practice.

Overview of Findings for Research Question 1

The findings addressing this research question reflect the difficulties students have with incorporating representations, as shown by the hierarchy of problem scenarios that students produced. The hierarchy had three major levels. The levels came from comparing the problem scenarios students created and then identifying the difficulties that the qualities of the problem scenarios suggested.

At the lowest level, students created problem scenarios without representations. At the middle level, students created problem scenarios with one representation. At the top level, students created problem scenarios with multiple representations.

Within each of the three major levels, there were also sub-levels. At the lowest level, there were two sub-levels. There were problem scenarios that did not include representations at all (first sub-level), and ones that did not include representations in posing their problem scenarios, but that included representations in their solutions (second sub-level). The difficulties identified in the first sub-level suggested to me that some students might not understand much about representations, let alone how to include them in a problem scenario. Difficulties identified in the second sub-level suggested that even when students have some understanding about how to use representations to solve problems, it may require additional understanding to know how to include a representation into the actual problem. This suggests that new problem posers may need to develop this understanding.
At the middle level of the hierarchy (i.e., problem scenarios that included a single representation), there were three sub-levels. In the first sub-level, students provided a representation, but the representation was not mathematically meaningful. At the second sub-level, students provided a mathematically meaningful representation but did not provide an explicit purpose or reason for the representation. These two sub-levels suggested to me that including a mathematically meaningful representation into a problem scenario and making the purpose explicit are additional understandings that new problem posers may need to develop. At the third sub-level, students provided a mathematically meaningful representation and an explicit purpose for the representation. However, for the third sub-level, I only found cases where students provided a representation that was a table or a graph. Therefore, I concluded that at the third sub-level, students might have lacked understanding for how to provide mathematically meaningful equations that had an explicit purpose.

At the top of the hierarchy (i.e., problem scenarios that included multiple representations), there were again three sub-levels. At the first sub-level, students provided multiple representations, but there was no clear relationship between the representations. At the second sub-level, students provided one representation with a clear purpose and one with no explicit purpose. At the third sub-level, students provided multiple representations that were related and both with an explicit purpose. These sub-levels suggested that even when students included a representation into their problem scenarios, additional understanding may be required to include another representation. Further understanding may also be necessary to ensure that the additional representation is clearly related to the first representation and that explicit purposes exist for both.
Significance of the Findings for Research Question 1

These findings regarding student groups’ incorporation of representations into problem scenarios is significant, because the hierarchy reveals difficulties students may have, and in turn, may reveal new aspects of understanding representations. For instance, just because students may know how to use a representation to solve a problem scenario, does not mean that they necessarily understand how to incorporate the same representation into a problem scenario. The difficulties students had also suggest that even beginning problem posers who do incorporate a representation may not necessarily understand how to include an explicit purpose for it and how to make it mathematically meaningful. Thus, this study makes a contribution to the representations literature because it distinguishes itself from the existing literature on the use of representations to solve problems or to understand mathematical ideas.

Second, the hierarchy of problem scenarios and the associated difficulties can be used as a guide for instructors who want to try problem posing with their students. Recall that the NCTM has called for the use of problem posing in classrooms, as well as for students to develop an understanding of representations (NCTM, 2000). My study can guide instructors on where students fall within the hierarchy of incorporating representations into problem scenarios and can shed light on the understandings students may and may not have about incorporating representations into problem scenarios.

Implications for Practice from Research Question 1

Based on the findings from Research Question 1, there are a number of implications for future practice. First, teachers may have to provide additional scaffolding to help students understand what they are being asked to do. For example,
students may conflate what it means to be asked to include a representation in a problem scenario and being asked to use a representation to solve a problem scenario. Thus, teachers may need to make it the distinction more clear or explicit. One way that teachers could do this is by providing examples of a representation being provided or asked for in the problem scenario and non-examples of a representation only in the solution. This may help students clarify their understanding of the task of incorporating a representation.

Second, instructors could provide additional supports to make the incorporation of a representation into a problem scenario more productive. For example, teachers could emphasize to students what it means for a representation to have an explicit purpose. One way to do this would be to make the role of representations—to display a mathematical relationship or to solve a problem—more explicit and to show examples of how representations play such.

