INVESTIGATION OF SOUND AND VIBRATIONAL PERFORMANCE OF SANDWICH COMPOSITE STRUCTURES THROUGH WAVE NUMBER ANALYSIS

by

James J. Sargianis

A thesis submitted to the Faculty of the University of Delaware in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering

Spring 2012

© 2012 James J Sargianis
All Rights Reserved
INVESTIGATION OF SOUND AND VIBRATIONAL PERFORMANCE OF SANDWICH COMPOSITE STRUCTURES THROUGH WAVE NUMBER ANALYSIS

by

James J. Sargianis

Approved: __________________________________________________________

Jonghwan Suhr, Ph.D.
Professor in charge of thesis on behalf of the Advisory Committee

Approved: __________________________________________________________

Anette M. Karlsson, Ph.D.
Chair of the Department of Mechanical Engineering

Approved: __________________________________________________________

Babatunde A. Ogunnaike, Ph.D.
Interim Dean of the College of Engineering

Approved: __________________________________________________________

Charles G. Riordan, Ph.D.
Vice Provost for Graduate and Professional Education
ACKNOWLEDGMENTS

I would like to thank my parents, Jim and Carol, and my sister, Kristin, for their constant love and support. Secondly, I would like to thank my advisor, Dr. Jonghwan Suhr, for his help and support throughout the last two years. I would also like to thank Dr. Hyung-ick Kim and Erik Andres for their help with mechanical testing, fabrication and SEM imaging. Also, I would like to thank Dr. Gopal P. Mathur, Dr. Hongbin Shen and the M.C. Gill Corporation for materials and helpful insight. Finally, I would like to thank the University of Delaware and the Center for Composite Materials for funding and support.
# TABLE OF CONTENTS

LIST OF TABLES ........................................................................................................ vi
LIST OF FIGURES ...................................................................................................... vii
ABSTRACT ................................................................................................................ ix

Chapter

1 INTRODUCTION ......................................................................................................... 1
  1.1 The Sandwich Composite-Noise Radiation Problem ........................................ 1
  1.2 Structural Damping for Sandwich Composites ................................................ 3
  1.3 Natural Material Based Sandwich Structures ................................................... 4
  1.4 Goals and Thesis Organization ......................................................................... 5

2 METHODS AND MATERIALS .................................................................................. 7
  2.1 The Basics of Vibration .................................................................................. 7
  2.2 Fundamentals of Acoustics and its Relationship to Wave Numbers .............. 9
  2.3 Wave Number Analysis: Experiment .............................................................. 11
  2.4 Wave Number Analysis: Analytical Modeling .............................................. 16
  2.5 Damping Analysis ......................................................................................... 17
  2.6 Experimental Setup ...................................................................................... 19
  2.7 Flexural Bending Stiffness ............................................................................. 20
  2.8 Materials ....................................................................................................... 21
    2.8.1 Face Sheet Materials ........................................................................... 21
    2.8.2 Core Materials .................................................................................... 23

3 RESULTS ................................................................................................................... 27
  3.1 Wave Number Analysis: Experimental Results ............................................. 27
    3.1.1 Core Thickness/Bending Stiffness Effect ............................................. 28
      3.1.1.1 Coincidence Frequency Analysis ............................................. 28
      3.1.1.2 Wave Number Amplitude Analysis ....................................... 32
3.1.2 Core Material Effect ................................................................. 34
  3.1.2.1 Coincidence Frequency Analysis ...................... 35
  3.1.2.2 Wave Number Amplitudes Analysis .................... 37

3.1.3 Natural Materials ................................................................. 39
  3.1.3.1 Coincidence Frequency Analysis ...................... 39
  3.1.3.2 Wave Number Amplitude Analysis .................... 45

3.2 Wave Number Analysis: Analytical Results ...................... 50
3.3 Damping Properties ................................................................. 53
  3.3.1 Core Thickness/Bending Stiffness Effect .................. 53
  3.3.2 Core Material Effect ......................................................... 55
  3.3.3 Natural Material Based Sandwich Composites ............ 57

