SUPPORTING NOVICE MATHEMATICS TEACHER EDUCATORS TO TEACH AMBITIOUSLY VIA CONTINUOUSLY IMPROVED CURRICULUM MATERIALS

by

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A dissertation submitted to the Faculty of the University of Delaware in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Education

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ABSTRACT

Mathematics teaching in the United States has notoriously and historically been classified as skill and drill instruction where the main focus of mathematics teaching is procedural fluency across all grade levels. In order to change this norm, we need to seriously consider how we prepare prospective teachers and in turn, how we support those responsible for preparing prospective teachers. In this study, I investigated how a set of continuously improved curriculum materials supported novice mathematics teacher educators in teaching an elementary content course for pre-service teachers for the first time. Results show that the two instructors were able to productively set up rich, cognitively demanding tasks and respond productively to pre-service teachers’ questions and comments—even those that were unanticipated by the curriculum materials. The main curricular aspects that supported these productive task set-ups and responses were targeted learning goals, descriptions of what successful students would be saying and doing during class, “scripts,” rationales for each task, anticipated comments, questions, and difficulties, helpful ways to respond to students, and associated PowerPoint slides. These findings indicate that curriculum materials designed through gradual continuous improvement methods have the potential to support novice mathematics teacher educators to teach ambitiously.
Chapter 1

HOW CAN CURRICULUM MATERIALS SUPPORT IMPROVEMENTS IN MATHEMATICS TEACHING?

The nature of mathematics teaching in the United States has remained remarkably consistent over the past century, even in the face of various reform attempts. Teaching seems to rely on fact-centered, procedural skills, where the majority of class time is spent reviewing previous relatively unchallenging material (Fey, 1978; Hiebert et al., 2005; Hoetker & Ahlbrand, 1969) and emphasizing basic arithmetic skills in elementary grades (Plinta, Belsky, Houts, & Morrison, 2007). In other words, the core of teaching mathematics—how the teacher and students interact around mathematical content—has not changed over the years.

This stable trend has resulted in little improvement in the quality of teaching over the years (Fey, 1978; Hoetker & Ahlbrand, 1969; Morris & Hiebert, 2011) even in the midst of several national reform attempts. Given the limited success of reform efforts, how do we improve the quality of mathematics teaching on a wide scale? Furthermore, how do we improve the quality of mathematics teaching for all students? Integrating these two questions, we arrive at the critical question: How do we help to ensure that all students receive equal access to a quality mathematics education?
This is a difficult question requiring a complex answer. There will likely be many things that we will have to improve within and outside of our educational system to completely address this question of equal access to quality education. In this paper, I focus on one aspect of this complex system: how can curriculum materials support improvements in mathematics teaching?

The purpose of this theoretical paper is to present a new perspective on a prior curriculum framework proposed by Stein, Remillard, and Smith (2007) in order to address this question. In particular, I extend this framework in two ways: (a) I expand upon the idea of the written curriculum and the intended curriculum (Remillard, 2005) and (b) I narrow the focus of the curriculum transitioning through the phases of the curriculum transformation process to thinking about learning opportunities transitioning through the various phrases of the curriculum transformation process. Further articulating the differences between the written curriculum and the intended curriculum may help researchers and curriculum developers understand how to support teachers through curriculum materials. In addition, focusing on the construct of learning opportunities may help to further understand the relationship between curriculum materials and teachers and how to provide all students with a high quality mathematics education.

I begin by defining the construct of curriculum and several assumptions regarding the relationship between the written curriculum and teachers, followed by briefly reviewing the curriculum framework proposed by Stein, Remillard, and Smith (2007). I then elaborate upon the idea of the author’s intended curriculum, the author’s written curriculum, the teacher’s intended curriculum, and the teacher’s written curriculum. My
expanded view of Stein et al.’s framework will be presented next, which adds these additional phases to the framework. Finally, I discuss the differences between curriculum and learning opportunities and how a more narrow focus on what is transitioning across the phases of the enactment process might benefit future researchers, curriculum developers, and reformers. I conclude by providing recommendations to guide future researchers, curriculum developers, policy-makers, and reformers in thinking about how curriculum materials can be used to improve the quality of mathematics teaching for all students.

**Definition of Curriculum**

In general, I perceive the construct of curriculum very broadly as involving the content taught to students, the sequencing structure of that content, and the learning goals associating with that content (Remillard, 2005). According to Remillard (2005), there are three different types of curricula: the written curriculum, the intended curriculum, and the enacted curriculum. The *written curriculum* (or “curriculum materials”, which I sometimes refer to as “curriculum” for short) refers to the written content, learning goals, activities, and resources the teacher uses to plan lessons, such as a textbook or teacher’s guide. The *intended curriculum* refers to the teacher’s aims, learning goals, and intentions for students. Finally, the *enacted curriculum* refers to what happens in the classroom between teachers and students.

**Underlying Assumptions about the Teacher-Curriculum Relationship**

In order to address the question of how curriculum materials can support teachers, we must explicitly address underlying assumptions and beliefs held regarding the
teacher-curriculum relationship. First, is the relationship between teachers and the written curriculum even worthy of study? In other words, do mathematics teachers even interact with the written curriculum? It turns out that mathematics teachers have been found to rely heavily on textbooks and curriculum materials to guide their planning (Remillard, 2005; Remillard, Herbel-Eisenmann, & Lloyd, 2009; Stein, Remillard, & Smith, 2007). Therefore, we can safely assume that teachers will use curriculum materials in some way. However, it is unclear how they will interact with these materials (e.g., Collopy, 2003; Ding & Carlson, 2013; Remillard, 2005). Therefore, another important aspect of the teacher-curriculum relationship is how the teacher uses the curriculum.

**Assumptions about How Teachers Use the Curriculum**

Several researchers have studied how teachers use curriculum materials and have shown that there are a number of ways teachers can use curricula (e.g., Remillard, Herbel-Eisenmann, & Lloyd, 2009), such as following the text closely by offloading the lesson onto the curriculum, adapting the curriculum by making some edits to the curriculum, or improvising the lesson by editing the lesson entirely (Brown, 2002 as cited in Remillard, 2005). In addition, several studies have investigated how teachers’ beliefs (e.g., Collopy, 2003; Remillard & Bryans, 2004; Sleep & Eskelson, 2012) and knowledge (e.g., Borko et al., 1992; Charalambous & Hill, 2012; Lewis & Blunk, 2012) affect the ways in which they use and interpret curricula. This line of research captures the complexity of how teachers use curriculum materials to plan their lessons and factors that influence how teachers interpret text. Therefore, we cannot safely assume that teachers will use curricula in the same ways.
Assumptions about the Role of the Curriculum

Finally, we need to consider what we expect teachers to do as they use and interact with the written curriculum. In other words, what is the purpose of the written curriculum and what role does it play in the teacher-curriculum relationship? Remillard (2005) has captured some of this discussion in terms of conceptions of curricular use. In particular, she specifies four ways in which teachers may be conceived of as using the written curriculum: (a) following or subverting the text, (b) drawing on the text, (c) interpreting the text, and (d) participating with the text. These categories delineate different purposes of the written curriculum and how we might expect teachers to interact with the written curriculum. In addition, different conceptions of the purpose and/or role of the written curriculum are associated with different underlying assumptions.

The role of curriculum as a blueprint. For example, in some cases, the purpose of the written curriculum might be conceived of as a blueprint for teachers to follow. In line with this view, some have investigated fidelity of implementation and the degree to which teachers follow the written text (Fullan & Pomfret, 1977; O’Donnell, 2008) to determine whether teachers implemented the text as intended. In these cases, the purpose of the written curriculum would align most closely with Remillard’s (2005) conception of curriculum use as following or subverting the text. The underlying assumption is that the text represents the written form of the enacted curriculum and teachers should use the written text to produce the enacted curriculum. Thus, the role of the written curriculum is a “blueprint” to be followed accurately.
**The role of curriculum as a resource.** In other cases, the written curriculum might be conceived of as a set of suggestions, a guide, or a resource for teachers to consider while planning their lessons. In these situations, researchers and curriculum developers would likely expect teachers to alter the written text and adapt the curriculum for their own needs. It would appear that many textbook authors design curriculum under this assumption because textbooks have been well-known for including much more content than can typically be covered in a single course (Remillard, 2005) and teachers rarely use all of the content within a single textbook (Chval, Chavez, Reys, & Tarr, 2009). In this case, we might also expect teachers to also make adaptations in the moment while teaching (Ben-Peretz, 1990; Morris, 2012) in addition to making alterations to the intentions of the lesson. The role of curriculum as a resource aligns most closely with Remillard’s (2005) conception of curricular use as drawing on or interpreting the text because the written curriculum is conceived of as a supporting resource for teachers (drawing on) that may influence their intentions depending on how they interpret the text (interpreting text).

**The role of curriculum as an archive.** Finally, the purpose of the written curriculum might be to capture and store valuable knowledge about teaching in a written artifact that can then be improved over time (Morris & Hiebert, 2011). The role of the written curriculum in this case would be to accumulate knowledge in a database or archive that would be accessible for other teachers. Perceiving of curriculum materials for this purpose is often seen in contexts such as Japanese lesson study (Lewis, Perry, & Hurd, 2009), continuous improvement systems in teaching (Bryk, 2015; Bryk, Gomez,
Grunow, & LeMahieu, 2015; Morris & Hiebert, 2009), and curriculum development programs where teachers pilot the curriculum and provide feedback to the curriculum authors. The role of curriculum as an archive regards the written text as a living document that is constantly changing. The focus is on how to improve the written curriculum by understanding how the written curriculum supports or hinders teachers’ enactments and how to improve the written curriculum over time to better support teachers (Morris, 2012). This conception aligns most closely with Remillard’s (2005) conception of curriculum use as participating with the text because of the integrated relationship between the teacher and the text. In these cases, the text is thought to influence the teacher (and hence the teaching) while the teacher is also expected to influence the text (to improve and refine the text).

All of these conceptions can be useful for various reasons and are not mutually exclusive. For example, Morris (2012) uses teachers’ adaptations and lack of fidelity in order to make improvements to the written text. The focus on expecting teachers to make adaptations and perhaps alter the intentions of the text and using these instances to refine the text blurs the lines between the two conceptions of the role of curricula as a “resource” and as an “archive.” In addition, Quinn and Kim (2017) blur the lines between considering the text as a “blueprint” and as a “resource” by investigating when it may be more appropriate for teachers to aim for fidelity to the written text or to adapt the text. In another example, Brown, Pitvorec, Ditto, and Kelso (2009) make the distinction between teachers following the literal text (“blueprint” conception) and teachers following the author’s intentions behind the text (“resource” conception). In some cases, teachers who
followed the literal text did not enact a curriculum aligned with the author’s intentions. Similarly, teachers’ enactments that aligned with the author’s intentions did not necessarily follow the literal text.

These examples reveal the complexity of the purpose and roles of the written curriculum, an assumption that should not be taken for granted. As researchers and curriculum developers, it is important to address this assumption explicitly in order to understand the relationship between the written curriculum and teachers and to anticipate how teachers may use the text. Stein, Remillard, and Smith (2007) unpack this process of the relationship between the teacher and the written curriculum in a progression from written curriculum to intended curriculum to enacted curriculum.

**Phases of the Curriculum Enactment Process**

Mary Kay Stein and colleagues (2007) propose a curriculum framework that depicts the curriculum enactment process as beginning with a written curriculum (see Figure 1). The teacher interacts with this written curriculum and forms the teacher’s intended curriculum. This represents the first transition during the entire enactment process: the written curriculum transitioning into the teacher’s intended curriculum. The teacher’s intended curriculum may look different from the written curriculum for a variety of reasons. In fact, several studies have documented teachers using the same textbook, but implementing very different lessons (Brown, 2009; Brown, Pitvorec, Ditto, & Kelso, 2009; Collopy, 2003; Lewis & Blunk, 2012; Thompson & Senk, 2014). These outcomes are likely due to differences in the teacher’s intentions for students compared to the author’s intentions for students (e.g., teachers’ beliefs about mathematics, about
teaching, and about students; teachers’ mathematical content knowledge; etc.). The
teacher may not understand the purpose of the task proposed in the written curriculum or
may disagree with that purpose for her own students, and thus change the task or omit the
task entirely. The teacher may also have different learning goals for students or sequence
the content differently than the written curriculum (e.g., Chval, et al., 2009; Remillard &
Bryans, 2004). There are a variety of reasons for why the teacher’s intended curriculum
may look similar to or different from the written curriculum.

Figure 1. Stein, Remillard, and Smith’s (2007) curriculum framework.

Framework depicting the curriculum enactment process and factors influencing
each phase.
Following this transition, the teacher’s intended curriculum then transforms into the enacted curriculum once the teacher begins teaching the lesson. I define teaching to mean the ways in which students and the teacher interact around mathematical content (Morris & Hiebert, 2011). Defining teaching in this way means that students play a key role in the enacted curriculum because they are involved in the interactions. As such, students may “disturb” the transition from the teacher’s intended curriculum to the enacted curriculum (e.g., Stein, Grover, & Henningsen, 1996; Stein & Lane, 1996). In other words, students questioning the teacher or engaging in a task in unexpected ways may disrupt the teacher’s plans and aims during enactment. Therefore, the teacher’s intended curriculum might look very different compared to the enacted curriculum. In conclusion, the curriculum transforms along this process and may change at each transition point for various reasons (see Stein et al., 2007 for a discussion of influencing factors at each node).

The final transition involves the enacted curriculum affecting student learning. The curriculum that students experience during the teaching of the lesson is the curriculum that will directly affect their learning. Therefore, the written curriculum is not a fair representation of students’ learning because the written curriculum will have transitioned through these different phases before reaching students and may be a very different curriculum from what the students experience.

**The Intended and Written Curriculum**

Because mathematics teachers across all grade levels have been found to rely heavily on the textbook (Mary Kay Stein et al., 2007), the transition between the written
curriculum and the teacher’s intended curriculum is important to study in depth. If we want to influence teaching practices through curriculum materials, then we must seriously examine the relationship between the written curriculum and the teacher’s intended curriculum and how teachers *use* the written curriculum.

Remillard (2005) suggests that the teacher’s lesson plans can be considered the teacher’s intended curriculum because these plans represent the teacher’s intentions. However, because lesson plans are a written artifact, I would consider lesson plans as a type of written curriculum. Yet this written curriculum is not the same as the written curriculum the teacher used to write these lesson plans. Therefore, it seems helpful to make this distinction and clarify between the *author’s* written curriculum and the *teacher’s* written curriculum. In my view, the teacher’s intended curriculum exists in his/her mind and it would be impossible to write down all of the teacher’s intentions. I regard the intended curriculum as a sort of “black box” that is difficult to uncover. Therefore, this suggests that we must further differentiate between what is a “written” curriculum and an “intended” curriculum.

A written curriculum consists of a written artifact, or written text, that articulates in general the content, learning goals, and sequencing of such content for students’ learning. An intended curriculum represents someone’s intentions for students in terms of the content, learning goals, and sequencing of such content. The “intended curriculum” does not necessarily have to be the *teacher’s* intended curriculum. Many people have intentions for students: curriculum developers, reformers, parents, and so on. Since the
focus of this paper is on how curriculum materials can support improvements in mathematics teaching, I suggest that the following distinctions be made.

The curriculum enactment process begins with the curriculum author’s intended curriculum (see Figure 2). The author has some intentions for students regarding the learning of some mathematical content in a particular sequence. The author then translates these intentions into a written artifact. Through translating the author’s ideas into written text, some of the author’s intentions will necessarily be lost due to the nature and limitations of written artifacts (Brown, 2009). Just as a musician cannot fully transcribe exactly how to deliver a quality performance through written sheet music, a curriculum developer cannot fully transcribe exactly how to enact a high-quality mathematics lesson through curriculum materials. The musical performance, like the curriculum enactment, requires a subjective being in the moment to read the audience or students, interact with them, and make adjustments along the way to achieve a quality performance or enacted lesson. These kinds of decisions cannot be simply expressed through writing to cover all possible scenarios as there will always be some uncertainty in teaching (Floden & Clark, 1988). As Carpenter et al. (2004) suggest, teaching “is complex, and complex practices cannot, in principle, be simply codified and then handed over to others with the expectation that they will be enacted or replicated as intended” (p. 10). Therefore, it is impossible to include every intention of the author in a written form. If we want to address how curriculum materials can support improvements in teaching, understanding how to write intentions into a written text in order to support changes in teaching is crucial, along with understanding the limitations in doing so.
Figure 2. Modified version of Stein, Remillard, and Smith’s (2007) framework.

This framework includes an additional phase at the beginning and distinguishes the author’s written and intended curriculum from the teacher’s written and intended curriculum.

After the curriculum author translates his or her intentions into a written text, this text is then given to teachers. The teacher then interacts with the author’s written curriculum and produces the teacher’s intended curriculum. This is the transition that Stein and colleagues begin with in their framework.

Following this transition, the teacher may or may not write down lesson plans. These lesson plans represent the teacher’s written curriculum. In writing these lesson plans, the teacher’s intentions may change. Therefore, the transition between the teacher’s intended curriculum and the teacher’s written curriculum is not necessarily a linear process. They likely continually depend upon each other and operate in a circular process. In addition, the teacher’s lesson plans are typically not the starting point of the next transition phase because they are not typically directly given to anyone else. Thus, the teacher’s intended curriculum and the teacher’s written curriculum can be considered
a set of intertwined intentions for students—where one version exists completely in the mind, and part of these intentions exist within the written artifact.

The teacher’s intended curriculum then transforms into the enacted curriculum. Finally, the enacted curriculum influences student learning. Notice in Figure 2 that the curriculum enactment process moves the curriculum author’s intentions to a written artifact and then from the teacher’s intentions to an enacted lesson. There is a constant transition between someone’s intentions and the actions that result from those intentions. It is difficult to include all of one’s intentions in a written artifact. Likewise, it is difficult to implement a lesson perfectly aligned with one’s intentions due to the complex interactions between teachers, students, and the mathematical content of a lesson. As a result, the curriculum that authors hope for students to experience can sometimes be very far from the curriculum that students experience.

In order to address the question of how to provide equal access to a quality mathematics education for all students, thinking of the curriculum transforming through these different phases may help to understand the relationship between teachers and the written curriculum. In order to provide students with an equal access to a quality mathematics education, all students should be offered high quality learning opportunities where they have opportunities to productively struggle, make connections between concepts and procedures, and reason about mathematical ideas in order to develop mathematical understanding. In addition, students should have opportunities to practice procedures and apply their understandings to familiar and unfamiliar problems in order to develop procedural fluency and complete mathematical competency. In order to offer all
students these kinds of learning opportunities, it is helpful to think about key stakeholders’ intentions and how those intentions lead to actions.

The two stakeholders considered within this framework are curriculum authors and teachers. However, there are a multitude of stakeholders that can be added onto this framework to extend our ideas in thinking about how to offer equal access to all students. I narrow the focus to curriculum authors and teachers because curriculum authors are well positioned in the United States to support teachers in some direct way, while teachers are positioned to directly support student learning. I will explain next how this framework can help address the question of how to offer all students a high quality mathematics education.

First, regarding the curriculum author as a key player within the curriculum enactment process emphasizes the fact that the written curriculum is produced by a subjective being with certain intentions, beliefs, values, and goals for students (and teachers). Understanding the curriculum author’s intentions as apart from the written curriculum will help us to realize that the written curriculum may or may not fully represent the curriculum the author hopes for students to experience. For example, the curriculum author may hope for students to engage in reasoning and argumentation around whether the quadratic formula can be applied to solve any quadratic equation; yet the written curriculum may not fully represent this intended learning opportunity. For example, if the written curriculum suggested that the teacher ask students whether the quadratic formula can be used to solve any quadratic equation, it is not clear whether every student in class would have an opportunity to reason about this question and to
argue and defend their position. Therefore, conceiving of the author’s intentions as a separate entity from the written curriculum can help us to explore whether curriculum authors agree with reform beliefs and values and/or whether the written curriculum does not manifest those beliefs and values in terms of students’ learning opportunities.

If curriculum authors’ intentions align with reform intentions, then the focus of reform efforts can be on supporting curriculum authors to fully convey those intentions and represent those intentions appropriately through written text. If curriculum authors’ intentions do not align with reform intentions, then the focus of reform efforts can be to convince curriculum authors of the value of reform intentions. This difference is a key difference in addressing the question of equal access to a quality mathematics education for all students.

Secondly, teachers are also subjective beings in this process who bring their own set of beliefs, values, intentions, expectations, and expertise while planning and enacting their lesson. By viewing the teacher’s intentions as a separate entity from the written curriculum, we can study the ways in which written curricula support teachers to offer students equal access. We can address whether efforts should focus on altering teachers’ beliefs, values, expectations, intentions, and so on, and/or whether efforts should focus on supporting teachers with more content knowledge, pedagogical knowledge, knowledge of students, and so on in order to support their enactments. These differences that can be seen as different phases of this curriculum framework can help focus our efforts on the transitions that need more support and attention. This framework provides a visualization
of intentions transitioning throughout this system and vocabulary to help pinpoint exactly where and in what ways curriculum authors and teachers require support.

Furthermore, this vision of curriculum helps us to accept that conceiving of the role of curriculum as a blueprint for teachers is unproductive. This conception is unproductive because teachers and curriculum authors have their own beliefs which may or may not align. Thinking about teachers as following the authors’ suggestions the way a carpenter would follow a manual to assemble a table does not take into account the fact that teachers’ beliefs and expectations may lead the teacher to a different understanding of the intended learning opportunities for students. The carpenter’s beliefs do not significantly impact his decisions or abilities to follow the manual to assemble a table. However, a teacher’s beliefs do significantly affect students’ learning opportunities.

Furthermore, the role of curriculum as a blueprint does not take into account teachers’ expertise. This view does not value teachers as curriculum designers themselves and consider moments where the written curriculum does not anticipate students doing something unexpected. This view does not support teachers in these moments of uncertainty if they are solely expected to follow the curriculum. Using the worker analogy, assembling the table comes with everything needed and there are hardly ever any uncertainties within the process of assembling the table. Using curriculum as a blueprint to follow may support teachers to set up tasks ambitiously but does not support teachers in moments of uncertainty or unanticipated moments where the teacher will have to make a decision while teaching. Therefore, viewing the role of curriculum as a blueprint to follow is unproductive.
Considering the role of the curriculum materials as a resource or an archive to store and improve knowledge about teaching is more productive. These views allow for the conception of the teacher as a curriculum designer (Brown, 2009) and place more authority with the teacher. In particular, conceiving of the curriculum as a resource allows us to study how teachers use this resource, whether they gather additional resources to support their enactments and why. This conception helps us to address the question of how teachers interpret text which helps us to understand how the teacher’s intended curriculum develops while using the written curriculum as a resource of support. Viewing the written curriculum as containing learning opportunities for teachers can help to study whether teachers need to be provided with certain learning opportunities through text in order to offer all of their students high quality learning opportunities.

Likewise, envisioning the written curriculum as part of an archive can help to incorporate teachers’ expertise into the written artifacts. This conception values teachers’ knowledge and focuses on how to use that knowledge to improve the written artifact over time. By using teachers’ enactments as a means of empirical evidence, the written curriculum can be improved to include productive ways the teacher responds to student questions, anticipations for what students will say and do throughout the lesson, and possible misconceptions and difficulties that may need to be addressed during the lesson. Viewing the role of curriculum as an ongoing artifact that can be continually improved allows us to study the continuous effects of a written resource in supporting teachers to offer all students high quality learning opportunities. This conception also includes studying the kinds of curricular features that support teachers. In other words, this
conception focuses on understanding the kinds of learning opportunities teachers are provided via text and whether those learning opportunities lead to teachers providing equal access to a quality mathematics education to all students.

While these conceptions may be more productive than the role of curriculum as a blueprint, they are not without issues. These conceptions open many conversations about nontrivial assumptions and how we can support teachers. For instance, if curriculum authors assume that the teachers will use the text as a resource and will interpret the text in different ways, then how can we expect teachers to deliver similar high quality lessons to students in order to promote equal access to education? Like Michael Brown’s analogy, sheet music might be used as a resource or a guide, and different musicians might play the song that is written a little differently, but in the end the musicians are all still playing the same song. How can this become a reality in teaching? If we consider the written curriculum as a resource that teachers will interpret differently, how can we support them to deliver high quality lessons to students and essentially “play the same song?” One suggestion that may help to answer this question is to narrow our lens of the curriculum enactment process to consider the construct of learning opportunities transitioning through the temporal phases of the process.

**The Curriculum Enactment Process from a Narrower Perspective**

In this section, I expand upon Stein, Remillard, and Smith’s (2007) theoretical framework for the curriculum enactment process by narrowing the view of curriculum transitioning through the temporal phases to considering learning opportunities transitioning through the temporal phases. As a reminder, the critical question is: How do
we help to ensure that all students receive equal access to a quality mathematics education? Curriculum materials are part of the answer to this question because they influence teachers and teaching practices, who in turn influence students. Therefore, to deeply address this question, we must consider the relationship between curriculum materials and teachers. I will first define precisely what I mean by equal access and learning opportunities.

Equal access to education means that all students receive similar learning opportunities. I define learning opportunities in ways similar to how Stacy Brown and colleagues and Jim Hiebert have defined opportunity to learn (Brown et al., 2009; Hiebert, 2003; Hiebert & Grouws, 2007). As Hiebert (2003) explains, providing an opportunity to learn means “setting up the conditions for learning that take into account students’ entry knowledge, the nature and purpose of the tasks and activities, the kind of engagement required, and so on” (p. 10). A learning opportunity is therefore more than just divulging information to students; a learning opportunity takes into account a student’s current knowledge and how to help that student achieve a learning goal. Hiebert and Grouws (2007) explained that a second-grader would not have an opportunity to learn calculus even if a teacher delivered an excellent lecture on calculus to the child because the child is not yet ready to learn calculus. A learning opportunity therefore considers the student’s zone of proximal development (Norton & D’Ambrosio, 2008) and sets up the student to construct knowledge within or just outside of that zone.

A learning opportunity takes into account two main objectives: (a) what content students are aiming to learn and (b) how students are expected to engage in learning that
content. For example, students may have an opportunity to learn about adding fractions by using fraction strips and completing several exercises involving adding fractions. The content in this case would be adding fractions, while the engagement expected would be students using fraction strips to model several problems. Under this definition of learning opportunities, I define *equal access* to education to mean that all students receive similar learning opportunities.

My perspective of the curriculum enactment process is thus depicted in terms of learning opportunities (see Figure 3). To begin the process, the curriculum author has some intended learning opportunities in mind. These intended learning opportunities become operationalized in the form of the written curriculum. The teacher then interacts with the written curriculum and develops intended learning opportunities for students (part of the teacher’s intended curriculum). The teacher then provides certain learning opportunities for students (part of the enacted curriculum). The process ends with students taking advantage of (or not taking advantage of) those learning opportunities for various reasons.

To focus on equal access, I propose viewing the curriculum enactment process from the perspective of learning opportunities in every phase. The goal is for similar learning opportunities to be provided to students. Therefore, the goal is for the enacted curriculum to look the same *in terms of learning opportunities* across different teachers. This construct is analogous to musician’s playing the same song even though it may sound somewhat different across different musicians. In the end, if students receive
similar learning opportunities, this is the crucial determining factor influencing what students learn.

Learning opportunities are different from the construct of curriculum because the unit of analysis is smaller. The way I define a learning opportunity coincides with the grain size of a day’s lesson, whereas the construct of curriculum can span an entire course or even several grade levels. While this broader view is still important, I wish to place the spotlight on the construct of learning opportunity as a complimentary smaller grain size that may help to provide a fuller picture (and hence answer) to the question of how curriculum materials can support improvements in mathematics teaching.

The construct of curriculum also tends to place emphasis on what content to cover, as opposed to also valuing how that content is “covered.” This emphasis is problematic for several reasons, but I will explain two primary reasons. First, “covering” content does not necessarily take into account whether students are ready to study that
content. Any teacher can “cover” content by lecturing about a specific topic, just like a teacher can give a great calculus lecture to a second grader. However, this does not consider the student’s zone of proximal development and whether the student is prepared to engage in learning that content. Yet the construct of learning opportunity does take into consideration the student’s current level of knowledge because it focuses on whether students have an equitable opportunity to learn that content. Therefore, the emphasis on covering the curriculum fades when considering learning opportunities.

Second, because curriculum has the tendency to focus on “what” is being covered, the question of “how” that content is covered is typically undervalued. Yet how content is covered is equally as important as what content is covered. Engaging students in learning about fraction multiplication by showing an example of the procedure for multiplying the numerators and denominators to obtain the final answer and then requiring students to practice the procedure on 50 exercises is a very different enactment compared to engaging students in learning about fraction multiplication by folding pieces of paper to visualize how the multiplier acts upon the multiplicand to produce the product and why this process always leads to the procedure of multiplying the numerators and denominators to obtain the product. Many of us would agree that these are very different ways to enact a lesson on multiplying fractions. However, curriculum does not tend to consistently capture this difference. For some who study and write curriculum, how content is taught is equally as important as what content is taught, but this has not always been the case. With a view on learning opportunities however, it is impossible not to
consider both “how” and “what” because how content is taught changes a student’s learning opportunity.

Because of these differences, I believe studying the construct of learning opportunities and how learning opportunities transition through the curriculum enactment process would progress the field of curriculum studies forward in further unpacking the relationship between teachers and curriculum materials and promote novel ways of thinking about how curriculum materials can support improvements in mathematics teaching. To demonstrate the usefulness of this construct, I will discuss how to view each temporal phase of the curriculum enactment process (see Figure 3) through the construct of learning opportunities. The process begins with a curriculum author’s set of intended learning opportunities for students, which then transform into the author’s written learning opportunities for students and teachers, which then transform into teacher’s intended learning opportunities for students, and ends with the enacted learning opportunities students experience. Each phase of this transformation will be described in more detail next.

The Author’s Intended Learning Opportunities

Beginning with the author’s intended learning opportunities, the author develops activities, problems, and exercises for students that he/she believes align with a certain set of learning goals. The learning opportunities consist of what students will learn and how they are expected to engage in learning. Therefore, the activities that the author develops for students are a direct manifestation of the learning opportunities the author intends for students to experience. However, the activities alone do not completely
describe students’ learning opportunities. The rest of the picture relies on the how the activities are set up and implemented, as Stein and colleagues elaborate (Stein, Grover, & Henningsen, 1996; Stein & Lane, 1996; Stein & Kim, 2009). The ways in which tasks are set up and implemented involve how students are expected to engage in learning that content.

For instance, tasks can be set up at a high cognitive demand level for students or a low cognitive demand level for students (e.g., Stein & Lane, 1996). An example of a task involving high cognitive demand would be having students productively struggle (Hiebert & Grouws, 2007) to make connections between the visual process of multiplying fractions and the procedure of multiplying the numerators and denominators to determine the product. An example of a lower cognitively demanding task would involve the teacher showing students the procedure for multiplying fractions and asking students to repeat this procedure on a series of exercises. These two enactments contrast each other due to the way students are expected to engage in learning the content (in this case, multiplication of fractions). The tasks given in a written curriculum for this topic could have been identical, yet the way teachers expect students to engage in learning the topic are vastly different. This difference changes the learning opportunity. Therefore, the author’s intended learning opportunities are manifested through the activities the author plans for students and how the author intends for those activities to be set up by the teacher.

This implies that the author must speak to the teacher through the written text in order to explain the author’s intentions for students to the teacher. In providing
information for the teacher, the author is providing the teacher with learning opportunities as well. The author likely has a different set of intended learning opportunities for teachers than for students. Perhaps the author intends for teachers to have an opportunity to visualize possible student work to help the teacher understand the intended learning opportunities for students. Therefore, the curriculum author translates his or her intended learning opportunities for both teachers and students into a written text.

**The Author’s Written Learning Opportunities**

The written curriculum should represent the author’s intentions. The written curriculum is crucial because it communicates the author’s intended learning opportunities to the teacher. During this translation from the author’s mind to physical artifact, some of the author’s intentions will be lost. Given the above examples of how a task can be set up differently, to maintain the main aspects of a learning opportunity, the written curriculum should not only include the content that the author intends for students to learn, but also the ways in which the author intends for students to engage in learning the content and why. This might include directions for how to set up a task in certain ways, such as “allow students to work on problems 1 thru 3 on their own; they are expected to have to struggle productively while working so try not to answer their questions” and advice for teachers for how to enact the task while students are working (e.g., how to respond to student questions).

These two aspects of a learning opportunity are the foundations of expressing the author’s intended learning opportunity. However, these are not the only features that need
to exist in a written curriculum. Because teachers are subjective, active players in the enactment process, teachers will bring their own values, beliefs, and knowledge to the table when interacting with the written curriculum. Therefore, the written curriculum represents more than just the author’s intended learning opportunities; it represents an opportunity for the teacher to learn.

If certain features exist in the written curriculum, the teacher may be able to expand his/her knowledge and/or change or solidify his/her beliefs and values. The written curriculum has the opportunity to help the teacher learn about content, pedagogy, students, the author’s intended learning opportunities for students and why the author intends to provide those specific learning opportunities for students. To provide rich learning opportunities for the teacher, the curriculum author should make his or her intentions and rationales for those intentions visible and explicit. Curriculum materials designed in this manner have been referred to as transparent curricula (Stein & Kim; 2009) because the goal is for the author to make his/her intentions as transparent as possible. Clarifying these intentions focuses on articulating the author’s deep, underlying assumptions and decisions that he or she made regarding the content, learning goals, tasks, sequencing and so forth for students. This will empower the teacher to adapt the curriculum if necessary with the same learning opportunities in mind for students as the curriculum authors’ intended learning opportunities.

Since teaching always involves uncertainties (Floden & Clark, 1988), transparent curricula which offer teachers rationales for certain decisions may help teachers to make decisions in the face of said uncertainties. Thus, transparent curricula give teachers more
authority over content, value their professional judgement about content and their students, and acknowledge their role as curriculum designers (Brown, 2009). As Stein and Kim (2009) lament,

Just as students need connection to meaning in order to perform well under situations of uncertainty, so do teachers. A teacher who follows a set of activities for which the rationale or purpose is not apparent can be viewed as acting just as mechanically as a student who follows an algorithmic procedure without connecting it to underlying concepts. When students experience procedures in this way they become prisoners of them, not understanding when and how to apply them in novel situations or how to respond when they fail them (p. 51).

The kinds of curricula that afford teachers these kinds of learning opportunities are best characterized as educative curricula (e.g., Davis & Krajcik, 2005). Educative materials are hypothesized to contain written features such as precise learning goals, explicit rationales, anticipated student comments and questions, along with helpful ways to respond to students and why. These kinds of features provide learning opportunities for the teacher to learn about the goals of the lesson, the purpose of the task, how students are expected to engage in learning, why the goals, purpose, and task are important, and why the task will help students to achieve the learning goals. These kinds of features are most likely to influence teachers’ opportunities to learn from the text. However, to date, we know little about exactly what kinds of features make a curriculum “educative” (Davis & Krajcik, 2005; Drake, Land, & Tyminski, 2014).
The Teacher’s Intended Learning Opportunities

The next phase of the enactment process is the teacher’s intended learning opportunities for students. As previously explained, the written curriculum represents an opportunity for teachers to learn. Thus, by interacting with the written curriculum, the teacher has the potential to learn more about content, pedagogy, student thinking, the author’s intended learning opportunities, and so on. As the teacher likely possesses his/her own beliefs about content, students, and pedagogy, the teacher’s views, values, and beliefs can be affirmed or challenged through interacting with the text. Perhaps the text contains anticipated student misconceptions about a topic that the teacher has witnessed in prior teaching experiences. Or perhaps the text contains anticipated student misconceptions that the teacher would not expect because he/she has never witnessed them during prior teaching experiences. The text may align with the teacher’s views and beliefs or the text may contradict the teacher’s views and beliefs.

The teacher’s intended learning opportunities consist of the same features of the author’s intended and written learning opportunities discussed previously: the activities, how to set up the activities, the purpose of the activities, the learning goals for students, etcetera. The curriculum author has an opportunity to influence the teacher’s intended learning opportunities through the written curriculum. If the written curriculum contains certain features such as those educative features described above, it is more likely that the teacher’s intended learning opportunities will align with the author’s intended learning opportunities. The more features that influence the learning opportunities for students that are included in the written curriculum, the more likely the teacher’s intended curriculum
will include many of those intentions. Thus, the more likely the teacher will develop similar intended learning opportunities for students compared to the curriculum author’s intentions.

**The Enacted Learning Opportunities**

The enacted learning opportunities involves both the teacher and students. These learning opportunities are partially controlled by both players. The part controlled by the teacher represents the teacher’s attempt at enacting his/her intended learning opportunities for students. The part controlled by the students (engagement in learning the content) may alter the teacher’s fidelity to his/her own intend learning opportunities. The enacted learning opportunities consists of the mathematical content the activity addresses, the ways in which the activity was set up, and how students engage with the activity.

All of these events in the classroom come together to form the students’ learning opportunities. A question asked by the teacher may subtly change the learning opportunity. The way the task is set up by the teacher may change the learning opportunity. The expectations the teacher has of the students can change the learning opportunity. All of these kinds of actions may change either the content students are expected to learn or the way students are expected to engage in learning that content, which changes the learning opportunity for students. Therefore, the enacted curriculum contains complex events that interact with one another simultaneously to influence students’ learning opportunities.
As teachers and students navigate the enacted curriculum together, both players influence the learning opportunities for students. When the students engage in the learning opportunity in ways undesired by the teacher, the teacher must somehow adapt the enactment (sometimes by introducing an entirely new task or sometimes with a simple question as the students work) to redirect the student towards the desired learning goals. In addition, it is possible for the teacher to unknowingly adapt the lesson (in productive or unproductive ways) depending on how the student responds to the teacher’s actions, which brings us to the final stage in the curriculum enactment process: student learning.

**Student Learning**

The final phase in the enactment process concentrates on how students take up (or do not take up) the learning opportunities provided to them during enactment. Student learning occurs when the learning opportunities provided to students are fair and students take advantage of the those learning opportunities. If the learning opportunities are fair, then each student would be ready to learn and have an opportunity to engage in learning the same content. A learning opportunity would be unfair if students were not ready to learn due to having not met certain prerequisite skills or knowledge. In other words, if they are not mathematically prepared to engage in learning the content, then having a learning opportunity presented to them would be unfair.

If the learning opportunity is fair, students may take advantage of these opportunities by engaging with activities in ways intended or unintended by the teacher. The way the student engages in the task promotes certain kinds of learning. Note that I
define equity not in terms of student learning but in terms of students’ learning opportunities. The main reason for this distinction is because we, as teachers, researchers, and curriculum-developers have some control over the enacted learning opportunities and can strive to reduce variation in the learning opportunities provided to students. However, in the end, the students have control over whether they take advantage of these opportunities. Those who choose not to take advantage of said learning opportunities still had equal access to those opportunities. The challenge is getting to the point where all students are provided with similar learning opportunities and have equal access to engaging in those learning opportunities.

Conclusion

Equity of education is one of the most important issues facing our society today. I defined equity to mean equal access to education, and I define equal access to mean that students are provided with similar high-quality learning opportunities. This means that if all students were to have equal access to mathematics education, then all students would be afforded the opportunity to learn similar content in similar cognitively demanding ways. However, not all students are afforded the same quality of education in mathematics, which limits some students’ access to careers in fields such as science, technology, engineering, and mathematics (STEM). Therefore, this is an issue that deserves substantial attention from researchers, teachers, curriculum developers, parents, and policy-makers.

I propose that one way of partially addressing this issue is by considering how the written curriculum can support improvements in mathematics teaching. One way to
address this question is to focus on learning opportunities transitioning through the curriculum enactment process. An emphasis on learning opportunities can help to narrow analyses regarding the relationship between the curriculum and teacher. Thinking about how to design transparent, educative curricula in order to provide teachers with rich learning opportunities through the text is one way to progress the field. A view on learning opportunities also makes it easier to discuss how to help achieve equal access to a quality mathematics education for all students. It is my hope that the proposed modified framework will benefit curriculum developers, researchers, policy-makers, and reformers in explicitly thinking about how to provide learning opportunities to teachers and students to improve mathematics teaching on a wide scale.
Chapter 2

SUPPORTING NOVICE MATHEMATICS TEACHER EDUCATORS TO TEACH AMBIOUSLY VIA CONTINUOUSLY IMPROVED CURRICULUM MATERIALS

Students’ mathematical achievement is related to the types of learning opportunities they receive during instruction (Hiebert et al., 2005; Stein, Grover, & Henningsen, 1996; Stein & Kim, 2009; Stein & Lane, 1996). Furthermore, students who have opportunities to productively struggle or grapple with mathematical ideas and make connections between mathematical concepts and procedures have been shown to achieve greater mathematical understanding (Hiebert & Grouws, 2007; Mary Kay Stein & Lane, 1996). Therefore, students who are not afforded these kinds of learning opportunities can be considered not as mathematically prepared as their peers who do receive these kinds of learning opportunities, which is inequitable.

While many factors influence the kinds of learning opportunities students receive, the tasks students engage in during class have a substantial impact on students’ learning opportunities (Doyle, 1983, 1988; Hiebert & Wearne, 1993; Wilhelm, 2014). When a task affords a higher level of cognitive demand for students, students are likely to achieve deeper mathematical understanding (e.g., Stein & Lane, 1996). According to Stein and Lane (1996), cognitively demanding tasks involve connecting concepts to procedures,
applying procedures to non-routine problems, or disciplinary activities such as justification, explanation, or analysis. Tasks involving lower levels of cognitive demand typically involve applying procedures to routine problems without making connections to underlying concepts or pure memorization.

In order to achieve equitable education, all students should be afforded similar, high-quality learning opportunities. Since the task greatly influences students’ learning opportunities, some may conjecture that one way of achieving equitable education is to provide all students with the same task. However, students may not be equally mathematically prepared to engage in the task, perhaps due to differences in students’ zones of proximal development (Norton & D’Ambrosio, 2008). In other words, the same mathematical task may be extremely challenging for some students and fairly easy for other students within the same classroom. This implies that the same task for students may not provide all students with similar high-quality learning opportunities. Hence, it is no easy feat to provide students with the similar, high-quality learning opportunities.

One way to improve upon this potential idea (of providing all students with the same task with the hopes of affording students similar, high-quality learning opportunities) is by supporting teachers to respond productively to students as students work on the task. Since the teacher mediates the relationship between the task and students’ learning, the teacher plays a key role in influencing students’ learning opportunities. Indeed, teachers (and students) have been found to change the learning opportunities for students as students work on the task (Hiebert et al., 2005; Mary Kay Stein & Lane, 1996; Wilhelm, 2014). For example, when students question the teacher
regarding the task, the way the teacher responds has the potential to possibly raise or lower the cognitive demand of the task for students. Therefore, the cognitive demand level intended by the task is not always the cognitive demand level experienced by students during enactment.

The fact that the cognitive demand of the task can be changed by how the teacher responds to students as they work implies that responding to students’ questions, comments, or work is an important, nontrivial skill for teachers. A productive response by the teacher should maintain the cognitive demand level of the task; align with the learning goal(s) of the task; align with the learning theory of the course; and involve mathematically correct language, symbols, and/or expressions. An unproductive response by the teacher would violate at least one of those aspects (i.e., a response that does not align with the learning goal(s) of the task would be considered unproductive).

I hypothesize that teachers may respond differently based on whether the student question/comment was anticipated or unanticipated by the teacher prior to the enactment. If the question/comment was anticipated, teachers are more likely to respond productively to the student. If the question/comment was unanticipated, teachers are less likely to respond productively to the student. Because not all questions/comments can be anticipated in advance, the teacher is expected to have to make some adaptations to lesson as the lesson unfolds. However, being able to adapt the lesson in the face of unanticipated student questions/comments is a very challenging skill. These kinds of challenging skills have been captured under the construct of ambitious teaching.
Ambitious Teaching

“Ambitious teaching requires that teachers teach in response to what students do as they engage in problem solving performances, all while holding students accountable to learning goals that include procedural fluency, strategic competence, adaptive reasoning, and productive dispositions” (Kazemi, Franke, & Lampert, 2009, p. 11). Thus, one aspect of ambitious teaching involves responding to students’ questions and comments in productive ways. This would involve responding to students in different ways based on how the student is engaging in the task, the knowledge the student currently has, and the goals of the task. Furthermore, teaching ambitiously requires that teachers productively adapt the lesson when necessary, such as in the face of unanticipated student questions, comments, or work.

In order to hold students accountable to learning goals that promote procedural fluency, conceptual understanding, adaptive reasoning, and productive dispositions towards mathematics (Kazemi et al., 2009; Lampert et al., 2013; Lampert, Beasley, Ghousseini, Kazemi, & Franke, 2010), students must first be afforded tasks that promote these kinds of learning opportunities. A necessary component of ambitious teaching thus involves students working on cognitively demanding problems—problems that involve connecting concepts to procedures, and problems that promote justification, explanation, and reasoning. Thus, to teach ambitiously, teachers need to productively set up cognitively demanding tasks.

Because ambitious teaching involves the interactions between teachers and students when students are engaged in cognitively demanding problem-solving tasks,
there are two important enactment components that define this kind of teaching: task set-ups and student-teacher interactions. These two components align with Stein and Lane’s (1996) components of measuring the cognitive demand level of tasks as they are enacted: (a) task set-ups and (b) task implementation (interactions between the teacher and students as students work). In order to engage in ambitious teaching, teachers must set up tasks in a productive manner and respond to students in a productive manner. As such, I narrow my analyses in this study to focus solely on these two aspects of ambitious teaching.

When students work on these types of mathematical tasks in a sustained and deep manner (as opposed to quickly executing procedural skills and memorized facts), the types of classroom interactions shift from the common cultural interactions seen in many US mathematics classrooms to different kinds of interactions. In particular, students begin to do most of the cognitive work through discovery or inquiry learning. In doing so, they tend to struggle and look to the teacher to alleviate that struggle. The way the teacher responds has the potential to either promote high cognitive demand levels in ways aligned with ambitious teaching or to lower the cognitive demand levels and digress back to the commonly accepted cultural norms in US mathematics classrooms (Hiebert et al., 2005). Since it is more common in the US for mathematics teachers to respond in ways that lower the cognitive demand levels (Hiebert et al., 2005; Mary Kay Stein et al., 1996; Mary Kay Stein & Lane, 1996), it is crucial that we seek ways to support US mathematics teachers to set up high-quality tasks and respond productively to students as students work.
While there are many reasons for why teachers struggle to teach ambitiously (e.g., teachers’ content knowledge, and teachers’ beliefs about student learning), the factor I focused on is the level of support provided to teachers once in the field. By support, I mean factors that help teachers develop knowledge, beliefs, skills, and competencies that enable them to teach ambitiously and thus provide rich, high-quality learning opportunities to students. Teachers can be supported through many forms: professional development, coaching, curriculum materials, and so on. For this study, I focused on curricular support for teachers.

**Purpose of Study**

The purpose of this study was to investigate a unique set of curriculum materials that have been continuously improved by and for mathematics teacher educators for an undergraduate elementary mathematics content course. This curriculum was written and improved with two foundational learning opportunities in mind for each lesson: pre-service teachers should have opportunities to productively struggle with mathematical concepts and to make explicit connections between concepts and procedures (Hiebert & Grouws, 2007; Morris, 2012). The tasks in this curriculum were designed to be cognitively demanding by engaging pre-service teachers in nonroutine problems, making connections between concepts and procedures, and focusing on analyzing, explaining, and justifying solution strategies (Stein & Lane, 1996).

This study explored the intentions and enactments of two instructors teaching a course *for the first time* using this set of curriculum materials. Although the participating instructors had experience teaching high school mathematics, neither instructor had
taught an elementary content course, or elementary mathematics, or worked with pre-
service teachers (PSTs), or used a curriculum such as this one prior to this experience.
Thus, for the purposes of this study, both instructors can be considered “novice”
 mathematics teacher educators (MTEs) regarding their experiences teaching this specific
course. The main purpose of this study was to investigate whether this unique set of
curriculum materials could support two novice MTEs to teach ambitiously.

As mentioned previously, I focused on the two primary components of ambitious
teaching: task set-up and responses to pre-service teachers. Because my hypotheses also
included whether PSTs’ comments, questions, and work were anticipated by the
curriculum materials, my research questions reflect this aspect as well. The main research
questions were: (a) Do novice MTEs set up tasks productively when using this
curriculum? (b) Do novice MTEs respond productively to anticipated PSTs’ questions,
comments, or work? (c) Do novice MTEs respond productively to unanticipated PSTs’
questions, comments, or work? And (d) What features of this written curriculum appear
to predominantly influence how novice MTEs set up tasks and respond to PSTs? Taken
as a whole, this set of questions focuses on whether features of this unique curriculum
can support novice MTEs to teach ambitiously.

The focus of this paper draws primarily on literature regarding curriculum and
continuous improvement systems because the curriculum materials under study have
been improved via continuous improvement methods. Therefore, I begin by briefly
reviewing the literature base of these two fields. Because this study also focuses on
teaching and the connections between teaching and curriculum materials, I will also
discuss the construct of fidelity. Finally, I will review the differences between
experienced teachers and novice teachers and explain why I chose to focus on novice
mathematics teacher educators.

Literature Review

Conceptual Framework of Curriculum

By curriculum materials, I refer to any set of written materials that a teacher or
instructor might use to plan a lesson, such as textbooks, lesson plans, teacher guides,
student workbooks, etc. I conceptualize different kinds of curriculum as Remillard (2005)
does as either a written curriculum, an intended curriculum, or an enacted curriculum. I
use the terms written curriculum, curriculum, and curriculum materials interchangeably,
referring broadly to a set of written artifacts teachers might use during lesson planning.
The intended curriculum refers to someone’s intentions for students (likely either the
curriculum author’s intentions or the teacher’s intentions for students). Finally, the
enacted curriculum refers to the implementation of the lesson, the way the lesson is
enacted by both the teacher and students.

The curriculum enactment process begins with the curriculum author’s intentions
which transforms into a written curriculum. The teacher then interacts with the written
curriculum to produce the teacher’s intended curriculum for students. The teacher’s
intended curriculum then transforms into the enacted curriculum. This conceptual
framework depicting the curriculum enactment process and the temporal phases and
transitions of curricula guide this research study (see Suppa, in preparation for a more
detailed explanation of this theory).
In the US, it has been found that many mathematics teachers rely heavily on textbooks to plan their lessons (Mary Kay Stein et al., 2007). Yet, several research studies have shown that teachers using the same textbook do not necessarily offer their students similar learning opportunities (e.g., Brown, Pitvorec, Ditto, & Kelso, 2009; Collopy, 2003; Lewis & Blunk, 2012; Thompson & Senk, 2014). In fact, many reform efforts have acted under the assumption that if we can change textbooks then teaching methods will change as a result (Mary Kay Stein et al., 2007). However, while textbooks have seemingly changed substantially, teaching has changed very little (e.g., Baker et al., 2010).

Because of the evidence showing mathematics teachers’ reliance on textbooks, it seems worthy of investigation whether the ways in which textbooks are written and designed, the ways in which the textbook speaks to the teacher, and the content of textbooks has the potential to affect teaching methods. For instance, while the content of mathematics textbooks may have changed substantially, perhaps the ways in which textbooks speak to teachers have not changed. In addition, since teaching is a cultural activity (Stigler & Hiebert, 1999), the textbook may need to be designed and written in drastically different ways to significantly affect teaching.

Some researchers have speculated that in the past, textbook authors attempted to speak through teachers in order to directly impact student learning (Remillard, 2005). This theory does not regard the teacher as an active player in the curriculum enactment process, one who brings her own set of beliefs about the subject, pedagogy, and students to the table when planning lessons. Because teachers actively interpret the text when
engaging with curriculum materials (Remillard, 2005), the ways in which textbooks are designed may be more likely to have a significant impact on teaching methods if textbook authors operate from the stance that they are speaking to teachers. Notably, curriculum materials that have been continuously improved by and for instructors using them tend to speak directly to instructors.

It may also be the case that the kind of content that has been changing in textbooks does not incorporate enough knowledge for the teacher to understand the textbook author’s intentions. For instance, if the textbook only contains mathematical content without much explanation for why this content is important for students, then teaching will likely remain unchanged. Textbooks that fail to include rationales explaining the purposes of each task, the main learning goals being targeted, and pedagogically appropriate ways to target these learning goals leave teachers to infer the purposes, learning goals, and appropriate teaching methods in order to implement the task. As a result, the teacher is unlikely to infer these meanings in substantially different ways than the teacher already implements tasks due to the cultural influences of teaching (Stigler & Hiebert, 1999) and the teacher’s beliefs (Remillard & Bryans, 2004). Therefore, teaching will remain unchanged if the desired change is attempted through the mechanism of curriculum materials, unless curriculum materials are designed differently.

It is notable that the curriculum materials under investigation contain these types of features (e.g., precise learning goals, rationales, suggested ways to set up tasks, and suggested ways to respond to PSTs as they work).
Finally, if textbooks were designed in ways that were *educative* (Davis & Krajcik, 2005), we may be more likely to see a significant change in teaching methods in the United States. Educative curriculum materials speak *to* the teacher and educate the teacher on various content, such as subject matter knowledge, pedagogical content knowledge, knowledge of the students, and knowledge of the tasks (Ball, Thames, & Phelps, 2008). It is hypothesized that educative curriculum materials are more likely to influence teaching methods because the goal is to provide teachers with as much relevant knowledge as possible in order to influence the teacher’s *intended curriculum*, which is more likely to influence the enacted curriculum. This theory differs from prior reform attempts that set out to influence the enacted curriculum directly via the written curriculum (Remillard, 2005). However, the most direct link to influencing the enacted curriculum is the temporal phase that occurs immediately prior to the enacted curriculum, which is the teacher’s intended curriculum.

Educative curriculum materials may be more likely to support changes in teachers’ beliefs (and hence intended curricula) if teachers are given opportunities to learn specifically about content knowledge, pedagogical content knowledge, and how students will likely interact with the content. While the literature on educative curriculum materials is still emerging, I claim that the curriculum materials under investigation in this study can be conceived as educative for MTEs because they contain features such as rationales, anticipated PST comments, questions, and misconceptions, and a history of the changes made to the written materials with associated reasons for each change. These curriculum materials offer instructors opportunities to learn about mathematical content,
pedagogical decisions, pedagogical content (ways to respond to students when students are struggling or confused), and information about the task and how the task has changed over time and why. Therefore, I regard the curriculum materials under study as educative curriculum materials.

**Continuous Improvement Systems**

The curriculum materials in this study were created by and for mathematics teacher educators through a system of continuous improvement. A *system of continuous improvement* of teaching typically involves a network of teachers cooperating together to write, share, and improve lesson plans in a cyclical process (Bryk, 2015; Bryk, Gomez, Grunow, & LeMahieu, 2015; Lewis, 2015; Lewis, Perry, & Hurd, 2009; Lewis, Perry, & Murata, 2006; Morris & Hiebert, 2011; Raudenbush, 2009). The cyclical process in a general sense typically involves planning a lesson, enacting the lesson, studying the enactment of the lesson, and writing changes into the lesson plan to improve the lesson for future enactments.

This process resembles plan-do-study-act cycles or PDSA cycles commonly found in various fields including medicine, automobile industries, and education (e.g., Bryk et al., 2015; Lewis, 2015; Senechal, 2015). PDSA cycles involve planning an intervention (Plan), implementing the intervention (Do), studying the implementation (Study), and then revising elements of the intervention, such as hypotheses or theories, to improve future implementations to obtain desired results (Act). PDSA cycles focus everyone’s attention on a particular goal and involve collaboration with an intentional focus on gradually improving in order to achieve the goal (Bryk et al., 2015). This
systematic method of improvement involves small tests of small changes in order to effectively improve (Hiebert & Morris, 2012; Lipsey, 1993).

A system of continuous improvement around curriculum materials can also be conceptualized as a type of network improvement community or NIC (e.g., Bryk, 2015; Bryk et al., 2015) since teachers (of all levels of experiences and backgrounds) typically work together in a collaborative social network to improve the written curriculum. All members of the system focus on the same goals and gradually improving curriculum materials to achieve those goals. As a result of this community of people collaborating and working towards the same goals, various features of the curriculum will become more and more precise and detailed over time.

For example, the learning goal for a lesson may initially be stated quite vaguely and in informal, general terms (e.g., “Students will understand division of fraction.”). Over time, this learning goal will become more precise and well-defined as participants work to continuously improve the goals of the lesson and how to achieve those goals. This initially vague and ill-articulate learning goal may eventually be decomposed into many smaller learning goals (e.g., “Students will understand the meaning of the reciprocal as how many copies of the divisor fit into 1;” and “Students will understand the repeated subtraction meaning of division;” and “Students will understand the ‘flip’ part of the invert-and-multiply algorithm by referring to their understanding of the meaning of the reciprocal.”). With each iteration, features such as learning goals and rationales become increasingly well-specified as a result of everyone’s focused effort on improving the materials and working towards the same goals. In addition, each iteration
will result in more and more precisely anticipated student comments, questions, and solution strategies as teachers write into the materials the ways students engage with each task.

**Curriculum materials under study.** The curriculum materials in this study were created by and for MTEs of varying levels of experience (from tenured faculty members to graduate students) through a system of continuous improvement. In this particular study, I studied instructors implementing the curriculum materials for the first elementary mathematics content course in a sequence of three required courses for elementary PST education majors. The curriculum materials include written lesson plans, PowerPoint slides, handouts, and worksheets for PSTs. The worksheets are printed in a large packet, referred to as the “course packet,” in which PSTs complete all of their classwork and homework.

Since 2001, the lesson plans and course packet for this course have undergone intensive scientific improvement cycles in a highly supportive and collaborative School of Education at a large northeastern university. The lesson plans for this course contain features such as precisely targeted learning goals for PSTs, descriptions of what PSTs should be saying and doing if they are achieving the learning goals (which I term descriptions of successful student behavior or DSS for short), explicit and detailed rationales for why each activity was selected to achieve the learning goals and why previously selected activities failed to achieve the learning goals, and anticipated PST comments and questions and suggested ways for instructors to respond. These features were collaboratively written into the curriculum materials based on empirical processes
involving observations of students’ learning and teachers’ enactments. These analyses were conducted by the teachers themselves who were teaching during the same semester. Because the lesson plans contain these types of features, the lesson plans are highly transparent (Stein & Kim, 2009) in terms of the curriculum authors’ intentions.

These lesson plans can also be considered an educative curriculum (Davis & Krajcik, 2005) for instructors. As a result of small tests of small changes over time, and treating teaching as “experiments” (Hiebert, Morris, & Glass, 2003) in order to minimize variation in outcomes (student learning), the ways PSTs engaged with the tasks became much more predictable over time. Treating each lesson as an experiment and carefully observing student behavior allowed instructors throughout every cycle of improvement to specify, with more precision each time, exactly how PSTs would engage with each task, thus allowing instructors to gain knowledge about PSTs’ thinking and to plan how to respond to PSTs in ways more and more aligned with the learning goals. Therefore, the lesson plans are educative for instructors in terms of subject matter knowledge, pedagogical content knowledge, knowledge of the PSTs, and knowledge of the tasks.

Finally, it is important to note that each semester, the instructors currently teaching the course were the ones revising and editing the curriculum materials. In particular, nearly every semester there was an experienced full-time tenured mathematics teacher educator participating in analyzing data and improving the lesson plans with the other instructors. Therefore, this set of curriculum materials was created and improved by and for the instructors in the system. These lesson plans were written by the very people
closest to the enacted curriculum: the instructors themselves. As such, the lesson plans were designed in a way that speaks to future instructors of this course (Remillard, 2005).

For an example of a full lesson plan, see Appendix A. This lesson will be used to illustrate various claims in the results. It is helpful to thus define common features of each lesson plan to aid in the interpretation of findings. The main features of this curriculum that appear in nearly every lesson plan are (a) the learning goals; (b) descriptions of successful student behavior, (c) rationales for the task, (d) “scripts” that can be used to introduce a task to PSTs, (e) anticipated PST comments, questions, and misconceptions, and (f) suggested ways to respond to these anticipations. Appendix A provides examples of these six different features and where they commonly appear within each lesson plan. What follows is a brief description of each curricular feature.

The learning goals of each lesson are always stated at the top of the first page of the lesson plan. These are the main goals for PSTs for the lesson. Most lesson plans contain at least two or three learning goals, although some lessons contain one learning goal or more than three. The goals are more mathematical in nature than pedagogical or goals focused on addressing PSTs beliefs (e.g., “PSTs will understand how to write story problems for multiplication” versus “PSTs will explain their reasoning for writing story problems for multiplication” or “PSTs will confront their belief teaching mathematical concepts means teaching a procedure with very clear and accurate explanations”).

The descriptions of successful student behavior operationalize the learning goals for instructors. These descriptions typically immediately follow the learning goals and describe precisely what PSTs would be saying or doing if they were achieving the
learning goals of the lesson. For instance, one learning goal for lesson 3 (see Appendix A) states “The pre-service teachers need to unpack their understanding of the Hindu-Arabic system to figure out the properties of the system” (p. 1). The descriptions of successful student behavior state “Pre-service teachers will develop and show these understandings by (a) carrying out the following mathematical actions and (b) giving explanations that involve these actions: …The pre-service teachers will be able to count in the base five numeration system” (p. 1). These descriptions are more pedagogical in nature and describe how students should be achieving the learning goals, whereas the learning goals define what content students should be attempting to learn.

The rationales for each task are typically given at the beginning of the task description and are labeled as “rationale.” However, many purposes of the task are intermixed with the task description as well. Therefore, the rationale for the task is a more implicit feature that can appear anywhere in the lesson plan.

Some lessons contain “scripts,” which are literal statements that the MTE can use during class to introduce a task. These scripts typically appear immediately after the rationale for the task and are usually italicized or written in quotes. The lesson plan also sometimes states “Introduce the following activity by saying something like…” followed by the script. However, as can be seen in lesson 3 (see Appendix A), some activities simply italicize the statements MTEs can directly say during enactment to set up tasks, and this design continues throughout every lesson plan.

Finally, anticipated PST comments, questions, and misconceptions sometimes appear within a two-column table with anticipations in the left column and suggested
ways for MTEs to respond in the right column (see pages 7-8 in the lesson plan in Appendix A). This is not the case in every lesson plan, but this design is the most common way to include these anticipations and suggestions in these lesson plans. As evident in the lesson plan for lesson 3, there are sometimes anticipations written in various locations throughout the lesson plan, such as in the rationale. In this lesson plan for example, the rationale concludes by stating “groups tend to invent different numeration systems that have different properties” (p. 5). Anticipations also vary in precision from more general anticipations, such as the example just provided, to more specific questions that PSTs may ask, such as in the two-column table on pp. 7-8 (Appendix A). It is more common among the lesson plans to include PST misconceptions and difficulties within the rationale for the task or the description of the task because it is more natural to explain these misconceptions while describing the purpose of the task in addressing those misconceptions.

**Fidelity of Implementation**

This study seeks to explore relationships between curriculum materials and teachers’ enactments. Many researchers that have investigated these kinds of relationships by focusing on measuring teachers’ fidelity of implementation of curricula. There has been a history of debate regarding measuring fidelity of implementation in terms of teachers enacting written curricula. Historically, in the case of most curriculum studies, fidelity of implementation has been measured in terms of whether teachers “followed the text” (Remillard, 2005). Critiques of this construct argue that asking
teachers “to blindly follow” a text devalues their expertise and deprofessionalizes the field (Eisner, 1984; Remillard, Herbel-Eisenmnn, & Lloyd, 2011).

As a result of numerous critiques over the value of this construct, fidelity of implementation regarding curricula has recently been defined in several ways breaking from the historical definition and conception of fidelity. For instance, Chval, Chavez, Reys, and Tarr (2009) suggest a more useful construct to capture the relationship between written curricula and enactment could be textbook integrity, which measures the degree to which the teacher uses the curriculum (or textbook) as a primary resource to plan lessons. In another empirical study, Stacy Brown and colleagues reconceived of fidelity in terms of faithfulness to the literal text and faithfulness to the intended learning opportunities for students (Brown et al., 2009). In these studies, the goal was to uncover how teachers used the written curriculum materials to enact lessons and how the text supported or hindered teachers’ decisions.

The debate regarding fidelity appears to revolve around the dichotomy of maintaining faithfulness to the written text or making adaptations (Carvalho et al., 2013; Hall & Loucks, 1978; Harold, 2018; Quinn & Kim, 2017). This debate appears to boil down to the issue of supporting teachers to implement written curricula in ways that honor the curriculum authors’ intentions, yet still value the teachers’ expertise in making adaptations. This dichotomy does not need to involve making a decision about whether fidelity or adaptation is more important. Both constructs can be important for different purposes and at different times. For example, Quinn & Kim (2017) found that teachers who attempted to implement a new curriculum with high fidelity the first time using it
and to later (in the following years) make adaptations where necessary led to higher productive adaptations and faithfulness to the authors’ intentions.

In another example of diluting this dichotomy, Anne Morris explored how curriculum materials can be improved by studying teachers’ lack of fidelity of implementations to the written text and teacher’s productive adaptations (Morris, 2012). Morris observed novice MTEs teaching a lesson for the first time. The focus of her study was to measure MTEs’ faithfulness to the intentions of the text, and to use these results to consider ways to improve the written curriculum in order to support future MTEs in making productive adaptations to the lesson when adaptations to the written curriculum are necessary during enactment.

The purpose of my study complements Morris’s (2012) focus by deeply investigating specific features of written curricula that support novice MTEs to teach ambitiously. Therefore, while this study seeks to uncover relationships between curriculum materials and teachers’ enactments, the aim is not to rival fidelity of implementation against adaptations, but instead to understand how the text supports instructors to make decisions about when to follow the literal text and when to make adaptations while still honoring the curriculum authors’ intentions.

**Differences between Expert and Novice Teachers**

There are significant differences between the enactments of expert and novice teachers. These differences in performance can be attributed to differences in the ways expert and novice teachers store and access their knowledge. In particular, experts possess bodies of knowledge that are highly integrated and interconnected (Alexander,
Experts also access and recall that knowledge with minimal effort and appropriately apply that knowledge during enactment (Alexander, 2003; Berliner, 2001; Carter, Sabers, Cushing, Pinnegar, & Berliner, 1987). Experts know how to apply this knowledge appropriately because they interpret classroom events differently than novice teachers (Kersting, 2008) by recognizing opportune moments in students learning and capitalizing on these opportunities to help students progress toward achieving the learning goals.

Because experts store and access knowledge differently, they are more likely to make productive adaptations while teaching (Charalambous & Hill, 2012; Morris, 2012). For instance, if an expert mathematics teacher observes students engaging with a task differently than expected, he/she possess substantial mathematical knowledge for teaching (Ball et al., 2008) and can quickly access it to flexibly adapt the lesson in ways that redirect students’ engagement towards achieving the learning goals without lowering the cognitive demand of the task for students.

Because ambitious teaching involves setting up cognitively demanding tasks and productively responding to students as they work on these tasks, it is very challenging for novice teachers to teach ambitiously. Novice teachers have difficulties responding to unanticipated student questions, comments, or work, and orchestrating productive discussions due to the ways their knowledge is stored and not easily accessed during enactment (Berliner, 2001; Carter et al., 1987; Mary Kay Stein, Engle, Smith, & Hughes, 2008). Novices are less likely to productively adapt the lesson as it unfolds due to this inflexibility (Borko & Livingston, 1989; Westerman, 1991). Therefore, it is clear that
novice teachers need different kinds of supports in order to teach ambitiously compared to expert teachers.

What kind of supports do novice teachers and instructors need? Thus far, we know very little about the kinds of curricular features that would best support novice teachers and instructors to teach ambitiously. However, several studies have recently begun to document the kinds of knowledge and skills necessary for novice teachers to teach ambitiously (Ball et al., 2008; Borko et al., 1992; Kazemi et al., 2009; Lampert et al., 2013, 2010). The results of these studies show that some possible ways to support novice teachers to teach ambitiously is by explicitly developing pedagogical content knowledge in prospective teachers and providing them with opportunities to deliberately practice structured teaching rehearsals during their preparation programs. While these studies focus on preparing teachers to teach ambitiously prior to entering the field, my study focuses on investigating the kinds of curricular features that will likely support pre-service and in-service teachers.

Although there is very little, if any, empirical evidence that suggests which kinds of curricular features are most helpful for teachers, several theories about the kinds of curricular features and specific ways to design curriculum to support ambitious teaching have been suggested (Ball & Cohen, 1996; Davis & Krajcik, 2005). This study represents a first step in empirically investigating the kinds of curricular features that will most likely support teachers to teach ambitiously. As I will explain next, I chose to study college instructors. However, I believe that the results of this paper can be used as a
starting point for investigating the kinds of curricular features that will likely support K-12 teachers to teach ambitiously.

**Supporting Mathematics Teacher Educators**

Although I chose to investigate novice MTEs, the theory and ideas within this paper can likely be extended and modified to apply to instructors of fields other than mathematics teacher education and likely K-12 teachers. I chose to focus on MTEs because the population of MTEs consists of professionals with a range of experiences from graduate students and part-time instructors to full-time instructors and tenured-track instructors (Masingila, Olanoff, & Kwaka, 2012). In addition, the majority of MTEs across various institutions do not possess K-12 teaching experience themselves. Furthermore, a majority of MTEs report a lack of training, support, and preparedness teaching these courses, let alone the *first* time teaching these courses (Masingila et al., 2012).

In order to improve the quality of mathematics education in the United States, we must improve the quality of preparation of our PSTs. However, because the knowledge, backgrounds, and expertise of MTEs varies substantially, even within a single teacher education program, this is not an easy feat. Therefore, we must seriously consider how we support MTEs if we hope to significantly impact mathematics teaching methods in the US. How can we support MTEs, especially novice MTEs, with a variety of educational backgrounds and experiences to effectively prepare PSTs? One way could be to provide them with specific resources in the form of written curriculum materials to provide them
with opportunities to learn about the kinds of knowledge required to effectively prepare PSTs.

While I do believe that the ideas and results of this paper can inform instructors of other disciplines and K-12 teachers, teaching prospective teachers also has unique challenges that imply there are unique challenges to supporting MTEs. In particular, teaching prospective teachers is unlike teaching a “typical” undergraduate mathematics course for a number of reasons. First, the content differs in important yet subtle ways. Students need to learn mathematical content; PSTs need to learn mathematical content knowledge for teaching (Ball et al., 2008). Second, students typically do not possess substantial knowledge of the mathematical content in which they are learning, while PSTs typically have already had extensive exposure to the mathematical content they are learning, typically limited in terms of mainly procedural knowledge (Thanheiser, 2009, 2018). Therefore, MTEs face the unique challenge of preparing PSTs to teach content that they believe they already know. Therefore, effectively preparing PSTs includes addressing their beliefs about what it means to learn mathematics and to teach mathematics for understanding. Thus, supporting novice MTEs is a serious task we must undertake with unique challenges to address.

**Research Questions**

The purpose of this study is to investigate whether this type of curriculum materials can support novice MTEs to teach ambitiously. I hypothesize that these curriculum materials will support novice instructors to productively set up tasks and respond productively to anticipated PST comments, questions, and misconceptions.
However, I predicted that instructors would likely not respond productively to unanticipated PST comments, questions, and misconceptions because the instructors were teaching this course for the first time. To address these hypotheses, the following research questions drive this study:

1) Do novice MTEs set up tasks productively when using this curriculum?
2) Do novice MTEs respond productively to anticipated PSTs’ questions, comments, or work?
3) Do novice MTEs respond productively to unanticipated PSTs’ questions, comments, or work?
4) What features of this written curriculum appear to predominantly influence how novice MTEs set up tasks and respond to PSTs?

These four questions serve to address the hypotheses about whether and which features of this unique curriculum can support novice MTEs, and by extension, other instructors and K-12 teachers, to teach ambitiously.

**Methods**

**Participants and Setting**

The two participating MTEs for this study were teaching an elementary mathematics content course for pre-service teachers for the first time. At the time of data collection, both instructors were graduate students in a mathematics education doctoral program at the same large northeastern university. Both instructors had prior experience teaching high school. One instructor taught high school for seven years and had two years of experience teaching undergraduate mathematics courses, such as intermediate algebra.
and statistics. The other instructor had six years of experience teaching high school and three years of experience teaching undergraduate mathematics courses such as intermediate algebra and precalculus.

Both instructors claimed that prior to teaching this course, their teaching methods primarily involved direct instruction and traditional lecture format lessons with some group work and group discussions. The majority of their lessons focused on procedural fluency due to pressures of state tests. Neither instructor reported incorporating components of ambitious teaching in their former teaching experiences. They both claimed that the few connections they tried to make to concepts while teaching were superficial, and they directly told students about the connections, as opposed to allowing students to wrestle with the mathematical ideas to discover certain connections on their own. They both claimed that students were not given time to really think deeply on their own, the tasks were not ambitious, and students were expected to repeatedly practice whatever procedure the teacher just explained to them. Thus, these two instructors were recruited for this study because they were considered novice MTEs since they had no experience teaching PSTs, no experience teaching elementary mathematics, no experience teaching elementary content courses for PSTs, and no experience with ambitious teaching methods.

Both instructors were teaching the first elementary content course in a sequence of three content courses for PSTs during a regular semester (Fall or Spring semester). At this school, the social atmosphere while teaching is highly collaborative and supportive in terms of accessing prior instructors for these courses, the primary authors of this
curriculum (those involved in several improvement iterations), and expert mathematics teacher educators. Typically, instructors teaching the same course meet weekly to discuss how to enact upcoming lessons and debrief about the success of the prior week’s lessons. These meetings can help calibrate instructors’ understandings and interpretations of the lesson plans and purposes of each lesson. Thus, for the purposes of this study, instructors were asked not to meet weekly to discuss lesson plans.

By suspending the weekly instructor meetings during the time of data collection, I was attempting to minimize the influence of expert instructors supporting the novice instructors. Since I was mainly interested in the curricular support provided to instructors, controlling for support through mentorship or weekly meetings was critical to ruling out alternative hypotheses. Although instructors were asked not to meet weekly, the course coordinator still had to meet with instructors to discuss logistical items, such as quiz and exam dates and material. I attended and audio recorded all of these meetings to ensure the lesson plans under investigation were not discussed at these times. Furthermore, instructors were asked to copy me on any email conversations between themselves and the course coordinator or any other expert instructor.

Finally, it is important to note that both instructors had offices in the same vicinity as other graduate students, some of whom had taught these courses during prior semesters. Therefore, although instructors were asked not to seek advice on how to enact lessons or interpret curriculum materials, I cannot be confident about the conversations that may have occurred between the participating instructors and other graduate students. However, I am confident that the conversations that occurred during participating
instructors and the course coordinator, an expert MTE, did not influence instructors’ interpretations or enactments of any lessons under investigation. In addition, I asked the primary curriculum authors if the instructors contacted them with any questions and they confirmed that they made no contact during the time of data collection.

**Data Collection**

The data collection procedures involved classroom observations, interview data, and copies of the curriculum materials the instructors used (lesson plans, PowerPoint presentations, and course packets). I observed one instructor teach 17 lessons and the other instructor teach 15 lessons. I planned to observe 17 lessons for both instructors, but one instructor had a substitute cover his class on two different occasions. I did not analyze all lessons. Instead, I purposefully selected four lessons to study in more depth to conduct an exploratory qualitative analysis. The selected lessons were chosen based on the level of support provided in the lesson plan (the quality of the six features described earlier) because I wanted to investigate the types of curricular features that supported novice MTEs to teach ambitiously. Before conducting any classroom observations, I chose four lessons that had varying levels of curricular support (such as lessons with detailed rationales and lessons with less detailed rationales; and lessons with a high number of anticipations and lessons with fewer anticipations).

Every classroom observation was video and audio recorded. There were three main reasons for observing more lessons than I planned to analyze. First, attending nearly every class allowed me to understand the classroom culture for each instructor so that when the instructor referenced certain practices during the interviews, I would be aware
of to what he was referring. Second, I conjectured that not every lesson may perfectly end in time and certain lessons may carry over to the next class. So for example, if I planned on analyzing lesson 4, but the instructors did not finish lesson 4 during the intended class time on that day, they would likely finish lesson 4 at the start of lesson 5. Planning for this possibility by observing every lesson allowed me to capture all aspects of each lesson that was to be analyzed.

The four lessons that were analyzed were part of all of the lessons that were recorded. The purpose of recording the lessons under study was to investigate whether instructors exhibited clear connections to the curriculum materials while setting up tasks and while responding to students. For instance, if instructors used the scripts to set up tasks, this would be a clear connection that the curriculum materials influenced the ways instructors set up tasks. In another example, if instructors used the suggested responses to PST comments, questions, and difficulties, this would indicate influence of this curricular feature on the ways instructors responded to students. The interview data would serve to corroborate or disaffirm the influences observed during enactment.

For these four lessons, I conducted semi-structured open-ended interviews (Clement, 2000; Ginsburg, 1997; Seidman, 2006) with both instructors separately before and after teaching each lesson. I interviewed each instructor approximately one hour before they had to teach the lesson. The pre-interviews lasted approximately 25 – 45 minutes (see Appendix B for both the pre- and post-interview protocols). In every possible case, I conducted interviews as close to the lesson as possible to ensure the instructors had completed all desired planning prior to teaching. Similarly, the post-
interviews were conducted immediately after teaching (see Appendix B for the post-
interview protocol). The post-interviews lasted approximately 5 – 20 minutes. The
purpose of the pre-interviews was to investigate whether the MTEs used the curriculum
materials and what features they claimed predominantly supported them in preparing how
to set up each task and how to respond to PSTs during enactment and why they felt
supported by such features, if any. The purpose of the post-interviews was to investigate
which features of the written curriculum materials, if any, influenced instructors to
respond to PSTs in certain ways and why. Again, all of the interview data were used to
corroborate or disaffirm the findings from the classroom enactment analyses.

I contacted the primary curriculum author to obtain the most up-to-date versions
of the curriculum materials that the instructors would be using. Recall, these curriculum
materials undergo continuous improvement cycles every semester and so they constantly
change. During the time of data collection, these improvement cycles were paused for the
purposes of this study. I was given electronic copies of the lesson plans, the handouts
students would be working on (the classwork), the homework, the homework solutions,
the PowerPoint slides associated with all 27 lessons for the entire semester, and a
hardcopy of the course packet for PSTs. I obtained these materials in order to analyze
features of the curriculum materials and question both MTEs during the interviews
regarding various features of each lesson.

Finally, instructors were asked to invite me to any meetings they planned with the
course coordinator (an expert MTE for this course) and to copy me on any email
exchanges between themselves and the course coordinator and/or the primary curriculum
authors. Recall that the instructors had no interactions with the primary curriculum authors. However, they did hold several meetings with the course coordinator. I attended all of these meetings and audio recorded the conversations. I also saved electronic copies of the email exchanges between the instructors and the course coordinator. The purpose of collecting these data was to ensure that interactions between expert MTEs and the participating novice MTEs were not influencing the novice MTEs’ decisions and plans for setting up tasks or responding to PSTs prior to teaching any lesson under investigation.

**Data Analyses**

The data were analyzed in a specific order to avoid bias. First, I analyzed the classroom observations in terms of whether instructors set up tasks productively or unproductively and whether instructors responded productively or unproductively to students (this analysis supports evidence to answers research questions 1, and partially questions 2 and 3). Next, I analyzed the lesson plans carefully and determined which aspects of the classroom enactment were anticipated and which were unanticipated according to the lesson plan (in order to fully answer research questions 2 and 3). I then analyzed the interview data to determine whether and how the curriculum materials supported instructors’ enactments (to corroborate or disaffirm results from the enactment analysis). I was interested in whether the instructors claimed that the curriculum materials were the main influencing factor in how they set up tasks and responded to students and if so, what *specifically* about the materials supported them (these data would also provide evidence to answer research question 4). Finally, I listened to the audio recordings of the
meetings involving both instructors and the course coordinator and reread the email exchanges between them. In no instance was there evidence of the course coordinator helping the instructors to interpret the purpose of the lesson or how to enact any lesson under investigation. This result provides evidence to reduce the likelihood of alternative hypotheses regarding the main influences on the ways instructors set up tasks and responded to PSTs. Next, I will explain in more detail the ways in which I analyzed the classroom enactments.

First, I fully transcribed all eight lessons (four per instructor) and divided each transcript into “segments.” A “segment” is a unit of analysis that marks either when the instructor was setting up a task, engaged in a lecture or whole class discussion, or when an interaction between the instructor and PST(s) occurred (see Appendix C). Thus, the introduction to a task was coded as its own segment. A lecture by the instructor on a specific topic was coded as a new segment. Once students began working on a task, each time the instructor spoke with a new group was considered a new segment (unless the topic of discussion within that group interaction changed). In addition, whenever a PST asked a question of the teacher, either during a whole class discussion, lecture, group work, or individual work, this was coded as a segment as soon as the PSTs’ question was resolved. If PSTs asked more questions aligned with an initial question that were all about the same topic, this entire interaction was coded as a single segment. If a PST asked a question about a different topic at any point (during group work or whole class discussion for example), then this interaction was coded as a new segment.
These segments were created to allow for meaningful coding of the classroom enactment in ways that revealed task set-ups and student-teacher interactions (the two components involved in ambitious teaching). The segments that involved task set-ups were used to analyze whether instructors set up tasks productively or unproductively. The segments that involved lectures or whole class discussions and the segments that involved student-teacher interactions were used to analyze whether instructors responded productively or unproductively to PST comments, questions, and misconceptions. Lectures were captured in their own segments because PST might ask questions during lectures which would result in an interaction between the PST and the instructor.

In order to analyze the classroom video data, it was necessary to reduce the enactments into smaller portions, rather than attempt to analyze the entire transcript as the unit of analysis. Segments were used as the unit of analysis in particular because it was crucial that a part of the transcript of the classroom enactment fully capture the set-up of each task in its entirety, and interactions between the instructor and student(s) fully. Doing so allowed coders who were determining whether a task set-up or instructor response to a student was productive or unproductive to fully understand the context in which the task set-up or instructor response was taking place.

To ensure the definition of a “segment” was well understood, I randomly selected two different lessons out of the four possible under investigation and then randomly selected one instructor’s (out of the two possible instructors’) enactments for each lesson to create segments with a second coder. To physically create the segments, I printed out copies of the entire classroom transcripts for each lesson and we individually drew
Horizontal lines within the transcripts to show where we would create new segments. Across two different classroom enactments, reliability for creating segments was 95% agreement (55 agreements out of 58 total decisions). The disagreements were discussed and resolved through comparisons and competitive argumentation (Vanlehn, Brown, & Greeno, 1984). After reliability was established, I segmented each lesson enactment for the other six classroom transcripts.

Each segment was then coded as a productive or an unproductive (a) task set-up or (b) response (see Appendix D). A productive task set-up or response is defined to accomplish all of the following: (a) aligns with the learning goal(s) of the task; (b) aligns with the learning theory of the course; (c) maintains the intended cognitive demand level of the task (does not relieve students’ struggle too early while they are working); (d) uses mathematically correct language, symbols, and expressions; (e) focuses students’ attention on concepts or connections between concepts and procedures (uses pictures or concrete materials to connect concepts to procedures); (f) focuses on students’ ways of reasoning; (g) corrects student language where appropriate (e.g., helps students use language such as “measuring units”); (h) provides students with concrete materials when appropriate (such as blocks or straws); and (i) does not violate a direct command in the lesson plan (such as “do not read 0.1 as zero point one, it should be read as one tenth”). An unproductive task set-up or response would be categorized as a task set-up or instructor response that does not obey at least one of the aspects above.

I randomly selected one lesson out of the four total lessons to code with another coder (a different coder from the previous step in analysis—regarding creating segments).
The second coder and I coded the enactments by the two instructors for the same lesson. Reliability for coding productive or unproductive task set-ups and instructor responses was 92% (34 agreements out of a total of 37 possible decisions across two enactments). The disagreements were discussed and resolved through comparisons and competitive argumentation (Vanlehn, Brown, & Greeno, 1984).

Finally, I determined whether each interaction segment contained anticipated PSTs comments/questions or unanticipated comments/questions (see Appendix E). Because these comments/questions were easily identifiable in the lesson plan, these segments were not coded by a second coder.

To analyze the interview data, I first fully transcribed all of the pre- and post-interviews. I then engaged in a method of open coding techniques (Clement, 2000; Strauss, 1987) to determine which features of the written curriculum materials participants claimed supported them while planning. I initially developed a set of codes regarding how participants discussed these “features” based on how participants described helpful parts of the curriculum materials. The categories for these features was therefore grounded in the ways participants spoke about the curriculum materials (Strauss & Corbin, 1994).

The features I hypothesized would be important prior to collecting data were the learning goals, rationales, anticipations, and suggested responses. Based on how the participants discussed the learning goals, I divided this category into two categories: the main learning goals and descriptions of successful student behavior. Furthermore, I initially predicted that participants would only refer to the lesson plans as supporting
them. As instructors continually referred to the PowerPoint slides and course packet, I also added these two features to my list of codes. Finally, although neither participant referred to the “scripted” parts of any lesson plan, I added this as a curricular feature due to the clear connection between the scripts in the lesson plans and the ways instructors introduced tasks during enactment. The final list of curricular features that was developed through a open coding analysis of the interviews was: (a) the learning goals, (b) descriptions of successful student behavior, (c) rationales, (d) scripts, (e) anticipated PST comments, questions, and misconceptions, (f) suggested responses to PSTs, (g) the PowerPoint slides, and (h) the course packet. This final list represents a saturation of all of the possible curricular supports instructors referred to during the interviews.

Once these features had been named and categorized, I then engaged in a method of axial coding (Strauss, 1987) to relate each of the features to the reasons instructors provided for the features either being helpful or unhelpful during their planning phases. I employed the constant comparative method of analysis (Fram, 2013; Glaser, 1965) to refine these reasons based on evidence that did and did not fit across instructors for why each feature was either helpful or not helpful.

To summarize, this analysis resulted in segments of classroom video transcripts that were coded as productive or unproductive task set-ups or productive or unproductive instructor responses to PSTs. Furthermore, each of the interaction segments was also coded as being anticipated in the curriculum materials or as being unanticipated in the curriculum materials. These resulting data were used to answer research questions 1, 2, and 3. Scrutinizing the segments for connections between the written curriculum
materials and the language instructors used to set up tasks and respond to PSTs was used to answer the fourth and final research question. Finally, the interview data served to corroborate or disaffirm these results.

**Results**

As a reminder, the main research questions driving this study were: (a) Do novice mathematics teacher educators set up tasks productively when using this curriculum? (b) Do novice mathematics teacher educators respond productively to anticipated pre-service teacher questions, comments, or work? (c) Do novice mathematics teacher educators respond productively to unanticipated pre-service teacher questions, comments, or work? And (d) What features of this written curriculum appear to predominantly influence how novice mathematics teacher educators set up tasks and respond to pre-service teachers?

The answer to the first question is yes; two novice MTEs set up almost every task productively across four lessons (see Table 1). According to the results, there was only one instance where a task was set up unproductively (i.e., Sam lesson 5). This one instance was coded as unproductive because it violated one of the aspects within the definition of a productive task set-up. Namely, Sam did not align with the learning theory of the course, which focuses on “actions on quantities” and drawing quantities (when appropriate) to represent numerical symbols. During this task set-up, Sam stated that a quantity of twenty-seven circles would represent a specific measuring unit, which was mathematically correct but instead of drawing twenty-seven circles, he wrote the symbols “[27].” Hence, this task set-up was coded as unproductive.
Table 1

*Total Number of Productive and Unproductive Task Set-Ups for Each Instructor*

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Sam – Task Set-Up Segments Only</th>
<th></th>
<th>Aaron – Task Set-Up Segments Only</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total Segments</td>
<td>Productive</td>
<td>Unproductive</td>
<td>Total Segments</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>4/4 100%</td>
<td>0/4 0%</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>6/6 100%</td>
<td>0/6 0%</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>4/5 80%</td>
<td>1/5 20%</td>
<td>5</td>
</tr>
<tr>
<td>16</td>
<td>5</td>
<td>6/6 100%</td>
<td>0/5 0%</td>
<td>6</td>
</tr>
</tbody>
</table>

You may notice that both instructors had the same exact number of total task set-up segments for each lesson (see Table 1). It turns out that both instructors set up the same tasks that the lesson plan suggested in every lesson. Therefore, instructors were setting up the same tasks for their respective students.

The answer to the second and third question is also yes. A majority of all responses to PSTs (anticipated and unanticipated comments, questions and work) by both instructors were rated as productive (see the gray columns in Table 2). Indeed, at least
70% of all interaction segments by both instructors were rated as productive across every lesson analyzed. Furthermore, of all the interaction segments across all four lessons, 78% of Sam’s were rated as productive, while 95% of Aaron’s were rated as productive.

Table 2

**Total Number of Productive and Unproductive Response Segments for Each Instructor**

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Sam – Response Segments Only</th>
<th>Aaron – Response Segments Only</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total Segments</td>
<td>Productive</td>
</tr>
<tr>
<td>3</td>
<td>55</td>
<td>47/55 85%</td>
</tr>
<tr>
<td>4</td>
<td>18</td>
<td>13/18 72%</td>
</tr>
<tr>
<td>5</td>
<td>37</td>
<td>26/37 70%</td>
</tr>
<tr>
<td>16</td>
<td>14</td>
<td>11/14 79%</td>
</tr>
<tr>
<td>Total</td>
<td>124</td>
<td>97/124 78%</td>
</tr>
</tbody>
</table>

Notice from Table 2 that the total number of segments in each class differs, sometimes substantially. For instance, let’s compare the first row of data in Table 2, representing the enactments of lesson 3 for both instructors. Sam’s enactment contained 55 total segmented interactions; whereas Aaron’s enactment only contained 22 total segmented interactions. Of Sam’s total 55 segmented interactions, 47 of them were rated as productive interactions (85% of all interactions in lesson 3). Similarly, of Aaron’s total
22 segmented interactions, 21 of them were rated as productive interactions (95% of all interactions in lesson 3).

These differences are expected because each “response segment” marks when an instructor is interacting with a PST, group of PSTs, or engaged in an interactive lecture or whole-class discussion. Therefore, the number of response segments can vary substantially in each classroom depending on how students interact with the instructor. For example, in Sam’s lesson 3 enactment, there are 55 total response segments compared to Aaron’s 22 total response segments. When reviewing these lessons, there were more response segments in Sam’s classroom because PSTs had many questions about the homework that Sam decided to address. While Aaron’s class had several questions about the homework, Aaron chose not to address all of them during class and instead reminded PSTs to visit him during office hours.

Both novice MTEs were able to respond productively to PSTs in most of the anticipated cases and most of the unanticipated cases (see the gray columns in Tables 3 and 4). As a reminder, these two instructors had no prior elementary mathematics backgrounds and had never taught pre-service teachers before. The data were also coded highly conservatively (i.e., if a coder was unsure of what code to apply, the segment was coded as unproductive). The cells in the second column of data for Tables 3 and 4 (gray column labeled “Anticipated Productive”) were calculated by determining the total number of productive responses to anticipated PST comments, questions, and work by each instructor divided by the total number of anticipated PST comments, questions and work. Similarly, the cells in the third column of data in Tables 3 and 4 (white column
labeled “Anticipated Unproductive”) were calculated by determining the total number of unproductive responses to anticipated PST comments, questions, and work by each instructor divided by the total number of anticipated PST comments, questions and work. The fourth and fifth columns of data were calculated in an analogous manner for unanticipated PST comments, questions, and work. This calculation allows us to quickly determine the percent of total anticipated PST comments, questions, and work that were responded to productively (and unproductively) and the percent of total unanticipated PST comments, questions, and work that were responded to productively (and unproductively).

To ensure accuracy of the table values, the numerators of the cells in the second and fourth columns (both gray columns) of data should add up to the numerator provided in the first column of data. The first column of data re-displays the total number of productive responses out of the total number of interaction segments for each instructor. Therefore, the total number of productive responses (both anticipated and unanticipated) should remain consistent if we sum the number of anticipated productive responses (column 2) and unanticipated productive responses (column 4).

Notice that for both instructors, the percentage of productive responses to anticipated PST responses were all greater than or equal to 67% (first gray column of data in Tables 3 and 4). Similarly, the percentage of productive responses to unanticipated PST responses was greater than or equal to 70% in all cases, except for one (lesson 4 for Sam) across both instructors (second gray column in Tables 3 and 4). Furthermore, across all four lessons Sam responded productively to 82% of anticipated
PSTs’ responses and 71% of unanticipated PSTs’ responses. Likewise, across all four lessons, Aaron responded productively to 93% of anticipated PSTs’ responses and 94% of unanticipated PSTs’ responses. These percentages imply that the majority of anticipated PST comments, questions, and work were responded to in a productive way by both instructors; and that the majority of unanticipated PST comments, questions, and work were responded to in a productive way by both instructors. Therefore, the answers to the second and third research questions are both yes.

Table 3

Sam’s Responses to Anticipated and Unanticipated PST Responses

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Total Number of Productive Responses</th>
<th>Anticipated PST Responses</th>
<th>Unanticipated PST Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Anticipated Productive</td>
<td>Anticipated Unproductive</td>
</tr>
<tr>
<td>3</td>
<td>47/55 85%</td>
<td>40/47 85%</td>
<td>7/47 15%</td>
</tr>
<tr>
<td>4</td>
<td>13/18 72%</td>
<td>11/13 85%</td>
<td>2/13 15%</td>
</tr>
<tr>
<td>5</td>
<td>26/37 70%</td>
<td>12/18 67%</td>
<td>6/18 33%</td>
</tr>
<tr>
<td>16</td>
<td>11/14 79%</td>
<td>4/4 100%</td>
<td>0/4 0%</td>
</tr>
<tr>
<td>Total</td>
<td>97/124 78%</td>
<td>67/82 82%</td>
<td>15/82 18%</td>
</tr>
</tbody>
</table>
Table 4

Aaron’s Responses to Anticipated and Unanticipated PST Responses

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Total Number of Productive Responses</th>
<th>Anticipated PST Responses</th>
<th>Unanticipated PST Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Anticipated Productive</td>
<td>Anticipated Unproductive</td>
</tr>
<tr>
<td>3</td>
<td>21/22</td>
<td>14/15</td>
<td>1/15</td>
</tr>
<tr>
<td></td>
<td>95%</td>
<td>93%</td>
<td>7%</td>
</tr>
<tr>
<td>4</td>
<td>22/24</td>
<td>17/18</td>
<td>1/18</td>
</tr>
<tr>
<td></td>
<td>92%</td>
<td>94%</td>
<td>6%</td>
</tr>
<tr>
<td>5</td>
<td>20/20</td>
<td>5/5</td>
<td>0/5</td>
</tr>
<tr>
<td></td>
<td>100%</td>
<td>100%</td>
<td>0%</td>
</tr>
<tr>
<td>16</td>
<td>8/10</td>
<td>3/4</td>
<td>1/4</td>
</tr>
<tr>
<td></td>
<td>80%</td>
<td>75%</td>
<td>25%</td>
</tr>
<tr>
<td>Total</td>
<td>72/76</td>
<td>39/42</td>
<td>3/42</td>
</tr>
<tr>
<td></td>
<td>95%</td>
<td>93%</td>
<td>7%</td>
</tr>
</tbody>
</table>

Finally, the answer to the fourth question is mixed depending on whether the instructor is setting up a task or responding to PSTs. Since the fourth research question is the main question that connects my hypotheses and the other research questions, my main claims for this study were derived by answering this research question. My main claims are (a) instructors relied most heavily on the PowerPoint slides, scripts in the lesson plan, rationales for each task in the lesson plan, and learning goals in the lesson plan in order to
set up tasks productively; (b) instructors relied most heavily on the anticipated PST comments and questions, and suggested ways to respond to PSTs in order to respond productively to anticipated PST comments and questions; and likely relied to a lesser extent on the learning goals and the rationales in the lesson plan; and (c) instructors appear to rely on the learning goals, descriptions of successful student behavior, rationales, anticipated PST comments and questions, and suggested ways to respond in the lesson plan in order to respond productively to unanticipated PSTs comments and questions. Table 5 summarizes the curricular features that influenced instructors’ task set-ups and responses to PSTs.

Table 5

Curricular Features Influencing Instructors’ Enactments

<table>
<thead>
<tr>
<th>Curricular Features</th>
<th>Task Set-Up</th>
<th>Responding to Anticipations</th>
<th>Responding to Unanticipated Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>PowerPoint slides</td>
<td>Anticipations</td>
<td>Learning Goals</td>
<td></td>
</tr>
<tr>
<td>Scripts</td>
<td>Suggested Responses</td>
<td>Descriptions of Successful Student Behavior</td>
<td></td>
</tr>
<tr>
<td>Rationales</td>
<td>Rationales</td>
<td>Learning Goals</td>
<td></td>
</tr>
<tr>
<td>Learning Goals</td>
<td>Learning Goals</td>
<td>Rationales</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Anticipations</td>
<td></td>
</tr>
</tbody>
</table>
The interview data confirm these claims by uncovering why the two novice instructors found certain curricular features to be helpful and how they used these features to plan their lessons. The instructors claimed that these kinds of features were helpful in supporting them to deliver effective lessons. At times, they clearly favored certain features over others, as will be demonstrated throughout the presentation of the results. However, it is noteworthy that at no point during any interview did either instructor claim that a specific curricular feature was not helpful for them or did not support them in some significant way. In fact, both instructors tended to struggle answering the question “which one feature would you keep in the lesson plan if you had to remove all of the other features?” However, the resounding answer to the question of what should be added to these curriculum materials was almost always “nothing.”

Taken together, these results suggest that this unique kind of curriculum can support novice instructors to teach ambitiously. To support this claim, I will first present results on how instructors set up tasks and the primary influencing curricular features for their task set up. Then I will present results on how instructors responded productively to anticipated PST comments and questions and how the curriculum materials supported them. Finally, I will present results on how instructors responded productively to unanticipated PST comments and questions and how the curriculum materials supported them. Throughout the presentation of this evidence, I will refer to Tables 1-4 above. In addition, I structure each section by summarizing the main claim, followed by a representative example of the claim, and concluding with evidence from the interview data that corroborate, expand, or contradict the findings from the enactments.
Task Set-Ups

Results show that a majority of segments involving the set-ups of tasks were coded as productive (see Table 1). In fact, all instances of task set-ups except for one were rated as productive. When analyzing why the majority of tasks were set up productively, there is a clear connection between the curriculum materials and the ways the instructors set up tasks. In particular, the instructors relied heavily on the PowerPoint slides, scripts in the lesson plan, rationales for each task in the lesson plan, and the learning goals in the lesson plan in order to set up tasks productively. I will present an example from one lesson to illustrate this claim; however, this example is representative of how every task was set up by both instructors. I will follow this example with interview data that corroborate this finding.

Example of a productive task set-up. This example comes from the main task in lesson 3, which involves providing pre-service teachers with the opportunity to invent their own numeration system that only uses five symbols: A, B, C, D, and 0 (see Appendix F for the handout students receive in their course packets). The entire lesson plan is provided in Appendix A. The scripted parts of the lesson plan to introduce this activity are as follows:

Imagine that you are a member of a small tribe. Your tribe has a numeration system that uses the symbols A, B, C . . . Z. A represents [1], B represents [2], and so forth up to Z. Z represents [26]. This system has met your needs up to now. You were hunter-gatherers and you only needed to keep track of small amounts. Now your tribe has started farming and you have a need to keep track of larger
amounts. You need a more sophisticated numeration system. One day, a member of your tribe says that she has invented a new numeration system. She says that she can represent any quantity by using the symbols A, B, C, D, and 0. (Read ‘0’ as ‘zero’.) She shows you some blocks (hold up base five blocks), and says they are an essential part of her system for representing quantities. She says she will explain the system to you when she returns from a hunting trip. Unfortunately, she dies during the hunting trip. Now your tribe has to figure out what her system was. That’s your job. You need to figure out her numeration system. The blocks tell you something about the structure of the system and how she figured out how to represent any quantity with just the five symbols A, B, C, D, and 0…

Sam and Aaron introduce this activity using almost the exact language from the scripted part of the lesson plan. In addition, both instructors chose to use the associated PowerPoint slides to introduce this activity. The PowerPoint slides contain the same kind of language that appears in the script in the lesson plan. For example, Sam states:

So this activity, imagine that you are a member of a small tribe and you have a numeration system that uses the symbols A, B, C, all the way up to Z. Ok. A represents 1, B represents a quantity of 2, etcetera etcetera all the way up to Z. And Z represents 26. And this is on page 45 of your packet. And this system has met your needs up until now. You’re hunter-gatherers, and you only needed to keep track of small amounts, so that was your system. But now that you’re farming, you have to use a more elaborate system to help you guys understand how to keep track of. So one day, a member of your tribe says she has invented a
new numeration system. She said that she can represent any quantity by using the symbols: A, B, C, D, and 0. So only five symbols. She shows you the following blocks and says that they are essential part in her system to represent each quantity. She says she will explain the system to you when she returns from the hunting trip. Right so these are the only symbols that we're looking at here. So this numeration system is based off of those symbols—or those quantities. Ok, she dies during the hunting trip. I don't know who wrote this story but, now your tribe has to figure out what her system was. Ok so you need to make—you guys are going to make a numeration system with your groups…

Sam’s task set-up continues following the script in the lesson plan because he reads from the PowerPoint slides in order to set up this task. Aaron’s set-up follows almost identical language. It is therefore very evident that Sam and Aaron used the PowerPoint slides and the script in the lesson plan in order to set up this task productively.

The rationales for each task also influenced the ways both instructors set up tasks. The rationales helped instructors by explaining the purpose of each activity and why each task was selected to help PSTs achieve the learning goals of each lesson. In particular, before introducing each task, the instructors explained to PSTs the purpose of each activity. The language that they use mirrors the language in the rationale of each lesson plan.

For example, in this lesson, Sam explains to PSTs that the purpose of this activity is to “help you guys think through the ideas that underlie our system,” and to “help you
realize how complex our system is and why kids may have trouble understanding it.” He explains to PSTs that learning about different kinds of number systems will help them to develop a deeper understanding of the Hindu-Arabic system and better prepare them to support their future elementary students. Similarly, Aaron says to his PSTs: “we’re trying to put ourselves in the shoes of our future students,” and “this activity will also help you realize how complex our current system is and probably most importantly, why children might have difficulty learning our numeration system for the first time.” The language that both novice MTEs use to explain the purpose of the task aligns closely with the language used in the explicit “rationale” of this task in the lesson plan (see Appendix A). The interview data also corroborate the claim that instructors’ task set-ups were influenced by the PowerPoint slides, scripts, and rationales. However, the interview elaborate upon this finding by suggesting that instructors were also influenced by the learning goals of each lesson.

**Interview data corroborate and expand finding.** The claim that instructors relied upon the PowerPoint slides and scripts within the lesson plans is also evident based on instructors’ responses during the interviews:

*Sam:* Another reason I don’t say many of these things [in the lesson plan] is because there is so much to say. It’s a page [of learning goals]. I don’t want to read a page to anybody. That’s why I think the PowerPoint is pretty helpful. It keeps it streamlined. (Lesson 3 Pre-Interview)

*Aaron:* I’ll then read through the lesson plan because the lesson plan—obviously the PowerPoints were made from the lesson plan. But you know, you can’t
include everything that’s in the lesson plan in the PowerPoints...the reference PowerPoint has a lot of information that’s on that lesson plan. Like it unpacks everything. So I’ve found that pretty helpful. (Lesson 3 Pre-Interview)

According to both MTEs, the PowerPoint slides “keep it streamlined” and “unpacks everything,” which helps provide instructors with structure on how to set up each task and keep the lesson moving. This structure appears to be essential in supporting instructors to set up tasks productively because it focuses their attention on exactly what to say and do to set up each task. The script in the lesson plan provides an explicit example of exactly what instructors can say to set up each task. However, without the PowerPoint slides, it is not evident that either MTE would have used the script in the lesson plan to set up this task, because they both acknowledge the overwhelming amount of information in the lesson plan and claim that the PowerPoint slides help them to structure what exactly to say to PSTs during enactment. Therefore, both instructors found both the PowerPoint slides and the scripts to be helpful in setting up tasks.

The interview data also confirm that the rationales supported both instructors. For example, during the pre-interview for lesson 3, Sam refers to the rationales as being helpful for him, especially as a first-time instructor:

Sam: Rationales are also something that I did not particularly write down in my [notes]. It helps me get my head around why these things are important. Like I said, I don’t use them particularly in my [notes], it just helps me know what’s the point of this lesson plan.

Interviewer: So you mean you don’t plan on saying the rationale to the students?
Sam: In this lesson, not particularly…

Interviewer: So are these rationales more for you?

Sam: Somewhat. Yeah. I think it’s important especially as a first-time teacher to understand how everything connects because if we’re doing something and it seems disconnected then it’s not good for the teacher to not know why things are being done. Because I mean someone taught this 3 or 4 times, they can say oh yeah that’s why we do that. But if it’s a first-time teacher, I think these rationales are good because it gives us a sense of direction and how everything connects.

As Sam explains, the rationales helped him to understand the point of the lesson. Without knowing the point of the lesson, it is unclear that Sam would have set up this task (or any task) productively. He also points out that knowing the purpose of the task and how everything connects is especially important for novice instructors.

Aaron also uses the rationales to understand the purpose of each task. However, he explained the usefulness of the rationale differently than Sam. Aaron describes that he plans his lesson by first looking through the PowerPoint slides and completing the classwork in the course packet. He then reads the lesson plan and uses the rationales in the lesson plan to reaffirm his own interpretation of the purpose of each task. In an interview for lesson 4, Aaron states “So in some ways, there might be some differences in my rationales here [in my head] and the rationale there [in the lesson plan]. But for the most part, it was pretty reassuring that how I was interpreting things was how they were interpreting things, the authors of the lesson plan.” Therefore, both instructors found the
rationales helpful for setting up tasks productively and used the rationales to understand the point of each task.

The learning goals supported instructors to understand the ultimate goal and direction of the task and what PSTs are expected to know. While this is difficult to visualize from their enactments, it is evident by their responses in the interviews:

Sam: The learning goals are helpful for me to understand what I need to know to help them understand what they need to know and understand. These are things that I will state because they’re on the PowerPoint, but I will talk about them in general terms. They mainly help me to understand what the point of the lesson is and I will convey some of that in the PowerPoint. (Lesson 3 Pre-Interview)

Aaron: Everything that I’ll try to explain will go towards that learning goal. So that’s another thing that I think everything is sort of directed at [the learning goals] and if I decide to add anything like for example using a Hindu-Arabic example, it would be in my opinion to aid in accomplishing that learning goal if students aren’t quite getting it or something. So everything really is directed towards those [learning goals]. (Lesson 3 Pre-Interview)

Based on their responses, the learning goals directed them to the “point of the lesson.” Everything “is directed towards those” learning goals. So instructors keep the learning goals in mind when making every decision during enactment. Having explicit learning goals at such a precise and detailed level allows instructors to focus their every decision on whether this decision will help students to achieve those specific goals. Without the learning goals, it is not evident that either MTE would have set up this task productively.
because they would not have understood what PSTs “need to know and understand,” and ultimately, where the lesson was heading.

In sum, these same patterns of both instructors using the PowerPoint slides, mirroring the language used in the scripts and rationales, and using the rationales and learning goals to understand the point and the direction of the lesson continued to be evident in every task set-up for every lesson. Thus, the main curricular features influencing these two novice MTEs’ task set-ups were the PowerPoint slides, the scripts in the lesson plan, the rationales in the lesson plan, and the learning goals in the lesson plan.

**Responding to Anticipations**

A majority of anticipated student comments, questions, and work was handled in a productive manner by both instructors (see Tables 3 and 4). Instructors primarily relied on the anticipated PST comments and questions, and suggested ways to respond in order to respond productively to PSTs. Instructors also likely relied to a lesser extent on the learning goals and the rationales in the lesson plan to respond to anticipated PST comments and questions. Indeed, in most anticipated cases, the ways instructors responded to PSTs mirrored the suggestions in the lesson plans. These kinds of responses were not included in any PowerPoint slides for any lesson; they were only written in the lesson plans.

To illustrate this finding, I will first present an example of Sam responding productively to anticipated PSTs’ comments and questions. This example is illustrative of the majority of cases where instructors responded productively to PSTs. Following this
example, I will explain how the instructor used the features in the lesson plan (the anticipations and suggested responses) to respond productively to anticipated PSTs’ responses based on the classroom observations. I will conclude by presenting interview data from both instructors that corroborate this finding and also suggest that instructors relied on the learning goals and rationales to respond productively to anticipated PST comments, questions, and work.

**Example of a productive response to anticipated comments.** To illustrate this claim, I will present one example from lesson 16, a lesson on multiplication of decimals. In this lesson, there is an activity where students compare two story problems side by side (see below). These stories are meant help PSTs address the misconception that multiplication always results in a product that is bigger than the multiplicand. In addition, the phrasing of the story problems alert PSTs to misconceptions about using phrases such as “more than” to model multiplicative comparison stories. The lesson plan includes the following:

Jose and Miguel have two separate bottles of water. Jose’s bottle has 6.3 ounces of water. Miguel’s bottle has three tenths more water than Jose’s. How much water does Miguel have?

Jose and Miguel have two separate bottles of water. Jose’s bottle has 6.3 ounces of water. Miguel’s bottle has three tenths as much water than Jose’s. How much water does Miguel have?

Focus the discussion around the difference in the wording “three tenths more water than” and “three tenths as much water as.” In particular, the second story
problem could be solved with the number sentence $0.3 \times 6.3 = ?$. However, the wording of the first story problem is problematic. Does the pre-service teacher want the referent to be the entire amount of water in Jose’s bottle? Then the number sentence would be $6.3 + (0.3 \times 6.3) = ?$. Does the pre-service teacher want the referent to be one ounce? Then the number sentence would be $6.3 + 0.3$. Ask the pre-service teachers what would need to be included in the third sentence in order for story problem #1 to be solved with $6.3 + 0.3$ (…three tenths of an ounce more than…). In either case, story problem #1 would not be solved with the number sentence $0.3 \times 6.3 = ?$ so it is not correct…

For the side-by-side story problems, some students may think that both stories would be solved with $0.3 \times 6.3$. It has been found that many pre-service teachers equate “more than” with “as much as.” Many pre-service teachers believe that multiplication always “makes bigger” and so the “more than” language fits this misconception. Moreover, it has been found in previous semesters that some pre-service teachers also think the first story problem can be solved with $6.3 + 0.3 = ?$ because they think the three-tenths is referring to three tenths of an ounce. This should also be discussed; the wording of the problem must make it clear whether the numeral is an operator (e.g., we have to find three tenths of the whole, which in this problem is 6.3 ounces) or the numerical value of an amount of stuff (three tenths of the BMU).
In this example, the lesson plan anticipates how PSTs will respond to the task and what misconceptions they likely hold regarding multiplication (believing that the product in any multiplication number sentence is always bigger than the multiplicand). While the lesson plan in this example does not necessarily “script” the discussion, the lesson plan does provide suggestions for ways instructors can respond to PST comments (such as “Ask the pre-service teachers what would need to be included in the third sentence in order for story problem #1 to be solved with 6.3 + 0.3”). Both instructors applied this suggested response in their enactments.

For example, after presenting the task to PSTs and allowing them time to think about it, Sam engages in a whole-class discussion of the issues surrounding these story problems. The following excerpt is a brief section of that discussion:

T: Any shot at what this number sentence might be [for the incorrect story problem]? We said it was additive comparison. Yeah?
S: 6.3 plus .3?
T: Ok so this is 6.3 plus 3 tenths. Anybody interpret it a different way? Three tenths more water than Jose’s. Yeah?
S2: Wouldn’t it be plus 3 tenths of 6.3?
T: Ok. So some people might think of it that way. Here’s Jose’s. Then it says 3 tenths more water than Jose’s. Number 3, what makes this first story problem kind of confusing? Why is it so confusing? Yeah?
S3: Because when you see ‘more’ you typically add.
T: We typically add. We’re good there. But why is it not clear about why are we not in agreement here?
S: Because we don’t know what quantity we’re using.
T: Good because we don’t know what quantity we’re working with. It’s not that clear is it. When we say 3 tenths more water than Jose’s. Some people might think okay 3 tenths of a BMU, and S2 thought 3 tenths of what Jose had. So what could we do in this problem to make it more clear what we are talking about?
S: Um you have to say 3 tenths of what…
you’re adding.
T: Okay good so 3 tenths of what we’re talking about. So if you want to talk about this [circles 6.3+.3], what can we add in there to make it clear that it’s just 3 tenths. Three tenths of?
S: Of an ounce?
T: Of an ounce! Okay good. We said 3 tenths of an ounce more water, or if you want to talk about this way, 3 tenths of Jose’s. So the point of this is to be clear when you’re writing your story problems so that when somebody else reads it, they understand what you’re talking about with your quantities.

In this excerpt, Sam productively responds to every comment and question. Sam’s responses during this episode were clearly influenced by the suggested responses in the lesson plan because he used those suggestions during this interaction (e.g., “So what could we do in this problem to make it more clear what we are talking about?” and later “So if you want to talk about this [circles 6.3+.3], what can we add in there to make it clear that it’s just 3 tenths. Three tenths of?”). In addition, all of the comments PSTs
made were anticipated in the lesson plan, allowing Sam to envision how PSTs would engage with this task. Aaron’s implementation of this task unfolded in a very similar way and he also responded to PSTs during this discussion by using the suggestions in the lesson plan.

The learning goals and rationale for each task also helped both instructors to respond to anticipated PST comments, questions, and work, but to a lesser extent than the anticipations and suggested responses in the lesson plan. As with their task set-ups, it was evident that instructors used the learning goals and rationales when responding to a majority of anticipated PST responses because instructors would use language that mirrored the learning goals or rationales.

For example, in lesson 16, the lesson plan explains the learning goals of this lesson explicitly (“Pre-service teachers will be able to write multiplication story problems that involve multipliers less than one”) and the purpose of this task very clearly (“These story problems alert them to some of the problems associated with writing correct multiplication story problems—in particular to the importance of using the correct referent for the multiplication and the importance of correct wording”). It is evident that Sam and Aaron relied on the learning goal and the rationale of the activity by their closing remarks of this task:

Sam: So the point of this is to be clear when you’re writing your story problems so that when somebody else reads it, they understand what you’re talking about with your quantities.
Aaron: This is why we gotta be sticklers to detail and be unambiguous about to what we are referring: this 3 tenths piece. Because is it 3 tenths of an ounce of a basic measuring unit? Or is it 3 tenths of the quantity of his friend’s water bottle that we’re tacking on. So you gotta be specific right? So that’s what we are getting at here.

In both of their closing remarks, the two MTEs refer to the learning goal of writing story problems correctly and to the rationale which focuses on using the correct referent in the story. The interview data also show that instructors claim to rely primarily on the anticipations and suggested responses, and less heavily on the learning goals and rationales.

**Interview data corroborate finding.** While it is clear from the classroom observation data that both novice MTEs used the anticipations and the suggested responses in order to respond to PSTs in every lesson, the interview data also corroborate this finding. Notably, the lesson plans for lesson 4 and 5 do not contain hardly any anticipated PST responses or suggested ways to respond to PSTs. The interview data reveal that instructors mentioned how helpful the anticipations and suggested responses were only for the interviews of lessons 3 and 16:

Sam: The questions that could be asked are things I wrote down in my PowerPoint notes as things that I can potentially ask…I wrote down mainly the questions in italics that were there because I saw them as the most useful, practical, to say. The questions on page 9. Those are the ones that I mainly focused on. (Lesson 3 Pre-Interview)
Sam: The student responses I see as beneficial to help me anticipate what they may be thinking and like I said before hand, I don't anticipate me being able to think through every possible misconception that students may have. But the questions I see on page 9 I see as beneficial as something practical to implement right there and then because those questions themselves connect with the gist of where things are going in lesson 4, getting them to think about the base 5 system. (Lesson 3 Pre-Interview)

Interviewer: Did something in the lesson plan lead you to respond that way?

Sam: Yes. I think the questions on page 9, like I said earlier. I did not word the questions exactly like that and I did not have the sheet right in front of me but I think that at least helped to get an idea of where to go with my questions to help me to pull out their thinking about the number of symbols compared to the place value, multiplicative property, etcetera, and getting them to think about how to develop a base 5 system without forcing them to. I wanted to get them to think about that without being told how to do it. (Lesson 3 Post-Interview)

Aaron: I don't know, it's hard for me to predict what they will do. (Lesson 4 Pre-Interview)

Aaron: I mean, if I just had this [the course packet without the lesson plans and PowerPoint slides], I would infer that the learning goal would be to write story problems and the underlying learning goal, or the sub learning goal would be to relate all of these topics because it’s all in here. I wouldn't know the
misconceptions and things like that that was in the lesson plan and not in the task. That was really helpful. (Lesson 16 Pre-Interview)

Aaron: Well. I know what the point is. If I didn’t know the main learning goal and I just had access to the course packet, I know what the point is. I don’t think I’d be able to try to deliver that point as well without all of this support. For example, the common student misconceptions that are all over this. So that’s helpful in creating and hopefully enacting a more quality lesson that gets at that learning goal better than if I didn’t have this. That’s for sure.

Interviewer: So if you were going to rewrite the lesson plan and keep only one thing about it to support you, what would you keep and remove everything else?

Aaron: I would keep the common student misconception stuff, which is not really a section of it here but it’s in there a lot. A lot of stuff I’ve highlighted is that sort of ‘many students will incorrectly word the second problem, blah blah blah blah blah, if that happens, do this or think about this,’ and that kind of stuff. I would do that. If I had that, if I had to choose one thing to keep, I would keep that. Because all of the other stuff is also important but I think I could piece that together myself if I had to. But I don’t know this stuff [the common misconceptions]! So that’s helpful. (Lesson 16 Pre-Interview)

In lessons 3 and 16, the interview data strongly suggest that instructors prefer to have anticipations and suggested responses whenever possible. The anticipations help them to understand how the lesson will unfold and predict how PSTs will engage with each task. Indeed, in lesson 4, Aaron admits that “it’s hard for me to predict what they will do”
because anticipations were not included in this particular lesson plan. The suggested responses also appear to have primarily influenced both instructors in responding to PSTs in anticipated cases, which is evident in the fact that they claimed that they would try to use these responses during enactment and the fact that they did use these responses during enactment in a majority of cases.

It is also likely that instructors relied on the learning goals and rationales for each task to support them in responding productively to anticipated PST responses. However, it is difficult to infer when instructors referred to these curricular features during the interviews whether they were claiming these features were helpful for them to set up tasks or to respond to PSTs. They claimed that the learning goals and rationales helped them to understand the direction and purpose of each task. This could infer that they used this knowledge to productively set up tasks and to productively respond to PSTs as they worked. Since it is unclear, it is likely that these features helped instructors to do both during enactment. As such, I will refer readers to the aforementioned statements instructors made during the interviews regarding the usefulness of learning goals and rationales in the prior section on task set-ups. In conclusion, instructors relied primarily on the anticipations and suggested responses in the lesson plans to respond productively to PST comments, questions, and work; and likely relied to a lesser extent on the learning goals and rationales for each task in the lesson plan to support them in responding to PSTs in anticipated cases.

**Responding to Unanticipated Events**
Results show that the majority of instances of instructors responding to unanticipated PST comments, questions, or work were rated as productive (see Tables 3 and 4). The case of unanticipated situations was more difficult to analyze in terms of which features predominantly supported instructors to respond productively. Unlike the task set-ups and the cases of anticipated situations, the ways instructors responded to PSTS in unanticipated moments do not always closely reflect the specific language used in any of the primary curricular features under study. However, instructors’ productive responses do reflect the ideas grounded within: the theory of the course, the learning goals, and the rationales of each task.

Because it was difficult to find explicit connections between the language instructors used during enactment and the language used in the curriculum materials, the interview data were the main sources of analyses for answering the fourth research question with respect to these cases. Therefore, these claims are not as strongly supported as the prior claims. Furthermore, during the interviews, instructors rarely differentiated exactly how the curricular features supported them when they claimed certain features helped them while planning. The only times this differentiation was evident was in post-interviews when instructors were explicitly asked about why they responded in certain ways during enactment. As such, we cannot be certain as to whether the curricular features instructors reference during the interviews imply that the features supported instructors to set up tasks, respond to PSTS, or both. It seems most likely that the features they claimed helped them supported them to do both, but we cannot confirm this. However, one thing that is certain is any time instructors were asked about what
influenced them to respond a certain way to PSTs (whether in anticipated or unanticipated cases), instructors always said the lesson plans. Therefore, the lesson plans were the predominant factor influencing instructors’ responses, but exactly which features of the lesson plans influenced instructors is less certain.

Based on the interview data, it appears that instructors primarily relied upon the learning theory of the course, the learning goals, the descriptions of successful student behavior, the rationales of each task, anticipated questions, comments, and misconceptions, and the suggested instructor responses to anticipated situations in order to respond productively to PSTs in unanticipated moments. This kind of information was not included on the PowerPoint slides, only within the lesson plans. To illustrate this claim, I will first present an example of how an instructor responded to an unanticipated PST question during class. Then I will present interview data that suggest the main curricular features that influenced instructors’ responses in these moments were the ones just listed.

**Example of a productive response to an unanticipated question.** This example comes from lesson 16 again, a lesson on writing story problems for multiplication of decimals. The main activity for this lesson asks students to draw diagrams to model number sentences and then write story problems reflecting their diagrams and the meaning of the number sentence. In this example, a group of students in Aaron’s class is struggling with how to model the number sentence: $0.6 \times 3 = ?$ The handout in the course packet for this lesson gives students three numbers: 0.6, 1.8, and 3, and asks
students to form a number sentence and then model it on graph paper (see Appendix G). So in this example, the students are already aware of the product being 1.8.

The question that arises is how to make an appropriate selection for representing the numeral “1” on graph paper. The quantity chosen to represent 1 is commonly referred to as the basic measuring unit, or BMU, in this course and in the curriculum materials. To visualize why this selection can be difficult, most students typically chose one square on graph paper to represent the BMU. While this selection is always appropriate when working with solely whole numbers and operations, it becomes an inconvenience when working with decimal numbers and operations. In this example, \(0.6 \times 3 = ?\), the more appropriate choice for the BMU would be ten blocks on graph paper. This decision would result in the numeral “3” being represented by a quantity of thirty blocks on graph paper that can easily be cut up into tenths in order to find six tenths of three. The choice of one block to represent “1” on graph paper in this example would be inappropriate as it would result in “3” being represented by three blocks on graph paper, which is difficult to nicely divided into ten equal parts.

Aaron approaches a group of six PSTs working on modeling this number sentence. At the beginning of the exchange, a PST asks the following unanticipated question: “How do you know what to make your BMU?” Aaron asks her in response “How did you choose the BMU for this problem?” The PSTs say they chose ten blocks on graph paper and Aaron asks them why. One PST suggests because they already know the answer is 1.8 (because they were given three numbers, 0.6, 3, and 1.8 and asked to form a correct number sentence). They already knew the answer involved tenths so they
needed to represent tenths in their measuring units. Aaron asks them what if they did not know the answer already? One PST suggests using trial and error until a guess for the BMU works out. Aaron acknowledges this strategy and says there is a more efficient way to decide on the BMU.

Another PST suggests that because one number in the number sentence has a measuring unit of size one tenth (the operator of 0.6), then the BMU should be ten squares because that’s the smallest measuring unit they will need to show. This strategy does not generalize to all multiplication number sentences though. Aaron helps PSTs to understand this by creating a different problem: 0.6 × 0.5 = ? He asks students what BMU they would choose for this problem and he gets a piece of paper and starts working on this new problem with the group of PSTs, sitting down at their table. The following exchange occurs:

S1: Because it’s in the well, I don't know I was thinking it would be in your smallest place value.

T: Right so there you would say because it’s .6 times .5—So that doesn’t make sense right? So S2, you’re saying that since these are both tenths

S1 and 2: Right.

T: Then you’re thinking that the BMU?

S1: Should be in tenths.

T: Should be ten?

S1: Yeah.
T: Ten blocks. Because what we are trying to avoid here is when we carry these problems out, we don’t want to have to cut into a tiny [S1: Right.] single unit cube right?

[Students all nod.]

T: Well when we start to model this though, this is find 6 tenths of .5 or 5 tenths.

S1: Right.

T: If your BMU is this [ten blocks], then that makes .1 [measuring unit] one block. So your quantity, .5, would look like what?

S1: Five blocks.

T: Five of those single blocks, right? One two three four five, so then like that? [Draws five blocks on graph paper and writes “= .5” next to them]

Ss: Mmhmm.

T: So now, this is why this is so hard.

S3: But you can’t break that up into ten [pieces].

T: Yeah! Like you gotta break that guy up into ten equal pieces. And you only have five equal pieces to work with. So you’re forced to say, crap, a half of a block is now one tenth of this thing and we want to try to avoid that.

S1: Right.

T: So the key is when you look at the measuring unit or the place of the quantity, the quantity is in tenths [underlines .5 in the number sentence] and you need to cut that up into tenths [underlines .6 in the number sentence].

S1: Oh!
T: This tenths place should have enough room to make ten equal pieces. So that tenth place, which is the .1, should be those ten blocks because then you’ll never run out.

S1: Right.

T: And then you can build the BMU through that times ten relationship to be whatever you need it to be. In this case one hundred [blocks].

In this instance, Aaron productively responds to PST comments and questions throughout this entire interaction by maintaining the cognitive demand of the task for PSTs, leading PSTs towards the learning goal of the lesson, and aligning with the learning theory of the course. In this interaction, Aaron appears to have relied on this lesson plan and prior lesson plans to respond to PSTs because prior lesson plans focused on modeling decimal multiplication number sentences. For instance, the lesson plan for lesson 13 includes the following in the descriptions of successful student behavior describing what a successful modeling procedure entails:

First, .7 × 8 means, ‘Find seven groups of one-tenth of 8 (or ‘Find seven tenths of 8 or ‘Find seven one-tenths of 8’). I chose my BMU to be 10 squares on the graph paper. Then a measuring unit of size .1 is 1 square; I drew and labeled these measuring units here. Now I can represent the quantity 8 as 8 BMUs, or 80 squares. Now I need to find one tenth of 8, so I partitioned 8 into 10 equal parts (shows the partitioning by partitioning the 80 squares into 10 equal parts). One of these parts is one tenth of 8. So, one tenth of 8 is 8 squares. I redrew 8 squares here and labeled it ‘one tenth of 8.’ (I can also see that one tenth of 8 is .8 because
8 mus of size .1 fit into 8 squares.) Now I need seven groups of one tenth of 8, which is 56 squares. So, I drew them here and labeled the 56 squares ‘seven tenths of 8.’ Now this quantity is my answer. But I need to find the numerical value of this quantity. I assign a numerical value by seeing how many measuring units of each type fit in. Five BMUs fit in, so this area has a numerical value of 5. I showed how 5 BMUs fit in by circling and labeling them each with ‘BMU’ (or measuring unit of size 1). Six measuring units of size 0.1 fit into the remaining amount. I showed how 6 MUs of size 0.1 fit in by circling and labeling them each with ‘0.1.’ So $.7 \times 8 = 5.6.$

This knowledge from the prior lesson plan apparently influenced Aaron’s response to focus on aspects of this process and how to show actions on quantities (one of the key factors in the learning theory for this entire course). In addition, the learning goal for lesson 16 (“Pre-service teachers will be able to write multiplication story problems that involve multipliers less than one.”) and the descriptions of successful student behavior that follow likely influenced Aaron to focus on using the meaning of the number sentence to determine the BMU:

Given three numbers (e.g., 0.4, 6, 15) that can be related in a multiplication number sentence, pre-service teachers will (a) arrange them into two valid equations (e.g., $0.4 \times 15 = ?$ and $15 \times 0.4 = ?$), (b) write the meaning of the equations (e.g., “find 4 groups of one tenth of 15,” “find four one tenths of 15,” “find four tenths of 15,” “find 15 groups of 0.4”), (c) make graph paper models
of the equations, and (d) use the graph paper models and the meaning of the equations to help write a story situation that involves the equation.

These curricular aspects, including the rationale for this activity, all led Aaron to focus on using the meaning of the number sentence (finding 6 tenths of 5 tenths in his example of $0.6 \times 0.5$) to determine what one measuring unit should be (the measuring unit of size $0.1$ should be ten blocks because we have to find tenths of tenths) and then building the BMU from there (so if one tenth is ten blocks, then the BMU should be one hundred blocks). Many of the examples of instructors responding productively to PSTs’ unanticipated questions and comments appear to be primarily influenced by the learning theory of the course, the learning goals of the lesson, descriptions of successful student behavior, and the rationales of each task. However, the interviews reveal that instructors also appeared to rely upon anticipated questions, comments, and misconceptions, and suggested responses for anticipated situations within the lesson plan.

**Interview data corroborate and elaborate upon finding.** In this specific example, when questioned about this instance during the post-interview for this lesson, I ask Aaron why he responded in the ways that he did. Aaron explains, “I think it was in the lesson plan,” which was the most common response from both MTEs when asked why they responded in certain ways during enactment. From his response, I could not tell to what he was referring so I asked him what in particular was the in lesson plan? Our conversation during the interview continues:

Aaron: Focusing on the action of the operator. That type of language, thinking about the type of operator and how that implies a certain action to take on the
multiplicand. Like you have to think about what that operator is telling you to do to that quantity of three. Which is more complex when the quantity is a value like five tenths. When you’re operating on a whole number quantity, the operator implies that you’re cutting something into a certain number equal pieces. In this case six tenths means cutting something into ten equal pieces and then choosing six of them. Cutting a whole quantity into tenths is easier. In this particular situation probably because of what the students were saying, I thought that it would elicit some meaningful struggle on their part if the quantity in which they were cutting was tenths. So that they were cutting tenths into tenths.

Interviewer: Did you anticipate students asking you this question?

Aaron: I did not anticipate this question nor did I anticipate students guessing and checking their BMU choice. I do remember thinking about how I would choose my BMU in a situation like this and then thinking about how an operator less than one affects the product and that motivated me to attack that student’s question in the moment the way that I did.

Interviewer: So do you think that you would have responded this way without the lesson plan?

Aaron: No, and lesson plans before this one burned into my brain this idea of operating on a quantity and modeling such operations. If that wasn’t explicitly a goal of these lesson plans, I don’t think I would have thought this critically about it and I probably would have guess and checked just like they did. Maybe I would have figured out more efficient ways to select your MUs [measuring units] but I
don’t know if I would have been as clear in scaffolding their work to help them figure that out in this particular moment.

Interviewer: Had you thought about this before?

Aaron: I thought about this for the first time last week.

Interviewer: Wow.

Aaron: This is a testament to the lesson plan, just making that available, and then I can think about it.

Therefore, Aaron again acknowledges the lesson plan for this lesson and prior lesson plans and how much he felt supported by having this kind of information available. He admits that he does not believe he would have been able to respond this effectively to that unanticipated question if he did not have these curriculum materials.

During this interview, Aaron also points to a paragraph in the lesson plan that states:

These types of wording problems reflect the students’ lack of understanding of how an operator less than one affects the product. The instructor can assist students by contrasting the wording of the story problems in part (a) and the wording of the story problems in part (b). Students may also have difficulties even writing a story problem for a number sentence with an operator less than one.

He claimed that this particular part of the lesson plan informed him of the fact that PSTs have difficulty with operators that are less than one and the suggested way to help PSTs was to do the first activity (contrasting the two story problems displayed earlier). Aaron also inferred from prior lessons leading up to this point that the connection between
actions and quantities is the most important concept to emphasize throughout the course. Thus, his interpretations of how the operator acts on the multiplicand led to this productive interaction with students beginning with an unanticipated question. This instance was representative of the majority of cases for both instructors that involve productive responses to unanticipated PST comments and questions in that both instructors attribute their successful responses to the lesson planning materials, suggesting that the curriculum materials supported both of them even during unanticipated situations.

In several other interviews, Sam and Aaron both mention how helpful they found the learning goals, descriptions of successful student behavior, rationales, anticipations, and suggested responses while preparing to teach each lesson. As mentioned earlier, although the instructors do not refer explicitly to these features helping them to respond to PSTs in unanticipated moments, it seems most likely that these features did help them to respond productively in these moments. The learning goals, descriptions of successful student behavior, and the rationales gave instructors insights into the direction, flow, and purpose of each task, which I believe allowed instructors to productively navigate unanticipated moments because they kept the learning goals and purpose of each task in mind while making decisions.

Sam: Because I mean someone taught this 3 or 4 times, they can say oh yeah that’s why we do that. But if it’s a first-time teacher, I think these rationales are good because it gives us a sense of direction and how everything connects.

(Lesson 3 Pre-Interview)
Aaron: I think the learning goals help me see the big picture about why this lesson is important and things that it should perhaps serve as a foundation towards in the future…The rationales for each of the activities help me just remind me why we're doing them. And I think though I probably won't explicitly mention every little piece of the rationale to my students as they engage with the activity, because I'm buying into it, I think it will convey in my instruction why we're doing it. (Lesson 3 Pre-Interview)

Sam: I think this uh this part right here on this is page 125 of the lesson plan. I really think clung to this a, b, c, d. It was really broken down for me to have these little mini subgoals that kind of summed up writing problems and modeling them. So, arrange them into two valid equations, write the meaning of the equations, this is something they are already used to, make graph paper models, and use your graph paper models to write a story situation. So I think these points right here really kind of stuck in my mind as I was going through this lesson. That we are building up to them writing a problem. And then once they write a problem, they can break it back to what everything means from that problem. So yeah I think the major learning goals and then this.

Interviewer: Like a description of what they should do?

Sam: Yeah like a description of what they should do. Right. Exactly. The rationale is good too. It's just big. I was trying to underline some stuff. It was helpful but it was just a lot so it was a lot to process. So yeah. I think this part of
the lesson plan was really the most helpful for me [description of goals] for
delineating what they need to do. (Lesson 16 Pre-Interview)

The interviews also reveal that instructors relied upon the anticipations and the
suggested responses within the lesson plan to respond to unanticipated moments. The
instructors claim that having suggested ways to respond to students helped them to get a
feel for productive ways to respond to PSTs in any situation. Seeing examples of how to
respond to PSTs and exactly what to say to help PSTs in certain moments influenced
instructors’ responses when PSTs said or did unanticipated things:

Interviewer: Did something in the lesson plan lead you to respond that way?
Sam: Yes. I think the questions on page 9, like I said earlier. I did not word the
questions exactly like that and I did not have the sheet right in front of me but I
think that at least helped to get an idea of where to go with my questions to help
me to pull out their thinking about the number of symbols compared to the place
value, multiplicative property, etcetera, and getting them to think about how to
develop a base 5 system without forcing them to. I wanted to get them to think
about that without being told how to do it. (Lesson 3 Post-Interview)

Aaron: The prompting part of the lesson plan made me think about how in
general, if kids had a question, how I could not answer it but still help them
hopefully guide them a little towards the conclusion that they’re trying to get to,
but not answer it explicitly. That’s probably a bad way of putting it, but yeah. The
prompts in the lesson plan, like students might do this you should say this, I think
like that. Though I don’t know if I said anything verbatim, I really don’t know if I
did or not. It definitely helped me get in the mindset of don’t just tell them, hey well that’s wrong, that’s wrong, that’s wrong, you want to do this, this and this. Instead, maybe focus their attention like, explain to me this ‘AB.’ Focus their attention on a particular spot and hopefully they’ll realize where they went wrong. (Lesson 3 Post-Interview)

Aaron: Um they were just helpful to read to sort of put my--to think about how the lesson might unfold as they start to work on the Alphabetian project. It was helpful for me to look at the teacher responses to the student responses…Um you know, like not directing them towards a right answer and just really focusing on having them figure it out and having them take the time to um come up with their own system, regardless in the end if it works or not. But really try to figure it out on your own. So the student responses are helpful to have in the back of my mind, like I can anticipate a little bit…I think it's helpful to see what the students have done in the past. (Lesson 3 Pre-Interview)

Aaron: And the teacher and student responses, this is helpful because I've never taught this before and I really don't know what to expect from students. So seeing tendencies is helpful. And then the teacher responses, though, I'll be honest that's probably the biggest thing to me right now is I don't want to screw up the teacher responses. Because there are just so many things! Like if this then that, if this then that. And I certainly don’t have them memorized, nor should I have them memorized I think. Like I understand the goals of the lesson, so I guess I just kind of came full circle. I understand the goals, I understand why we're doing it, and I
hope I can support them as the authors of this curriculum would hope and the teacher responses kind of at least give me a little bit of support in how I should support them. So it's helpful to read through these and I hope that when the time comes, I can use them. (Lesson 3 Pre-Interview)

It appears that the suggested responses in the lesson plan (the “prompts”) helped instructors to visualize what a productive response looks like in any situation. Both instructors also apparently claim that getting into a certain “mindset” for how the lesson is supposed to unfold helped them to respond to PSTs in unanticipated cases. This mindset was influenced by everything in the lesson plan. Therefore, it is difficult to delineate exactly which features predominantly influenced instructors in these situations. Instead, it seems most likely that the accumulation of coherent learning goals and rationales focused on connecting concepts to procedures and allowing PSTs to productively struggle (the learning theory of the course) built into instructors’ minds the overall aim of the course. In unanticipated moments, these influences led to instructors typically responding productively to PSTs. Therefore, this set of lesson plans for each lesson and the learning goals, descriptions of successful student behavior, rationales, anticipations, and suggested responses appear to predominantly influence instructors in unanticipated cases.

The learning goals set clear goals for PSTs that instructors aim for during implementation. The descriptions of successful student behavior align very closely with the learning goals. However, these descriptions represent more closely what the learning goals would look like “in action.” They operationalize the learning goals and bring to life
what PSTs should be doing and saying if they are achieving the learning goals. The rationales support this visualization even more by explicating the purpose of each task in helping students to achieve the learning goals. This kind of triangulation of the intentions of each lesson and the coherence and consistency of these intentions across lessons supported instructors to respond to unanticipated PST comments and questions.

In addition, the anticipations help instructors to predict how PSTs will engage with each task and the suggested responses help instructors to infer appropriate ways to respond to PSTs even in unanticipated situations. It is not evident that instructors would have been able to respond productively to unanticipated situations in a majority of cases without all of these curricular features. Therefore, instructors appeared to rely upon the learning theory of the course, the learning goals, descriptions of successful student behavior, the rationales, anticipations, and suggested responses in order to respond productively to unanticipated PST comments and questions.

Discussion

Ambitious teaching requires that teachers teach in response to what students do as they work on cognitively demanding tasks. It requires that teachers set up cognitively demanding tasks for students to work on and that teachers respond productively to students as they work, and if necessary, adapt the lesson to help students achieve the learning goals of the lesson while still maintaining the cognitive demand of the work for students. Results of this study suggest that curriculum materials that contain certain features may support novice mathematics teacher educators (MTEs) to teach ambitiously. Specific features of this curriculum supported two novice MTEs to productively set up
tasks and respond to pre-service teachers (PSTs) in anticipated and unanticipated
scenarios.

The main curricular features that supported the novice MTEs were (a) precise
learning goals, (b) descriptions of successful student behavior, (c) rationales for each
task, (d) scripts for introducing a task, (e) anticipated PSTs’ comments and questions, (f)
suggested ways to respond to PSTs’ comments and questions, and (g) associated
PowerPoint slides. More specifically, the MTEs primarily relied on the PowerPoint slides
and scripts in the lesson plan, and to a lesser extent, rationales and learning goals in order
to set up tasks productively. Instructors relied primarily on the anticipated PST comments
and questions and helpful ways to respond, and to a lesser extent, learning goals and
rationales in order to respond productively to anticipated PST comments and questions.
Finally, instructors appear to rely primarily on the learning goals, descriptions of
successful student behavior, rationales, anticipations, and suggested responses in order to
respond productively to unanticipated PST comments and questions.

Recall that the two instructors participating in this study had no prior experience
teaching elementary mathematics, no experience working with PSTs, and no experience
teaching an elementary mathematics content or methods course for PSTs. The fact that
both of these novice MTEs were able to productively set up tasks and productively
respond to PSTs in cases of anticipated and unanticipated PST comments, questions, and
work across several lessons is remarkable. Furthermore, given that they always attributed
their success to the curriculum materials, writing and designing curricula that contain
these types of features appears to be a fruitful method for supporting practitioners.
Both instructors were found to implement the same tasks the lesson plan suggested in every lesson. Instructors did not make any edits to the tasks and did not plan to make edits to the tasks according to their interview responses. Some may interpret this to mean that instructors simply “followed the curriculum.” However, instructors were not asked to “follow the curriculum” for the purposes of this research study. Thus, this finding in itself is unique because many other curriculum studies show that teachers do not “follow the written curriculum,” but instead make adaptations to the suggestions in the written curriculum, omit certain parts of the tasks, and omit entire activities and sometimes replace them with activities they designed (e.g., Brown et al., 2009; Brown, 2009; Penuel, Phillips, & Harris, 2014; Stein & Kim; 2009; Thompson & Senk, 2014). In fact, there are several well-known theoretical frameworks regarding how teachers use written curricula that include actions such as omissions and adaptations to the text (Brown, 2009; Remillard, 2005). Thus, it is unique that neither instructor changed any of the planned tasks in written curriculum.

I believe that the curricular features that appeared to support instructors were not doing so in isolation. In other words, I am not sure that instructors would have been able to productively set up almost every task if they were only given scripts in the lesson plan. It is not evident that the instructors would have made similar decisions about how to set up each task if they were not also provided with the learning goals and the rationales for each task. In fact, prior studies show that teachers may interpret the same “script” very differently and thus enact very different learning opportunities for students (Collopy, 2003). Therefore, it seems more likely that all of the features amalgamate to support
instructors and it is difficult to separate the influence of one specific curricular feature on the instructors’ enactments.

I believe that having pre-made PowerPoints for them to implement in addition to having the learning goals and rationales all contributed in significant ways to supporting instructors. It seems likely that the learning goals and rationales educated instructors on the purpose and goals of each task and why to incorporate the suggestions, while the script provided them ideas on how to introduce each task in the ways suggested. Therefore, while these results may seem obvious, prior studies show that simply telling teachers what to say to introduce a task or respond to a student does not always result in productive task set-ups or responses (Brown et al., 2009; Collopy, 2003), especially if they are not provided with information on why to introduce a task a certain way or respond to students in certain ways. Thus, results suggest that in order to support practitioners to teach ambitiously, curriculum materials should include precise learning goals and descriptions of what students will be doing and saying if they are achieving those learning goals; explicit rationales; suggested scripts for introducing each task; anticipated student responses, and helpful suggestions for ways to respond to students.

**Alternative Hypotheses**

There are several likely alternative hypotheses that should be considered while interpreting these findings. First, while the instructors both claimed that the curriculum materials supported them throughout every interview, it could be that other factors within this environment are mainly responsible for supporting instructors to teach ambitiously. For example, the environment in which both instructors were teaching was a highly
collaborative and supportive school in terms of the faculty, staff, and other graduate students. In addition, the physical office space in which both instructors worked included graduate students who had previously taught the course they were both teaching. While this is a possible reason explaining why the enactments of both instructors were mainly rated as productive, it does not seem likely that this environment had a greater influence on instructors’ enactments than the curriculum materials since the instructors never mentioned the importance of working in such an environment to aid in their teaching.

I attempted to control for the social environment influencing instructors’ interpretations of the curriculum materials by requesting that the weekly meetings of all course instructors (which are typical of this school) be suspended for this semester for the purposes of this study. In addition, the course coordinator and primary curriculum authors of these materials reported no significant contact with the two instructors that could have influenced their interpretations of the curriculum and hence their enactments. As a result, this hypothesis seems unlikely to explain the main reason for the observed results. However, it is unclear whether different results would have emerged had the instructors been permitted to meet each week with the expert instructor to discuss the upcoming lessons.

Another possible explanation for the observed results is that instructors assumed that they were supposed to follow the curriculum. The social context within this School of Education may have led them to believe that their job as a graduate student teaching their own class of pre-service teachers is to “obey” the curriculum and do as the written lesson plans tell them. A variety of factors could have led to this assumption, including
the social environment in this school, the fact that there are common assessments, and the fact that the course coordinator teaches using these curriculum materials. For instance, the social environment from other individuals, from tenured faculty to other graduate students, may have led both instructors to assume that they are expected to follow the written materials.

In line with this alternative hypothesis, it is also possible that instructors attempted to follow the curriculum materials because they were participating in this research study and being observed every day, knowing that the researcher was studying this specific curriculum. Although they were never explicitly told to follow the curriculum by the researcher, they may have assumed that the purpose of the research study was to investigate fidelity of implementation of this specific curriculum. Thus, instructors may have attempted to maintain fidelity to the written curriculum materials based on both of these factors: (a) the social context of the school influencing their understanding of their job, and (b) the fact that a researcher was observing them every day and studying the curriculum in which they were using. In my view, this still suggests that teachers who wish to maintain fidelity to the curriculum authors’ intentions will be better positioned to do so when provided with these types of curricular features.

Another possible competing hypothesis is that these two novice MTEs are simply good teachers. It could be that they would have performed similarly if they were handed any curriculum materials and thus, there is nothing special about this particular curriculum. While this is plausible and difficult to rule out, I would argue that these instructors would likely not have responded in ways so highly aligned with the
curriculum materials had they not be given these materials. The ways the instructors set up each task mirrored almost exactly the wording provided within the scripts of each lesson plan. In addition, the ways in which instructors responded to PSTs in many cases very closely reflected the wording suggested in the lesson plans. Furthermore, the instructors themselves claimed that the lesson plans supported them a great deal during almost every interview and disclosed that they spent hours studying these materials while planning each lesson. Therefore, while this alternative hypothesis is difficult to rule out, it seems more likely that the instructors are not solely good teachers but also relied heavily on the curriculum materials to deliver effective lessons.

Finally, it should be noted that both instructors were pursing PhDs in mathematics education during their time teaching this course. At the time of data collection, both instructors had completed one year of their doctoral program. Therefore, this could have played a substantial role in how they enacted each lesson. However, I hypothesize that the main effect of their doctoral experiences would most likely influence their beliefs and philosophies regarding teaching. Simply altering one’s beliefs about teaching though is not enough to directly affect one’s teaching practice (e.g., Erickson, 1993). Therefore, it does not seem likely that one year in a doctoral program resulted in both instructors being able to teach a new course ambitiously.

Given the arguments above, it seems most likely that the curriculum materials (the lesson plans and PowerPoint slides) were the primary factors influencing instructors’ enactments. Next, I will highlight some of the limitations of this study and suggest possible directions for future research.
Limitations and Future Research

This study was an exploratory study that employed qualitative methods to investigate whether a unique set of curriculum materials could support novice instructors to teach ambitiously. However, there are several limitations of this study worth nothing. First, results show that it is possible for these types of curricular features to support novice instructors to teach ambitiously. However, the results of this study are not yet generalizable. In particular, because this study was conducted with only two participants across four lessons, it is not clear whether these curricular features would help the vast majority of mathematics teacher educators. These two instructors also come from a distinct population (that of doctoral mathematics education students). It is not clear that we would see the same results across different populations of instructors or with K-12 teachers. Future studies could provide similar curriculum materials (containing these kinds of features) to K-12 teachers and other populations of instructors and replicate these methods to test this hypothesis. Future studies might also consider studying more participating instructors/teachers.

Second, I restricted my analyses solely to the effects of the written curriculum on instructors’ enactments. However, it is likely that a range of factors influenced instructors’ enactments. Future studies could allow educational continuous improvement systems to continue conducting “business as usual” while teaching and study the enactments of novice instructors in the system. In my study, I requested that these kinds
of collaborative meetings be suspended for the entire semester. However, these types of meetings are an essential aspect of creating and improving the written curriculum materials. Thus, to study these written materials in their authentic context and development, future studies could not limit analyses to only the ways in which the written curriculum affected instructors, but also the ways in which other factors affected instructors. In this case, I suggest including interview questions about how helpful instructors find the weekly meetings to be, or having common assessments, or having access to the primary curriculum authors, or access to prior instructors of the same course, or other aspects of the social environment that may support instructors to teach ambitiously.

Third, a main outcome of this study was not only the empirical results, but also the methodology used to attain these results. Much of the time and energy spent conducting this study was directed at developing an appropriate methodology for supporting claims regarding the relationship between curriculum materials and enactment. In the past, many research studies focused on connecting the written curriculum to student learning (Remillard, 2005; Mary Kay Stein et al., 2007). Only in recent years have these kinds of studies surfaced where the main focus is on very specific connections between the written curriculum and the enacted curriculum (Brown et al., 2009; Hill & Charalambous, 2012; Lewis & Blunk, 2012).

This type of methodology is not yet well developed. I scrutinized classroom enactments for very clear connections to the written curriculum by analyzing the exact language instructors used during enactment. While this methodology may contribute to
the field by suggesting one possible way of analyzing relationships between the written and enacted curricula, it is not the only way to study these relationships. Future studies may help to provide a fuller picture of how instructors use this type of curriculum by collecting various forms of data (other than interview and classroom videos), perhaps to include student data and tests of instructors’ knowledge before and after using the curricula in order to determine the ways in which these types of curricula educate instructors. Future studies might also include more quantitative methods, ethnographic methods, or mixed methodologies, just to name a few possible alternative methods for studying the relationships between written and enacted curricula.

While I investigated which features instructors found to be most supportive for helping them to teach ambitiously, the main focus of this study was not on explicitly ranking the helpfulness of each curricular feature. Instead, this study served as an exploratory study to lay the groundwork for the most common features instructors tended to rely upon and reveal some of the reasons as to why instructors relied upon those specific features. Future studies might focus on obtaining a much deeper understanding of the ways instructors plan their lessons when using this type of unique curriculum and deduct all of the possible ways instructors interpret each feature. This kind of information could be helpful in supporting curriculum developers to write and improve curricula that support instructors and teachers to teach ambitiously.

**Recommendations**

Based on these results, I suggest that curriculum developers consider ways to design curricula that include precise learning goals, descriptions of what exactly students
will be saying and doing if they are achieving the learning goals, explicit rationales, as many anticipations as possible in terms of what students will know, say, do, and where they will struggle or exhibit misconceptions, and finally suggested ways for instructors to set up tasks or respond to students in the form of exact statements or questions instructors can use during class. All of these features play important roles in educating instructors about the content of the lesson, what students will say and do, and where they will struggle, why this task was selected to help students achieve the learning goals, and how to help students achieve those learning goals.

The learning goals and rationales likely inform instructors of the ultimate goals for students in each lesson and the purpose of each task, which supports instructors to productively set up tasks and respond to students as they work. Furthermore, operationalizing the learning goals in the form of explicit descriptions of what students should be doing if they are successfully achieving the learning goals can support instructors to respond to students in both anticipated and unanticipated moments. Anticipating what students will say and do while they work can prepare instructors for what they can expect during class. And finally, suggestions for exactly what to say to set up tasks and to respond to students can support instructors to set up tasks productively and respond productively to students.

Yet having only one of these features is not enough to support instructors. Prior studies suggest having “scripts,” or suggested ways a discussion might unfold, without these other features can lead to unproductive enactments (Collopy, 2003). On the other hand, having only rationales for why certain pedagogical decisions or task selections
should be made may help to change teachers’ beliefs about how to enact a lesson is not enough to support them in enacting a productive lesson. Instructors and teachers should also be provided with examples of how to enact tasks, even if it is clear as to why the task or certain pedagogical decision are important. This was notable when instructors claimed that suggested responses in the lesson plan also helped them to respond in unproductive cases because they were able to see an example response of how to direct students toward the learning goals.

The fact that both of these novice MTEs primarily relied on the PowerPoint slides and the scripts in the lesson plan to set up tasks productively suggests that curriculum developers and authors might benefit from including explicit “scripts” in written materials and associated PowerPoint slides to aid in teachers’ enactments. In particular, the scripts used in these written lesson plans were also provided (although not fully) in the PowerPoint slides. These scripts were most useful in supporting instructors to set up tasks productively. The fact that they were made readily available to instructors in the form of PowerPoint slides which are immediately useable to implement “as-is” during enactment, is likely also a contributing factor to why these instructors were able to set up a majority of tasks productively.

Notable again is that the results of this study remain unclear about whether the anticipated student comments and possible instructor responses, which were sometimes presented in a “scripted” discussion format, would have supported instructors to respond productively to student comments and questions without all of these other curricular features. While anticipations and suggested ways to respond were included in these
lesson plans and had a primary influence on the ways these instructors responded, it is unclear that this influence would have been *productive* if the written curriculum did not also contain precise learning goals and explicit rationales. Therefore, curriculum developers and authors should consider when to use “scripted” task introductions and “scripted” discussions and how instructors will interpret them, as “scripts” can lead to unproductive enactments as well (see Collopy, 2003).

The anticipations and suggested ways to respond influenced instructors not only to respond to anticipated student comments and questions, but also surprisingly to *unanticipated* student comments and questions. By exemplifying ideal responses to PSTs in anticipated scenarios, instructors were provided an opportunity to infer the types of responses that would be ideal in any situation. Thus, the anticipations, suggested responses, learning goals and rationales appear to support instructors to *adapt* the lesson in-the-moment and ultimately teach *ambitiously* by responding to what students were saying and doing as they engaged in problem-solving tasks. This is highly noteworthy as these curricular features may potentially bridge the gap between the enactments of expert and novice instructors, as long as *all* of these curricular features are provided.

**Conclusion**

The results of this study suggest promising implications for effective ways to design curriculum materials in order to support ambitious teaching in novice instructors (and likely K-12 teachers). The main curricular features that supported instructors to teach ambitiously were: (a) precise learning goals, (b) descriptions of successful student behavior, (c) explicit rationales for each task, (d) scripts to introduce each task, (e)
anticipated student comments, questions, and difficulties, (f) helpful ways to respond to these anticipations, and (g) PowerPoint slides. These features can be written into all kinds of curricula, not just continuously improved curricula. However, the precision of these features and degree of detail is likely easier to create by gradually making small changes over time to the tasks and features.

These curriculum materials were developed and improved using empirical data from classroom enactments over time in order to gradually improve the curriculum materials for future mathematics teacher educators of this course. The degree of detail to which the learning goals, rationales, and anticipations were specified would not have reached such levels of depth without empirical evidence. Furthermore, the suggested ways instructors can respond to students was based on evidence from prior instructors responding to students in those ways and testing whether those exact responses helped students achieve the learning goals of each lesson. Therefore, continuously improved curricula developed through repeated cycles of empirical evidence and making small changes over time have higher likelihood of articulating these kinds of curricular features with great precision. As teachers, instructors, teacher educators, and curriculum developers, if we want to improve student learning, it would be wise to seek the most effective ways of supporting teachers and instructors to teach ambitiously through high-quality curriculum materials containing these kinds of features.
REFERENCES


Madison, WI. Retrieved from

and using information about students: A study of expert, novice, and postulant
https://doi.org/10.1016/0742-051X(87)90015-1

Carvalho, M. L., Honeycutt, S., Escoffery, C., Glanz, K., Sabbs, D., & Kegler, M. C.
(2013). Balancing fidelity and adaptation: Implementing evidence-based chronic
348–356. https://doi.org/10.1097/PHH.0b013e31826d80eb

Charalambous, C. Y., & Hill, H. C. (2012). Teacher knowledge, curriculum materials,
and quality of instruction: Unpacking a complex relationship. *Journal of Curriculum

related to conceptualizing and measuring textbook integrity. In J.T. Remillard, B.A.

A. E. Kelly & R. Lesh (Eds.), *Handbook of research design in mathematics and

mathematics textbook affected two teachers’ learning. *The Elementary School
https://doi.org/10.3102/0013189X034003003


https://doi.org/10.3102/0013189X14528039


Mathematics Teacher, 72, 490–504.


https://doi.org/10.3102/00346543047002335


Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on


http://www.jstor.org/stable/30034969

https://doi.org/10.3102/0034654307313793


https://doi.org/10.3102/0028312117717692


https://doi.org/10.3102/00346543075002211


https://doi.org/10.1007/s11858-014-0622-y


APPENDIX A

Lesson Plan for Lesson 3

**Topic: Numeration Systems II**

**Learning Goals:**

In this lesson, the pre-service teachers are given 5 basic symbols (A, B, C, D, and 0) and asked to invent a numeration system that has the same properties as the Hindu-Arabic numeration system (except for being base ten). To invent such a system, they will need to struggle to figure out the following features of the Hindu-Arabic numeration system.

1. The pre-service teachers need to unpack their understanding of the Hindu-Arabic system to figure out the properties of the system (additive, multiplicative, positional, place valued, zero property).
2. The pre-service teachers will need to struggle to figure out how the Hindu-Arabic system can represent any quantity by using ten basic symbols and by using only one digit in each place (unlike the Babylonian system). That is, they will need to figure out how the number of basic symbols and the sizes of the measuring units are related.

Imagine we are counting straws in the Hindu-Arabic numeration system and let the basic measuring unit be one straw. If we start with no straws and keep adding one more to a pile, we can assign the following numerals to the straws as we add them: “0, 1, 2, 3, 4, 5, 6, 7, 8, 9.” We have now “run out of” basic symbols. If we add one more straw, however, we do not need to create another basic symbol to represent the new amount. Instead, we bundle the ten straws into a measuring unit of size 10 straws, and represent this amount with the numeral “10,” where “10” means one measuring unit of size 10 straws and no measuring units of size 1 straw. Similarly, when we reach ninety-nine straws and then add one more, we exchange the ten measuring units of size 1 straw for one measuring unit of size 10 straws. So once again, we do not need a special basic symbol for [10] in the ones place. Now we have ten measuring units of size [10]. But once again, we do not need to create another basic symbol to represent the ten measuring units of size [10] in the tens place of the numeral. Instead we rebundle the ten measuring units of size [10] into one measuring unit of size [100], the next largest measuring unit. Hence we can represent this amount with the numeral “100,” which means one measuring unit of size [100]. Similar statements can be made for larger place values. In general, a modern base b place-valued numeration system can represent any quantity with only b symbols and one digit per place because (a) the basic symbols 0, 1, 2, 3, . . . , b – 1 can represent 0 of any measuring unit, 1 of any measuring unit, 2 of any measuring unit, . . . , and b – 1 of any measuring unit respectively, and (b) the sizes of the measuring units are [1], [b], [b groups of b], etc.
Pre-service teachers will develop and show these understandings by (a) carrying out the following mathematical actions and (b) giving explanations that involve these actions:

a. In the Alphabetician activity, the pre-service teachers will invent a base-five numeration system. If they do not accomplish this goal, then they will at least be able to use the base-five numeration system in the ways listed below after it is introduced by other pre-service teachers in the class or by the instructor.

b. The pre-service teachers will be able to count in the base five numeration system.

c. The pre-service teachers will be able to use the measuring units and basic symbols of the base-five system to measure and assign numerical values to physical amounts. For example, for the quantity of [777] dots, the pre-service teachers will be able to show how this quantity is bundled in this system by circling one measuring unit of size [625] dots, one measuring unit of size [125] dots, one measuring unit of size [25] dots, and 2 separate measuring units of size [1] dot each. After physically partitioning a quantity in this way, the pre-service teachers will be able to represent the measured quantity numerically (e.g., as AA0B) by using the basic symbols of the system to show how many measuring units of each type fit into the measured quantity.

d. The pre-service teachers will be able to make a place value chart that represents this process of measuring and assigning a numerical value to a quantity. The first row of the place value chart should show the measuring units as pictures of physical amounts. For example, a measuring unit of size [1] could be shown as one dot, a measuring unit of size [5] as 5 dots, a measuring unit of size [25] as 25 dots, etc. The place value chart should also show the multiplicative relationship between the measuring units; pre-service teachers should draw an arrow from each measuring unit in the place value chart to the next largest measuring unit and label the arrow “× 5” and be able to explain (and demonstrate with quantities or pictures of quantities) that this means that each measuring unit is 5 times as big as the measuring unit that is associated with the place to the right, that 5 copies of the smaller measuring unit will fit into the larger measuring unit, that we can find the larger measuring unit by making 5 copies of the smaller measuring unit, and that we can find the smaller measuring unit by partitioning the larger measuring unit into 5 equal parts. In the second row of the place value chart, the pre-service teachers should show the number of measuring units of each type that fit into the measured quantity, represented with the basic symbols (A, B, C, D, 0) of the system.

e. Given a quantity, the pre-service teachers will be able to write the base five numeral for the quantity, using the basic symbols A, B, C, D, 0.

f. The pre-service teachers will be able to identify the properties of their invented base-five numeration system, such as the fact that the highest digit is 4 (not 9) and the place values are powers of 5.

b) Descriptions of Successful Student Behavior
### Equipment:
- Base-five blocks: One of each type of block for each group (essential)
- 500 straws, 50 per group
- Poster paper and markers for each group

### Associated Files:
- L03_NumSystemsII_LP.doc
- L03_NumSystemsII_H1.doc
- L03_NumSystemsII_T.doc

### Associated Text:
- *Mathematics for Elementary School Teachers*, Bassarear – Section 2.3, pages 100 – 115

### Time: 0-45 min.

**Activity Flow – Part 1 – Whole group discussion of the homework**

#### Rationale
In the whole group discussion of the homework, the instructor’s emphasis on the idea of a measuring unit, the relationship between the sizes of the measuring units, and the idea of breaking a quantity into parts equal in size to the measuring units should (a) help students understand these ideas in the context of the Hindu-Arabic numeration system, and (b) increase the probability that students will apply these ideas during the Alphabetian activity. It is essential that the pre-service teachers learn how to make place value charts during the homework discussion. This will help them to organize their solutions and will provide a concrete representation of the meaning of a numeral, and both of these factors will allow a smooth transition to decimals in the next lessons.

#### Activity
Assign one homework problem to each of the groups. The groups should write their solutions on the board or a transparency. Let the group present their solution first. If necessary, re-explain their solution using the terms “measuring units,” “basic symbols,” and “times as big relationship between the measuring units.” Students should solve the problems as they were solved in the lecture in Lesson 2. **Emphasize the following actions on quantities: breaking a quantity to be assigned a numerical value into parts equal in size to measuring units, making a quantity x times as big by copying the quantity x times, fitting a quantity into another quantity, exchanging x groups of a quantity for a quantity x times as big. Develop**
their ability to talk about these ideas: Remember that the pre-service teachers will be teachers, so the instructor needs to develop their ability to explain concepts.

Be on the lookout for:

1. The nature of students’ explanations: Are they interpreting the symbols in terms of quantities? Or are they solving the problems in purely a numerical way? Students who are interpreting the symbols in terms of quantities draw pictures, use place value charts, talk about the relationship between the sizes of the measuring units, etc. Students who are solving the problems purely numerically might give an answer like this when they are trying to represent, for example, 780,021 circles with the Babylonian system: “On my calculator, I kept multiplying 60 by 60, trying to find something that would divide into 780,021. Then I subtracted that much off with my calculator. I got mixed up because of all the dividing and subtracting. I finally gave up.” Purely numerical approaches will not help them make sense of decimals later in the course.

2. Are students using exponents on the problems involving the Babylonian system? If a student is using exponents, encourage the student to represent the measuring units as amounts of stuff rather than numerals involving exponents. We have found that students who use exponents tend to adopt a mechanical approach to these problems; the student represents the quantities in the places as the base raised to a power (e.g., $6^x$) without recognizing the relationship between the places or thinking about the meaning of the symbols in terms of quantities (amounts of stuff). We have also found that these students tend to exhibit difficulties when we introduce measuring units smaller than the basic measuring unit later in this course. If they understand that measuring units are quantities, then place values that represent measuring units smaller than the basic measuring unit are not so hard to understand; they are quantities that are (the base) times as small as the next largest measuring unit, as opposed to, for example, “$6^x$.”

In the discussion of #8 and #9, ask the students why the numerals are hard to decode. [It is impossible to know which place value is represented.] The benefits of having a symbol for 0 should be elicited from the students. For example, ask, “Does the Hindu-Arabic system have this problem? Why or why not?” Point out the difference between the Babylonian placeholder and a true zero.

For #11, let the students lead and develop the discussion. If they cannot answer it as a class, just say, “You’re going to have to think about this some more.” If they cannot answer it now, they should be able to answer it after Lesson 5. Make sure this problem appears in some form on Midterm 1. (The instructor could modify it to make it easier to grade. For example, the instructor might ask students to explain how to represent 73/3600 of a fixed length with the Babylonian numeration system.)
After discussing the homework assignment, remind the students that solutions to Lesson 2 Classwork and Homework problems are in the appendix in the packet. They are also posted online on our Sakai site.

### Instructor Notes

**1. Theory of Learning Embodied in the Math 251 Lessons:**

a. The instructor should emphasize the following actions on quantities whenever it is possible to do so in the Math 251 course: partitioning quantities into parts (e.g., in constructing smaller measuring units from larger ones, breaking a quantity to be assigned a numerical value into parts equal in size to measuring units, finding a tenth of a quantity by breaking it into 10 equal parts, finding how many times a divisor fits into a dividend), making a quantity $x$ times as big by copying the quantity $x$ times (e.g., in constructing larger measuring units from smaller ones, in modeling multiplication), fitting a quantity into another quantity (e.g., in measuring quantities, in modeling division), treating a fraction of a quantity (e.g., one tenth of a quantity) as a quantity in itself (as opposed to part of a whole) by drawing it as a separate quantity and measuring it to assign it a numerical value, and exchanging $x$ groups of a quantity for a quantity $x$ times as big. This allows the pre-service teachers to recognize that these same actions underlie many of the concepts covered in elementary school mathematics. It also reflects our belief that the pre-service teachers develop meaning for many of these concepts by performing relevant actions on quantities. Homework problem #11 shows the potential power of these actions; by performing the same types of actions on quantities, students will potentially be able to re-invent how to represent quantities numerically when they are smaller than the basic measuring unit.

b. The learning theory on which the lessons are based is actually a pair of learning principles rather than a full theory. The two principles are especially relevant for learning goals that have a heavy conceptual component. As described by Hiebert and Grouws (2007), they are (1) conceptual relationships among mathematical ideas, representations, and procedures must be made clear, and (2) students must be given an opportunity to grapple or struggle with the critical mathematical concepts.

<table>
<thead>
<tr>
<th>Time: 45-50 min.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Activity Flow – Part 2 – Introduction to the Alphabetian project</strong></td>
</tr>
<tr>
<td><strong>Rationale</strong></td>
</tr>
</tbody>
</table>

This activity is designed to help the pre-service teachers achieve the major learning goals of the lesson. The pre-service teachers learned how to use the Hindu-Arabic system when they were young, so their understanding of the numeration system is often implicit. They need an explicit understanding of the Hindu-Arabic numeration system in order to teach it. This activity is designed to help them examine their own understandings of the Hindu-Arabic system and to independently figure out exactly how it works (as opposed to just telling them). This independent discovery has two added bonuses. First, it helps the pre-service teachers recognize that they do not understand their own system whereas they thought they did: The ability to accurately gauge one’s level of conceptual understanding is an essential attribute that we want them to develop. Second, because the Hindu-Arabic system has become so familiar to them,
they view it as transparent and easy to understand. Therefore, they assume children will readily understand the system (e.g., after a brief explanation by the teacher). This activity is designed to help them unpack all of the component ideas that are part of the system, and hence all of the different sites where children might experience difficulties. Finally the activity provides another context for exploring the properties and common structures of numeration systems; the groups tend to invent different numeration systems that have different properties.

**Activity**

In the last few days, we have looked at different numeration systems. We explored these systems because it helps you recognize and understand how the Hindu-Arabic numeration system works. Why would looking at other systems help you understand your own system? Imagine that my shirt is red, the desks are red, everything in this room is red; i.e., everything in your world is red. All your life, you have only seen red. Would you have a concept of color? The same is true for other concepts. If you have an opportunity to compare and contrast other numeration systems with the only numeration system that you have ever used, then it helps you develop a better understanding of your own system. It helps you to see the common structures underlying all numeration systems and how they represent quantities—e.g., that they all involve breaking a quantity into parts and representing the parts with a fixed set of measuring units.

In addition, you learned the Hindu-Arabic numeration system when you were young, and therefore you may have an implicit understanding of this system. You can use it, but you may not understand or be aware of all the concepts underlying the Hindu-Arabic numeration system. In order to teach it, you need to have an explicit understanding of this system and how it works. By looking at other numeration systems, it helps us to explicitly understand the features of numeration systems, including our own.

One of the goals of the next activity is to continue this process—to help you develop an explicit understanding of your own system. It is designed to help you unpack all the ideas that you already possess about your own numeration system, explore them, and make them explicit.

This activity will also help you to realize how complex the Hindu-Arabic numeration system is, and why children might have difficulties understanding this system. You may find that you do not explicitly understand how this system works—that it is not as simple as you think it is after years of using it. So, another goal is to help you experience what children experience when they try to understand and decode this system and to help you figure out exactly what they may have trouble with. [Put transparencies up: L04_NumSystemsII_T.doc]
Here’s the activity. [Distribute Handout 1, “The Alphabetian Project”.] Imagine that you are a member of a small tribe. Your tribe has a numeration system that uses the symbols A, B, C . . . Z. A represents [1], B represents [2], and so forth up to Z. Z represents [26]. This system has met your needs up to now. You were hunter-gatherers and you only needed to keep track of small amounts. Now your tribe has started farming and you have a need to keep track of larger amounts. You need a more sophisticated numeration system.

One day, a member of your tribe says that she has invented a new numeration system. She says that she can represent any quantity by using the symbols A, B, C, D, and 0. [Read ‘0’ as ‘zero.’] She shows you some blocks [hold up base five blocks], and says they are an essential part of her system for representing quantities. She says she will explain the system to you when she returns from a hunting trip. Unfortunately, she dies during the hunting trip. Now your tribe has to figure out what her system was.

That’s your job. You need to figure out her numeration system. The blocks tell you something about the structure of the system and how she figured out how to represent any quantity with just the five symbols A, B, C, D, and 0. Here’s another fact about her numeration system. Her numeration system has the same features as the Hindu-Arabic system (except for being base ten), but uses only the five given symbols. So again, her system works exactly the same way as the Hindu-Arabic system except for being base ten.

After you invent a system, ask yourselves, “Except for being base-ten, does this numeration system have the same features as the Hindu-Arabic numeration system? Does it work exactly the same way? If it doesn’t, keep trying to invent one that does. Ask yourselves: How does the Hindu-Arabic system work? If you really can’t figure out how to do this, then answer this question: How does your invented system differ from the Hindu-Arabic system and how is it the same?”

Hand out one set of base five blocks, and 50 straws and a few rubber bands, to each group. (The base-five blocks are the measuring units of the base-five system, so they provide a hint for students. The blocks suggest using measuring units that are related in a five times relationship and help them think about the relationship between the times relationship for the measuring units and the number of basic symbols in a modern based place valued numeration system. Instructors who fail to hand these out are leaving out the quantity component of the system, whereas the intent is to give the students both the basic symbols and the measuring units. They have to figure out how to use both of these elements in their system.)

Student Responses

1. The groups usually develop a variety of systems, including purely additive systems (like the Egyptian), subtractive and additive systems (resembling the Roman system), and base 10 place-
valued systems with multiple symbols in each place value (resembling the Babylonian system). Many groups cannot figure out how to use the 0 symbol and often use it as another non-zero value (e.g., A = [1], B = [2], . . . D = [4], 0 = [5]).

2. Most groups do not develop a place-valued system that has one digit per place (i.e., a base five system) because they do not recognize the relationship between the number of basic symbols in the Hindu-Arabic system and the magnitudes of the measuring units in the Hindu-Arabic system. Even though they have just worked with the Babylonian system, when they invent a place-valued system, they usually do not consider using a base other than 10; they use [1], [10], [100], [1000], etc. as the measuring units associated with their place values.

<table>
<thead>
<tr>
<th>Student Responses</th>
<th>Teacher Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students want the instructor's input on &quot;right&quot; or &quot;wrong&quot; ways, or on what they are &quot;allowed&quot; to do when inventing a numeration system.</td>
<td>Reassure the students that there are many possibilities. Encourage the students to try out their ideas and to reassess their developing ideas. Tell them to ask themselves, &quot;Does this work like the Hindu-Arabic system works?&quot;</td>
</tr>
<tr>
<td>&quot;Can we make 0 be the number after D?&quot;</td>
<td>Help them recognize that they are just creating a new set of basic symbols. Remind them that they should limit themselves to the given five basic symbols. It may help to use the Hindu-Arabic numeration system as an example. What would happen if I decided to add a symbol to our system? Like ~, and say it stands for [50]?</td>
</tr>
<tr>
<td>Students invent a base ten system. They recognize their system does not share a property of the Hindu-Arabic system; for many numerals, they have to use more than one digit per place value. The group addresses this problem by modifying the given symbols or their orientation so that they have more basic symbols. Some examples are:  • Students turn some symbols upside down or backwards.  • Students create symbols that are some combination of the original ones [ex: A – one straw, B - two straws, C - three straws, D - four straws, AD - five straws, BD - six straws, CD – seven straws, DD - eight straws, ADD - nine straws, BDD - ten straws].</td>
<td></td>
</tr>
</tbody>
</table>

Time: 50-65 min.

Activity Flow – Part 3 – Instructor prompting during the Alphabetian project

Rationale
In order to maximize the effectiveness of this activity and to develop the students’ understanding of the properties and features of place-valued numeration systems, at least one group must develop a base five system. Moreover, groups seem to get the most out of this activity when they develop a base five system, so the more groups that accomplish this, the better. Therefore, as the groups work, the instructor should make sure that as many groups as possible develop a base five system.

**Activity**

As the groups work, identify a group that appears to be moving toward a base five system. If they get stuck, give a prompt and leave. Return to monitor their progress and provide more prompts if necessary. Do this with as many groups as possible.

Here are some prompts that the instructor can use to help the group(s) develop a base five system.

1. After the group assigns quantities to the basic symbols (e.g., 0 = [0], A = [1], B = [2], etc.), ask the students to count objects with their basic symbols, and to represent each numeral in a place value chart that shows only the ones place. As they try to count, they start to recognize when they need a new place value (i.e., when they run out of basic symbols). When they recognize this, suggest that they try to count higher.

2. Groups often use a base ten system, and find they need to have more than one symbol in a place value for [5], [6], [7], and [8]. For example, students typically use A through D to represent the quantities [1] through [4], but run into trouble with their representations of [5] through [8]. The instructor can try (a) or (b).

   (a) The instructor can ask the following types of questions:

   *Why aren’t you satisfied with your system?*
   
   Sample response: Because we needed more symbols and now there is more than one symbol in each place value.

   *Why did you need to create additional symbols? Why were five symbols not enough for your system?*
   
   Sample response: Because we didn’t have enough to designate all the quantities between [0] and [9] so we had to create some or use additivity within the place values.

   *Why doesn’t the Hindu-Arabic system have this problem?*

   *You added basic symbols. Is there another way that you can try to resolve this issue of the number of symbols versus the size of the measuring units for your place values—that they don’t match?*

   *Why did you choose ten as the size of the next measuring unit? Does the size of the next measuring unit have to be ten?*
You are trying to maintain the size of the next measuring unit at [10] and do something with the symbols to accommodate the size of the measuring units. Is there another way to approach this problem? [Adjust the size of the measuring units instead.]

(b) The teacher can use straws to show the quantities [1] through [10] and ask the group to represent each quantity in a place value chart with their numeration system. Then ask the group to demonstrate the same type of process with the Hindu-Arabic system to help them recognize why they didn’t run into the same difficulty in the Hindu-Arabic system.

Instructor Notes

History of the lesson: The choice of the specific quantities that students are required to represent in their numeration system in Handout 1 was based on the following considerations:

- The choice of "five" is directed at the idea of a 5 for 1 exchange. In a base five system, this would be $10_{five}$.
- The choice of "ten" is to emphasize the similarity between the numeral 10 in base ten and the numeral $10_{five}$ in base five.
- The choice of "thirty-eight" requires generating three measuring units in a base five system.
- The choice of "seven hundred seventy-seven" has several benefits. Its representation in a purely additive system usually requires a large number of symbols, and its representation in a modified base ten system would require additivity within the place values. However, its representation in a base five system requires only four symbols and emphasizes the idea that each measuring unit is "five times as big or as small."
- The choice of "zero" emphasizes the two meanings of zero.

Time: 65-75 min.
Activity Flow – Part 4 – Making the Posters and Homework

When you see one group that has settled on a system, hand out the poster paper and pens to all groups. Remind the groups to put the sticky end at the top of their poster.

Each group will be presenting their numeration system to the class during the next lesson. Make a poster that explains how your system works. Your packet describes what you should put on your poster. If you do not finish, arrange to meet outside of class, or choose someone to make the poster at home.
APPENDIX B

Interview Protocols

Pre-Interview Protocol:
1. What do you view as the main idea behind this lesson?
2. What are your goals for this lesson?
3. What are you going to do to achieve these goals?
4. What do you think students have to do to accomplish these goals?
5. What do you think it’s important for students to do during this activity?
6. What do you view as the purpose of this activity [go through each activity]?
7. What is the purpose of each additional activity you created (if any were created)?
8. So you said your learning goals are…What are the things about the lesson plan that helped you decide on your main learning goals?
9. You’ve mentioned [these features of the lesson plan]…If you were going to rewrite the lesson plan and keep only one of them and remove the others, which one do you think would be the most important to keep?
10. You’ve mentioned [these features of the lesson plan]…Would you add another element that would have been most helpful?

Post-Interview:
1. How did the lesson go?
2. Is there anything that you thought went particularly well? Why do you feel that way?
3. Is there anything that did not go particularly well? Why do you feel that way?
4. Did something in the lesson plan influence you to react this way to that student’s question/comment?
5. Do you think your main goal for this lesson was achieved?
APPENDIX C

Coding Rubric for Creating Segments

A “segment” is the unit of analysis that marks either when the instructor is setting up a task, lecturing, or an interaction between instructor and student(s). The following categories are different “segments”:

- When the instructor gives **directions for a new task**, this is a new segment.
- Each **discussion/lecture** is a segment. The lecture may or may not include interactions with PSTs (such as the instructor asking the PSTs questions and PSTs answering).
  - Lectures differ based on the topic (see below for topic changes according to each lesson).
- When a **PST asks a question** during whole class discussion/lecture, this is a segment. The segment ends when the PST’s initial question is resolved. Note: The PST may ask more questions in line with his/her first question during this entire segment that should not be segmented. If the PST asks a question about a different topic, then this should be coded as a new segment.
- During individual PST work or group work on a task, every time the **instructor talks to a new** group/PST (unless a PST changes the conversation to a different topic and the instructor continues talking to that same group).

Examples:

- The instructor is modeling a multiplication number sentence and a PST asks, “If I wrote an addition number sentence, would that also be correct?” This would be the start of a new segment and the segment would end once this PST’s question is resolved.
- The instructor is modeling a division number sentence and a PST says “I’m confused about how you got the final answer.” (This is an implicit question of “How did you find the final answer?”)
- A PST asks, “Can I use the bathroom?” or a question unrelated to the course material—this should be coded as one segment and end after instructor resolves question.
- A PST answers the instructor’s question with an answer, but the tone is answered as if the PST is unsure of his/her answer so it sounds like a question. For instance
  - T: What is one interpretation? S: By place?

Non-examples:
The instructor is modeling multiplication and asking PSTs what the next step is during the entire process and PSTs are answering the instructor’s questions (correctly or incorrectly). As long as a PST does not ask a question, this would all be one segment.

A PST asks a question to clarify the instructor’s question but it is not clear that the PST is confused or has a misconception. In other words, the instructor asks a question, but then the PST answers with a question related to the instructor’s question (this does not count as a new segment). For instance:


Note: In general, segments should be coded **liberally** because the coders judging whether the segment is productive or unproductive are given the freedom to further divide a segment if they need to, but they are **not** given the freedom to combine segments.

Note: Topic changes for a lecture vary by lesson. The following specifies the topic changes for each lesson under study. These “changes in topics” were decided by reviewing the lesson plan for each lesson and how the lesson plan characterized different activities.

**Lesson 3 Topic Changes Include:**
- Discussion of the homework (each group presentation is a new segment or each new problem is a new segment, depending on the way the instructor structures the discussion).
- Discussion/lecture on properties of each numeration system (Tally, Egyptian, Babylonian, Hindu-Arabic). Any group work on any of these systems is a new segment.
- Introduction to the Alphabetian project.
- Group work on the Alphabetian project (each group conversation is a segment).
- End of class/concluding remarks or lecture by the instructor.

**Lesson 4 Topic Changes Include:**
- PSTs’ presentations of their Alphabetian projects (each group presentation is a new segment); or posters that the teacher brings in (each new poster is a new segment).
- Concluding remarks/discussion about their projects (if there is a conclusion/summary about the properties of place value systems or how their system works compared to the Hindu Arabic system, etc…this is one whole segment).
- Lecture/discussion of a “base five” system.
- Task intro to representing DB with straws
- Discussion/lecture of how to represent DB and concepts needed in order to do so (in general, not necessarily for this specific example).
- Task intro to the problem of adding C plus DB.
- Discussion/lecture of how to represent C+DB and concepts needed in order to do so (in general, not necessarily for this specific example).
Task intro to the problem of explaining the standard algorithm involved in doing C+DB (this may also be a discussion).
- Discussion of standard algorithm for C+DB and concepts needed to understand algorithm (in general, not necessarily for this specific example).
- Task intro to representing A00 straws.
- Discussion/lecture of representing A00 straws.
- Task intro to subtracting A00 – AB.
- Discussion of how to represent A00 – AB with straws and concepts needed in order to do so (in general, not necessarily for this specific example).
- Task intro to explaining standard algorithm involved in doing A00 – AB.
- Discussion/lecture of explaining standard algorithm for A00 – AB and concepts needed in order to do so (in general, not necessarily for this specific example).
- End of class/concluding remarks or lecture by the instructor.

Lesson 5 Topic Changes Include:
- Discussion of the homework (each group presentation is a new segment OR each new problem is a new segment, depending on the way the instructor structures the discussion).
- Introduction to base b place valued systems in general, why they will be working in different bases, and what the learning goals are for this lesson.
- Discussion/lecture about how measuring units are related in base b place valued systems.
- Discussion/lecture about numerals in base b place valued systems.
- Introduction to base five system (using symbols 0, 1, 2, 3, 4 instead of 0, A, B, C, D).
- Going over the base 3 example (either through interactive discussion, lecture, or group/individual work). Going through the entire process of knowing what symbols are in base 3, knowing how to make a PVC, knowing how to group the quantity of twenty Xs if BMU is one X, then knowing how to write this in PVC and as a base 3 numeral.
- Introduction to classwork.
- Discussion of each classwork problems (each new problem is a new segment).
- Discussion of having flexible understanding of “1” (includes relationship between quantities always being related to 1, and example of 4 away from my dorm room…4 what?)
- Introduction to problem with 2 Xs as BMU, asking students to bundle a quantity of Xs (example 4 in classwork).
- Discussion of example 4 (might all be one segment if interactive discussion about this problem as entire class).
- Introduction to remaining classwork problem (examples 5 and 6—two separate segments and any additional examples the T creates).
- Discussion of example 5 and 6 (two separate segments).
- End of class/concluding remarks or lecture by the instructor.
Lesson 16 Topic Changes Include:
- Differences between all-at-once versus by-place process involved in drawing diagrams (e.g., if we use the all at once interpretation, we would first need to take 7 and cut it into ten equal pieces and then copy that piece 15 times. If we use the by-place interpretation, we would first need to take 7 and make one copy of it, then cut 7 into ten equal pieces, and copy one of those pieces 5 times).
  o Differences between the two interpretations do not count as two different segments (e.g., all at once means find 15 tenths of 7, by place means find 1 group of 7 and 5 tenths of 7).
- General number sense about the size of the product in multiplication number sentences (when will the product be larger than b, or than 1, or smaller, etc…). Each time a new number sentence is discussed, this is a new segment (e.g., 2.3x3.1 versus .23x3.1).
- Misconception about multiplication always “making the answer bigger”
- Task intro to writing story problems involving multiplication (brief review of multiplication problem types included in this segment)
- Task intro to Miguel/Jose water bottle/of 1/of a quantity (all one segment)
- Discussion of Miguel/Jose/of 1/of a quantity (all one segment)
- End of class/concluding remarks or lecture by the instructor.
- Note: If a task intro is then followed by group work that lasts more than 60 seconds, the group work should be coded as its own segment.

Non-examples:
- Small changes such as if the BMU represents 2 blocks what is the MU of size 10 and then asking what is the MU of size 100.
- Mechanical knowledge versus understanding (such as discussing the meaning of multiplication and how to use it to draw an appropriate diagram for a multiplication number sentence)
- PSTs may use vague language that might sound like a topic shift but it may not be a topic shift (PSTs’ knowledge is still developing and will sound imperfect and mathematically incorrect at times).
- Shifting from general ideas to “specific” cases or specific ideas is not a topic change (such as understanding that a x b = c results in c being bigger than b when a is bigger than 1 and then giving a specific example to depict this abstract claim).
- Using pictures to show something with symbols or something abstract is not necessarily a topic change.


APPENDIX D

Coding Rubric for Enactments

Set Up
Set up of the task only includes the **introduction** of the task and **directions** for how PSTs should work on the task. It typically only involves the instructor talking, although PSTs may ask clarifying questions.

<table>
<thead>
<tr>
<th>Productive</th>
<th>Unproductive</th>
</tr>
</thead>
<tbody>
<tr>
<td>A productive task set up does ALL of the following:</td>
<td>An unproductive task set up does at least one of the following:</td>
</tr>
<tr>
<td>- Aligns with learning goal(s) of the task</td>
<td>- Does not align with learning goal(s) of the task</td>
</tr>
<tr>
<td>- Aligns with the learning theory of the entire course outlined in LP3 (see below)</td>
<td>- Does not align with the learning theory of the entire course outlined in LP3 (see below)</td>
</tr>
<tr>
<td>- Maintains cognitive demand level (does not lower the demand level)</td>
<td>- Lowers the cognitive demand level (substantial enough to change PSTs’ learning opportunities)</td>
</tr>
<tr>
<td>- Does not violate a direct command in the lesson plan (e.g., do not read “0.1” as “point one,” it should be read as “one tenth.”)</td>
<td>- Violates a direct command in the lesson plan</td>
</tr>
<tr>
<td>- Uses mathematically correct language, symbols, and expressions</td>
<td>- Uses mathematically incorrect language, symbols, or expressions</td>
</tr>
<tr>
<td>- Provides PSTs with concrete materials when appropriate (such as straws or blocks)</td>
<td>- Does not provide PSTs with concrete materials when appropriate (such as straws or blocks)</td>
</tr>
</tbody>
</table>

Instructor Notes
1. Theory of Learning Embodied in the Math 251 Lessons:
   a. The instructor should emphasize the following **actions on quantities** whenever it is possible to do so in the Math 251 course: partitioning quantities into parts (e.g., in constructing smaller measuring units from larger ones, breaking a quantity to be assigned a numerical value into parts equal in size to measuring units, finding a tenth of a quantity by breaking it into 10 equal parts, finding how many times a divisor fits into a dividend), making a quantity x times as big by copying the quantity x times (e.g., in constructing larger measuring units from smaller ones, in modeling multiplication), fitting a quantity into another quantity (e.g., in measuring quantities, in modeling division), treating a fraction of a quantity (e.g., one tenth of a quantity) as a quantity in itself (as opposed to
part of a whole) by drawing it as a separate quantity and measuring it to assign it a numerical value, and exchanging $x$ groups of a quantity for a quantity $x$ times as big. This allows the pre-service teachers to recognize that these same actions underlie many of the concepts covered in elementary school mathematics. It also reflects our belief that the pre-service teachers develop meaning for many of these concepts by performing relevant actions on quantities. Homework problem #11 shows the potential power of these actions; by performing the same types of actions on quantities, students will potentially be able to re-invent how to represent quantities numerically when they are smaller than the basic measuring unit.

b. The learning theory on which the lessons are based is actually a pair of learning principles rather than a full theory. The two principles are especially relevant for learning goals that have a heavy conceptual component. As described by Hiebert and Grouws (2007), they are (1) conceptual relationships among mathematical ideas, representations, and procedures must be made clear, and (2) students must be given an opportunity to grapple or struggle with the critical mathematical concepts (Lesson 3 lesson plan, pp. 4-5).

Instructor Responses to PSTs

Instructor responses are coded for how the instructor responds to PST comments/questions/difficulties/misconceptions while PSTs work.

<table>
<thead>
<tr>
<th>Productive</th>
<th>Unproductive</th>
</tr>
</thead>
<tbody>
<tr>
<td>A productive response by the instructor does ALL of the following:</td>
<td>An unproductive response by the instructor does at least one of the following:</td>
</tr>
<tr>
<td>- Aligns with learning goal(s) of the task</td>
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</tr>
<tr>
<td>- Aligns with the learning theory of the entire course outlined in LP3 (see above)</td>
<td>- Does not align with the learning theory of the entire course outlined in LP3 (see above)</td>
</tr>
<tr>
<td>- Maintains cognitive demand level (does not lower the demand level)—does not relieve PSTs struggle too soon, does not tell PSTs the answer too soon</td>
<td>- Did not address a PSTs’ misconception or misunderstanding in a way that relates back to actions on quantities (or conceptual meaning)</td>
</tr>
<tr>
<td>- Is mathematically correct</td>
<td>- Lowers the cognitive demand level (substantial enough to change PSTs’ learning opportunities)—relieves PSTs’ struggle too soon</td>
</tr>
<tr>
<td>- Focuses on concepts OR connections between concepts and procedures (uses pictures and diagrams OR uses pictures and diagrams along with symbols to connect concepts to procedures)</td>
<td>- Includes something that is mathematically incorrect</td>
</tr>
<tr>
<td>- Focuses on (or acknowledges) correct ways PSTs are reasoning, not ways the instructor is reasoning</td>
<td>- Focuses on purely procedures, algorithms, calculations, symbols, or equations</td>
</tr>
<tr>
<td>- Does not violate a direct command in the lesson plan (e.g., do not allow PSTs to use calculators)</td>
<td>- Pushes the way the instructor reasons about the task onto PSTs, when PSTs’ ways of reasoning differ from that of the instructor</td>
</tr>
<tr>
<td>- Corrects student language when appropriate (e.g., help PSTs use words like “measuring units”).</td>
<td></td>
</tr>
</tbody>
</table>

153
A productive response does NOT have to:
- Sound word-for-word like the suggested teacher responses in the lesson plan
- Answer PSTs with a statement (it can be a question or statement)
- Settle PSTs confusion

An unproductive response is NOT
- A “missed opportunity”. (A missed opportunity is not classified as productive or unproductive).
- Telling the PSTs that they can come to office hours to ask certain kinds of questions (that the teacher likely doesn’t have time to answer during class—especially regarding homework, quizzes, tests).

| |  
|---|---|
| | Suggesting an alternative method is not pushing the instructor’s reasoning on PSTs as long as the instructor acknowledges PSTs’ reasoning
| | Violates a direct command in the lesson plan (allows PSTs to use calculators)
| | Does not relate to the PST’s question/comment at all (doesn’t listen closely enough to PST’s confusion or misunderstands PST’s confusion)
APPENDIX E

Coding Rubric for Anticipations

Task set-ups will not be coded in terms of whether the set up was anticipated or not unless a new task has been created by the instructor. I assume that every task set-up, apart from any tasks created by the instructor, is thus anticipated because it is listed in the lesson plan.

Therefore, only interactions between the instructor and students will be coded as anticipated or unanticipated. The aspect that is being coded is what the student says or does. More specifically, every student question will be coded as either anticipated or unanticipated. Student comments will be coded as either correct, anticipated, or unanticipated. The reason for the addition of the code “correct” is because correct answers or statements typically do not pose the potential to interrupt the flow of class or change the intended learning opportunity. It is when students are incorrect or have errors in their reasoning that the intended learning opportunity has the potential to change. Therefore, the code anticipated or unanticipated will be used mainly for incorrect student comments. For the purposes of this study, comments coded as correct may be interpreted as anticipated in the lesson plan because instructors are assumed to have gained the necessary content knowledge from the lesson plan to determine whether student comments are correct.

Anticipation:
An anticipated student question or (incorrect) comment implies that it was written in the lesson plan (e.g., under a table of student responses, or written as a prediction in the description of the activity such as “students may try to use the repeated subtraction meaning of division”, etc). There may or may not be an associated suggestion for how instructors can reply to the student, but this does not affect whether the student comment or question was anticipated.

Unanticipated:
If the student’s question or (incorrect) comment is not written explicitly in the lesson plan, then the student comment or question should be coded as unanticipated.

Correct:
If a student’s comment or response to the instructor is mathematically correct, the comment is coded as correct. Student questions should never be coded as “correct.”
Lesson 3 – The Alphabetian Project

**The Problem**
Imagine that you are a member of a small tribe that lived thousands of years ago, when people were making the transition from being hunter-gatherers to farmers. You have a numeration system that is alphabetically based, so you are called Alphabetians. Like many other ancient peoples, your numeration system only works well up to a certain amount or quantity. So, for any amount greater than Z, you have no symbol at all.

<table>
<thead>
<tr>
<th>Amount</th>
<th>*</th>
<th>**</th>
<th>***</th>
<th>****</th>
<th>***</th>
<th>**</th>
<th>*</th>
<th>...</th>
<th>******</th>
<th>*******</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alphabetian numeral</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>E</td>
<td>...</td>
<td>J</td>
<td>K</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Amount</th>
<th>*******</th>
<th>...</th>
<th>*******</th>
<th>...</th>
<th>*******</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alphabetian numeral</td>
<td>L</td>
<td>...</td>
<td>Z</td>
<td>...</td>
<td>???????</td>
</tr>
</tbody>
</table>

Now that your tribe has settled down, you have “many” cows and “many” bags of wheat. Without an adequate numeration system, figuring out how many more cows you have this year than last year and determining each family’s share of the wheat harvest is very tedious. A young woman in your tribe had excitedly announced that she had invented a new counting system with which she can represent any amount using only the symbols A, B, C, D, and a new symbol she calls zero and writes as 0. Unfortunately for your tribe, she died on a hunting trip. So, your tribe still has a big problem and really needs a better way of representing quantities.

You need a new numeration system. Since the visionary member of your tribe is no longer with you, it is up to you to figure out her numeration system. Here is an additional fact about her numeration system: The numeration system that the young woman invented has the same features as our Hindu-Arabic numeration system (except for being base-ten). **It works exactly the same way, except for being base ten.**
The Task
Your job is to develop a system has the same features as the Hindu-Arabic system (except for being base-ten) but uses only the symbols A, B, C, D, and 0. You cannot use any other symbols. Be sure that you can explain how your system works.

The Poster
Each group will present their numeration system to the class during the next lesson. Make a poster for the class that presents your numeration system. The poster should include the following.

a) Count in your system from 1 to 50. That is, write out the numerals from 1 to 50 in your system.

b) Make a key that shows the basic symbols A, B, C, D, and 0 and the quantity (amount of physical stuff) that each symbol represents.

c) Make a place value chart that shows the measuring units for your system; include arrows between the measuring units that show the multiplicative relationship between the measuring units.

d) For each of the quantities below, show how your system would bundle them into measuring units and then write the corresponding numeral for these amounts.

(1) •••••• (We call this quantity “six.”)
(2) •••••••••••••••••• (We call this quantity “ten.”)
(3) •••••••••••••••••••••••••••••••••••••• (We call this quantity “thirty-eight.”)
(4) The quantity that we call “seven hundred seventy-seven.”
(5) The quantity that we call “zero.”

e) List the properties of your system (e.g., additive, multiplicative, place valued, etc.).

f) Respond to either (i) or (ii):

(i) Explain how your invented numeration system has the same properties as the Hindu-Arabic system.

OR

(ii) Explain how your invented numeration system differs from the Hindu-Arabic system and how it is similar.
Your poster should be organized exactly like the following template:

<table>
<thead>
<tr>
<th>Alphabetian Project</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part a) goes here</td>
</tr>
<tr>
<td>Part c) goes here</td>
</tr>
<tr>
<td>Part e) goes here</td>
</tr>
</tbody>
</table>
### APPENDIX G

**Handout in Student Course Packet for Lesson 16**

<table>
<thead>
<tr>
<th>(a) Write one multiplication number sentence that uses the three numbers above.</th>
<th>(b) Write a different multiplication number sentence that uses the same three numbers above.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>For the number sentence in part (a) above, write its meaning.</td>
<td>For the number sentence in part (b) above, write its meaning.</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>For the number sentence in part (a) above, use the grid below to model the multiplication. What is your BMU and what are your other measuring units? How did you represent ( b ) (in ( a \times b = ? ))? How did you represent how ( a ) operates on ( b )? How did you find the final answer from your model?</td>
<td>For the number sentence in part (b) above, use the grid below to model the multiplication. What is your BMU and what are your other measuring units? How did you represent ( b ) (in ( a \times b = ? ))? How did you represent how ( a ) operates on ( b )? How did you find the final answer from your model?</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Use the diagram and the meaning of your multiplication number sentence to write a <em>multiplicative comparison</em> story problem that can be solved with your number sentence from part (a). (The story should be about length of string.)</td>
<td>Use the diagram and the meaning of your multiplication number sentence from part (b) to write a multiplicative comparison story problem with the same context as the story to the left. (The story should be about length of string.)</td>
</tr>
</tbody>
</table>
APPENDIX H

IRB Approval
DATE: February 9, 2016

TO: Slobahn Young, BS in Mathematics, MS in Mathematics
FROM: University of Delaware IRB

STUDY TITLE: [858785-1] Analyzing Fidelity of Implementation of Instruction in a System of Continuous Improvement

SUBMISSION TYPE: New Project

ACTION: APPROVED

APPROVAL DATE: February 9, 2016

EXPIRATION DATE: February 8, 2017

REVIEW TYPE: Expedited Review

REVIEW CATEGORY: Expedited review category # (6,7)

Thank you for your submission of New Project materials for this research study. The University of Delaware IRB has APPROVED your submission. This approval is based on an appropriate risk/benefit ratio and a study design wherein the risks have been minimized. All research must be conducted in accordance with this approved submission.

This submission has received Expedited Review based on the applicable federal regulation.

Please remember that informed consent is a process beginning with a description of the study and insurance of participant understanding followed by a signed consent form. Informed consent must continue throughout the study via a dialogue between the researcher and research participant. Federal regulations require each participant receive a copy of the signed consent document.

Please note that any revision to previously approved materials must be approved by this office prior to initiation. Please use the appropriate revision forms for this procedure.

All SERIOUS and UNEXPECTED adverse events must be reported to this office. Please use the appropriate adverse event forms for this procedure. All sponsor reporting requirements should also be followed.

Please report all NON-COMPLIANCE issues or COMPLAINTS regarding this study to this office.

Please note that all research records must be retained for a minimum of three years.
Based on the risks, this project requires Continuing Review by this office on an annual basis. Please use the appropriate renewal forms for this procedure.

If you have any questions, please contact Nicole Farnese-McFarlane at (302) 831-1119 or nicolefm@udel.edu. Please include your study title and reference number in all correspondence with this office.