Third, to encourage students’ abilities to effectively incorporate various kinds of visual representations (e.g., graphs, equations, tables, pictorial representations) in mathematically meaningful ways with explicit purposes, teachers can provide opportunities for students to learn about the affordances and constraints of certain kinds of representations before and during a problem-posing activity. For example, teachers could help students understand that tables and graphs may better illustrate the covariational nature of a relationship than an equation. They could also help students understand that equations are better for finding the associated output for an input quantity that does not fall within a convenient domain of values, which may be impractical to represent with a graph or table. Hence, by reminding students of the
affordances and constraints of each kind of representation, students may be more equipped to provide an explicit purpose for how they can be used.

**Research Question 2**

The second research question was “Based on students’ productive and unproductive uses of the rate of change and the y-intercept, what difficulties do secondary students reveal about incorporating those concepts into their problem scenarios?” I will again provide a short overview for how I answered this question and the conclusions I drew from the findings. Then, I will discuss the significance of the findings and implications for practice.

**Overview of Findings for Research Question 2**

The findings addressing this research question revealed difficulties with how students attempted to include rates of change and non-zero y-intercepts into problem scenarios. One difficulty with including rates of change into linear function problems was revealed when I analyzed those problems scenarios that did not include a numerical value for the rate of change into their problem scenarios, that did not provide sufficient information to make the rate of change determinable, or that did not include a rate of change that was constant. These posed problem scenarios suggested to me that students who are beginners at problem posing may need to develop an understanding about what information is needed for a constant rate of change to be included in a linear function problem scenario.

A second difficulty with including rates of change into linear function problems was revealed when I analyzed student problem scenarios that involved more than one rate of change. Problem scenarios of this type contained a number of issues,
such as insufficient contextual information to relate the two rates to each other or trivial question about the slopes (e.g., find the slope). These problem scenarios suggested to me that students may need to develop an understanding about how to productively and non-trivially incorporate more than on rate into linear function problems.

A third difficulty with including rates of change into linear function problem scenarios was revealed when I looked across student problems that involved rates of change and noticed that one type of problem was missing. Specifically, no problem scenario gave the rate of change and a value of the dependent variable and asked for the corresponding value of the independent variable. This suggested that students may need to develop an understanding about how to create linear function problems that ask about a value of the independent variable.

A final difficulty with including y-intercepts into linear function problems was revealed when I noticed that students either did not include a non-zero y-intercept into their problem scenarios or that there was a mathematical issue with how they included a non-zero y-intercept (e.g., graphical inaccuracy, lack of sufficient context). This observation suggested that students may need to develop an understanding of how to incorporate non-zero y-intercepts into linear function problem scenarios.

Significance of the Findings for Research Question 2

These findings on incorporating the mathematical concepts of rate of change and y-intercept are significant for at least two reasons. First, the inclusion of mathematics concepts into problem scenarios in non-productive ways, illustrates that problem posing may be useful as an assessment tool to reveal student understandings and misunderstandings about particular concepts. For example, when students include
a rate of change into their problem scenario as a unit rate but then ask the solver to find the slope, such a problem scenario may reveal that the students lack an understanding that the rate of change and the slope are referring to the same quantity. Similarly, if students would represent the amount of a gratuity as the y-intercept for a problem scenario in which the amount of a gratuity is based on a percentage of a restaurant bill, this could be an indication that the students do not fully understand that the y-intercept of a linear functions represents as a fixed amount or the real-world quantities that the y-intercept can be used to represent.

These findings are also significant because they inform the field about beginning problem posing in a specific mathematical context. In other words, my study provides unique insights into posing problem scenarios that are specific to posing scenarios that involve linear functions. This is particularly significant, given that the linear function is so central to Algebra. Thus, the study provides exploratory insights into challenges beginning problem posers might face with incorporating some of the most important algebraic concepts into posed problem scenarios.

Implications for Practice from Research Question 2

These results regarding the incorporation of linear functions concepts into problem scenarios is important for teacher practice. Based on the evident challenges associated with incorporating a rate of change and y-intercept into a problem scenario, it may be necessary to help students develop an understanding of what it means to incorporate a rate of change and non-zero y-intercepts into their problem scenarios. For example, it may help for the teacher and students to compare several of students’ problem scenarios to see how different groups are trying to include rates of change and non-zero y-intercepts. This may help students understand the necessary
mathematical and/or contextual features to include in and exclude from problem scenarios, so that the rate of change and y-intercept is productively incorporated. Also, teachers could be very explicit with their students about the language to include in or exclude from problem scenarios so that their problem scenarios reflect rates of change that are constant, or at least, that those words are avoided if a constant rate of change is not intended.