4 CONCLUSIONS .............................................................................. 61
  4.1 Bending Stiffness Effect ........................................................ 61
    4.1.1 Acoustic Performance ................................................. 61
    4.1.2 Damping Performance ............................................... 62
  4.2 Core Material Effect .............................................................. 62
    4.2.1 Acoustic Performance ................................................. 62
    4.2.2 Damping Performance ............................................... 63
  4.3 Natural Materials ................................................................. 63
    4.3.1 Acoustic Performance ................................................. 63
    4.3.2 Damping Performance ............................................... 64

5 FUTURE WORK ............................................................................. 65

REFERENCES .................................................................................. 67

Appendix

A MATLAB CODE FOR WAVE NUMBER CALCULATION .................. 72
B BENDING STIFFNESS CALCULATION RESULTS ......................... 73
LIST OF TABLES

Table 2-1: Face sheet material properties ................................................................. 22
Table 2-2: Core material properties ........................................................................... 23
Table 3-1: Natural fiber beam stiffness-to-mass ratio analysis ................................. 42
LIST OF FIGURES

Figure 2.1: Beam deformations influencing vibrational responses. ............................. 9
Figure 2.2: Wave number response spectrum for aluminum reference beam. .............. 13
Figure 2.3: Wave number contour plot for aluminum reference beam. ....................... 14
Figure 2.4: Dispersion plot for aluminum reference beam. ........................................ 15
Figure 2.5: 3-Decibel method damping process [47] ................................................. 18
Figure 2.6: Experimental setup ................................................................................. 20
Figure 2.7: Close up photographs of (a) carbon-fiber face sheet with cork agglomerate core beam and (b) carbon fiber face sheet with Rohacell 110 IG core beam (Courtesy of H. Kim) ............................................. 24
Figure 2.8: SEM images of (a) cork agglomerate and (b) Rohacell 110 IG (Courtesy of H. Kim) ........................................................................................................... 25
Figure 2.9: Close up photographs of (a) cotton face sheet with pine wood core and (b) bamboo face sheet with balsa wood core (Courtesy of H. Kim) ................................................................. 26
Figure 3.1: Dispersion plot for core thickness study ...................................................... 29
Figure 3.2: Relation between bending stiffness and core thickness ............................ 30
Figure 3.3: Coincidence frequency as a function of core thickness ......................... 31
Figure 3.4 Wave number amplitude projection for the carbon fiber beam with (a) 5.9mm and (b) 18.4mm Rohacell 51 WF core ......................................................... 33
Figure 3.5: Dispersion curves for the core material effect experimentation ................ 35
Figure 3.6: Wave number amplitudes for the (a) Kevlar core beam and (b) Nomex core beam (c) Rohacell 51 WF core and (d) Rohacell 110 IG Core ...... 38
**Figure 3.7:** Dispersion curves for natural sandwich beams with (a) bamboo fiber face sheets and (b) cotton face sheets, each compared to carbon fiber with Rohacell 51 WF core. ................................. 40

**Figure 3.8:** Dispersion plot containing results for cork agglomerate core, compared to beams with cores of similar density as well as the aluminum reference beam .................................................. 44

**Figure 3.9:** Wave number amplitude plots of (a) Carbon fiber with 18.4mm Rohacell 51 WF core compared to cotton face sheets with (b) pine core (c) Rohacell 51 WF core and (d) Balsa core ......................... 46

**Figure 3.10:** Wave number amplitude plots of (a) Carbon fiber with 18.4mm Rohacell 51 WF core compared to bamboo face sheets with (b) pine core (c) Rohacell 51 WF core and (d) Balsa core ......................... 47

**Figure 3.11:** Wave number amplitudes comparison between carbon-fiber epoxy beams with (a) cork agglomerate core and (b) Rohacell 110 WF core ................................................................. 49

**Figure 3.12:** Kurtze and Watters model applied to the carbon fiber beam with 5.9mm thick Rohacell 51 WF core................................................................. 50

**Figure 3.13:** Analytical model applied to the carbon fiber beam with 18.4mm thick Rohacell 51 WF core................................................................. 52

**Figure 3.14:** Damping results for the core thickness experiments, shown with 95% confidence intervals ................................................................. 54

**Figure 3.15:** Damping results for the core material study with 95% confidence intervals ................................................................. 55

**Figure 3.16:** Damping results for the (a) bamboo face sheet and (b) cotton face sheet sandwich composite beams, shown with 95% confidence intervals ................................................................. 58

**Figure 3.17:** Damping values for carbon-fiber beams with cork agglomerate, Rohacell 110 IG and 110 WF cores. ................................................................. 59
ABSTRACT

Sandwich structures are utilized in many applications for their superior mechanical performance including strength and stiffness-to-weight ratios compared to metallic structures. Unfortunately as a result of these mechanical properties, sandwich composites are also excellent radiators of noise beginning at low vibrational frequencies. However, in applications which require the materials with the highest mechanical performance at a low-weight, such as aircraft, rotorcraft, wind turbine blades, or automobiles, acoustic performance remains a secondary design requirement. Current state-of-art methods involve additional vibration and sound absorbing layers, which are costly and increase manufacturing time. Therefore, solutions are sought to achieve improved acoustic and vibrational performance without the sacrifice of mechanical performance or weight.

This study employs a wave number approach to characterize the acoustic performance from the vibrational response of sandwich composite beams, along with their structural loss factor ($\eta$). It was first determined that the effect of changing the beam’s core thickness on the acoustic performance is non-linear, having a hyperbolic profile. Secondly, the core material’s specific shear modulus is inversely proportional to acoustic performance. Moreover, outstanding damping properties can help mitigate noise radiation over a broad range of frequency ranges, especially for frequencies under 1000 Hz.
With such a characterization, the wave number and damping properties of natural material based sandwich structures was explored. The purpose of investigating such materials is that they provide environmentally friendly alternatives to traditional, synthetic sandwich composite structures. Utilizing a cotton or bamboo fiber face sheet with a Rohacell foam core achieved 100% improvement in coincidence frequency, while using the same face sheets coupled with a balsa-wood core showed a 233% improvement in coincidence frequency. All beams showed substantial reductions in wave number amplitudes, which correlate to the level of noise radiation.

Finally, the use of a natural cork agglomerate as a core material showed unprecedented improvements in coincidence frequency; the coincidence frequency was not observed for vibrational frequencies up to 10 kHz. Along with such improvements, reductions in wave number amplitudes and increases in damping values were observed over all frequency ranges. Thus, it is concluded that a cork-agglomerate core sandwich composite could be a “noise free” sandwich structure with almost no compromise in flexural bending stiffness. Moreover, cork is another natural material which is not only a renewable resource, but only requires minimal carbon emissions during its fabrication and processing into an agglomerate.
Chapter 1
INTRODUCTION

1.1 The Sandwich Composite-Noise Radiation Problem

A sandwich structure consists of two thin, stiff face sheets and a thick lightweight core for a structure with overall superior stiffness and strength-to-weight ratios. Generally, composite materials such as carbon fiber and glass fiber based laminates are used as the face sheets, hence the term “sandwich composites”. Such a concept is similar to an I-beam, as the sandwich composite is optimally designed to carry bending moments by the face sheets and shear forces by the core material. From such desirable properties, sandwich composite structures are used in a wide variety of engineering applications such as civil, aerospace, automotive, marine, and wind turbine blades. However, the combination of such desirable properties gives rise to poor acoustic performance by efficiently radiating noise at much lower frequencies when compared to metallic structures. An understanding of the acoustic properties of sandwich composite structures is particularly important in the initial design stages of structural applications such as aircraft, automobile and ships. In these applications, it is necessary not to compromise the desirable mechanical properties and low weight to improve the acoustic performance. Therefore, current state-of-art solutions involve adding sound absorbing materials such as add-on acoustic treatments in later design
stages [1]. Combined with extra manufacturing time and cost, these additional materials can sum to large amounts of weight to their structures.

With such a problem at hand, considerable efforts have been made to study the sound and vibration properties of the sandwich composite structures by characterizing wave-speed properties and the transmission loss [1-15]. Peters and Nutt [3] studied the transmission loss and wave speeds in honeycomb core sandwich structures to evaluate their acoustic performance. They concluded that the properties of the core can have a direct effect on transmission loss, which is a commonly used method of determining acoustic performance. Palumbo and Klos [4] suggested a method to improve transmission loss of sandwich composites by creating voids in the core material to alter the wave propagation in the structure. Thamburaj and Sun [13-14] have studied optimization problems of sound transmission loss with design parameters including geometry and anisotropic material properties of the sandwich. Another study performed by Denli et al [15] showed the structural-acoustic optimization of sandwich composite structures with respect to anisotropic properties, cellular core geometry and boundary conditions.

The aforementioned studies involved the use of traditional methods such as transmission loss to quantify acoustic performance. The advantage to using a wave number analysis over traditional methods, such as transmission loss, is that a wave number analysis directly converts a structure’s vibrational response to acoustic performance. However, there are few studies in literature which involve the use of a wave number approach. He and Gmerek [8] have performed Statistical Energy
Analysis (SEA) along with experiments to obtain wave number characteristics including the amplitudes and the coincidence frequencies for sandwich beams. Nilsson [9] studied the dynamic and vibrational effects of sandwich panels, utilizing various numerical methods to estimate transmissions loss as well as wave number analyses.

1.2 Structural Damping for Sandwich Composites

A structure’s damping properties correlate with its ability to withstand and mitigate dynamic and vibrational loads [16]. Even low impact and cyclic stress loadings can break down structures over many cycles; thus having improved damping properties, which is often characterized by a loss factor ($\eta$), is important for improved fatigue performance and overall structural lifetime operation. As composites and sandwich composites are increasingly popular in structural applications, their damping properties have been well studied [9, 16-21]. Nilsson [9] performed work with characterizing the damping performance of sandwich composite panels, formulating that their properties are frequency dependent, and parabolic in profile. Moreover, he observed that the properties of the core material will often dictate the overall structures damping performance.

While the damping performance of sandwich structures is well understood, there are few studies which relate damping to acoustic performance. Such an understanding is important since vibration, damping, and acoustics may all be closely related. Therefore, the focus of this portion of the study will search for a correlation
between the acoustic performance (via a wave number approach) and the damping properties of a structure.

### 1.3 Natural Material Based Sandwich Structures

Besides finding a solution to the aforementioned problem, there is an increasing demand for materials that are more environmentally friendly. Over the last couple of decades, there have been many studies performed on natural material based sandwich composites [22-37]. These “natural” materials could essentially be grown for the purpose of sandwich composite fabrication, in turn providing benefits such as being both biodegradable and recyclable. Dwieb et al. [24] showed that structured sandwich panels made from natural resins and fibers can be coupled with recycled materials to maintain the structure’s superior stiffness. Bamboo is a popular fiber to be used when it comes to natural materials; it is obtained from bamboo shoots, which have superior stiffness in nature. Rao [29] compared different types of natural fibers, and found that bamboo-fiber based composites have superior strength amongst the natural fiber composites in the study. Kim et al [26] successfully demonstrated the feasibility of both wood and cotton fibers in polypropylene-based natural fiber composites at various fiber volume percentages.

Another viable option for a natural material is cork, as it is a natural product obtained from removing the outer bark of the cork tree [32]. Rather than having to chop down the tree, such as in harvesting wood, the outer bark of the cork tree
replenishes to its full thickness every nine to twelve years. Thus cork can be harvested as a natural, renewable resource. Cork has been well studied and reported for its extraordinary and remarkable mechanical performance [32-39], showing intriguing properties including nonlinear elasticity, unusual dimensional recovery capability, and outstanding energy absorption. Besides its desirable mechanical properties and low weight, other attractive features of cork involve better thermal insulation properties [38-40] and impermeability to gases or liquids. During fabrication, cork is generally formed into cork agglomerate, which is created by chopping the cork into fine granules, and then bonding them together with a resin under elevated temperatures and pressures [39]. This simple process requires significantly less energy, and thus reduces the carbon footprint, in comparison to other core material manufacturing processes, such as synthetic foams; thus this is another promising feature for cork. Although boasting many ideal properties, cork agglomerate is surprisingly an inexpensive material when compared to other cores such as synthetic honeycomb structures and foams. Thus from these favorable properties, this study will employ cork agglomerate as a core material in the sandwich structure, and consequently the structure’s sound and vibrational properties will be explored.

1.4 Goals and Thesis Organization

The first goal of the study is to understand how the geometry and materials of the core will affect the acoustic and damping properties of sandwich composites.
structures by employing a wave number analysis. Specifically, this study will use sandwich composite beams, as opposed to panels or other structures. As it will be described that vibrational properties are controlled by the beam’s bending stiffness and the properties of the core material, the core can alter both of these properties, hence why it is one of the main focuses of this study. With such an understanding of the wave number method, it will provide the necessary information and background to better understand the acoustic and damping behavior of the sandwich composites, which will consequently help us look for solutions to the sandwich structure-noise radiation problem.

As Chapter 1 described motivation for the investigation of the sandwich structure-noise radiation problem, Chapter 2 discusses the necessary background, experimental methods and materials in this work. Concepts such as vibration, sandwich composites, acoustics, and damping are discussed with respect to wave number analysis. Chapter 3 discusses the results of several studies within this work, such as analyzing the effects of the sandwich composites’ core thickness and core material on acoustic and damping performance. Also in Chapter 3, the investigation of sound and vibrational performance of natural material based sandwich composites is explored. The results are then summarized and discussed again in Chapter 4.
Chapter 2

METHODS AND MATERIALS

Chapter 2 covers a broad range of topics; it begins with the basics of vibration and acoustics, and then leads into the wave number approach and experimental methods, as well as an analytical model used to predict wave numbers. Next, it discusses the methodologies and background in measuring structural damping and flexural bending stiffness. The chapter concludes with descriptions of the materials used in this study.

2.1 The Basics of Vibration

A motion which repeats itself after an interval of time is called a vibration [41]. Generally, vibration involves some type of amplitude (whether it is displacement, velocity or acceleration) as a function of time or frequency. As a structure is subjected to vibration, it will deform since it is under a mechanical load. However as this load varies with frequency and/or time, vibrational waves will begin to propagate within the structure. From this motion, there are several properties of interest, including wave length ($\lambda$), vibrational frequency ($\omega$) and wave speed ($v$), which are simply related as [41]: 
However, the approach which will be taken involves the “wave number, \(k\)”, which is defined as [41]:

\[
\lambda = \frac{v}{\omega} \tag{1}
\]

\[
k = \frac{\omega}{v} \tag{2}
\]

which is simply the inverse of the wave length. Since sound is caused by vibration, the “wave number” analysis is a way to characterize acoustic performance based upon the vibrational response of the structure. This will be discussed in more detail in the following section.

Vinson [42] discuss that, in its simplest form, a sandwich structure’s vibrational response depends upon its flexural bending stiffness corresponding to the appropriate loading direction. For a typical beam, flexural stiffness dictates vibrational responses including resonant frequencies. However, such an assumption is only valid for low vibrational frequencies (approximately less than 1000 Hz) for sandwich composite beams. As vibrational frequency increases, the response begins to be influenced by the shearing of the core material, commonly known as transverse shear deformation. Thus for higher frequencies, properties of the core material (i.e. shear modulus and density) will play an effect on vibrational performance. See Figure 2.1 below which explains the difference between flexural bending and transverse shear deformation.
As the vibrational frequency increases significantly, the face sheet bending about its own neutral axis can begin to influence vibrational response.

2.2 Fundamentals of Acoustics and its Relationship to Wave Numbers

Pierce [11] along with Renji and Nair [43] describe the basics of acoustics and sound; sound is caused by changes in pressure that travel through a medium, and is often a result of vibratory or oscillatory motions. In order for sound to be heard, a pressure sensor is needed. The frequency and amplitude of sound which can be heard depends on the ability of the sensor; thus sound is something which is considered
subjective. Age and genetics play vital roles in a human’s ability to hear sound, while animals, such as dogs, can hear a broader frequency and amplitude range of sound [44]. Thus instruments such as microphones are used to measure the frequency and amplitude (commonly converted to an RMS value, and then normalized to create a decibel (dB) level) of the acoustic pressure waves which are traveling through the air. [44]

However, this study takes a different approach to the common methods of sound measurement; here, a wave number method is employed. Rather than measuring the changes in pressure resulting from vibrations, a wave number method characterizes acoustic performance from the vibrations themselves. Renji and Nair [43] amongst others [1,8,9,11] discuss the wave number method. When the vibrational wave speed (or wave number) of the structure is equal to that of the speed of sound in air, the structure is said to be in “coincidence.” Thus the frequency at which coincidence occurs is called the coincidence frequency. It is known [1,9,11,43] that a structure’s sound radiation and transmission properties are linked to coincidence frequency. Structures will have high acoustic radiation efficiency near this coincidence frequency, and above this frequency the structure efficiently radiates the vibrational energy into the surrounding area. This emission of vibrational energy results in the radiation of noise from the structure. Thus, measuring and improving coincidence frequency is an essential component to understanding and characterizing acoustic performance in sandwich composite structures.
2.3 Wave Number Analysis: Experiment

Wave number analysis is becoming increasingly popular in the field of acoustics and structural mechanics. As mentioned in Chapter 1, traditional methods of characterizing acoustic performance of materials, especially sandwich composites, involve measurements of transmission loss. This method quantitatively measures how much noise is absorbed, reflected and transmitted through a structure. However a wave number analysis involves utilizing a structure’s frequency response function to understand the vibrational waves within the structure. The idea is to capture the frequency in which the wave speeds become supersonic; it is at this frequency, which is called the coincidence frequency, where the structure will begin to radiate sound much more efficiently. Thus by increasing a structure’s coincidence frequency, acoustic performance is improved by widening the frequency range in which the structure does not radiate sound efficiently.

To obtain wave number measurements, one must record the spatial-frequency response function of a structure under vibration. Such data is obtained by measuring the frequency response function at multiple, equidistant points along the beam. Then, a Fourier transform is performed to transform the data from a spatial-frequency domain to a wave number-frequency domain [8]. In order to have a high resolution, as well as mitigate any local, minor flaws from manufacturing defects, the response was
measured at 64 equidistant points along the beams in this study. All beams in this study are 505mm in length, with the exception of the aluminum reference beam.

Once compiled, the data is formed into a matrix where each column is a frequency response function at a location (i.e. the matrix is an N x 64 matrix, where N represents the number of measured frequencies). Thus choosing a frequency (f) and a location (x) yields an amplitude (A), which will be denoted as $A(f,x)$. In order to obtain the response in a wave number (k) domain, that is $A(f,k)$, a Fourier Transform must be performed as follows:

$$A(f, k) = \int_{-\infty}^{\infty} A(f, x)e^{-jkx} dx$$  \hspace{1cm} (3)

This process was performed via MATLAB, and the script can be seen in Appendix A. The purpose of bringing the frequency response function into the wave number domain is that wave speeds can be understood, since a vibrational response’s wave number (k) is equal to the frequency (f) divided by the wave speed (c); see Equation (2) in Chapter 2.1. Such a response in the wave number domain looks like Figure 2.2 below, which is a 6061 aluminum beam that is 1.09 meters in length. Such a beam was first measured to confirm the accuracy of the experimental setup; it should be noted that all wave number domain experimentation utilized clamped-clamped boundary conditions.
Figure 2.2: Wave number response spectrum for aluminum reference beam.

Note in Figure 2.2, the amplitudes of the surface plot are both a function of frequency and wave number. From this plot two key metrics are determined; coincidence frequency and amplitudes. The concept of coincidence frequency was explained earlier in this chapter, however the wave number amplitudes correlate to the level of noise which is radiated from the structure. Thus lower wave number amplitudes correspond to lower levels of noise radiation. The downside to the wave number approach is that this metric is a relative one. That is, it cannot be converted to decibels, or some unit relating to noise. However by comparing the wave number amplitude responses from different structures, it may be concluded which one is quieter relative to the other. While characterizing and comparing wave number
amplitudes is rather straightforward, additional steps are required to determine coincidence frequency. To find this frequency, a contour plot of the response first needs to be created; see Figure 2.3 below for the contour representation of the surface plot in Figure 2.2.

![Wave number contour plot for aluminum reference beam](image)

**Figure 2.3:** Wave number contour plot for aluminum reference beam.

By plotting the highest peaks in this contour plot on a separate plot, a dispersion plot is formed; see Figure 2.4 for this dispersion plot. A dispersion plot, or dispersion curve, is a simpler representation of the contour plot since amplitudes are not included. In order to find the coincidence frequency, one must also plot the speed of sound in air
with the dispersion plots. Since the speed of sound is a constant \( c = 344 \text{ meters per second at } 20^\circ \text{C}, \) it appears as a line of constant slope on the dispersion curve.

Experimental data points which lie above this line represent where the vibrational wave speeds of the structure are sub-sonic, while data points below are supersonic. Therefore finding the frequency at which the speed of sound and the dispersion plot data intersects yields the coincidence frequency.

**Figure 2.4:** Dispersion plot for aluminum reference beam.

As mentioned, at the coincidence frequency the structure will begin to radiate noise; thus a structure with a higher coincidence frequency is, in one sense,
acoustically superior. Again in order to verify the experimental setup, a 6061 aluminum beam was tested as a reference beam. Since the structure is homogenous, its coincidence frequency can be easily calculated by Eq. 4 below [45];

\[ f_c = \frac{c^2}{2\pi\sqrt{K}} \]  

\[ K = \frac{Eh^2}{12\rho(1 - \nu^2)} \]

Where E is the elastic modulus, \( \nu \) is the Poisson ratio, \( \rho \) is the mass density, \( h \) is the thickness of the beam, and \( c \) is the speed of sound in air at room temperature (344 m/s). Utilizing typical properties for 6061 aluminum, along with the thickness as 2.29 mm, the coincidence frequency is calculated as 5200 Hz, which matches the values in literature [8] and experimental, as seen in Figure 2.4.

2.4 Wave Number Analysis: Analytical Modeling

Kurtze and Watters [10] developed an analytical model to predict the dispersion curves of symmetric sandwich beams. As the vibrational response of a beam is dominated by different responses in certain frequency ranges, three equations (Eq. (5)-(7)) are needed to completely predict the dispersion curve. \( k_s \) represents the waver number contributed by the shearing of the core, \( k_b \) represents the wave number
contributed the flexural bending, and \(k_{bf}\) represents the wave number contributed by the bending of the face sheet.

\[
\begin{align*}
    k_s^2 &= \frac{m\omega^2}{Gd} \quad (5) \\
    k_b^4 &= \frac{m\omega^2}{D_1} \quad (6) \\
    k_{bf}^4 &= \frac{m\omega^2}{2D_2} \quad (7)
\end{align*}
\]

\[
D_1 = \frac{Ed^2h}{2(1 - \nu^2)} \quad D_2 = \frac{Eh^3}{12(1 - \nu^2)}
\]

where \(G\) is the shear modulus of the core, \(d\) is the core depth, \(m\) is the mass per unit area, \(\omega\) is the frequency (Hz), \(k\) is the wave number, \(E\) is the longitudinal (\(E_1\)) modulus of the face sheet, \(h\) is the thickness of the face sheet, and \(\nu\) is the Poisson ratio of the face sheet (taken as 0.3). Since the Kurtze and Watters model assumes that the beam is one-dimensional, only the value of \(E_1\) (the laminate’s longitudinal modulus) is needed.

### 2.5 Damping Analysis

From the same frequency response functions used in the wave number analysis, the structural damping loss factor \(\eta\) can be calculated. A structure’s damping value correlates to its ability to mitigate undesirable dynamic and vibrational loads,
leading to improved operational lifetime of the structure. Thus, it is important for a structure to have the highest damping values as possible. ASTM E756-05 [46] was used to properly measure such damping values. The basis for calculating loss factors for a structure is to analyze the frequency response function around the resonant frequency. Such an example can be seen in Figure 2.5 below:

![Diagram](image)

**Figure 2.5:** 3-Decibel method damping process [47]

Taking the bandwidth and diving it by the “center” frequency, which is the resonant frequency, yields the value of the loss factor, η, i.e.:

\[ \eta = \frac{f_2 - f_1}{f_0} \]  

(8)

Since damping values (along with overall structural/mechanical properties) determine the vibrational behavior of a structure, and the vibrational properties determine
acoustic performance, it is desired to investigate a correlation between damping values and wave number properties.

2.6 Experimental Setup

Both the wave number and damping data are obtained through the vibrational response of a structure. The experiment setup involves imposing a clamped-clamped condition on the beams investigated. The clamps were placed upon silicone rubber pads, which are attached to a vibration isolation table to ensure undesirable ground vibrations were mitigated. The electrodynamic shaker (Labworks LW126-25 System with PA141 Shaker) was also placed upon vibration isolation pads and securely fastened to the table. In order to accurately compare each result, the frequency response function needs to be normalized by the input force applied by the shaker. This input force is measured by an impedance head, which is attached to the end of the shaker. To obtain the frequency response function, the shaker was excited with a random vibration ranging from 20 to 4000 Hz (up to 10,000 Hz for some experiments). A micro-accelerometer, with a mass of 0.6 grams, was used to measure the frequency response function at the equidistant points along the beams investigated in the study. The software of Vibration View (Vibration Research Corporation) was used to excite the shaker and record the data. See Figure 2.6 below for a schematic of the experimental setup.
Flexural Bending Stiffness

Flexural stiffness influences vibrational responses in low-frequency responses [42], and thus its measurement will be crucial in latter analyses. For homogenous materials, it is known that this stiffness is proportional to the product of the elastic modulus (E) and its moment of inertia (I). However for sandwich composite structures (and composite structures in general), bending stiffness is more difficult to calculate. Vinson [42] provides methodologies for calculating these bending stiffness values, often referring to as the “D” matrix. For this study, the value of interest is $D_1$, and when multiplying by the width (b) of the beam, b multiplied by $D_1$ becomes the flexural stiffness. If it is assumed that the structure is a three-lamina composite (i.e. a sandwich structure), the calculations algebraically reduce to the following:
Subscript “c” refers to the core material of the sandwich composite, while subscript “f” refers to the face sheet. Also, E is the elastic modulus, h the height of the core, t the thickness of the face sheet, and ν is the Poisson’s ratio. If it is assumed that the core is much thicker and weaker than the face sheets, then Equation 9 reduces to

\[ bD_1 = b \frac{1}{2} \frac{E_f h_c^2 t_f}{(1 - \nu_f^2)} \left[ 1 + \frac{1}{6} \frac{E_c h_c (1 - \nu_c^2)}{E_f t_f (1 - \nu_f^2)} \right] \]  

Measurements utilized ASTM D7250 [48] for flexural stiffness calculations. This involved measuring force-displacement data for the beam under quarter-span and third-span four-point loading conditions. Any non-linearity (i.e. low force/displacement) was ignored in the analysis.

2.8 Materials

The materials section has been divided into two sub-sections, comparing face sheet materials separately from core materials.

2.8.1 Face Sheet Materials

Table 2-1 at the end of this section summarizes the face sheet materials used in all of the analyses. The carbon fiber-epoxy sheet was supplied as a fully cured
lamine, and thus no further curing or processing was required. However for the two natural fiber face sheets, the method of manufacturing was Vacuum Assisted Resin Transfer Method, commonly known as VARTM. A stainless steel table was cleaned and prepared with the fiber sheets cut and oriented for infusion (see Table 2-1 for laminae orientation). The entire layup was then bagged and placed under a vacuum to have the resin properly infused. The resin used in this fabrication process was Derekane 510-A Vinyl Ester Resin, mixed with Methyl Ethyl Ketone Peroxide, Cobalt Napthalene, and 2, 4-P Acetalacetone. After about 24 hours, the bag was removed, and the part was left exposed to the atmosphere to finish venting the styrene gas produced from the curing resin.

Table 2-1: Face sheet material properties

<table>
<thead>
<tr>
<th>Face Sheet Material</th>
<th>Elastic Modulus (E1, GPA)</th>
<th>Density (kg/m³)</th>
<th>Laminate Thickness (mm)</th>
<th>Laminate Structure</th>
<th>Fabrication</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon Fiber-Epoxy</td>
<td>100</td>
<td>1600</td>
<td>0.38</td>
<td>0°-90°</td>
<td>Supplied by M.C. Gill [49]</td>
</tr>
<tr>
<td>Cotton-Vinyl Ester</td>
<td>2.90</td>
<td>1300</td>
<td>2.60</td>
<td>0°-90°-90°-0°</td>
<td>VARTM</td>
</tr>
<tr>
<td>Bamboo-Vinyl Ester</td>
<td>2.38</td>
<td>1150</td>
<td>1.90</td>
<td>0°-90°-90°-0°</td>
<td>VARTM</td>
</tr>
</tbody>
</table>
2.8.2 Core Materials

Similar to the previous section, