IMPACTS OF SHORT-TIME SCALE WATER COLUMN VARIABILITY ON
BROADBAND HIGH-FREQUENCY ACOUSTIC WAVE PROPAGATION

by

Justin Eickmeier

A dissertation submitted to the Faculty of the University of Delaware in Partial
fulfillment of the requirements for the degree of Doctor of Philosophy in
Oceanography

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Acoustical oceanography is one way to study the ocean, its internal layers, boundaries and all processes occurring within using underwater acoustics. Acoustical sensing techniques allows for the measurement of ocean processes from within that logistically or financially preclude traditional in-situ measurements. Acoustic signals propagate as pressure wavefronts from a source to a receiver through an ocean medium with variable physical parameters. The water column physical parameters that change acoustic wave propagation in the ocean include temperature, salinity, current, surface roughness, seafloor bathymetry, and vertical stratification over variable time scales. The impacts of short-time scale water column variability on acoustic wave propagation include coherent and incoherent surface reflections, wavefront arrival time delay, focusing or defocusing of the intensity of acoustic beams and refraction of acoustic rays.

This study focuses on high-frequency broadband acoustic waves, and examines the influence of short-time scale water column variability on broadband high-frequency acoustics, wavefronts, from 7 to 28 kHz, in shallow water. Short-time scale variability is on the order of seconds to hours and the short-spatial scale variability is on the order of few centimeters. Experimental results were collected during an acoustic experiment along 100 m isobaths and data analysis was conducted using available acoustic wave propagation models. Three main topics are studied to show that acoustic waves are viable as a remote sensing tool to measure oceanographic parameters in shallow water.
First, coherent surface reflections forming striation patterns, from multipath receptions, through rough surface interaction of broadband acoustic signals with the dynamic sea surface are analyzed. Matched filtered results of received acoustic waves are compared with a ray tracing numerical model using a sea surface boundary generated from measured water wave spectra at the time of signal propagation. It is determined that on a time scale of seconds, corresponding to typical periods of surface water waves, the arrival time of reflected acoustic signals from surface waves appear as striation patterns in measured data and can be accurately modelled by ray tracing.

Second, changes in acoustic beam arrival angle and acoustic ray path influenced by isotherm depth oscillations are analyzed using an 8-element delay-sum beamformer. The results are compared with outputs from a two-dimensional (2-D) parabolic equation (PE) model using measured sound speed profiles (SSPs) in the water column. Using the method of beamforming on the received signal, the arrival time and angle of an acoustic beam was obtained for measured acoustic signals. It is determined that the acoustic ray path, acoustic beam intensity and angular spread are a function of vertical isotherm oscillations on a time scale of minutes and can be modeled accurately by a 2-D PE model.

Third, a forward problem is introduced which uses acoustic wavefronts received on a vertical line array, 1.48 km from the source, in the lower part of the water column to infer range dependence or independence in the SSP. The matched filtering results of received acoustic wavefronts at all hydrophone depths are compared with a ray tracing routine augmented to calculate only direct path and bottom reflected signals. It is determined that the SSP range dependence can be inferred on a time scale of hours using an array of hydrophones spanning the water column. Sound speed
profiles in the acoustic field were found to be range independent for 11 of the 23 hours in the measurements. A SSP cumulative reconstruction process, conducted from the seafloor to the sea surface, layer-by-layer, identifies critical segments in the SSP that define the ray path, arrival time and boundary interactions. Data-model comparison between matched filtered arrival time spread and arrival time output from the ray tracing was robust when the SSP measured at the receiver was input to the model. When the SSP measured nearest the source (at the same instant in time) was input to the ray tracing model, the data-model comparison was poor. It was determined that the cumulative sound speed change in the SSP near the source was 1.041 m/s greater than that of the SSP at the receiver and resulted in the poor data-model comparison.

In this study, the influences on broadband acoustic wave propagation in the frequency range of 7 to 28 kHz of spatial and temporal changes in the oceanography of shallow water regions are addressed. Acoustic waves can be used as remote sensing tools to measure oceanographic parameters in shallow water and data-model comparison results show a direct relationship between the oceanographic variations and acoustic wave propagations.
Chapter 1

INTRODUCING WATER COLUMN VARIABILITY

1.1 Problem Foundation

The study of thermocline and water column variability can be traced back decades to several key contributors to the field. Studies of temperature fluctuations over short-time scales (minutes to hours) began in the late 1950’s. In shallow water, these fluctuations were found to be caused by internal waves following low and high tide cycles [1]. The first studies of radiation scatter over a homogeneous background environment with a superimposed inhomogeneous perturbation were conducted in the mid-1960's [2]. During this same period, an initial analysis of acoustic rays propagating through layered media with a changing index of refraction was conducted. The arrival of the ray(s) at a given point downrange of the source was determined to be a function of launch angle and the number of bottom bounces. In the absence of ray arrivals, the concept of a geometric shadowing zone was introduced [3].

Over the following years, the study of temperature influence on ocean acoustics was expanded. Preliminary modelling was limited by computational capabilities at the time, but early ray tracing results were obtained using a 3 layer model and geometric optics. Refraction was seen in the mid-water column and estimates of transmission loss through the thermocline were calculated [4]. In the early 1980’s a simple yet accurate formula for computing sound speed was introduced and was proven to be accurate at ocean test sites worldwide. Calculation of consistent SSPs using the same formula across ocean basins allowed for increased validity in data-model comparisons [5]. At the start of the 1990’s advances in computer processor design and digital storage media made numerical models for calculating transmission loss through acoustic fields
possible. This period also focused on internal waves and other fluid phenomena that can cause isopycnal displacement. The numerical models were driven by data collected from towed thermistor chains, CTD casts and acoustic backscatter measurements [6]. By the mid 1990’s a major shift in the field of ocean acoustics occurred when government and academic institutions began to study high-frequency acoustics in shallow waters after decades of lower frequency studies in the deep ocean basins.

The most significant challenge in the move to shallow water, is that water column variability exists in 3-D, is subject to both upper and lower boundaries and is subject to change over short-time periods from several seconds to several hours. The definition of “water column variability” includes, but is not limited to: evolving surface wave spectrum, temperature changes, oscillation of isotherms, fluctuations in the slope of the SSP at various depths, internal wave activity from single or multiple sources, mixing and / or turbulence issues as well as range dependence or independence of the SSP. Unmistakably, there are many complex layers involved in approaching the 3-D problem of high-frequency acoustic propagation in shallow water. However, all of the above factors are also pertinent to 2-D shallow water acoustic modelling.
Figure 1.1: Short-time scale variability with a) “range dependence restricted only to the rough surface” and b) range dependence some $N$ minutes later for numerous aspects of the shallow water 2-D acoustic field.

Fig. 1.1.a displays a typical set-up of a shallow water high-frequency experiment. With two stations typically separated by 1-10 km ($r$) with environmental measurements collected at both ends and ideally a third set of measurement collected at $r/2$. In Fig. 1.1.a, excluding the rough surface, all of the properties of the water column are range independent. The SSPs and wave spectra are identical at each station and a strong, deep isotherm is measured at a constant depth. In Fig. 1.1.b the time has stepped some $N$ minutes forward. At each station separated by range $r$, a unique SSP and surface spectrum are measured for this instance in time. At $r/2$ the passage of an internal wave depresses the isotherm and drives mixing / turbulence. Considering the multiple points of variability, the 2-D problem must be considered as range dependent in Fig. 1.1.b.

Each of the environmental variations shown in Fig. 1.1 has a unique influence on propagating high-frequency acoustics. The nature of such influences are studied
through coherent surface reflections from a rough surface, the angular spread of acoustic beams using beamforming, determining range dependence / independence of sound speed in the acoustic field and conducting a layer-by-layer reconstruction of SSPs to identify critical refractive layers that define acoustic ray path, arrival time and boundary interactions.. The purpose of this dissertation is to use the collective sum of the observations, modeling and analysis pertaining to the topics above and move the acoustical oceanography community towards a fuller understanding of how high-frequency acoustic are impacted by short-time scale water column variability in 2-D. Where knowledge of the 2-D problem can be expanded, the approach to the 3-D shallow water problem can be refined.

In Chapter 2, high-frequency experiments are analyzed and discussed as a prelude to the Kauai Acomms MURI 2011 (KAM11) experiment examined in this dissertation. Chapter 3 examines coherent surface reflections of 10 kHz center frequency linear frequency modulated (LFM) signals from transducers affixed to a pair of bottom mounted tripods. The tripods sit on the seafloor with a separation of 1 km and the bi-directional reflections from the sea surface are measured at a ship suspended monitoring hydrophone. Chapter 4 analyzes the angular spread of acoustic beams and wavefront arrival time of 25 kHz center frequency LFM signals sent from the UDel tripod at STA05 and beamformed with the 8-element vertical hydrophone array on the UDel tripod at STA07. Chapter 5 introduces a new source / receiver (multiple) geometry set and studies concepts of range independence and dependence of sound speed across multiple receiver depths with an overall focus on the influence of slope changes in the SSP.
Chapter 2

BACKGROUND: PREVIOUS EXPERIMENTS

2.1 HFA97 and HFA2000

Shallow water, high-frequency acoustic experiments were conducted in 1997 and 2000 in Delaware Bay (HFA97 and HFA2000 respectively). HFA97 studied the coherence of broadband acoustic waves with frequencies from 0.6-18 kHz transmitted between two stable tripods (each with a 3 hydrophone vertical array) [7]. This experiment was focused on the growing need for high baud rate, high-frequency underwater acoustic communications. Limits on the performance of underwater communication systems are strongly dependent on environmental variability. The impact of this variability is magnified in shallow water environments and can cause amplitude and phase fluctuations in received acoustic signals. The use of broadband transmissions adds an additional layer of complexity to the problem of predicting performance in underwater communications. The HFA97 experiment revealed the importance of further study of shallow water environmental fluctuations.

HFA2000 replicated the transmissions from HFA97 in Delaware Bay (15 m depth) during December 2000 (Fig. 2.1). A third tripod was added to the experimental infrastructure to study acoustic interactions with subsurface inhomogeneous water masses evolving from fluctuations in the water column temperature and current profile.
Reciprocal transmissions of LFM (0.345 s duration) were sent between three bottom mounted source / receiver tripod stations separated by varying ranges. Environmental data was collected at a nearby oceanographic platform; simultaneously, a shipboard ADCP and CTD were deployed. Analysis of direct path station-to-station arrival times (between Dec 18th 00:00 and Dec 19th 10:00 UTC, during which 126 acoustic transmissions each consisting of 29 chirps were sent) revealed significant deviation from arrival time patterns established during previous tidal cycles. Examination of the signal intensity over time revealed distinctive fluctuations in the direct path signal. Independent ADCP data showed an East-West current profile with velocity variations during the period of intensity fluctuations.
2.2 HFA2000 Observations

Figure 2.2: Environmental data collected during the HFA2000 experiment. A major storm event occurred between hour 24:00 and 48:00 indicated by a significant drop in a) atmospheric pressure and b) high wind speeds. c) Displays wind direction as measured from the nearby weather station. d) A period of irregular sound speed fluctuations is outlined in red following the storm event.
A shipboard ADCP was deployed throughout the duration of the HFA2000 experiment and collected environmental data critical to the analysis of acoustic measurements from the system of tripods. In Fig. 2.3 the top 7 m of data were removed from the data set to eliminate the “surface zone” and accentuate the current structure of the middle and bottom layers. The highlighted areas on the plot indicate periods of velocity reversal occurring between 8 and 12 m in depth. Development of the initial occurrence is outlined between 54:00 and 60:00. Looking 6 hours forward (66:00), the distortion has developed further and another rapid change in current speed and direction is observed at 72:00. The period of irregular sound speed fluctuation at 10 m depth was measured between hours 62:00 and 74:00 in Fig. 2.2.d, which overlaps the second current reversal event.
Figure 2.4: Arrival time deviation of a) single surface reflection and b) direct path signal from station C to A. Arrival time of c) single surface reflection and d) direct path signal from station C to B (See Fig 2.1 for geometry).

For the direct path signals sent from station C in Fig. 2.4, arrival time measured at stations A (DP C → A in Fig. 2.4.b) and B (DP C → B in Fig. 2.4.d) show arrival time fluctuations that deviate from the previous 20 hours of smooth oscillations. These rapid fluctuations in arrival time are driven by the periods of rapid current velocity fluctuation shown in Fig. 2.3.
2.3 Discussion

Figure 2.5: Intensity fluctuations in the direct path signal for the three cases of single station transmitter to two receiver stations. The time of current reversal and deviation as measured by the ADCP fall within the highlighted periods.

Reciprocal, direct path transmissions along the short 70 m path between stations A and B show minimal intensity fluctuations from hour 36:00 onwards (Fig. 2.5.a and Fig. 2.5.b, green plot line). For the long reciprocal transmission paths, 353 m between stations C and A (Fig. 2.5.a and Fig. 2.5.c, blue plot line) and 334 m between stations C and B (Fig. 2.5.b, B \rightarrow C, blue plot line and Fig. 2.5.c, C \rightarrow B, green plot line) there are multiple points of intensity loss in the direct path signal, ranging from -10 to -30 dB. There are multiple 10 dB intensity drops in the C to A direct path transmission (Fig. 2.5.c, blue plot line) that occur between 42:00 and 47:00 hours. These intensity drops are not detected in receptions from the C to B path (Fig. 2.5.c, green plot line). The times of peak intensity fluctuations fall within the periods of rapid current velocity reversal measured by the ADCP in Fig. 2.3.
Observations and analysis of HFA2000 data shows that short-time scale changes in the properties of the water column impact both the intensity and arrival time of high-frequency acoustics. A lack of resolution in environmental data restricted further analysis; however, future experiments would expand upon these initial observations and model shallow water, high-frequency acoustics utilizing high resolution water column and surface spectrum measurements. The primary experiment investigated in this study is the KAM11 experiment. KAM11 was designed to exceed the acoustic and environmental resolution of HFA2000.
Chapter 3

COHERENT SURFACE REFLECTIONS

3.1 Background

Time varying sea surface roughness introduced by surface gravity waves can have significant effects on acoustic wave scattering and reflections. When sound waves in water are incident on a rough ocean surface, scattered waves arise. The scattering occurs in the specular direction as a coherent wave and over a wider angular interval as incoherent components. The path of coherent elements is directly related to the angle between the direction of the surface wave propagation and the source-receiver track (azimuthally dependent on the source-receiver geometry). The problem is inherently 3-D and introduces an anisotropic upper layer boundary condition for the acoustic wave scattering and reflection. This problem has been of interest in the underwater acoustics community for some decades. In 1970, Fortuin [8] gave a comprehensive review of scattering and reflection of sound waves at the ocean surface. The review focused on Rayleigh’s and Uretsky’s methods for interaction with a sinusoidal surface. Rough surface predictions, conducted by Eckart were also reviewed. Eckart utilized the Kirchoff approximation and introduced the inverse problem of measuring the wave spectrum via low-frequency radiation. Diffraction at a rough surface was defined as a function of time, signal frequency and geometry.

From 1970 to the late 1980’s several experiments and theoretical studies were conducted on understanding and developing the sea surface reflection and scattering problem ([9], [10], [11] and [12]). In 1988, Thorsos [13] determined the limits of validity of the Kirchoff approximation for predicting scattering strength versus exact
integral solutions. The importance of surface correlation length was defined for high grazing angles as well as additional dependence of scattering strength on root mean square slope for low grazing angles. Williams, in 1994 [14] applied high-frequency approximations to forward scattering and examined the separate influences of geometric acoustics, interference and diffraction on spatially scattered pressure. In 1996, Dahl [15] conducted an experiment on the spatial coherence of sound that had been forward and incoherently scattered within a single surface bounce channel. A data-model comparisons using only wind speed to estimate the mean square large scale slope, proved satisfactory for matching normalized coherence length. Notable exceptions were found for the lowest wind speeds of the experiment at ~ 1.5 m/s.