Research Question 3

The third research question was, “What does group problem posing engagement and their interactions look like for beginner problem posers?” I will again provide a short overview for how I answered this question, conclusions that can be drawn from the findings, as well as discuss the significance of the findings and implications for practice.

Overview of Findings for Research Question 3

To address this research question, I provided a rich description of the general characteristics of engagement for one group of three beginning problem posers, as well as particular instances of their engagement, non-engagement and interactions. The students will be called Peter, Zeke, and Jocelyn. With respect to characteristics of engagement, Zeke faithfully participated in the problem posing activities throughout the entire intervention; Peter, began by participating actively, but was inconsistent; and Joselyn seldom participated, and when she did, it was in a passive manner. Such findings suggest it can be challenging for students to maintain active roles in group problem posing.
Even though variations existed regarding how much and how often students participated, each student was able to successfully create at least one problem scenario on his or her own. This shows that the activity of problem posing was within these students’ zone of proximal development (Vygotsky, 1987). However, these problem scenarios were almost always completed individually, that is, without students sharing in the intellectual work of creating the scenario. This suggests that sharing in the intellectual work of problem posing can be challenging.

With respect to the interactions between peers, I focused on the interactions between Peter and Zeke. I interpreted Peter and Zeke’s decision to take turns as the creator and solver on Days 1 and 2 as an indication that beginning problem posers may not fully understand how to share in the intellectual work of problem posing or that their conceptualization of sharing in the intellectual work may differ from that of the teacher. In my study, with the help of the teacher, Peter and Zeke were finally able to reach a point at the end of the intervention where the role of “creator” was shared between them as they created a single problem scenario. Yet, the students still did not engage with each other in the development of the problem, but rather worked on their part of the scenario independently. This finding further illustrates that for beginning problem posers to move toward sharing in the intellectual work of problem posing may be a challenge.

With respect to non-engagement, beginning problem posers like Jocelyn may struggle to participate in group problem posing at all. Jocelyn’s reliance on her group members and question to the teacher about her role, illustrated that beginning problem posers may benefit from having others, particularly, their teacher, as guides for how to participate in the activity beyond the general elements I included in my intervention,
such as asking student to choose roles and providing criteria for problem scenarios. However, because Jocelyn was able to successfully create one problem scenario and engage in discussion about a fellow group member’s problem that connected to her interest in a food item, it also seems it may help beginning problem posers to engage in group problem posing when the problem involves something of personal interest.

Significance of the Findings for Research Question 3

These findings about the characteristics of student interactions, engagement, and non-engagement in group problem posing is significant for at least two reasons. First, although many studies on individual student problem posing already exist, no studies to my knowledge have examined student interactions and engagement in group problem posing. Therefore, this study adds to the literature on problem posing with new insights into the way students interact with one another, as well as the challenges that may exist when students try to share in the intellectual work of group problem posing. Future research could build on my study by examining other classroom conditions and student populations.

Second, this study is significant because it examined which elements of an Activity Theory, may be more or less supportive for beginning problem posers participating in group problem posing. Peter and Zeke’s identification of their roles as creator and solver, and their choice not to choose to share their roles but to take turns, even when they consistently had the opportunity to share the same role, may suggest that each student playing a different role may not be beneficial for shared engagement. Also, Jocelyn’s choice of a food items that interested her may suggest that the Activity Theory component of mediating artifacts, namely cultural artifacts, may indeed support struggling students in participating in group problem posing. Future research
could continue to examine which elements of Activity Theory to keep, remove, or build upon to further maximize students’ opportunities to share in the intellectual work of problem posing.

Implications for Practice from Research Question 3

There are a number of implications for teaching practice based on this study. My study suggests that teachers need to be mindful that the development of student understandings about what it means to pose problems in groups may take time and significant teacher scaffolding. For example, if students within a group initially take turns creating a problem scenario, teachers may need to first have the students practice creating different aspects of the same problem scenario before having them share in the intellectual work of creating the entire scenario together. In this way, students may become more comfortable with the transition between taking turns and sharing in the intellectual work, as well as understand the difference between these activities.

Second, teachers may need to change the way they address the Activity Theory-based perspective of roles, because when students chose their own roles, their choices at times appeared to go counter to the goal of sharing in the intellectual task of posing a problem scenario. Additional scaffolding, such as sample questions and responses, as well as students viewing videos of others successfully collaborating in the process, may help to show students how to effectively share, build on, explain, and critique one’s own and other’s ideas. Furthermore, students can position themselves within a group in ways that are detrimental to group collaboration (Wood, 2013). Therefore, teachers may need to provided time for students to practice collaborating with one another, such as by using question-and-response scaffolds so that students an build confidence in their ability to engage with others.
Third, teachers need to be mindful of how personal interest can support student levels of participation in group problem posing. Hidi and Renninger (2006) proposed four stages of students’ interest in an activity: triggered situational interest, maintained situational interest, emerging individual interest, and well-developed individual interest. It may help for teachers to find out what their students are interest in and choose cultural artifacts that not only align with the mathematical content about which they are creating problems but choose artifacts that align to students’ personal interests. If student interest is merely triggered but not maintained, teachers can also have students share in choosing artifacts for themselves of which they are more personally interested, but still mathematically align with the content, so as to increase student engagement.

**Limitations and Future Steps**

This study is significant to mathematics education and has many implications for teacher practice. However, some limitations of my study do exist. First, for the analysis of posed problem scenarios, my study only examined five groups of Integrated Math 2 students in a small, mid-Atlantic town. Furthermore, I only examined the engagement patterns and interactions of only three students. Moving forward, I would like to replicate my study with other student populations (e.g., urban, rural) to determine the commonalities and differences regarding what students understand about group problem posing.

Second, I only grouped students heterogeneously according to gender and other mathematical characteristics (i.e., self-reported mathematical disposition and performance). However, I would like to conduct my study based on other types of groupings, as doing so may foster more quality problem scenario creation and student
interactions. For example, grouping in pairs rather than in groups of 3 or 4 in a shorter period of time may force students to engage more deeply by sharing in the intellectual work of creating a problem scenario. Additionally, having students grouped according to their shared personal interests may also contribute to greater maintained interest and thus more sharing, building, and critiquing of each other’s ideas.

Third, I was not the official classroom teacher. Thus, I did not have a pre-existing relationship with the students, nor did I fully understand their academic abilities and the social dynamics of the classroom. Having such understandings may have helped with grouping the students and conducting mathematics lessons so that students had greater opportunities to learn, as well as may have helped them be more comfortable with participating in such a new activity.

Fourth, some elements that I hypothesized would maximize the creation of problem scenarios that incorporated representations and mathematics concepts and that would maximize the shared intellectual work of problem posing did not promote as much change as I expected. For one, I did not realize that students did not know how to create scenarios that asked for the unit rate and the independent variable. Also, I did not realize that students did not understand what it means for problems to be structurally different from each other. Thus, I would like to revise my intervention regarding what it means to pose an original problem by having students understand the structural similarity and non-similarity by first using simpler mathematical contexts, such as addition and subtraction word problems. It may be the case that as students more deeply understand structural difference from simpler mathematical contexts that they can carry such understanding to creating linear function problems where the unknown value is not the dependent variable. Finally, regarding interactions, I did not
anticipate that students would choose roles that would not contribute to their collaboration. In the future, I would like to conduct a more scaffolded study where students first take turns making a problem and solving it. Next, each student makes one aspect of the problem. Then, students view videos of shared intellectual engagement before they are actually asked to share in the intellectual work.

A final limitation is that the study focused very narrowly on only certain mathematics concepts and conditions. For example, the study was primarily on linear functions. For future research, I would like to conduct a similar intervention for another Algebra topic, such as the concepts of the maximum/minimum value and the x-intercept for quadratic functions. I would also like to explore the effects of teacher-chosen or student-chosen artifacts on students’ triggered or maintained interest in the activity, as well as how artifacts that do or do not correspond to the mathematical subject matter impact the quality of posed problem scenarios. In addition, peer feedback was not an element of the study. Therefore, I am interested in conducting qualitative studies on how the feedback of peers on problem scenarios created from other groups influences problem scenario quality and student engagement in the activity. Finally, I would also like to include larger sample sizes and conduct quantitative studies to compare the effects of different interventions to show what kind of intervention elements for different student populations would best support students with posing quality problems and participating in group problem posing.

Conclusion

In conclusion, my five-day intervention has provided insights into secondary students’ challenges with both the process and product of group problem posing. With regard to the products, that is, the posed problem scenarios, my findings can help
practitioners and researchers understand the difficulties associated with the incorporation of mathematical features (single/multiple representations, rate of change, y-intercept), and what kinds of instructional supports may be implemented to foster such features within and beyond linear functions. Regarding the process, that is, the phenomenon of group problem posing, my findings can help practitioners be mindful of students’ understandings of what it means to collaborate with one another during group problem posing. Such a study lays a foundation for future research for how to engineer conditions in various classroom settings that support student understandings and interactions during group problem posing. More research on various classroom conditions and with different student populations is needed to gain additional insights into the phenomenon of group problem posing.
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## Appendix A

### DAILY CRITERIA

#### Day 1

Your task:
- Create at least one mathematics problem for a peer to solve involving a mathematical representation (graph, table, equation, pictures, etc.)
- Use information from the ShopRite advertisement to create your problem
- Use the "I Notice, I wonder" chart to write down information from the ShopRite ad and to brainstorm problem ideas

#### Day 2

Your task:
- Create at least one mathematics problem involving slope, y-intercept for a peer to solve
- Incorporate at least one mathematical representation into the problem (graph, table, equation, pictures, etc.)
- Use the ShopRite advertisement

Be clear with your words if you want the solver to use the representation
- Try to include more than one representation or a different kind of representation

Refer back to the roles sheet from yesterday to remind yourselves of your roles.
Day 3

Your task:
*Create a problem involving slope, y-intercept for a peer to solve

*Incorporate two mathematical representations (graph, table, equation, pictures, etc.)
  *Be clear if solver is to use or create the representation
  *Include a different kind of representation not used before
  *Have solver create a representation from another given representation OR have the solver use two given representations of the same data

*Use ShopRite advertisement

*Remind yourselves of your roles (roles sheet)

Day 4

Your task:
*Create a problem involving slope, y-intercept and adding/subtracting linear functions for a peer to solve

*Incorporate two mathematical representations (graph, table, equation, pictures, etc.)
  *Be clear if solver is to use or create the representation
  *Include a different kind of representation not used before
  *Have solver create a representation from another given representation OR have the solver use two given representations of the same data

*Use ShopRite advertisement and/or the Wendy’s Menu

*Remind yourselves of your roles (roles sheet)

Day 5

Your task:
*Create a problem involving slope, y-intercept, adding/subtracting linear functions, and parallel/intersecting functions

*Incorporate two mathematical representations (graph, table, equation, pictures, etc.)
  *Be clear if solver is to use or create the representation
  *Include a different kind of representation not used before
  *Have solver create a representation from another given representation OR have the solver use two given representations of the same data

*Use ShopRite advertisement

*Remind yourselves of your roles (roles sheet)
Appendix B

CULTURAL ARTIFACT #1: SHOPRITE ADVERTISEMENT  
(SAMPLE PAGE)
## Appendix D

**ORGANIZER: “I NOTICE, I WONDER” CHART**

| I notice... | I wonder... |
Appendix E

ROLE NEGOTIATION WORKSHEET

Roles:

In what ways did any of individual roles (roles that only one person plays) help with engaging group members to equally and meaningfully participate in the problem-posing process? Explain in what ways they helped to do so and why.

In what ways did any individual roles (roles that only one person plays) not help today with helping group members to equally and meaningfully participate in the problem-posing process? Explain in what ways they did not help to do so and why.

In what ways did any shared roles help today with helping group members to equally and meaningfully participate in the problem-posing process? Explain in ways they did not help to do so and why.

In what ways did any shared roles help today with helping group members to equally and meaningfully participate in the problem-posing process? Explain in ways they did not help to do so and why.
As a group, think about how you would like to re-assign roles, if you so choose to do so. Then, each person will write down their name as well as what role or roles they will play for the next problem-posing experience. (Remember, roles can be individual or shared)

Name: ___________________________ Role(s): ______________________________

Name: ___________________________ Role(s): ______________________________

Name: ___________________________ Role(s): ______________________________

Name: ___________________________ Role(s): ______________________________

Name: ___________________________ Role(s): ______________________________

Name: ___________________________ Role(s): ______________________________
Appendix F

DAY 1 PRESENTATION: “WHAT IS PROBLEM POSING?”

Three Types of Problem Posing:

1.) The creation of a *simpler* problem (perhaps analogical in nature) for the goal of understanding and/or solving a *more complex* problem.

Cancer/Radiation Problem (Duncker, 1945, in Gick & Holyoak, 1980)

“Suppose you are a doctor faced with a patient who has a malignant tumor in his stomach. It is impossible to operate on the patient, but unless the tumor is destroyed, the patient will die. There is a kind of ray that can be used to destroy the tumor. If the rays reach the tumor all at once at a sufficiently high intensity, the tumor will be destroyed. Unfortunately, at this intensity the healthy tissue that the rays pass through on the way to the tumor will also be destroyed. At lower intensities the rays are harmless to the healthy tissue, but they will not affect the tumor either.

What type of procedure might be used to destroy the tumor with the rays, and at the same time avoid destroying the healthy tissue?”
Military Problem (Gick & Holyoak, 1980)

“A general wishes to capture a fortress located in the center of a country. There are many roads radiating outward from the fortress. All have been mined so that while small groups of men can pass over the roads safely, any large force will detonate the mines. A full-scale direct attack is therefore impossible.

What can the general do to capture the fortress?”

Three Types of Problem Posing:

1.) The creation of a simpler problem (perhaps analogical in nature) for the goal of understanding and/or solving a more complex problem

2.) The creation of a problem after solving another problem by changing the conditions of the original problem (What if?)
“What if?”

- A customer in Delaware purchases a $17 meal and also pays a 20% tip. How much money does the customer pay in total?

- What if the restaurant was located in Pennsylvania? How much money would the customer pay in total?

Three Types of Problem Posing:

1.) The creation of a simpler problem (perhaps analogical in nature) for the goal of understanding and/or solving a more complex problem.

2.) The creation of a problem after solving another problem by changing the conditions of the original problem (What if?)

3.) The creation of problems from new situations:
   - Create a problem from a given graph, equation, or context
   - Come up with a new context (basketball, gaming, etc.)
Appendix G

LESSON MATERIALS (DAYS 2-5)

Day 2 (Lesson 1): Slope and Y-Intercept

**Tiled Patios**

Consider the tiled patios below, which consist of white and gray 1 x 1 sq. ft tiles.

Assuming the pattern continues, show what the next two patios in the sequence will look like.

Now, let’s create a table of our data.

<table>
<thead>
<tr>
<th>Patio #</th>
<th># of Gray Tiles</th>
<th># of White Tiles</th>
<th>Total # of Tiles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>14</td>
<td>18</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>16</td>
<td>21</td>
</tr>
</tbody>
</table>
Describe any patterns or relationships you can find regarding the number of gray tiles.

*The # of gray tiles increases by 1.*

Describe any patterns or relationships you can find regarding the number of white tiles.

*The # of white tiles increases by 2.*

Describe any patterns or relationships you can find regarding the total number of tiles.

*The total # tiles increases by 3.*

Are the data increasing or decreasing?

*increasing*

Can a patio be created using exactly 22 white tiles? Why or why not?

*Yes; 22 fits the pattern in the table*

Can a patio be created using exactly 35 tiles in all? Why or why not?

*No; 35 does not fit the pattern in the table*
Now, let's observe the graphs for each of the three data sets in relation to the patio number (# of gray tiles, # of white tiles, total # of tiles).

1. How are any of the three graphs the same?
   - the lines are of increase
   - two of the lines (# white tiles and total # of tiles) intersect the y-axis at the same place

2. How are any of the three graphs different?
   - they have different slopes (rates of change)

3. What are the rates of change?
   \[ m = 1, \ m = 2, \ m = 3 \]

What do they mean in context of this tiles situation?

As the patio # increases by 1:
- the # of gray increase by 1 (m = 1)
- the # of white increase by 2 (m = 2)
- the total # increases by 3 (m = 3)
4. Let's develop equations based on slope-intercept form. What would be our three equations?

**Function notation:**

- **Gray:** \( y = 1x + 0 \) or \( y = x \)  
- **White:** \( y = 2x + 6 \)  
- **Total:** \( y = 3x + 6 \)

5. What would the y-intercepts mean in context of this problem?

*When the patio # is 0:*

- the # of gray used is 0  
  \( (0, 0) \)  
- the # of white used is 6  
  \( (0, 6) \)  
- the total # used is 6  
  \( (0, 6) \)

6. Which one of these three y-intercepts makes the most sense?

- *When the patio # is 0, the # of gray used is 0*  
  \( (0, 0) \)  
- *It does not make sense to use 6 white tiles or 6 total tiles when the patio # is 0 (no patio created)*

7. a.) If we use 8 gray tiles, how many total tiles would be used to create the patio?

\[
\begin{align*}
\text{patio # is 8 (} x = 8 \text{)} & \quad t(x) = 3x + 6 \\
\text{} & \quad t(8) = 3*8 + 6 \\
\text{} & \quad = 24 + 6 \\
\text{} & \quad = 30
\end{align*}
\]

b.) ...how many white tiles would be used?

\[
\begin{align*}
\text{patio # is 8 (} x = 8 \text{)} & \quad w(x) = 2x + 6 \\
\text{} & \quad t(8) = 2*8 + 6 \\
\text{} & \quad = 16 + 6 \\
\text{} & \quad = 22
\end{align*}
\]
Day 3 (Lesson 2): Slope and Y-Intercept

Suppose the current rates for taxicabs in New York are:

<table>
<thead>
<tr>
<th>Service</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Fare</td>
<td>$2.50</td>
</tr>
<tr>
<td>Each 1/10 of a mile (4 blocks)</td>
<td>$0.50</td>
</tr>
<tr>
<td>Peak Surcharge (4 pm - 8 pm, M-F)</td>
<td>$0.50</td>
</tr>
<tr>
<td>Night Surcharge (8 pm - 6 am, daily)</td>
<td>$0.50</td>
</tr>
<tr>
<td>New York State Tax</td>
<td>$0.50</td>
</tr>
<tr>
<td>Additional Riders</td>
<td>FREE</td>
</tr>
</tbody>
</table>

1. What would be the cost for each of the rides below? Assume that all trips are taken during regular hours, i.e., during 7 am to 4 pm.

<table>
<thead>
<tr>
<th>Number of Miles</th>
<th>Cost for Ride</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2.50 + 0.50 + 2.50 = 5.50</td>
</tr>
<tr>
<td>2</td>
<td>$2.50 + 0.50 + 2(2.50) = 8.00</td>
</tr>
<tr>
<td>3</td>
<td>$2.50 + 0.50 + 3(2.50) = 11.50</td>
</tr>
<tr>
<td>4</td>
<td>$2.50 + 0.50 + 4(2.50) = 13.00</td>
</tr>
<tr>
<td>5</td>
<td>$2.50 + 0.50 + 5(2.50) = 15.50</td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>
2. Find a pattern in the table. How does the cost change for each additional mile?

\[ \text{goes up by } \$ 2.50 \]

3. Write an equation to represent the relationship between the number of miles traveled and the cost for the ride.

\[ y = mx + b \]

\[ y = 2.50x + 3 \]

4. Use your equation to calculate the cost for the following rides:

a.) a 10-mile trip from your house to a museum

\[ y = 2.5(10) + 3 = 25 + 3 = 28 \]

b.) a 20-mile trip from the airport to your house
Day 4 (Lesson 3): Adding and Subtracting Functions

Tiled Patios Revisited

We would like to tile a patio. To do this, we would like to keep the following type of pattern.

![Patio Patterns](image)

Here are some graphs that model the pattern for the # of gray tiles, \( g(x) \), and the # of white tiles, \( w(x) \), where \( x \) represents the patio #.

![Graphs](image)

How could we use only these graphs to create a graph for the total # of tiles?
Within the same coordinate plane above, show how we can use only the graphs to find the total number of tiles used for the 6th patio.

\[ 6 + 18 = 24 \]

Use the newly created graph to find an explicit rule for \( t(x) \), a function that models the total number of tiles for the \( x \)th patio. (any patio number)

\[ t(x) = 3x + 6 \]

Based on the graphs, what would be the explicit rules for \( g(x) \) and \( w(x) \)?

\[ g(x) = x \]
\[ w(x) = 2x + 6 \]

Use both the \( g(x) \) and the \( w(x) \) explicit rules to evaluate the total number of tiles for the 6th patio.

\[ g(6) = 6 \]
\[ w(6) = 2 \cdot 6 + 6 = 18 \]

\[ 6 + 18 = 24 \]
What operation would we use with these two rules to algebraically show how to find the explicit rule for $t(x)$? Show your work.

\[
t(x) = g(x) + w(x) = x + 2x + 6 \quad \frac{t(x)}{x} = 3x + 6 \quad x = 6
\]

Now, use the new function $t(x)$ to evaluate the number of tiles for the 6th patio. Show your work.

\[
t(6) = 3(6) + 6 = 18 + 6 = \boxed{24}
\]
Day 5 (Lesson 4): Distance over Time (Parallel and Intersecting Functions)

Suppose \( f(x) = x + 2 \) and \( g(x) = 3x - 4 \) (again, each block represents 1 unit). What is the solution to this problem? Use algebra and a graph to justify your answer.

\[
\begin{align*}
  x + 2 &= 3x - 4 \\
  -2x &= -6 \\
  x &= 3
\end{align*}
\]
Suppose f(x) represents your distance from school (in miles) and g(x) represents your friend Gina’s distance.

a.) At what point in time, x, (in hours) would both of you be at the same distance from school? Prove your answer algebraically and graphically.

\[ x = 3 \text{ hours} \]
\[ \text{we already did it} \]

b.) What would be this distance from the school? Prove your answer algebraically and graphically.

\[ y_f = 5 \text{ miles} \]
\[ f(x) = x + 2 \]
\[ f(3) = 3 + 2 = 5 \]

c. Suppose your friend Harry begins 2 miles ahead of you and runs at the same rate as you. What function would represent Harry’s distance over time?

\[ f(x) = \text{you} \]
\[ h(x) = \text{Harry} \]

d. Assuming that you and Harry run at the same rate forever, when will you meet up with Harry? Explain.

\[ \text{No, we will not meet} \]
e. When will Harry meet up with Gina? Justify your answer.

\[ h(x) \quad g(x) \]

(4, 8) dist.

- used the graph

f. At this meeting point, how far will both Harry and Gina be from the school? Explain or show your work to prove your answer.

8 miles

looked at the ordered pair (4, 8)
Appendix H

DAY 3 AND DAY 4 SAMPLE PROBLEM SCENARIO

(same problem scenario as in Figures 15 and 37, but here also including problem solution)

\[ y = mx + b \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
</tbody>
</table>

ShopRite is having a Sale on artichokes. One bag is \$3. 2 bags are \$6.
Make a \( y = mx + b \) equation if \( p \) is the price and \( x \) is the number of artichoke bags.

How much would it cost if you buy 5 bags of artichokes?

\[ p = \frac{6}{3} \]
\[ p = 2 \]
Appendix I

DAY 5 SAMPLE PROBLEM SCENARIO

(same as Figure 25)
Appendix J

IRB/HUMAN SUBJECTS APPROVAL

DATE: December 20, 2017

TC: Robert Micell
FROM: University of Delaware IRB

STUDY TITLE: [1155274-2] Investigating Collaborative Problem Posing of Secondary Students in a Linear Functions Intervention

SUBMISSION TYPE: New Project

ACTION: APPROVED
APPROVAL DATE: December 20, 2017
EXPIRATION DATE: December 19, 2018
REVIEW TYPE: Expedited Review
REVIEW CATEGORY: Expedited review category # (6,7)

Thank you for your submission of New Project materials for this research study. The University of Delaware IRB has APPROVED your submission. This approval is based on an appropriate risk/benefit ratio and a study design wherein the risks have been minimized. All research must be conducted in accordance with this approved submission.

This submission has received Expedited Review based on the applicable federal regulation.

Please remember that informed consent is a process beginning with a description of the study and insurance of participant understanding followed by a signed consent form. Informed consent must continue throughout the study via a dialogue between the researcher and research participant. Federal regulations require each participant receive a copy of the signed consent document.

Please note that any revision to previously approved materials must be approved by this office prior to initiation. Please use the appropriate revision forms for this procedure.

All SERIOUS and UNEXPECTED adverse events must be reported to this office. Please use the appropriate adverse event forms for this procedure. All sponsor reporting requirements should also be followed.

Please report all NON-COMPLIANCE issues or COMPLAINTS regarding this study to this office.

Please note that all research records must be retained for a minimum of three years.
Based on the risks, this project requires Continuing Review by this office on an annual basis. Please use the appropriate renewal forms for this procedure.

If you have any questions, please contact Nicole Farnese-McFarlane (302) 831-1110 or Nicolefm@udel.edu. Please include your study title and reference number in all correspondence with this office.