A new ray tracing technique, the wavefront method, was introduced in 2002 [16] that allowed the evaluation of the acoustic field on both the illuminated and shadow side of caustics as well as resolving the focusing points of two caustics. In 2004, Preisig and Deane [17] conducted an experiment in the surf zone adjacent to Scripps Pier in San Diego, CA and identified caustics in an evolving folded wavefront diverging into multiple arrivals. In 2009, a tank experiment was conducted along with a wavefront model to resolve experimental and modeling results of surface wave focusing [18]. Variations in arrival time, amplitude and phase from surface wave reflections showed close agreement between theory and experiment. In 2012, a shallow water (70 m) experiment was conducted at the Martha’s Vineyard Coastal Observatory with surface displacement measured by upward looking sonar [19]. Resolving acoustic pulses scattered from the sea surface in a deterministic matter was met by some challenges, including the need for a more complete knowledge of the surface wave field (contributions to power spectral density at higher frequencies) and indications of out of plane scattering.

Because of the complexity in modeling 3-D acoustic wave interactions with the rough sea surface, PE modeling is selected to obtain a better understanding.
However, outside of the last decade, PE computer modeling has been time prohibitive due to heavy computational requirements. Recently a 2-D PE model was used to model observed surface reflections [20]. The results show strong striation patterns with extended delay in surface reflected arrivals, due to acoustic interaction of LFM signals with surface gravity waves. In this chapter, the rough surface conditions (wave height and wavelength) that yield striation patterns (caustics) are defined using the BELLHOP ray model. The role of surface wave spectrum expansion (adding high-frequency surface waves to a rough surface) is also examined. The ray equation is introduced and solved for ray travel time. Acoustic and surface wave measurements during KAM11 are shown along with the mechanism forming coherent reflections using the BELLHOP ray model with a moving rough surface.

### 3.2 Ray Theory

A mathematical derivation [22] of the ray equations begins with the Helmholtz equation in Cartesian coordinates $x=(x,y,z)$:

$$\nabla^2 p + \frac{\omega^2}{c^2(x)} p = -\delta(x - x_0) \tag{3.1}$$

In Eq. 3.1, $c(x)$ is the sound speed and $\omega$ is the angular frequency of the source located at $x_0$. The ray equations will arise from a solution to the Helmholtz equation in the form of:

$$p(x) = e^{i\omega r(x)} \sum_{j=0}^{\infty} \frac{A_j(x)}{(i\omega)^j} \tag{3.2}$$

Eq. 3.2 is known as a ray series and is generally divergent; however, asymptotic approximations to the exact solution exist. Making use of the first and second
derivative of the ray series, the first term in the Helmholtz equation can be expressed as:

\[
\nabla^2 p = e^{i\omega \tau} \left[ \left(-\omega^2 |\nabla \tau|^2 + i\omega \nabla^2 \tau \right] \sum_{j=0}^{\infty} \frac{A_j}{(i\omega)^j} + 2i\omega \nabla \tau \right.

\left. \sum_{j=0}^{\infty} \frac{\nabla A_j}{(i\omega)^j} + \sum_{j=0}^{\infty} \frac{\nabla^2 A_j}{(i\omega)^j} \right)
\]

Substituting Eq. 3.3 back into the Helmholtz equation and equating terms of like order in angular frequency yields an infinite set of equations for \(\tau(x)\) and \(A_j(x)\):

\[
O(\omega^2) \quad |\nabla \tau|^2 = c^{-2}(x)
\]

\[
O(\omega) \quad 2\nabla \tau \cdot \nabla A_0 + (\nabla^2 \tau)A_0 = 0
\]

\[
O(\omega^{1-j}) \quad 2\nabla \tau \cdot \nabla A_j + (\nabla^2 \tau)A_j = -\nabla^2 A_{j-1}
\]

The equation for \(O(\omega^2)\) for \(\tau(x)\) is known as the eikonal equation. \(O(\omega)\), an equation of order omega and \(O(\omega^{1-j})\) for \(A_j(x)\) are the transport equations. Eq. 3.4 contains a nonlinear PDE (eikonal) and infinite series of linear PDEs (transport), which can be solved by introducing rays perpendicular to the wavefronts of \(\tau(x)\). These rays define a coordinate system in which the eikonal equation reduces to a linear, ordinary differential equation.
\[ \mathbf{\nabla}\tau \text{ is a vector perpendicular to the wavefronts and } s \text{ is the arclength along the ray. The ray trajectory } \mathbf{x}(s) \text{ is expressed as:} \]

\[ \frac{dx}{ds} = c\mathbf{\nabla}\tau \quad 3.5 \]

To give the tangent vector in the differential equation above “unit length”, a factor of \( c \) is added.

\[ \left| \frac{dx}{ds} \right|^2 = c^2 |\mathbf{\nabla}\tau|^2 \quad 3.6 \]

With the tangent vector having unit length, then \( \left| \frac{dx}{ds} \right| = 1 \). At this point, \( \tau(\mathbf{x}) \) is unknown, so differentiation of the x component with respect to \( s \) allows the ray equations to be written in a form that only involves \( c(\mathbf{x}) \). Taking the derivate of 3.6 with respect to \( s \) yields:
The eikonal equation can be used to replace \((\frac{\partial \tau}{\partial x})^2 + (\frac{\partial \tau}{\partial y})^2 + (\frac{\partial \tau}{\partial z})^2\) in Eq. 3.7 which yields:

\[
\frac{d}{ds} \left( \frac{1}{c} \frac{dx}{ds} \right) = \frac{c}{2} \frac{\partial}{\partial x} \left( \frac{1}{c^2} \right) \tag{3.8}
\]

For ray trajectories, the following vector equation is obtained [22]:

\[
\frac{d}{ds} \left( \frac{1}{c} \frac{dx}{ds} \right) = -\frac{1}{c^2} \nabla c \tag{3.9}
\]

The BELLHOP ray tracing method requires a solution of the ray equations in order to solve for the ray coordinates. The model assumes azimuthal symmetry and solutions are only developed in the 2-D range depth plane. Implementing a cylindrical coordinate system with \(r\) equal to the horizontal range and \(z\) equal to the depth, \(r = r(s)\) and so the \([r(s), z(s)]\) coordinate of the ray is a function of the arclength \(s\) and \(c(r, z)\) is the sound speed. In this new coordinate system, the second order differential ray equation may be decomposed into coupled first order terms by introducing the auxiliary variables \([\xi(s), \zeta(s)]\) [21]:

\[
\frac{dr}{ds} = c\xi(s) \tag{3.10}
\]

\[
\frac{d\xi}{ds} = -\frac{1}{c^2} \frac{\partial c}{\partial r} \tag{3.11}
\]
\[
\frac{dz}{ds} = c\zeta(s) \quad 3.12
\]

\[
\frac{d\zeta}{ds} = -\frac{1}{c^2} \frac{\partial c}{\partial z} \quad 3.13
\]

Initial coordinates for \(r(s), z(s), \xi(s)\) and \(\zeta(s)\) are given as follows:

\[
r(0) = r_0, \quad z(0) = z_0
\]

\[
\xi(0) = \frac{\cos \theta_0}{c_0}, \quad \zeta(0) = \frac{\sin \theta_0}{c_0} \quad 3.14
\]

Figure 3.2: Relation of vector \(t_{ray}\) to source and curve in cylindrical coordinates.
The tangent vector to a curve \([r(s), z(s)]\) in cylindrical coordinates is \(\frac{dr}{ds}, \frac{dz}{ds}\).

With auxiliary variables, the tangent vector to a ray becomes \(c[\xi(s), \zeta(s)]\). It is necessary to solve for the phase of the ray in order to reach an expression of travel time. This is accomplished by solving the eikonal equation in the coordinate system of the rays:

\[
\nabla \tau \cdot \frac{1}{c} \frac{dx}{ds} = \frac{1}{c^2} \tag{3.15}
\]

Eq. 3.15 can be rewritten as:

\[
\frac{d\tau}{ds} = \frac{1}{c} \tag{3.16}
\]

This is the eikonal equation as a function of ray coordinate \(s\) which is a linear ODE.

The solution to Eq. 3.16 follows:

\[
\tau(s) = \tau(0) + \int_{0}^{s} \frac{1}{c(s')} ds' \tag{3.17}
\]

In Eq. 3.17 above, the integral is the ray travel time.

### 3.3 The BELLHOP Model

BELLHOP is a ray tracing model that can produce a number of useful outputs including eigenrays, arrivals and received time-series. It allows for range-dependence in the top and bottom boundary (altimetry and bathymetry) as well as in the SSP
inputs [40]. Additional inputs can be specified for top and bottom reflection coefficients as shown in Fig. 3.3 below.

Figure 3.3: Structure of the BELLHOP ray tracing program including input / output files.

If the eigenray option is selected, then the output fan of rays is winnowed to include only the rays that bracket a specified receiver location. The rays provide a sense of how energy propagates in the acoustic channel. If the arrival time option is selected, then the time delay of each ray is calculated for a specified receiver location. For each output option the arrivals can be sorted and labeled based on the number of bottom and surface interactions. This allows for rays with similar arrival time but divergent paths and quantity of boundary interactions to be properly classified. In
this chapter, the BELLHOP model output is processed to show 4 types of rays: direct path, bottom reflected, surface reflected and bottom-surface reflected paths.

### 3.4 Observations: 07/10 - 07/11 (4th Deployment)

Figure 3.4: KAM11 Observations. a) Experimental geometry showing the angle between the surface wave propagation and the acoustic track. The average angles $\theta = 42^\circ$ for the acoustic track between STA07 and the monitoring hydrophone and $\theta = 12^\circ$ for the STA05 track. b) Measured wave spectrums 2 hours apart. c) SSPs, also 2 hours apart. d) Measured intensity impulse response from STA05 to the monitoring hydrophone. f) Measured intensity impulse responses from STA07 to the monitoring hydrophone.

The KAM11 experiment was conducted during the summer of 2011 in a 100 m shallow water region near Kauai Island, Hawaii [20]. Two seafloor tripod transceiver systems configured with an 8-element hydrophone array and top mounted transducer were deployed 1 km apart, at STA05 and STA07 as shown in Fig. 3.3.a, from July 10th-11th. Repeated 40 s chirp sequences (10 kHz center frequency, 6 kHz bandwidth, chirp duration of 48 ms, period of 144 ms with 277 chirps for each 40 s
sequence) were sent on a four-minute schedule with a two-minute offset from the two
tripods. A monitoring hydrophone was deployed at a depth of 25 m from the bow of
the *R/V Kilo Moana*, which was positioned approximately midway between the two
tripods (Fig. 3.4.a). Environmental data were collected by a 16-element thermistor
string and a Waverider buoy. Here the acoustic measurements made on July 10th,
2011 23:52-54 UTC are analyzed in-depth. Fig. 3.4.b and 3.4.c show power spectral
density (Sf) for surface wave spectra and SSPs respectively.

Fig. 3.4.d shows the measured impulse responses between the STA05 source
and the monitoring hydrophone. Fig. 3.4.e shows the arrival pattern from the STA07
transmission. The acoustic measurements in both subplots show coherent reflections
from moving surface waves. Different parts of the ocean surface can generate strong,
coherent reflections, which arrive at the receiving monitoring hydrophone at varying
arrival times (relative to range of the surface reflection from the receiver). These short-
time surface-interacting paths cover an extended span in arrival time. When the ocean
surface moves, some of the coherent surface returns show varying delay over time,
yielding a striation pattern. The striation pattern has a strong directionality, which
relates to the angle between the acoustic track and relative direction of the surface
wavefront \( \mathbf{k} \) (Fig. 3.4.a inset).

We define \( \phi \) as the angle between the acoustic track and the dominant surface
wave train such that when the surface wave crest propagates in parallel to the direction
of acoustic wave propagation, the angle is zero degrees, that is \( \phi = 0^\circ \). When \( \phi = 90^\circ \),
the acoustic track and surface wave train are perpendicular. Increasing \( \phi \) leads to a
longer “apparent” surface wave wavelength, \( \lambda_s \), projected in the 2-D plane (which
encompasses the acoustic source and receiver). The relationship is given by \( \lambda_s = \lambda / \cos \phi \),
where \( \lambda \) is the true surface wave wavelength. The average angle of offset is \( \phi = 42^\circ \)
between the acoustic track from STA07 to the monitoring hydrophone and the direction
of wave crest propagation. For the STA05 to monitoring hydrophone acoustic path, the average angle of offset is $\theta = 12^\circ$.

### 3.5 Mechanism of the Coherent Reflections

![Figure 3.5](image)

**Figure 3.5:** Coherent reflections from sinusoidal and random surfaces. a) BELLHOP ray trace diagram. In the ray trace, the black and grey dotted lines are the direct path and single bottom bounce rays respectively. The solid black line and solid grey line are the surface reflected and bottom-surface reflected rays respectively. The surface wave frequency, wavelength, and height are chosen at 0.11 Hz, 129 m, and 1.5 m, respectively. b) Arrival time patterns of the surface and bottom-surface reflections under a moving sinusoidal surface wave with the characteristics described in a) with $\theta = 0^\circ$. c) Similar to b), except $\theta = 50^\circ$ d) Measured surface wave spectra with high-frequency extension. e) Arrival time patterns of the surface returns for power spectral density wave bandwidths of 0.132 Hz and e) 0.198 Hz.

The BELLHOP ray model, combined with moving sea surface inputs, is implemented to study the mechanism of the observed coherent reflections. When the
surface progresses over time, the snapshots of the moving surface are fed to the BELLHOP model. The ray model calculates the acoustic rays under each rough surface snapshot. With multiple successive runs, the acoustic model generates time-varying acoustic arrivals.

Fig. 3.5.a shows the ray diagram under a snapshot of a moving sinusoidal surface where the source / receiver geometry matches the STA05 to monitoring hydrophone path. The properties of the sinusoidal surface are taken from the measured surface wave spectrum in Fig. 3.5.d, which has a power spectral density peak at 0.11 Hz. For this peak, the corresponding wavelength is 129 m and the significant wave height is 1.5 m. Three segments of the surface, one at a wave crest and two near the neighboring troughs, reflect the acoustic rays to the receiver, generating three micro-path surface reflected eigenrays. Due to the close proximity of the source to the seafloor, the same phenomenon affects the bottom-surface reflected path.

When the surface moves slightly, the reflections from three surface patches can still reach the receiver as coherent arrivals. The arrival patterns for the surface and bottom eigenrays are shown in Fig. 3.5.a (inset) under the moving sinusoidal surface for 40 s. We note that the ray arrival pattern is closely related to the butterfly structure that was previously reported in a very shallow waveguide [17].

Due to the source-receiver geometry, the butterfly shape resulting from a sinusoidal wave is not symmetric; rather, it is skewed as shown in Fig. 3.5.b. When the surface propagates from the receiver to source direction, the butterfly pattern tilts upward. When the surface propagates in the source to receiver direction, the asymmetry develops in the opposite direction. When source and receiver are at the same depth, a symmetrical butterfly is generated.

We define $\beta=H/\lambda_s$ as the ratio of the significant wave height $H$ to the surface wavelength $\lambda_s$ along the acoustic path in the 2-D acoustic propagation plane. The ratio
β is important to the formation of the caustics under the moving surface. For a given range and geometry, a threshold ratio exists above which the butterfly shape will form in the arrival pattern. A larger value of β is more likely to generate the butterfly shape since it corresponds to a steep surface wave shape. We note that the angle between the surface wave train relative to the acoustic path becomes important as it changes the projected surface wavelength λs. Computer simulations reveal that sinusoid surface waves with parameters θ = 50° and λs = 129 m narrowly allow for butterfly pattern formation in Fig. 3.5.c for a significant surface wave height of 1.5 m. For this geometry, the threshold for “butterfly” formation is β > 0.007. This corresponds to a projected wavelength of 215 m and significant wave height of 1.5 m. The threshold value for a different experimental set-up will vary from the above value. However, a new β can be quickly determined by ray tracing.

To reproduce the surface effects observed in the experiment, an evolving linear sea surface model generated inputs for BELLHOP [20]. We note that a single-frequency sinusoidal surface wave cannot create the caustic phenomenon. The extended delay is only possible under the surface wave with a wide spectrum spread. The addition of higher frequency components increasingly roughens the surface by introducing new waves with lower amplitude (with respect to the peak) and shorter periods. The smooth arrival pattern without caustics in Fig. 3.5.e results from running BELLHOP with a narrow spectrum with bandwidth = 0.132 Hz in Fig. 3.4.d. Inclusion of additional high-frequency components in the surface wave spectrum can increase the occurrence of the caustics. This is shown by extending the narrow spectrum by 50% to a bandwidth of 0.198 Hz. The resultant arrival time pattern for the surface path is shown in Fig. 3.5.f, where the butterfly formation appears at the beginning and fades out after 20 s. The loss of the caustics is caused by waves with lower amplitude which no longer satisfy β > 0.007.


3.6 Coherent Reflection Under a Linear Surface

A data-model comparison is performed by running BELLHOP based on the experimental setup and using the measured SSP and surface wave spectrum on July 10th, 2011 23:52-54 UTC, a different period than previously examined [20]. The surface wave spectrum was extended to 2 Hz in Fig. 3.5.d which rolls off in a manner $\sim \omega^{-5}$. If the spectrum is not extended, the measured spectrum (ending at 0.58 Hz) is insufficiently rough to yield caustics with extended arrival times such as those seen in Fig. 3.4.d and 3.4.e. This extension of the surface spectrum, yields caustics with 5 ms separation from the main body of the surface and bottom-surface reflections. Fig. 3.6 shows the data-model comparison results. Fig. 3.6.a shows the BELLHOP modeling...
results for the STA05 to monitoring hydrophone path, while Fig. 3.6.b displays this result overlaid on the measured impulse response for this acoustic-receiver track.

Fig. 3.6.b shows notable agreements between the measurements and modeled results. The direct and bottom paths are evident in the comparison (both data and model outputs). Multiple features of surface reflections are predicted by the ray model. First, the early surface arrivals from the measurements show undulations with oscillation of surface wave height. This is reflected in the model outputs. Secondly, coherent reflection patterns or “striations” with extended delay are calculated by the ray model. The trend in impulse response measurements is closely matched by the overlaid BELLHOP results in the 6-10 ms arrival time range. Third, both data and model show the surface and bottom-surface reflections extending into the 10-15 ms range in arrival time (intermittently) for the STA05 source. These late surface returns share the directionality (increasing arrival time delays with advancing time) of the earlier reflection patterns and are closely approximated by the BELLHOP outputs.

Fig. 3.6.c and 3.6.d show some similarity in surface and bottom-surface arrival time structure. The transmission direction is reversed and the receiver range is shorter at 460 m for the STA07 source, compared with 570 m for the STA05 source. As a result, the directionality of the striations are reversed, showing decreasing arrival time delays with advancing time. In the 8-12 ms range, there is good agreement between measured striation directionality and BELLHOP results. However, the data-model comparison using the STA05 source is more complete.

3.7 Summary

In this chapter, the analysis was focused on arrival time fluctuations of coherent reflections from surface gravity waves during the KAM11 experiment. A 2-D ray model with an evolving rough sea surface is used to explain the mechanism and formation of the deterministic striation patterns. The model also yields matches for the
arrival time fluctuation of the surface returns with a moving linear surface generated from a measured surface with a high-frequency extended spectrum. In the next chapter, we focus on the angular spread of direct path and bottom bounce signals between the UDel Tripods.
Chapter 4

BEAMFORMING IN THE WATER COLUMN

4.1 Background

The Hawaiian ridge is an excellent internal wave generator due to its steep bottom topography and local barotropic tidal flow perpendicular to the Hawaiian Ridge. The tidal energy cascade is from basin scale barotropic waves down to internal waves and finally turbulence. Internal tides are susceptible to dissipation, refraction and energy transfer to other frequencies from intersection with other internal wavefronts or thermocline interactions [23].

The effect of water column variability on the propagation of broadband acoustic signals has been of interest in the acoustical oceanography modelling community for more than 4 decades [24]. In the 1970’s broadband signals were first used in acoustic tomography studies on the high end of ocean mesoscale ranges. During this period, pulse propagation experiments achieved results that agreed with established internal wave theory. However, due to limitations in hydrophone count and / or aperture length, spatial resolution was limited and the acoustic influence of other internal wave characteristics could only be estimated [25]. In 1989, a 3 km long, 50-element vertical array was deployed at a range of 1000 km from a source in the Pacific. This array allowed for measurement of wavefront arrival time variance at a resolution of ~ 1 ms [26].

In 2000, an experiment with a 64-element array in 100 m of water using 18 kHz broadband LFMs and carrier signal phase key shifted communication signals, found measured receiver performance to be comparable to predicted performance in the absence of thermal fluctuations in the mid-water column. However, receiver
performance suffered when thermal fluctuations were restricted to the lower water column [27]. In 2009, spatial and temporal spreading from ray and modal theoretical estimates were compared with low-frequency PE based simulations in a deep ocean environment. Both spreading types were found to be largely controlled by the background sound speed structure through the ray based stability parameter [28].

Previous papers concerning the KAM11 experiment investigated coherent acoustic surface returns in the upper and lower water column and the mechanism of deterministic surface reflected striation patterns with an evolving rough sea surface at 10 kHz center frequency [20,29]. This chapter focuses on fluctuations in the angular beam spread and intensity of the direct path and bottom reflected rays. The UDel Tripod at STA05 is the source and a hydrophone array is mounted on the UDel Tripod at STA07 (experimental geometry shown in Fig. 4.1 below, on a sloping bottom). The time period examined here (3rd UDel Tripod Deployment on 07/06-07/07, 2011) is 72 hours prior to the period of coherent surface reflection study (4th UDel Tripod Deployment).
Figure 4.1: Experimental set-up during the 3rd UDel Tripod Deployment (07/06 02:00 to 07/07 02:00 UTC). Thermistor string locations at STA04 (WHOI) and STA08 (MPL VLA1). Source, UDel Tripod (STA05), at 100 m depth with hydrophone array, UDel Tripod (STA07), at 108 m depth. The ray paths of interest in this paper are defined as follows: The direct (non-refracted) ray path and bottom reflection are shown in black, with the downward refracted direct path shown in green. In yellow, a ray path is downward refracted beyond STA07. Surface reflections are shown in dashed red.
4.2 Observations: 07/06 18:22-20:37 UTC (3rd Deployment)

Figure 4.2: Environmental observations and measured acoustic receptions at STA07. a) Thermistor plot with 25.5° and 24.5° C isotherms from thermistor string at VLA1 (STA08) b) Beamformed impulse response measured on 8-element array at STA07, from chirp transmissions at STA05. c) through f) Measured impulse response at STA07 for each of the 4 times of interest (T1→T4).

Two seafloor tripod transceiver systems, UDel Tripod at STA05 and UDel Tripod at STA07 (Fig. 4.1), configured with an 8-element hydrophone array and top mounted transducer were deployed 1 km apart along the 100 m isobath from July 6th -
7th, 2011. Thermistor strings were deployed on both sides of the UDel Tripods with the WHOI TS at STA04 and the VLA1 TS at STA08. The WHOI TS measured temperature to a depth of 90 m while the VLA1 TS extended to 98 m. Reciprocal 20 s chirp sequences (25 kHz center frequency, 6 kHz bandwidth, chirp duration of 48 ms, period of 144 ms and 138 total chirp count) were sent on a four minute schedule with a two minute offset from the two tripods. In this chapter we focus on refraction and spreading of the direct path and bottom reflected signal, from a tripod mounted source at 95 m depth, as received on an 8-element hydrophone array. With the deepest temperature range, data from the VLA1 TS at STA08 is selected for modelling purposes.

For a period on July 6th, 2011 from 18:22-20:37 UTC, short period oscillations of isotherms (25.5° and 24.5°C) in the mid to lower water column show the propagation of medium to low amplitude (10 – 20 m), short period (~1 hour) solitons along or across the acoustic path. These physical characteristics are
consistent with past measurements of internal wave driven isotherms fluctuations in shallow water around the Hawaiian Islands [30]. The bathymetry shorewards to the 100 m isobath decreases by 300 m over a range of 2 km. This degree of bathymetric gradient has been shown to generate short period solitary waves from tidal flows [31].

Presently, we analyze and model three times of interest and discuss the measurements taken at a 4th point in time. The following times were selected to span a period of observed fluctuations in direct path and bottom reflected wavefront arrival time and angular beam spread of the 25 kHz center frequency LFM sequence: T1 at 18:33, T2 at 19:18, T3 at 19:48 and T4 at 20:19 UTC.

At T1, in Fig. 4.2.a, the 25.5°C and 24.5°C isotherms show upward oscillation in the water column with the 24.5°C isotherm reaching a depth of 70 m. At T2, this isotherm has been depressed to 90 m by the propagation of a short period soliton. At T3, the water column recovers and the isotherm is elevated to 75 m. Downward oscillation of the isotherms at T4 show a component of periodicity to the temperature structure of the water column.

In Fig. 4.2.b the results of beamforming across the 8-element hydrophone array are shown. The intensity threshold (lower) for measuring the angular spread of an acoustic beam is set at 50 dB (just above the level of ambient or signal processing induced noise). A total intensity range of 60 dB is used for all measured and modeled results in this study to allow for valid comparison and analysis (this excludes the intensity range of the single frequency PE field figures, which have a different dB range to reveal beam features). For T1 in Fig. 4.2.b the angular beam spread of the beamformed impulse response ranges from -4° to 4°. With the depression of the isotherms at T2, the angular spread has expanded in range from -6° to 5°. The soliton passes the thermistor string at T3 and the angular spread narrows to a range of -4° to 4°. The intensity threshold for measuring the spread of arrival time is 35 dB.
For T1, in Fig. 4.2.c the wavefront arrival time spread of the direct path reception as measured on channel 5 of the tripod array at STA07 is 1.0 ms. This hydrophone is positioned at the vertical midpoint of the array, 2.5 m from the seafloor. The depression of the isotherms at T2 increased the arrival time spread of the direct path to 1.25 ms. At T3, the elevation of the isotherms yields a measured arrival time spread of 0.75 ms.

For T4, the 25.5° and 24.5°C isotherms are again depressed into the lower water column with measured angular and arrival time spread comparable to the measurements at T2. This depression in the water column following the upward recovery of the temperature profile at T3 has a period of ~ 1 hour from T2.

4.3 Beamforming Theory

Angular beam spread in the direct path / bottom bounce signal is measured and modeled using beamforming. A beamformer is essentially a spatial filter used to increase the signal-to-noise ratio (SNR) by taking advantage of signal coherence, over the incoherence of noise and summing across all elements. The goal of beamforming is to isolate plane wave signals arriving from varying angles with different arrival times. While collecting data for beamforming, a large overall array length is desired with uniform spacing of elements so the 3 dB (half power) beamwidth is small. Ideally, the element spacing should be less than ½ the acoustic wavelength of the highest frequency in the broadband signal to avoid aliasing [41] which causes duplicates of the main lobe to appear in the directivity pattern. In linear beamforming the array can be shaded / filtered in amplitude and phase at each hydrophone to achieve a desired change in the directivity sensitivity pattern. This process is known as array steering [33] and can reduce the intensity of sidelobes at the expense of widening of the main lobe.
One implementation of linear wave beamforming is known as a delay-sum beamformer. Through the application of phase delays to the input channels, the main lobe of the directivity pattern can be steered to a desired direction. The form of the phase delays are implemented on the input channels:

\[
\phi_n = -\frac{2\pi(n - 1)l \cos \phi'}{c}
\]  

4.1

A uniform linear array of hydrophones implementing the delays from Eq. 4.1 is a discrete receiving aperture with a directivity pattern [36]:

\[
D(f, \Phi) = \sum_{n=1}^{N} e^{j \frac{2\pi f(n-1)l (\cos \phi - \cos \phi')}{c}}
\]  

4.2

Above, \( N \) is the total number of hydrophones, \( l \) in the uniform inter-element spacing (see Fig. 4.3). The directivity pattern’s main lobe has moved to the direction \( \Phi = \Phi' \). The phase shift in the frequency domain can be implemented by applying time delays to the hydrophone inputs for a single-frequency. The delay for the \( n^{th} \) hydrophone is expressed as:

\[
\tau_n = \frac{(n - 1)l \cos \phi'}{c}
\]  

4.3

Eq. 4.3 is the travel time of the plane wave between the \( n^{th} \) hydrophone and the reference element. To compensate for spatial aliasing, a Gaussian weight, which like a Gaussian distribution, approximates the exact binomial distribution of events, is implemented on the delay-sum beamformer. The resulting weights are complex:
\[ w_n(f) = \frac{1}{N} e^{-j \frac{2\pi f}{c} (n-1) l \cos \theta} \quad 4.4 \]

The output of the hydrophone array is the sum of the weighted channels:

\[ y(f) = \frac{1}{N} \sum_{n=1}^{N} x_n(f) e^{-j \frac{2\pi f}{c} (n-1) l \cos \theta} \quad 4.5 \]

In Eq. 4.5, \( x_n(f) \) is the input signal and equivalently, in the time domain:

\[ y(t) = \frac{1}{N} \sum_{n=1}^{N} x_n(t - \tau_n) \quad 4.6 \]

In Eq. 4.6, \( y(t) \) is the output of the delay-sum beamformer.

![Diagram of a plane wave beamformer and grating lobe exaggeration due to aliasing when \( l \) is greater than \( \frac{1}{2} \) the acoustic wavelength.]

Figure 4.4: A plane wave beamformer and grating lobe exaggeration due to aliasing when \( l \) is greater than \( \frac{1}{2} \) the acoustic wavelength.

As discussed in previous experiments [20], a set number of hydrophones are deployed on each of the UDel Tripod arrays, 8-elements in the case of KAM11. At each tripod array the 8 hydrophones are separated by 50 cm over a total array length of 3.5 m. The LFM signal used during this deployment has a maximum frequency of
28 kHz where ½ the acoustic wavelength is 2.675 cm. As a result, duplicate zones of high intensity form at the wrong bearings and arrival times. Ideally, beamforming is conducted with the smallest separation of hydrophones possible to minimize aliasing [34]. During KAM11 a Gaussian weighting function is applied to the received signal [7]. The process of Gaussian weighting in beamforming is a probabilistic estimate of the pressure for all point couples.

4.4 2-D Parabolic Equation Model

The 2-D PE rough surface model is ideally suited to modeling results from KAM11 were spatial aliasing in beamformed data occurred due to large element spacing. In this PE model it is possible to set the vertical grid spacing to less than 50 cm, reducing spatial aliasing when beamforming. PE runs are conducted with a range independent SSP calculated from the VLA1 thermistor string. A downward sloping bottom is used with a flat surface. With the first arrival group (direct path and bottom reflected signals) as the focus of this chapter, the inclusion of a rough surface is unnecessary. The PE is set to use 512 frequency bins over the 6 kHz bandwidth of the high-frequency 22-28 kHz LFM. The horizontal grid spacing is set at 1 m over a range of 1 km. The vertical grid is set at 2000 elements over 108 m of depth, with 5.4 cm separation, which is approximately equal to the acoustic wavelength of a 28 kHz signal (at 5.48 cm) at a reference sound speed of 1535 m/s. To beamform at ½ the acoustic wavelength and satisfy Nyquist criterion, twice the number of vertical elements would be required, exceeding fast Fourier transform size limits in this model. However, beamforming with 5.4 cm element spacing across multiple geotimes shows no evidence of energy at the wrong bearings. In the absence of sidelobe intensification, the Gaussian weight in Eq. 4.4 has no application.
The angles of any surface reflected signals incident upon the artificial beamformer (set to the same overall depth span as the actual 8-element array from 104 to 107.5 m) are outside the arrival time and angular envelope for the first arrival group. As such, the signals of interest are easily separable. The following is a brief overview of the formulation of the parabolic equation approximation.

The formulation begins with the Helmholtz equation in cylindrical coordinates [38]:

$$\frac{\partial^2}{\partial r^2} P + \frac{1}{r} \frac{\partial}{\partial r} P + \frac{\partial^2}{\partial z^2} P + k_0^2 n^2 P = 0$$  \hspace{1cm} 4.7

Pressure is expressed as a product of an envelope term $\psi$ and the Hankel function $H_0^1$.

$$P(r, z) = \psi(r, z) H_0^1(k_0 r)$$  \hspace{1cm} 4.8

Eq. 4.8 is plugged into the Helmholtz equation and the sorting of terms reveals a Bessel equation solved by the Hankel function with a zero sum. Considering a far field ($k_0 r >> 1$) in which $H_0^1$ can be approximated as:

$$H_0^1 \approx \frac{2}{\sqrt{\pi k_0 r}} e^{i(k_0 r - \frac{\pi}{4})}$$  \hspace{1cm} 4.9

The far-field Helmholtz equation can now be written as:

$$\left[ \frac{\partial^2}{\partial r^2} + 2i k_0 \frac{\partial}{\partial r} + k_0^2 \left( n^2(r, z) + \frac{1}{k_0^2 \frac{\partial^2}{\partial z^2}} - 1 \right) \right] \psi(r, z) = 0$$  \hspace{1cm} 4.10
Eq. 4.10 can be further expanded using operators defined as:

\[
R = \frac{\partial}{\partial r} \quad Q = \left( n^2(r,z) + \frac{1}{k_0^2 \frac{\partial^2}{\partial z^2}} \right)^\frac{1}{2}
\]

Substitution of the R and Q operators into Eq. 4.10 and factoring yields:

\[
[R^2 + ik_0 - ik_0Q][R + ik_0 + ik_0Q] + ik_0QR - ik_0RQ
\]

The assumption of weak range dependence of \( n \) allows \( R \) and \( Q \) to commute. The forward travelling term in 4.12 can now be expressed as:

\[
\frac{\partial}{\partial r} \psi(r,z) = -ik_0 \left[ 1 - \left( n^2(z) + \frac{1}{k_0^2 \frac{\partial^2}{\partial z^2}} \right)^\frac{1}{2} \right] \psi(r,z)
\]

Eq. 4.13 is known as the PE approximation. Next, the approximate solution to the square root operator allows the index of refraction to be moved outside the square root. The approximate solution is given by:

\[
Q \equiv \left( 1 + \frac{1}{k_0^2 \frac{\partial^2}{\partial z^2}} \right)^\frac{1}{2} - n + 1
\]

Using \( Q \) in the PE approximation:

\[
\frac{\partial}{\partial r} \psi(r,z) = ik_0 \left[ 1 - \left( 1 + \frac{1}{k_0^2 \frac{\partial^2}{\partial z^2}} \right)^\frac{1}{2} - n + 1 \right] \psi(r,z)
\]
The operations on the index of refraction and depth variable are separated and Eq. 4.15 is the wide-angle parabolic equation (WAPE) [39]. The PE solution in the numerical domain steps forward from a known value \( \psi(r = r_0, z) \):

\[
\psi(r+\Delta r, z) = e^{-ik_0\frac{\Delta r}{2}}U_{op}(r+\Delta r, z) \ast FFT\{e^{-ik_0\Delta rT_{op}(k_z)} \ast IFFT\{e^{-ik_0\frac{\Delta r}{2}U_{op}(r, z)} \ast \psi(r, z)\}\}\tag{4.16}
\]

Calculations involving the index of refraction are performed in the physical domain, while those involving depth are performed in the wavenumber domain. The operators \( U_{op} \) and \( T_{op} \) are defined as follows:

\[
U_{op} = -n + 1 \tag{4.17}
\]

\[
T_{op} = 1 - (1 - \frac{k_z^2}{k_0^2})^\frac{1}{z} \tag{4.18}
\]

### 4.5 Analysis

The results of beamforming with the Gaussian filter on the downslope transmission from STA05 to STA07 are compared with the beamformed PE output. The arrival time delay window spans 1.2 ms and the angular range is from -10° (down from the surface) to 10° (up from the bottom). The full range acoustic pressure field calculated by the PE is shown with an inset corresponding to an enhanced view closer to the receiver. The SSP for T1, T2 and T3 are shown below in Fig. 4.4 and contrasted.
Figure 4.5: Data-model comparison. a) Beamformed result of STA07 of direct path (measured data) impulse response at T1, 18:33. B) Beamformed PE result. c) PE acoustic TL field for STA05 to STA07 with center frequency 25 kHz. d) SSP from VLA1 at T1. Matching figures as designated above respectively in e), f), g) and h) for T2. Matching figures as designated above respectively in i), j), k) and l) for T3. m) Color enhanced zoom view of ray refraction in acoustic field at T1, n) for T2 and o) for T3. p) Comparison of measured SSPs from VLA1 for T1, T2 and T3.
At T1 (18:33 UTC) in Fig. 4.5.a a Gaussian filter has been applied to the beamformed impulse response of measured data across the array and shows low intensity spreading over a total angular spread of 8° defined by a transmission loss (TL) minimum threshold of 50 dB. The beamformed PE result in Fig. 4.5.b shows a total angular spread of 8° with the same 50 dB TL threshold. In Fig. 4.5.c, downward refraction in the PE acoustic field (inset view in Fig. 4.4.m) occurs between 70 and 80 m depth with beam focusing at 75 m (divergence occurs below 80 m and above 65 m). In the measured SSP (Fig. 4.5.d) the steepest slope in sound speed change (velocity per meter of depth in the water column) begins at 65 m depth and extends to 75 m.

At T2 (19:18 UTC) in Fig. 4.5.e the beamformed result shows separate regions of local maxima and minima intensity. The angular separation of regions of high and low intensity in the direct path signal is unique to the depressed isotherm structure at this time. The total angular spread in the data is 11°. The PE beamformed result in Fig. 4.5.f also has separate regions of varying intensity with a total angular spread of 10°. The PE acoustic field in Fig. 4.5.g (inset view in Fig. 4.5.n) shows downward beam refraction below 80 m and the strongest beam convergence at multiple depths between 92 and 108 m. In the measured SSP shown in Fig. 4.5.h the highest magnitude slope in sound speed begins at 82 m depth and extends to 92 m.

At T3 (19:48) in Fig. 4.5.i the beamformed result of measured data shows a total angular beam spread of 8°. The PE result in Fig. 4.5.j has a total angular spread of 7°. The PE acoustic field in Fig. 4.5.k (inset view in Fig. 4.5.o) shows downward refraction between 70 and 90 m with beam divergence above and below. In the measured SSP shown in Fig. 4.5.l the highest magnitude slope in sound speed is found between 70 and 80 m depth. On average the angular beam spread calculated in the PE is 1° less than that of the measured result.
4.6 Discussion

In Fig. 4.5.p, the SSPs at T1, T2 and T3 are plotted against each other, with T1 and T3 sharing similar profiles. As a result, the beamformed acoustic data, 2-D PE acoustic field and beamformed PE output are a close match in terms of angular and temporal spread. The temperature profile at T1 and T3 are both times of elevation of the 24.5 °C isotherm in the water column. At T2, the time of greatest isotherm depression, the measured SSP shows greatest deviation from the T1 and T3 profiles. This occurs between 60 m and 98 m reaching a maximum of +3.5 m/s at 85 m depth. This deviation in the SSP drives strong downward refraction of rays with an initially positive launch angle.

Fluctuations in the temperature profile are shown to influence ray path refraction and angular spread in the direct path and bottom bounce rays over short periods of time. It is shown that in both measured data and PE modeled beamforming results that the angular beam spread and intensity of the direct path and bottom reflected receptions in the lower water column are a function of isotherm depression and elevation. The steep slope of bathymetry shorewards and internal waves propagating towards the acoustic path from different source directions are driving factors of isotherm oscillations over short-time periods.

4.7 Summary

The angular, temporal and spatial spread of the received signal are functions of the water column temperature structure. This vertical structure of isotherms rise and fall in depth over periods of several minutes to 1 hour. The vertical movement of isotherms changes the SSP by altering the slope of sound speed in the thermocline. A positive slope is downward refracting while a negative slope refracts rays towards the surface. The magnitude of sound speed slope in a layer of the water column is
indicative of the refractive strength in that layer. The KAM11 experiment is dominated by a downward refracting thermocline.

The next chapter introduces a new source / receiver geometry with multiple receiver depths at a range greater than that used to study coherent surface reflections or angular beam spread. This allows the study of short-time scale water column variability to include depth variation. Concepts of sound speed range independence and dependence in the water column are also investigated. The influence of slope changes in a stratified water column are examined using ray tracing techniques.
Chapter 5

SOUND SPEED FLUCTUATIONS AND ARRIVAL TIME STRUCTURE

Figure 5.1: Full geometry of the KAM11 experiment

5.1 Background

The simultaneous deployment of multiple thermistor strings and hydrophone arrays during the KAM11 experiment (specifically during the 3rd UDel Tripod Deployment) presented a unique opportunity to investigate short-time scale water column variability at multiples depths and ranges outside of the STA05 to STA07 1 km path previously studied. To date, it has been sufficient (in terms of accuracy of data-model comparisons) to use the VLA1 thermistor string at STA08 for modelling transmissions between the UDel Tripods. The VLA1 thermistor string offers greater depth resolution in the lower half of the water column than the WHOI thermistor string at STA04. With the study of angular spread in the previous chapter focused on refraction in the lower water column, the deep temperature measurements were
considered more important than using the thermistor string closest to the source; a hypothesis that will be tested later in this section.

Chapter 4 introduced the concept of slope in the SSP, a property that has both spatial and temporal variation. Also, the choice of a single thermistors string for modelling in previous chapters assumed range independence in the sound speed profile. For modelling below the thermocline at STA07, at ranges not exceeding 1 km, model results have validated this selection. Chapter 5 introduces a new source / receiver (multiple) geometry set, studies concepts of SSP range dependence across multiple receiver depths with an overall focus on how slope changes in the SSP influence the arrival time structure of multiple ray paths.

Figure 5.2: Bathymetry off the Northeast coast of Kauai. Stations relevant to this study along the 100 m isobath are identified. Five km seaward from the 100 m isobath the bottom bathymetry exceeds 600 m in depth as the slope of the seafloor steepens.

The source in this chapter is the UDel Tripod at STA05 during the 3rd tripod deployment (unchanged from the beamforming analysis; however, 23 hours of the
deployment are studied here). The 16-element VLA1 hydrophone array at STA08 (a straight line range of 1480 m from STA05), which vertically spans the water column from 35.4 to 91.3 m in depth (Fig. 5.1) is used to measure LFM transmissions from STA05. Between the WHOI and VLA1 thermistor string (TS) there is sufficient difference (and lack of consistent features) between the temperature profiles measured at STA04 and STA08 (for all measured times) to indicate time dependent mixing / turbulent activity occurring along the acoustic path. As such, a range independent SSP may not suffice for accurate modelling at all times. In addition to modelling refraction and arrival time in BELLHOP with range independent SSPs (from WHOI TS and VLA1 TS), range dependent SSPs between STA04 and STA08 (2 km separation) are input to BELLHOP for all geotimes of interest assembled using multiple interpolation methods: linear, spline and cubic (with Gaussian or geometric rays). This allows for range dependent SSPs to be used in the data-model comparison when range independent SSPs in BELLHOP (from either WHOI or VLA1) yield arrival time structures divergent from measured data at multiple depths. Arrival time “structure” refers both to the type of arrival (direct or bottom reflected rays), temporal separation from other arrivals and any geometrically shadowed rays. A ray “trace” refers to a series of eigenrays that correspond to the ray arrival time plot.
The selection of the STA05 to VLA1 transmission path as the geometry of interest was not arbitrary and required consideration of all additional arrays deployed during the 3rd Deployment. Transmissions from the UDel tripods at STA05 and STA07 to VLA2 at STA16 have ranges of 5500 m and 4500 m respectively. These receptions have insufficient intensity (for matched filtering of data) and lack the required environmental data resolution to model direct path and bottom bounce arrivals at these extended ranges. However, the VLA1 array has a range of 1480 m from STA05 and 500 m from STA07. For the downward refracting SSPs typical of this deployment, direct path and bottom reflected signals (without surface interactions) reach their turning depths at ranges exceeding 500 m. Therefore, the STA07 to VLA1 transmissions are not examined in detail. However, the STA05 to VLA1 transmissions have a straight line separation of 1480 m which is an ideal
balance between the range to thermistor strings and the complexity of the observed arrival time structure from matched filtering.

The objective is to determine sound speed range dependence as a function of time, a property of the water column indeterminable with data from the thermistor strings deployed in KAM11. For both eigenray and arrival time modes, sound speed range independent run options include sound speed data from the WHOI or VLA1 array using Gaussian or geometric rays. Sound speed range dependent run options include; interpolation between the WHOI and VLA1 thermistor strings using linear, cubic and spline type interpolation using Gaussian or geometric rays.

5.2 Data-Model Comparison

With the source / receiver pairing determined, a method of interpolation must be selected to rebuild the water column. A flat surface is selected (as surface reflections have been analyzed in Chapter 3) and a simple sandy bottom built by a bathymetry file built using the depth of moorings nominally located along the 100 m isobath (STA05 is on the seafloor at 100 m, STA07 is downslope at 108 m and the VLA1 mooring site is upslope from STA07 at a depth of 106 m). Resultantly, transmissions with an initially negative launch angle may encounter both up and downslope conditions.

BELLHOP runs were programmed to study the distribution and refraction of ray paths with no surface interactions. Correspondingly, only direct path and bottom bounce rays are analyzed. The latter instance refers to a case where an eigenray has at least one bottom reflection (but no surface interaction) and may be refracted back to the bottom one or more additional times before reaching the range of the receiver array. When BELLHOP is run in .ARR or arrival time mode it is possible to distinguish the arrival time of the direct path rays from those of the single or multiple
bottom reflected rays. Each arrival time calculated by the BELLHOP model can represent a solitary ray or a number of “stacked” rays with a different path but nearly identical arrival times. The differentiation can only be made by running BELLHOP in eigenray mode (presented later in this study). To make a visual arrival time comparison using BELLHOP, the single arrival time points are plotted repeatedly over an arbitrary geotime (Tg). This allow the data-model comparison to be expressed both quantitatively and through the use of figures.

The thermistor profiles for VLA1 and WHOI are displayed in Fig. 4.3. There are several short-time period (~hour) depressions or elevations in temperature (small amplitude solitons) that are visible in the VLA1 thermistor record (highlighted in Fig. 4.2.a). However, they are not reflected in the WHOI thermistor record indicating that the propagation of these solitons is not along the acoustic path. At 12:00 UTC in Fig. 4.3.a the VLA1 thermistor profile shows an upward fluctuation in the thermocline, while little change is observed in the WHOI temperature record in Fig. 4.3.b. Hours later, the WHOI thermistors measure four small amplitude solitons centered on 19:00 UTC. The temperature response at VLA1 shows a single main soliton centered at 19:00 UTC with a broader time span. On July 7th, at 00:00 UTC a warm water intrusion (24.5°C from 50 to 80 m depth) is measured at VLA1. At the WHOI station a deep, cold water layer is measured at this time and persists through the end of the experiment. Considering these observations, it is difficult to infer any directional characteristics of the water column. However, the following data-model comparison and analysis shows elements of periodicity in the water column, which would typically be measured with an ADCP.

Matched filtering of the recorded data from the VLA1 array (receiving transmissions from STA05) spans 16 hydrophones from a depth of 40.15 to 96.4 m and an arrival time span of 5 ms is shown for each channel. For the shallowest hydrophones it is possible to identify refracted direct path and multiple bottom
reflected arrivals for 2-3 ms from the first arrival of significant energy (above the level of ambient noise) before reflections from the rough surface arrive. At 68.8 m depth, surface reflected signals begin to exceed 4 ms separation from the direct path and bottom reflection. As such, additional refracted paths can be identified as surface reflections fall outside the selected arrival time window with increasing depth.

The primary focus is on determining range dependence or independence of sound speed in the acoustic field. This determination is made through data-model comparison of BELLHOP modeled arrival time structure (for each of the range independent / dependent SSPs). A “best fit” is determined based on 3 weighted criteria. The first criteria is agreement in modelled arrival time with measured data across the 16 channels. If BELLHOP calculates arrivals with significant time offset from those measured, the fit is considered to be poor. If there is initial arrival time agreement, then the grouping of multiple rays with consideration to matched filtered intensity can be examined. Multiple ray arrivals at times of high intensity across the 16 channels satisfies the 2nd criteria. The 3rd criteria requires that the arrival time structure of the BELLHOP output, show multiple ray types (direct, bottom reflected and multiple bottom reflected) spread over arrival time (when visible in the matched filtered results). Without initial agreement in arrival time, ray grouping and temporal spread of multiple arrival types are unimportant. All three criteria must be satisfied to say that a certain SSP has driven a “good fit” in BELLHOP.

The figures that follow show the BELLHOP arrival time structure overlaid upon the matched filtered impulse response from each of the 16 hydrophones on VLA1 received from STA05. Here the interest is only on the influence of the water column on ray path and arrival time, so BELLHOP has been programmed to exclude surface bounce rays from the model output. There are 3 types of rays in the overlaid BELLHOP results. A solid red line indicates a direct path arrival, a dashed (- -) black line indicates a single bottom bounce and a dashed-dot (-.) cyan line indicates 2 or
more bottom bounces. In the figures below, some groups of direct path or bottom bounce rays appear to be thicker than others. This is not an indication of the intensity of a single reception, rather it is indicative of several individual rays with differing paths but with small separation in arrival time.

The VLA1 array has its deepest thermistor at 97 m, while the WHOI thermistor array terminates at 88 m. This is somewhat problematic for ray based calculations using WHOI data as rays with low positive launch angles will travel a significant range before encountering refractory effects. CTD casts taken throughout the KAM11 experiment show a bottom layer sound speed more consistent with the deeper VLA1 measurements. To compensate, the WHOI temperature profile is extended from 88 to 97 m using the VLA1 slope of temperature change in this depth range (before the calculation of sound speed). Below 97 m, both profiles have a sound speed equal to that measured at 97 m (consistent with KAM11 CTD casts).
Figure 5.4: Legend for acoustic and environmental data for comparison with BELLHOP model results.
In Fig. 5.4 the reception of a 22-28 kHz LMF sequence (from STA05) on each of the 16 VLA1 hydrophones is matched filtered. Thermistor plots from both VLA1 and the WHOI thermistor string are shown, along with the SSP at each site for the time of transmission. The shallowest hydrophone (35.4 m) shows the direct path / bottom reflect arrival group followed by surface reflected signals. As the depth of the hydrophones increase, the arrival time delay for the surface reflected signals increases until the reflections fall outside the arrival time delay window.
Figure 5.5: Overlay of BELLHOP arrival time results on acoustic data with legend.
The acoustic data shown in Fig. 5.5 is replotted with an overlay of the BELLHOP output in arrival time (.ARR) mode. The BELLHOP model was executed 16 times, once for each of the 16 hydrophone depths on VLA1. A legend indicates the line style pertaining to direct path, bottom reflected and multiple bottom reflected rays. In this example, no arrivals with more than 1 bottom reflection were calculated.

A summary of the data-model comparison in Fig. 5.6 to Fig. 5.28 is shown in Table 5.1. Each subsequent figure from Fig. 5.6 onwards represents a 1 hour step in geotime beginning on July 6\textsuperscript{th}, 2011 at 03:16 UTC and ending on July 7\textsuperscript{th}, 2011 at 01:16 UTC. In addition to the primary goal of determining sound speed dependence, a record of the periodicity of such change in the acoustic field is desired. When a range independent SSP yields the most robust arrival time structure comparison between BELLHOP and matched filtered experimental data, the SSP dominates the acoustic field over the 1.48 km range.
Table 5.1: Summary of data-model comparison using BELLHOP

<table>
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<tr>
<th>Time</th>
<th>BELLHOP Fit</th>
<th>Ray Type</th>
<th>Sound Speed Field</th>
<th>Figure</th>
</tr>
</thead>
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<td>Gaussian</td>
<td>Range Independent</td>
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<td>Range Independent</td>
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<td>Range Dependent</td>
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</tr>
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<td></td>
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<td>Range Independent</td>
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</tr>
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<td>Gaussian</td>
<td>Range Independent</td>
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</tr>
<tr>
<td>07/07 1:16:00</td>
<td>WHOI Data</td>
<td>Geometric</td>
<td>Range Independent</td>
<td>4.24</td>
</tr>
</tbody>
</table>

Note: The VLA1 hydrophone at 87.5 m experienced malfunctions during the period of interest.
Figure 5.6: Matched Filtered Impulse Response (MFIR) on the 16 VLA1 hydrophones on 07/06 03:16 UTC. Thermistor plot for VLA1 (top thermistor plot) and WHOI (bottom thermistor plot) with time of transmission indicated. SSPs for both thermistor strings. BELLHOP match at this time is achieved using a Gaussian rays and Spline type interpolation between WHOI and VLA1 SSPs (Range Dependent).
Figure 5.7: MFIR on 16 VLA1 hydrophones on 07/06 04:16 UTC. BELLHOP model generated with Gaussian rays and linear interpolation between WHOI and VLA1 SSPs (Range Dependent).
Figure 5.8: MFIR on 16 VLA1 hydrophones on 07/06 at 05:16 UTC. The BELLHOP match is achieved using Gaussian rays and the SSP measured at VLA1 (Range Independent).
Figure 5.9: MFIR on 16 VLA hydrophones on 07/06 at 06:16 UTC. The BELLHOP match is achieved using Gaussian rays and the SSP measured at VLA1 (Range Independent).
Figure 5.10: MFIR on 16 VLA1 hydrophones on 07/06 07:16 UTC. BELLHOP model generated with Gaussian rays and cubic interpolation between WHOI and VLA1 SSPs (Range Dependent).
Figure 5.11: MFIR on 16 VLA1 hydrophones on 07/06 07:16 UTC. BELLHOP model generated with Gaussian rays and spline interpolation between WHOI and VLA1 SSPs (Range Dependent).
Figure 5.12: MFIR on 16 VLA1 hydrophones on 07/06 at 09:16 UTC. The BELLHOP match is achieved using Gaussian rays and the SSP measured at VLA1 (Range Independent).
Figure 5.13: MFIR on 16 VLA1 hydrophones on 07/06 at 10:16 UTC. The BELLHOP match is achieved using Gaussian rays and the SSP measured at VLA1 (Range Independent).
Figure 5.14: MFIR on 16 VLA1 hydrophones on 07/06 11:16 UTC. BELLHOP model generated with Gaussian rays and spline interpolation between WHOI and VLA1 SSPs (Range Dependent).
Figure 5.15: MFIR on 16 VLA1 hydrophones on 07/06 12:16 UTC. The BELLHOP match is achieved using Gaussian rays and the SSP measured at VLA1 (Range Independent).
Figure 5.16: MFIR on 16 VLA1 hydrophones on 07/06 13:16 UTC. The BELLHOP match is achieved with Gaussian rays and the SSP measured at WHOI (Range Independent).
Figure 5.17: MFIR on 16 VLA1 hydrophones on 07/06 14:16 UTC. The BELLHOP match is achieved using Gaussian rays and the SSP measured at VLA1 (Range Independent).
Figure 5.18: MFIR on 16 VLA1 hydrophones on 07/06 15:16 UTC. BELLHOP model generated with Gaussian rays and linear interpolation between WHOI and VLA1 SSPs (Range Dependent).
Figure 5.19: MFIR on 16 VLA1 hydrophones on 07/06 16:16 UTC. BELLHOP model generated with Gaussian rays and linear interpolation between WHOI and VLA1 SSPs (Range Dependent).
Figure 5.20: MFIR on 16 VLA1 hydrophones on 07/06 17:16 UTC. BELLHOP model generated with Gaussian rays and cubic interpolation between WHOI and VLA1 SSPs (Range Dependent).
Figure 5.21: MFIR on 16 VLA1 hydrophones on 07/06 18:16 UTC. BELLHOP model generated with Gaussian rays and cubic interpolation between WHOI and VLA1 SSPs (Range Dependent).
Figure 5.22: Matched Filtered Impulse Response on the 16 VLA hydrophones on 07/06 19:16 UTC. At this point in time there is no range independent or dependent SSP that yields an arrival time structure match in BELLHOP. Significant divergence between data and all models exists in both the upper 8 and lower 8 hydrophones.
Figure 5.23: Matched Filtered Impulse Response on the 16 VLA hydrophones on July 6th, 2011 at 20:16 UTC. At this point in time there is no range independent or dependent SSP that will yield an arrival time structure match in BELLHOP. The deviation between data and model runs in this paper is greater than at time 19:16 UTC.
Figure 5.24: Matched Filtered Impulse Response on the 16 VLA hydrophones on July 6th, 2011 at 21:16 UTC. At this point in time, deviation in arrival time pattern between measured data and BELLHOP model output is still visible at depths of 57.5 m and 62.2 m, also at hydrophones deeper than 72.5 m. However, spline type interpolation between the WHOI and VLA1 thermistor string (Range Dependent) yields a viable match for 9 of the 16 hydrophones, which is a significant improvement over times 19:16-20:16 UTC, when there was no fit between data and model.
Figure 5.25: MFIR on the 16 VLA1 hydrophones on 07/06 22:16 UTC. The BELLHOP match is achieved using the WHOI SSP measured at this time using Gaussian rays (Range Independent). Deviation in arrival time pattern between measured data and BELLHOP model output is still visible at depths of 57.5 m, 62.2 m and 91.3 m. However, viable data-model comparisons have been calculated for 13 of 16 hydrophones.
Figure 5.26: MFIR on 16 VLA1 hydrophones on 07/06 23:16 UTC. The BELLHOP match is achieved using Gaussian rays and the SSP measured at VLA1 (Range Independent).
Figure 5.27: MFIR on 16 VLA1 hydrophones on 07/07 00:16 UTC. The BELLHOP match is achieved using Gaussian rays and the SSP measured at VLA1 (Range Independent).
Figure 5.28: MFIR on 16 VLA1 hydrophones on 07/07 01:16 UTC. The BELLHOP match is achieved using Geometric rays with SSP measured at WHOI (Range Independent).
5.3 Analysis

The observations and BELLHOP results from the above sections highlight several areas that warrant further analysis. Table 5.1 shows that BELLHOP calculates solutions to the measured arrival time structure alternating between range dependent and range independent SSPs. From 07/06 03:16:00 to 07/06 10:16:00 UTC, the sound speed field type alternates every 2 hours. At 07/06 11:16 the SSP becomes range dependent and is followed by 3 hours of range independence from 07/06 12:16 to 14:16 UTC. Four hours of range dependence in the sound speed field follows from 07/06 15:16 to 18:16 UTC. For the next 2 hours none of the 10 possible SSPs available for input to BELLHOP yield a good match in arrival time structure from 07/06 19:16 to 20:16 UTC. At 07/06 21:16 UTC a partial match using a range dependent SSP, generated using a spline type interpolation and Gaussian rays in BELLHOP is achieved. In the final 4 hours of the study, from 07/06 22:16 to 07/07 01:16 UTC range independent SSPs are input to BELLHOP.

To explain the periodic behavior of the sound speed field on an hourly time scale, the characteristics of internal wave generation around Kauai must be examined. The influence of the dominant semidiurnal M2 internal tide with large scale subtidal flow can be represented as an ensemble of background fields in a domain centered on the Hawaiian Ridge. Due to the interference of waves from multiple directions (i.e. generation sites), the internal tide is both variable and spatially inhomogeneous. Barotropic tidal currents are periodic water motions accompanying the tidal changes in sea level. Tidal currents flowing over topography in a stratified ocean can give rise to periodic isopycnal oscillations which are known as internal or baroclinic tides which can drive flow in different directions at different depths.
In some regions, the currents associated with internal tides can be much stronger than the currents associated with the surface or barotropic tide. The internal tidal currents may not be in phase with the barotropic tidal currents. As a result, surface currents may not consistently flow toward shore during rising or flood tides. Accordingly, surface current direction will lack consistency during falling or ebb tides. Tidal currents contribute to mixing, in some cases as the dominating force, and thus influence distribution of water properties including sound speed. The variations in depth in coastal areas can result in variations in tidal mixing which can lead to formation of fronts. Residual, or mean, circulations can also be generated through interaction of tides with bottom topography [25].

Data-model comparison with BELLHOP can be grouped into two main categories where the arrival time structure calculated by BELLHOP (including direct path, bottom bounce and multiple bottom bounces) is an excellent match for the arrival time structure measured by matched filtering of the data using: 1) Range Independent SSPs from the VLA1 or WHOI thermistor string 2) Range dependent SSPs using interpolation between the WHOI and VLA1 thermistor string (2 km) apart. When a suitable arrival time match is achieved using a range independent SSP, only 3 of the 11 times (hours) of range independence make use of thermistor data from the WHOI thermistor string at STA04 (which is closest to the source at STA05). The other 8 times of established range independence using the BELLHOP model make use of thermistor data from VLA1, 1480 m down range from the source and yielding the superior data-model comparison.

5.4 Ray Theory in Stratified Media with Bottom Interactions

When working with a layered medium, the scaling between the acoustic wavelength and layer thickness is important. When the layer thickness is small relative to the acoustic wavelength, the layered medium is substituted with an
effective medium with properties determined by Backus averaging which is a method of averaging the properties of a stack of thin layers so they are similar to average properties of a single thick layer [35]. However, when the acoustic wavelength is very small (high-frequency) compared to the layer thickness in the water column, ray theory is used to compute reflection and refraction responses.

Figure 5.29: a) Single ray with multiple bottom reflections. b) Single ray with no bottom interactions. c) A ray element.

Various rays from a point source \( (z = 0, r = 0) \) will arrive at a hydrophone designated \( P(r, z) \). In Fig 5.29.a above, we designate \( \chi_0 \) as the launch angle, \( N \) as the number of bottom reflections and \( \Delta(\chi_0) \) as the horizontal distance traveled by the ray between two consecutive reflections from the bottom boundary. \( g(\chi_0, z) \) is the projection on the plane \( z = 0 \) of the ray path as it travels from \( P_0(r_0, z_0) \) (initial launch point) with bottom reflection(s) to \( P(r, z) \). The range \( r \) is calculated differently based on \( P(r, z) \) being on the rising (Eq. 5.1) or falling crest (Eq. 5.2) of the reflection \( (N = 1, 2, 3 \ldots) \).
\[ r = N\Delta(X_0) + g(X_0, z) \quad 5.1 \]

\[ r = (N+1)\Delta(X_0) - g(X_0, z) \quad 5.2 \]

From Fig. 5.29.c, for a single ray element, \( dr = \frac{dz}{\tan X} \). By connecting \( g(X_0, z) \) with \( \Delta(X_0) \) Eq. 5.3 can be written as follows:

\[ g(X_0, z) = \int_0^z \frac{dz}{\tan X} \quad 5.3 \]

Using Snell’s law, we connect the initial launch angle \( X_0 \) with the angle of inclination of the ray \( X \) as it leaves the bottom layer.

\[ \frac{\cos X}{c(z)} = \frac{\cos X_0}{c_0} \quad \text{or} \quad n \cos X = \cos X_0 \quad 5.4 \]

Where \( n = c_0/c(z) \) is the index of refraction. Now, finding \( \tan X \) in Eq. 5.4 we can now solve for \( g(X_0, z) \) and \( \Delta(X_0) \).

\[ g(X_0, z) = \cos X_0 \int_0^z \frac{dz}{\sqrt{n^2 - \cos^2 X_0}} \quad 5.5 \]

\[ \Delta(X_0) = 2g(X_0, z_{max}) \quad 5.6 \]

In Eq. 5.6, \( z_{max} \) is the maximum depth (measured from the bottom up) reached by a ray with initial angle \( X_0 \). At \( z_{max} \), the turning depth of the ray, the angle \( X = 0 \) so Eq. 5.4 is reduced:
Now, substitution of Eq. 5.5 into Eq. 5.6 yields:

\[ \Delta(X_0) = 2 \cos X_0 \int_0^Z \frac{dz}{\sqrt{(n^2 - \cos^2 X_0)}} \]  

5.8

Allowing \( N \) to take on increasing numbers of bottom reflections (\( N=1,2,3\ldots \)) in Eq. 5.1 and Eq. 5.2, angles for \( X_{0N} \) at which the ray must leave the source if it is to arrive, after a number of reflections, at the observation point \( P(r, z) \) are determined. Many rays can arrive at \( P(r, z) \), however under certain conditions there is no solution to Eq. 5.1 or Eq. 5.2 [3]. When this occurs, no rays arrive at \( P(r, z) \) which indicates that the point is in the region of a geometric shadow.

In Fig. 5.29.b, a single ray is shown with no bottom interactions. This ray is the direct path or refracted direct path signal. That is to say that \( N = 0 \) for this scenario and it is necessary to rewrite Eq. 5.1 and Eq. 5.2 for the rising and falling crest respectively as:

\[ r = g(X_0, z) \]  

5.9

\[ r = \Delta(X_0) - g(X_0, z) \]  

5.10
Figure 5.30: The separation of a multiple bottom bounce ray from source \((r, z_0)\) to \(P(r, z)\) into ray segments to calculate travel time.

In analogy with Eq. 5.5 and using the ray segments defined in Fig. 5.30:

\[
g(z_0) = \cos \chi_0 \int_{z_0}^{z_{\max}} \frac{dz}{\sqrt{\left(n^2 - \cos^2 \chi_0\right)}} \tag{5.11}
\]

\[
g(z) = \cos \chi_0 \int_{z}^{z_{\max}} \frac{dz}{\sqrt{\left(n^2 - \cos^2 \chi_0\right)}} \tag{5.12}
\]

The time required to travel over an element of length \(ds\) is \(dt = \frac{ds}{c(z)}\) and \(ds = \frac{dz}{\sin \chi}\)

Using Snell’s law \(n(z) = \frac{c_0}{c(z)}\), \(c_0 = c(z_0)\) and \(dt\) is written above as:

\[
dt = \frac{1}{c_0} \frac{n^2 dz}{\sqrt{\left(n^2 - \cos^2 \chi_0\right)}} \tag{5.13}
\]

The time required to travel over the segment of the ray from \(z_0\) to the vertex is:

\[
t(z_0) = \frac{1}{c_0} \int_{z_0}^{z_{\max}} \frac{n^2 dz}{\sqrt{\left(n^2 - \cos^2 \chi_0\right)}} \tag{5.14}
\]
While, the time required to travel over the segment of the ray from \( z \) to the vertex is:

\[
t(z) = \frac{1}{c_0} \int_{z}^{z_{\text{max}}} \frac{n^2 dz}{\sqrt{(n^2 - \cos^2 \chi_0)}}
\]

Now given point \( P(r, z) \) as an example, the total travel time of the ray is expressed as:

\[
t = 3t(z_0) + t(z)
\]

### 5.5 SSP Reconstruction

#### 5.5.1 Background and Sound Speed Slope

The objective of a reconstruction of two SSPs is to determine a quantitative measure of refractive influence for a range independent SSP that yields a superior data-model comparison versus a divergent data-model comparison from a range independent SSP measured at the same instance in time but on the opposite end of the acoustic path. This analysis is performed through the partitioning of the SSPs over short spans of depth based on slope changes in sound speed. The focus is the data-model fit on July 6\textsuperscript{th} 10:16 UTC above (Fig. 5.13), where the fit is most robust over all 3 criteria for fitting BELLHOP arrival time output to matched filtering results over 16 channels. This fit is achieved using the range independent SSP from VLA1 with Gaussian rays in BELLHOP. The BELLHOP output for the range independent SSP from the WHOI TS using Gaussian rays at the same instance in time yields the
divergent data-model comparison. The following analysis has a prerequisite of range independence in the SSP profile, which has been established as a periodic feature of the water column in the previous section.

The Slope \( (I') \) of a layer \((i)\) in the SSP is defined as:

\[
I'_i = \frac{c_z \left( z_{\text{layer}}(i + 1) \right) - c_z \left( z_{\text{layer}}(i) \right)}{z_{\text{layer}}(i) - z_{\text{layer}}(i + 1)}
\]

The sound speed is \( c_z \) (m/s) and \( z_{\text{layer}}(i+1) \) is the depth (m) where the SSP (for VLA1 or WHOI) undergoes a change in slope from \( z_{\text{layer}}(i) \). These layer depths are determined by plotting the SSP for VLA1 against the WHOI profile for the same geotime. The cumulative sound speed change is the absolute value of \( \Delta c_z \left( z_{\text{layer}}(i) \right) \) measured from the bottom to depth \( z_{\text{layer}}(i + 1) \) and is written as:

\[
\Delta c_i \left( z_{\text{layer}}(i) \right) = \sum_{i=1}^{i} |I'_i \ast (z_{\text{layer}}(i) - z_{\text{layer}}(i + 1))|
\]

In the figures that follow the SSPs are reconstructed from the bottom up, layer-by-layer for a single geotime and input to BELLHOP to model acoustic ray arrival times and ray paths (for each additional layer of SSP reconstructed). The blue BELLHOP results are from the VLA1 SSP (which yielded the best arrival time structure match) at 10:16 UTC and the red BELLHOP results are from the WHOI SSP measured at the same time. The slope of each progressive layer is shown in the upper right hand corner of the figure, while the cumulative change of sound speed for
the WHOI and VLA1 SSPs over multiple layers can be found in the summary Table 5.2. The entire SSP reconstruction spans 36 layers; however, specific layers and groups of layers are highlighted and discussed to emphasize particular refractive effects. The maximum sound speed slope (with the strongest downward refractive effect) was measured at 0.341. The lowest sound speed slopes in the study occur in the well mixed upper layer of the water column and the minimum is measured at -0.022. Negative slope values indicate upward refraction; however, the negative slope is very weak and oscillates from negative to positive as the slope approaches 0 in the shallowest layer. With these maximum and minimum slope values defined, the reconstruction of the SSPs for WHOI and VLA1 can proceed layer-by-layer upwards from the seafloor.
In Fig. 5.31 a legend for the BELLHOP arrival time output is shown. A BELLHOP run is executed at each of the 16 hydrophone depths on VLA1 using the VLA1 measured SSP (blue arrival indicators) and again using the measured SSP at the WHOI thermistor string (red arrival indicators) for the same instance in time. The line style for each arrival type: Direct Path, Bottom Reflection and Double Bottom Reflection is shown to differentiate ray types. The progress of the SSP reconstruction is shown in the upper left corner of the diagram. The reconstruction begins at the bottom (108 m) and builds towards the surface one layer at a time. Here, this figure shows the reconstruction in the 11th layer which extends from 70 to 75 m in depth. The slope of both the VLA1 and WHOI SSPs in this layer are plotted in the upper right, with the value of each slope (velocity change per meter of depth) shown above. As an illustrative example, a diagram of the VLA1 array hydrophones is shown with the current layer indicated.
In Fig. 5.32 the ray trace generated by BELLHOP for all 16 hydrophones in eigenray mode is shown. Results are shown for SSPs from VLA1 and the WHOI thermistor strings. As the hydrophone depth increases, a green indicator shows the depth of the hydrophone in the ray trace. The line style to indicate the type of received ray is the same as that used for arrival time results. The ray trace plots often show rays that are concealed by numerous rays with similar arrival times. Each ray trace diagram corresponds directly to the arrival time plot for the matching hydrophone depth and SSP layer (11 in this case).
Table 5.2: Summary of SSP reconstruction for VLA1 and WHOI SSPs. Colored rows represent key characteristics/changes (legend at bottom) in refraction which are plotted/discussed below.

<table>
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<th>Layer</th>
<th>Depth Range (m)</th>
<th>Width (m)</th>
<th>SS Slope: VLA1 Slope (Normalized)</th>
<th>Cnl ASS (m/s)</th>
<th>Layer ΔSS (m/s)</th>
<th>SS Slope: WHOI Slope (Normalized)</th>
<th>Cnl ASS (m/s)</th>
</tr>
</thead>
<tbody>
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<td>1</td>
<td>108 to 96</td>
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<td>7.822</td>
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</table>

Legend: Initial Refraction Types | Multiple Arrival Geometric Cumulative Sound Speed | Final Layer of Influence

Initial Multiple Arrival Geometric Cumulative Final Layer of Refraction Types Shadowing Sound Speed Influence
In Table 5.2 a summary of all 36 layers used in this SSP reconstruction is shown. The temperature measurements from VLA1 and the WHOI thermistor strings are accurate to 3 significant digits, which bounds calculation of sound speed and sound speed slope to the same resolution. Sound speed slope in the various layers has also been normalized with respect to the slope of highest magnitude in layer 11 of the WHOI SSP (0.341, normalized: 1.000) which is indicative of a strong downward refracting layer. Any slope or normalized slope values with a negative value are upward refracting. The cumulative change in sound speed (Cml $\Delta SS$) for both the VLA1 and WHOI SSP is the absolute value of sound speed fluctuation summed across all layers (Layer $\Delta SS$).

### 5.5.2 Initial Refraction

The first 3 layers of the reconstruction spans from 108 to 88 m and shows a cumulative sound speed change of 1.480 m/s in the VLA1 profile and 1.500 m/s in the WHOI profile. This small cumulative difference arises from small variations in sound speed slope through Layers 2 and 3. Geometric shadow zones at single depths are first modeled in Layer 2 at 92.6 m driven by the VLA1 SSP reconstruction. A geometric shadow zone refers to certain depth or depths where no ray arrivals are measured. In Layer 3, a geometric shadow zone is modeled at 88.9 m in the WHOI reconstruction. Additionally, Layer 3 shows the first double bottom reflection in the reconstruction procedure at 92.7 and 96.4 m for both the VLA1 and WHOI profiles.
Figure 5.33: a) BELLHOP arrival time for Layer 1 (the bottom layer) extends from 108 to 96 m in depth. b) BELLHOP direct path and bottom reflection(s) eigenrays for Layer 1.
Although the seafloor at STA05 (100 m) and VLA1 (106 m) is not 108 m deep, the bathymetry file for BELLHOP has its deepest point at STA07 (108 m). Therefore, all the SSPs input to BELLHOP are extended to this depth. A linear interpolation is used between the three points in depth (100, 108 and 106 m) over the 1.48 km to build a range dependent bathymetry file which directly influences bottom reflections. With slope of 0, but offset in sound speed in Layer 1, the arrival time structures in Fig. 5.33.a from BELLHOP for both the WHOI and VLA1 SSPs have different arrival times, but are otherwise identical. In Fig. 5.33.b a second bottom reflected ray is modeled from 62.4 to 96.4 m (a span of 10 hydrophones). Green arrows indicate some of the depths where BELLHOP has calculated eigenrays that do not actually arrive at the depth of the hydrophone (at 96.4 m the second bottom reflection is real). These divergent rays are visible in Fig. 5.33.a as secondary bottom reflections and are an artifact of running BELLHOP with a limited number of rays (N=10000). Generally, divergent eigenrays are reduced with higher values of N; however, there is no guarantee that all divergent eigenrays will be properly resolved. As N increases, the computational time for one BELLHOP execution increases, so these few divergent eigenrays and their arrival times are ignored as an artifact of the ray tracer. As the number of layers in the sound speed profile reconstruction increases, fewer divergent rays are calculated.
Figure 5.34: a) BELLHOP arrival time for Layer 2 extends from 96 to 92 m in depth built upon the Layer 1 profile. b) BELLHOP direct path and bottom reflected eigenrays for Layer 2.
The slope of the WHOI and VLA1 SSPs are 0.158 (normalized: 0.463) and 0.160 (normalized: 0.469) in Fig. 5.34.a. Expressed as a percentage change from VLA1, the difference in slope in the WHOI profile within this layer is only 1.250%; however, it drives fluctuations in the arrival time structure of the 2 deepest hydrophones. At 92.6 m a geometric shadow zone is modeled using the VLA1 SSP reconstructed to 92 m in BELLHOP in Fig. 5.34.b. At 96.4 m (the deepest hydrophone), the VLA1 SSP yields two bottom reflected arrivals with no direct path. This is also visible in Fig. 5.34.a by the thickening of the bottom reflection mark over arrival time (representing 2 separate arrivals with partial overlap in arrival time).
Figure 5.35: a) BELLHOP arrival time for Layer 3 extends from 92 to 88 m in depth built upon the Layer 2 profile. b) BELLHOP direct path and bottom reflected eigenrays for Layer 3.
The slope of the WHOI and VLA1 SSPs in Layer 3 are again very similar in magnitude at 0.212 (normalized: 0.622) and 0.215 (normalized: 0.603) respectively. Above the 88.9 m hydrophone the VLA1 and WHOI arrival times and ray traces remain nearly identical. At 88.9 m only a single bottom reflection arrival is generated from BELLHOP using the VLA1 profile. At the same depth using the WHOI profile reconstructed to 88 m, a geometric shadow zone exists as no rays reach the receiver (due to the small offset in sound speed slope). At the two deepest hydrophones (92.6 and 96.4 m) in Fig. 5.35.a VLA1 and WHOI BELLHOP modelling results yield a third type of acoustic ray, the double bottom reflection (represented by dash-dot line). The double bottom reflection is clearly visible for these hydrophone depths in Fig. 5.35.b.

5.5.3 Multiple Arrival Types

The reconstruction of the VLA1 and WHOI SSPs to Layer 6 (85 to 82 m) and Layer 11 (75 to 70 m) respectively, yields direct path, bottom reflected and double bottom reflected arrivals at the two deepest hydrophones. For the case of the VLA1 profile in Layer 6, this result is due to the cumulative sound speed change driven by the addition of a relatively weak sound speed slope in the layer at 0.041 (normalized: 0.122). For the case of the WHOI profile in Layer 11, BELLHOP results show all arrival types at the two deepest hydrophones where the slope of the WHOI SSP has the highest magnitude of this study at 0.341 (normalized: 1.000).
Figure 5.36: a) BELLHOP arrival time for Layer 6 extends from 85 to 82 m in depth built upon the Layer 5 profile. b) BELLHOP direct path and bottom reflected eigenrays for Layer 6.
In Layer 6, the slope of the WHOI SSP, 0.124 (normalized: 0.364), is 3 times the VLA1 slope for this layer (0.041, normalized: 0.120), indicating stronger downward refraction per meter of depth. However, relative to its value in Layer 5, the WHOI slope has decreased by 2/3. At this point, the cumulative effect of 6 sound speed layers are visible from 81.4 to 96.4 m (a span of 5 hydrophones) in Fig. 5.36.a. The WHOI SSP reconstructed to 82 m yields BELLHOP results that show no direct path arrivals at the deepest 4 hydrophones (96.4 to 85.1 m). The arrival time structure from BELLHOP using the VLA1 SSP reconstructed to the same depth, develops a specific arrival time structure at 92.65 and 96.4 m that persists throughout the rest of the SSP reconstruction. This arrival time structure forms from the addition of Layer 6 as a leading direct path ray, multiple single bottom reflections and a trailing double bottom reflections for both depths. At this level of reconstruction in the VLA1 profile, for the 92.65 and 96.4 m hydrophones, the arrival time and ray paths will not be influenced from the addition of shallower sound speed layers. None of the depths modeled using the WHOI SSP reconstruction have stabilized in arrival time or ray path at this point.
Figure 5.37: a) BELLHOP arrival time for Layer 11 extends from 75 to 70 m in depth built upon the Layer 10 profile. b) BELLHOP direct path and bottom reflected eigenrays for Layer 11.
With the SSP now reconstructed to a depth of 70 m, fluctuations in arrival time patterns at the 73.9 and 70.1 m hydrophones are now evident. Layer 11 is a strong, downward refracting layer where each SSP has a high magnitude slope acting over 5 m of depth in this layer. The relative sound speed change in this layer for the WHOI and VLA1 profiles are 1.705 and 0.700 m/s respectively. The slope of the VLA1 SSP is 0.140 (normalized: 0.411) in this layer. The high magnitude slope of 0.341 (normalized: 1.000) in Layer 11 for the WHOI profile effectively changes the arrival time structure and ray paths for the lower half of the hydrophone array from 70.1 to 96.4 m (a span of 8 hydrophones). BELLHOP results using the WHOI profile reconstructed from 108 to 70 m, restores the direct path arrival to the 5 deepest hydrophones in Fig. 5.37.a. These 5 direct path arrivals have been shadowed since the addition of Layer 7 to the model. For the two deepest hydrophones (92.6 and 96.4 m) BELLHOP yields an arrival time structure from the WHOI profile very similar to that established using the VLA1 profile in Layer 6 at the same depths (multiple single bottom reflections enclosed by a leading direct path arrival and a trailing double bottom reflection). At the 73.9 and 70.1 m hydrophones, BELLHOP yields a single direct path arrival for both profiles. The slope of VLA1 in Layer 11 is nearly equal to its slope measured in Layer 10. As a result, all hydrophones from 77.6 to 96.4 m, no changes are evident in arrival time structure or ray trace.

5.5.4 Geometric Shadowing

From a depth of 67 to 57 m, 4 separate layers are defined by slope changes in the VLA1 and WHOI SSPs. Across the layers, multiple shifts in geometric shadow zone depth and span occur for both SSPs. These mid-water column shifts occur on the thermocline and yield shadow zones with coverage fluctuating from a single
hydrophone depth to 4 hydrophones spanning more than 10 m of the water column. The geometric shadow zones in Layers 13 through 16 are driven by fluctuations between mid and high slope values of the VLA1 and WHOI profiles in each layer on the thermocline.
Figure 5.38: a) BELLHOP arrival time for Layer 13 extends from 67 to 65 m in depth built upon the Layer 12 profile. b) BELLHOP direct path and bottom reflected eigenrays for Layer 13.
In Layer 13, the slopes of the VLA1 (0.101, normalized: 0.296) and WHOI profile (0.249, normalized: 0.730) are similar in magnitude to those measured in Layer 12. The shadowing effect of the cumulative SSPs for both layers has been shifted up the water column by the addition of Layer 13 to the reconstruction. BELLHOP calculates no arrivals for both profiles at 66.4 and 70.1 m. At 73.9 m only a single bottom reflection is calculated by BELLHOP using the VLA1 reconstructed SSP and at the same depth the WHOI SSP yields a shadow zone. Below 73.9 m the arrival structure and ray trace are consistent with results from Layer 12 using the WHOI profile. For the VLA1 profile in this layer, from hydrophone depths 77.6, 81.4 and 85.1 m there are changes in the direct path arrival structure and ray traces in Fig. 5.38.a. At 77.6 m two separate direct path rays are at their maximum temporal and spatial separation in Fig. 5.38.b. At 81.4 m this separation is less evident and at 85.1 m only the ray trace reveals that the direct path arrival in Fig. 5.38.a actually consists of 3 or more direct path rays with differences in arrival time on the scale of tenths of ms.
Figure 5.39: a) BELLHOP arrival time for Layer 14 extends from 65 to 62 m in depth built upon the Layer 13 profile. b) BELLHOP direct path and bottom reflected eigenrays for Layer 14.
In Layer 14, the slopes of the VLA1 and WHOI profiles are similar in magnitude, calculated at 0.119 (normalized: 0.349) and 0.146 (normalized: 0.428) respectively. The SSP reconstruction now spans from 108 to 62 m. At this point, the WHOI SSP based BELLHOP output for the deepest two hydrophones (96.4 and 92.6 m) have settled into their final arrival time structure and are not influenced by subsequent reconstruction of the SSP at shallower depths. At 62.6 m, BELLHOP yields only a single bottom reflected ray for the reconstructed VLA1 SSP and a geometric shadow zone for the WHOI SSP. At the 66.4 m hydrophone, results are unchanged from the previous layer as no ray arrivals are calculated and the shadow zone persists. Shadowed completely in the previous layer, BELLHOP yields a single bottom reflected arrival at 70.1 m for the VLA1 reconstructed profile, but no arrivals for the WHOI profile, unchanged from Layer 13. At 73.9 m, a direct path arrival is calculated using the VLA1 SSP with no changes on the deeper hydrophones. For the WHOI profile, 62.6 m is now shadowed which extends the shadow zone, vertically spanning 3 hydrophones in Layer 13, to 4 hydrophones (62.6 to 73.9 m) in Fig. 5.39.a.
Figure 5.40: a) BELLHOP arrival time for Layer 15 extends from 62 to 60 m in depth built upon the Layer 14 profile. b) BELLHOP direct path and bottom reflected eigenrays for Layer 15.
In Layer 15, the slopes of the VLA1 and WHOI profiles are calculated at 0.281 (normalized: 0.824) and 0.145 (normalized: 0.425) respectively. For the VLA1 and WHOI reconstructed SSPs at the 62.6 and 66.4 m hydrophones BELLHOP yields shadow zones. No significant variation from the previous layer is calculated using the VLA1 SSP for all deeper hydrophones in Fig. 5.40.a. Shadow zones at 70.1 and 73.9 m persist from the previous layer using the reconstructed WHOI SSP in BELLHOP. For this profile, BELLHOP continues to calculate arrival time pattern changes at the deepest hydrophones as the sound speed reconstruction proceeds. At 77.6 m, a second bottom reflection has an arrival time earlier than the direct path ray with a ray trace visible in Fig. 5.40.b. At 81.4 m, BELLHOP also yields a third single bottom reflected arrival; however, it is only visible in the ray trace diagram for this depth (masked by the direct path arrival in Fig 5.40.a). At 85.1 and 88.9 m, both hydrophones show a second direct path arrival.
Figure 5.41: a) BELLHOP arrival time for Layer 16 extends from 60 to 57 m in depth built upon the Layer 15 profile. b) BELLHOP direct path and bottom reflected eigenrays for Layer 16.
In Layer 16, the slopes of the VLA1 and WHOI profiles show a large difference in magnitude calculated at 0.264 (normalized: 0.774) and 0.059 (normalized: 0.173) respectively. Based on the reconstruction up to Layer 15, it would follow that the magnitude of the VLA1 SSP in this layer should yield significant changes to the arrival time pattern at many depths for a VLA1 BELLHOP run and fewer changes from the addition of the WHOI layer with a weaker sound speed slope. However, the BELLHOP model yields a significant shift from Layer 15 from the reconstructed WHOI profile. BELLHOP calculates no shadow zones for the WHOI profile where 4 consecutive zones where calculated in the previous layer. For the strongly refractive layer from VLA1 profile, the 66.4 and 62.65 m hydrophones remain within the shadowed area from the previous layer; however, the coverage of the shadow zone has expanded to the 58.9 m hydrophone. Below 70.15 m, the only fluctuation in arrival structure and ray trace is calculated at 88.9 m. Here, the introduction of Layer 16 from the VLA1 SSP into the BELLHOP model shows refraction of a direct path ray down to 88.9 m hydrophone. For the WHOI profile, the elimination of the shadow zones in Layer 16 has yielded multiple changes. At 58.9 m BELLHOP calculates only a single bottom reflection with no direct path arrival (present in Layer 15). The shadow zones from Layer 15 at 62.6, 66.4, 70.15 and 73.9 m have been replaced by nearly identical single direct path, single bottom reflected ray traces in Fig. 5.41.b. At the next two deepest hydrophones (77.6 and 81.4 m), BELLHOP calculates an additional direct path ray trace from the addition of Layer 16 from the WHOI SSP.
5.5.5 Cumulative Change in Sound Speed

Previous sections have covered slope in the sound speed profile over a vertical span of 2 m or more. The slope of a layer in the sound speed profile has a magnitude per meter of depth. Considering two layers with the same slope in sound speed, but different vertical spans in the water column, the layer with the greatest span will yield the greatest change in sound speed. To calculate the cumulative change in sound speed, the absolute value is taken for sound speed change in each level and summed. Taking the absolute value accounts for any negative changes in sound speed, as a value for total sound speed fluctuation is desired (otherwise the result would equal the difference in sound speed between the bottom of the deepest layer and the top of the shallowest). Here, the changes to arrival time structure and ray path that results from the addition of a 1 m layer to the SSP reconstruction are shown. Table 5.3 shows the sound speed slope and ray arrival types measured at 5 hydrophones in Layer 18. Table 5.4 shows the changes in ray arrival types from the addition of Layer 19 (1 m depth span).
Figure 5.42: a) BELLHOP arrival time for Layer 18 extends from 55 to 53 m in depth built upon the Layer 17 profile. b) BELLHOP direct path and bottom reflected eigenrays for Layer 18.
In Layer 18, the slopes of the VLA1 and WHOI profiles are calculated at 0.136 (Normalized: 0.399) and 0.269 (Normalized: 0.789) respectively. Layer 18 is shown as a reference to the changes induced by the addition of Layer 19 to the SSP reconstruction.

Table 5.3: a) Layer 18 slope, layer sound speed change and cumulative sound speed change. b) Ray arrival type over a span of 5 hydrophones.

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<td>WHOI</td>
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<table>
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<td>Dir, Bot Reflect</td>
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<td>Bot Reflect</td>
<td>Dir, Bot Reflect</td>
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<tr>
<td>66.4</td>
<td>Bot Reflect</td>
<td>Multi Dir, Bot Reflect</td>
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Table 5.4: a) Layer 19 slope, layer sound speed change and cumulative sound speed change. b) Ray arrival type over a span of 5 hydrophones. Changes from layer 18 are highlighted in blue for VLA1 and red for WHOI.

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Figure 5.43: a) BELLHOP arrival time for Layer 19 extends from 53 to 52 m in depth built upon the Layer 18 profile. b) BELLHOP direct path and bottom reflected eigenrays for Layer 19.
In Layer 19, from 53 to 52 m, the slopes of the VLA1 and WHOI profiles are calculated at 0.119 (normalized: 0.349) and 0.268 (normalized: 0.786) respectively. Although the slopes for VLA1 and WHOI are only measured over a 1 m layer, this thin layer and the cumulative effects of the previous layers do not preclude changes in arrival times or structure, particularly in the upper half of the water column. The most significant fluctuations in the BELLHOP results from the previous layer (Layer 18, see Fig. 5.43 and Table 5.3) occur in the VLA1 SSP reconstruction at 51.4, 58.9 and 66.4 m in Fig. 5.43.a (also see Table 5.4). Over 1 m in the water column the following variations from Layer 18 are calculated for the VLA1 profile: at 51.4 m, loss of direct path arrival, at 58.9 m, geometric shadow zone replaced by single bottom reflection and at 66.4 m, addition of direct path arrival to a previously calculated solitary bottom reflection. For the WHOI SSP reconstruction, the addition of Layer 19 yields the following: at 51.4 and 55.1 m, direct path and bottom reflections are geometrically shadowed.

5.5.6 Depth of Final Influence and Complete Reconstruction

From Layer 19 to Layer 26, the reconstruction approaches the well mixed surface layer of the water column. Layer 26 extends from 40 to 35 m and is the final layer addition that encompasses a hydrophone. The cumulative sound speed at Layer 26 for the VLA1 profile is measured at 7.853 m/s and 8.894 m/s for the WHOI profile. The difference between these two values is 1.041 m/s of fluctuation in sound speed between 35 and 108 m in depth. Above 35 m, the reconstruction of the SSPs yields no changes to the ray arrivals at the 16 hydrophone depths. As refraction in the water column is a function of sound speed change between layers, 1.041 m/s is a quantitative measure of total refractive influence of the WHOI SSP over the VLA1 SSP between
35 and 108 m. With the VLA1 SSP yielding the strongest data-model comparison (referencing the criteria in section 5.2) the greater total refractive influence in the WHOI SSP is a factor driving divergence from the VLA1 BELLHOP data-model comparison.
Figure 5.44: a) BELLHOP arrival time for Layer 26 extends from 40 to 35 m in depth built upon the Layer 35 profile. b) BELLHOP direct path and bottom reflected eigenrays for Layer 26.
In this layer, the slopes of the VLA1 and WHOI profiles are calculated at 0.043 (Normalized: 0.126) and 0.015 (Normalized: 0.044) respectively. For BELLHOP results using the WHOI profile, a direct path arrival is measured at 40.15 m with respect to Layer 25 where only a bottom reflection was calculated. At this point of the study, reconstruction of the SSPs has reached 35 m, passing the depth of the shallowest hydrophone at 40.15 m. During the KAM11 experiment the upper 30-40 m of water were well mixed, resulting in alternating weakly upward and downward refracting layers.
Figure 5.45: a) BELLHOP arrival time for Layer 36 extends from 10 to 1 m in depth built upon the Layer 35 profile. b) BELLHOP direct path and bottom reflected eigenrays for Layer 36.
Layer 36 extends from a depth of 10 m to the surface and shows the cumulative effects of all layers of both the WHOI and VLA1 SSPs. There are no shifts in arrival time, changes in arrival time pattern or ray trace between Layer 26 in the previous figure and Layer 36 shown here. All of the layers of the SSP reconstruction are included in the appendices for the sake of completeness.

5.6 Summary

A bottom-up reconstruction of differing SSPs for the same geotime is a new method in acoustical oceanography where the slope of the SSP is typically examined on the scale of the vertical span of the thermocline or on scales concerning micro turbulence. In order to reconstruct the SSP a data-model comparison that confirms the existence of a range independent SSP is required. Of the 23 times modeled in section 5.2, 11 of the SSPs were range independent and 10 times were range dependent (at 2 geotimes, no viable data-model comparison were calculated using the sound speed input methods described above). With a range independent sound speed in the water column prevailing for 47.8% of the total geotime investigated, an investigation of SSP fluctuations dependent upon this condition is a logical step.

As the layers of the SSP are built, there are several critical layer depths where key arrival time structure and ray trace characteristics diverge from those calculated in the previous layer. In Layer 2 the slope of the VLA1 and WHOI SSPs are similar in magnitude 0.158 (Normalized: 0.463) and 0.160 (Normalized: 0.469) (Fig. 5.34) yet at 92.6 m the VLA1 reconstruction yields a shadow zone from BELLHOP. This shows that small slope changes can yield arrivals of varying type and quantity. Expanding on the effects of small changes in slope between the profiles, in Layer 3 (Fig. 5.35) the slopes are 0.212 (Normalized: 0.622) and 0.215 (Normalized: 0.630) for the VLA1
and WHOI SSPs respectively. Here the addition of the 3rd sound speed layer yields a third arrival type, the double bottom reflection in VLA1 and WHOI BELLHOP modelling results at the two deepest hydrophones (92.6 and 96.4 m).

In Layer 6 (Fig. 5.36), which spans from 82 to 80 m, the cumulative refractive effect of the VLA1 profile establishes the final arrival time pattern of the 92.65 and 96.4 m hydrophones (direct, multiple single bottom reflection, double bottom reflection). No further changes occur at these depths as upward reconstruction continues with the VLA1 SSP. In Layer 11 (Fig. 5.37), the slope of the WHOI profile is 0.341 (the highest in the study) and this strongly refractive layer yields a direct path arrival for the above span of 5 hydrophones, which were shadowed in the previous layer.

Layer 13 (Fig. 5.38) shows evidence of spatial separation of the direct path signal for the VLA1 ray trace at 85.15 m. BELLHOP calculates no arrivals for both profiles at 66.4 and 70.1 m. At 73.9 m only a single bottom reflection is calculated by BELLHOP using the VLA1 reconstructed SSP and here the WHOI SSP yields a shadow zone. In Layer 14 (Fig. 5.39) the WHOI based BELLHOP output for the deepest two hydrophones (96.4 and 92.65 m) have developed their final arrival time structure and are not influenced by subsequent reconstruction of the WHOI SSP at shallower depths. Shadowed completely in the previous layer, BELLHOP yields a single bottom reflected arrival at 70.1 m for the VLA1 reconstructed profile, but no arrivals for the WHOI profile which is unchanged from Layer 13. For the WHOI reconstructed SSP in Layer 14, 62.6 m is now a shadow zone which expands the shadow zone spanning 3 hydrophones in Layer 13 to 4 hydrophones (62.6 to 73.9 m).
m of the water column. Layer 15 spans 2 m in the water column, from 62 to 60 m in depth where the slope of the VLA1 profile and WHOI profile in the layer are 0.281 (Normalized: 0.824) and 0.145 (Normalized: 0.425) respectively. In this layer the VLA1 profile yields a new shadow zone at 62.6 m. For the WHOI profile, four hydrophones depths are shadowed at 62.6, 66.4, 70.1 and 73.9 m (over 11.3 m of the water column). When Layer 16 (Fig. 5.35) is added to the reconstruction (from 60 to 57 m depth) the slope of the VLA1 and WHOI profiles in this layer are measured at 0.264 (Normalized: 0.774) and 0.059 (Normalized: 0.173) respectively. For VLA1 the layer-to-layer slope change is only 0.017, which results in only a small change in refractive strength. However, the cumulative refractive effect of adding Layer 16 results in three shadow zones from the VLA1 profiles at 58.9, 62.6 and 66.4 m. The four shadow zones calculated from the WHOI profile in the previous layer are replaced by a single direct path and single bottom reflected arrival.

Layer 19, the slopes of the VLA1 and WHOI profiles are calculated at 0.119 (normalized: 0.349) and 0.268 (normalized: 0.786) respectively. Although the slopes for VLA1 and WHOI only span a 1 m layer, this thin layer and the cumulative effects of the previous layers do not preclude changes in arrival times or structure, particularly in the upper half of the water column. These changes include the introduction of shadow zones from the WHOI SSP reconstruction in BELLHOP and loss / gain of direct path and single bottom reflections between 51.4 and 66.4 m from the VLA1 SSP.

Layer 26 (Fig. 5.44) spans from 40 to 35 m in the water column with a sound speed slope from the VLA1 and WHOI profiles of 0.043 (Normalized: 0.126) and 0.015 (Normalized: 0.44) respectively. Weak slopes in the upper 35 m of the water
column do not contribute additional arrivals at any hydrophone depth. As a result, the addition of layers 26 to 36 yield no cumulative refractive effects on arrival time structure or ray trace across the 16 hydrophone depths. The arrival time pattern for Layer 26 is the identical arrival pattern calculated by BELLHOP for the fully reconstructed SSP for both VLA1 and WHOI profiles in Layer 36 (Fig. 5.45).

The cumulative sound speed change of the WHOI SSP is 1.041 m/s greater than that of the VLA1 SSP from 35 to 108 m. The greater cumulative sound speed change in the WHOI SSP is a factor driving divergence in the data-model comparison.
CONCLUSION

Acoustic wave intensity and the wavefront time of arrival are affected by sound speed changes in the underwater environment, sea surface dynamics, and the sea floor roughness. Since both intensity and signal arrival time are directly measureable parameters and they are also affected by the changes in physical parameters of the water column such as the temperature and salinity, it is possible to study underwater sound speed variability primarily driven by oceanographic variation using acoustics. This method is known as acoustical oceanography and is a potential way of remote sensing the ocean from within. For complex environments, such as shallow water and coastal regions, this technique provides a unique and efficient way to study oceanography. However, the intricacies of the relationship between acoustics and variability in the underwater environment, such as sound speed, sea surface spectra and bottom topography, still need to be established. The spatial and temporal variability and the scales over which the physical parameters vary are wide; therefore, certain acoustic wavelengths need to be used to study a selected scale. Changing the acoustic wave frequency provides a variable acoustic wavelength to study different scales. Here, acoustic frequencies between 7 and 28 kHz are used to study short-time scale variability on the order of seconds to hours and the short-spatial scale variability on the order of few centimeters. This dissertation utilizes three sets of measured environmental and acoustic data which are centered on observations of short-temporal and spatial scale variability in the water column; in particular, fluctuations in surface wave spectra and sound speed profiles (SSPs) are accentuated. The acoustic data sets are analyzed with consideration towards coherent and incoherent surface reflections,
wavefront arrival time delay, the focusing and defocusing of intensity of acoustic beams and the refraction of acoustic rays.

First, acoustic waves reflected from a time varying rough surface were measured and analyzed using the method of matched filtering. Results from acoustic transmissions during an experiment in shallow water regions have been analyzed with a focus on arrival time fluctuations of coherent reflections from the surface gravity waves. A 2-D ray model with an evolving rough sea surface was used to explain the mechanism and formation of the deterministic striation patterns observed on the arrived acoustic wave field. With a moving linear surface wave generated from measured surface spectrum, the 2-D ray model was shown to provide qualitative matching results for the arrival time fluctuations of the surface returns. It is determined, that coherent surface reflections forming transient striation patterns evolve and dissipate on the scale of seconds, corresponding to typical periods of surface water waves and are accurately modelled by BELLHOP model raytracing.

Second, during a period of rapid isotherm depth fluctuations, the angular spread and intensity of measured acoustic beams oscillated. This dynamic behavior was significantly pronounced over a 2 hour period (between 70-90 m depth) during a 24 hour deployment. Over a range of 1 km, channel probing waveforms were sent between a stationary source (5 m above the seafloor) and a bottom mounted 8-element vertical hydrophone array. Delay-sum beamforming of measured impulse responses across all array elements and array steering with Gaussian weights revealed strong correlation between isothermal depth fluctuations and angular spread of direct path acoustic beam receptions. A 2-D Parabolic Equation model was used to calculate beam fluctuations as a function of the environmental changes. Measured sound speed
profiles during the experiment were used as input to the model and the output was beamformed across a vertical array with minimal element spacing for data-model comparison. It is determined that over time scale of minutes, intensity fluctuations, spatial ray path changes, and angular beam spreading of the direct path are correlated to the vertical oscillations of isotherms in the water column.

Third, a forward problem was introduced to infer short range (1.48 km) SSP changes using acoustic wave propagation. Comparison between the matched filtering results of the received acoustic wavefronts, measured on a vertical line array (VLA1), and the arrival time output from a ray tracing model was focused on the arrived signal time-spread. Figure 5.3 shows the geometry of this set up. Model results were obtained by input from the SSP measured at VLA1, the SSP closest to the STA05 source (i.e. the WHOI thermistor string), or an interpolation between the two profiles. Sound speed profiles in the acoustic field were found to be range independent for 11 of the 23 hours in the measurements. For any time of range independence in sound speed, SSPs can be separated into vertical layers based upon changes in slope of sound speed in the water column. A reconstruction process, conducted from the seafloor to the sea surface, layer-by-layer, identifies critical segments of each SSP that define the final ray paths, number of bottom reflections, and the ray arrival times. A single LFM sequence was selected where the data-model comparison between matched filtering results and ray tracing arrival time results using the SSP measured at VLA1 was particularly robust. For the same instant in time the data-model comparison using the SSP measured on the WHOI thermistor string, 2 km downrange from VLA1, was a poor fit. Each SSP was reconstructed layer-by-layer and repeatedly input to the ray tracing model to calculate ray arrival times and ray traces for each new SSP. It was
determined that the cumulative sound speed change in the SSP near the source was 1.041 m/s greater than that of the SSP at the receiver, resulting in the poor data-model comparison. As refraction in the water column is a function of sound speed change over depth, 1.041 m/s is a quantitative measure of total refractive influence of the SSP near the source over the SSP measured at VLA1.

In this study, the spatial and temporal changes in the oceanography of shallow water regions that can affect broadband acoustic wave propagation in the frequency range of 7 to 28 kHz have been addressed. Data-model comparison results show a direct relationship between the oceanographic variations and acoustic wave propagations. These results show that acoustic waves can be used as remote sensing tools to measure oceanographic parameters in shallow water. Further studies are required to develop full inverse techniques in order to enhance the capabilities of remote sensing to comprehensively study oceanography with acoustic waves.
REFERENCES


Appendix A

BELLHOP RESULTS WHOI / VLA1:
Figure A.1: a) BELLHOP arrival time for Layer 4 extends from 88 to 87 m in depth built upon the Layer 3 profile. b) BELLHOP direct path and bottom bounce(s) eigenrays for Layer 4.
Figure A.2: a) BELLHOP arrival time for Layer 5 extends from 87 to 85 m in depth built upon the Layer 4 profile. b) BELLHOP direct path and bottom bounce(s) eigenrays for Layer 5.
Figure A.3: a) BELLHOP arrival time for Layer 7 extends from 82 to 80 m in depth built upon the Layer 6 profile. b) BELLHOP direct path and bottom bounce(s) eigenrays for Layer 7.
Figure A.4: a) BELLHOP arrival time for Layer 8 extends from 80 to 78 m in depth built upon the Layer 7 profile. b) BELLHOP direct path and bottom bounce(s) eigenrays for Layer 8.
Figure A.5: a) BELLHOP arrival time for Layer 9 extends from 78 to 77 m in depth built upon the Layer 8 profile. b) BELLHOP direct path and bottom bounce(s) eigenrays for Layer 9.
Figure A.6: a) BELLHOP arrival time for Layer 10 extends from 77 to 75 m in depth built upon the Layer 9 profile. b) BELLHOP direct path and bottom bounce(s) eigenrays for Layer 10.
Figure A.7: a) BELLHOP arrival time for Layer 12 extends from 70 to 67 m in depth built upon the Layer 11 profile. b) BELLHOP direct path and bottom bounce(s) eigenrays for Layer 12.
Figure A.8: a) BELLHOP arrival time for Layer 17 extends from 57 to 55 m in depth built upon the Layer 16 profile. b) BELLHOP direct path and bottom bounce(s) eigenrays for Layer 17.
Figure A.9: a) BELLHOP arrival time for Layer 18 extends from 55 to 53 m in depth built upon the Layer 17 profile. b) BELLHOP direct path and bottom bounce(s) eigenrays for Layer 18.
Figure A.10: a) BELLHOP arrival time for Layer 20 extends from 52 to 50 m in depth built upon the Layer 19 profile. b) BELLHOP direct path and bottom bounce(s) eigenrays for Layer 20.
Figure A.11: a) BELLHOP arrival time for Layer 21 extends from 50 to 48 m in depth built upon the Layer 20 profile. b) BELLHOP direct path and bottom bounce(s) eigenrays for Layer 21.
Figure A.12: a) BELLHOP arrival time for Layer 22 extends from 48 to 47 m in depth built upon the Layer 21 profile. b) BELLHOP direct path and bottom bounce(s) eigenrays for Layer 22.
Figure A.13: a) BELLHOP arrival time for Layer 23 extends from 47 to 45 m in depth built upon the Layer 22 profile. b) BELLHOP direct path and bottom bounce(s) eigenrays for Layer 23.
Figure A.14: a) BELLHOP arrival time for Layer 24 extends from 45 to 42 m in depth built upon the Layer 23 profile. b) BELLHOP direct path and bottom bounce(s) eigenrays for Layer 24.
Figure A.15: a) BELLHOP arrival time for Layer 25 extends from 42 to 40 m in depth built upon the Layer 24 profile. b) BELLHOP direct path and bottom bounce(s) eigenrays for Layer 25.
Figure A.16: a) BELLHOP arrival time for Layer 27 extends from 35 to 33 m in depth built upon the Layer 26 profile. b) BELLHOP direct path and bottom bounce(s) eigenrays for Layer 26.
Figure A.17: a) BELLHOP arrival time for Layer 28 extends from 33 to 32 m in depth built upon the Layer 27 profile. b) BELLHOP direct path and bottom bounce(s) eigenrays for Layer 28.
Figure A.18: a) BELLHOP arrival time for Layer 29 extends from 32 to 30 m in depth built upon the Layer 28 profile. b) BELLHOP direct path and bottom bounce(s) eigenrays for Layer 29.
Figure A.19: a) BELLHOP arrival time for Layer 30 extends from 30 to 28 m in depth built upon the Layer 29 profile. b) BELLHOP direct path and bottom bounce(s) eigenrays for Layer 30.
Figure A.20: a) BELLHOP arrival time for Layer 31 extends from 28 to 27 m in depth built upon the Layer 30 profile. b) BELLHOP direct path and bottom bounce(s) eigenrays for Layer 31.
Figure A.21: a) BELLHOP arrival time for Layer 32 extends from 27 to 25 m in depth built upon the Layer 31 profile. b) BELLHOP direct path and bottom bounce(s) eigenrays for Layer 32.
Figure A.22: a) BELLHOP arrival time for Layer 33 extends from 25 to 23 m in depth built upon the Layer 32 profile. b) BELLHOP direct path and bottom bounce(s) eigenrays for Layer 33.
Figure A.23: a) BELLHOP arrival time for Layer 34 extends from 23 to 20 m in depth built upon the Layer 33 profile. b) BELLHOP direct path and bottom bounce(s) eigenrays for Layer 34.
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Appendix B

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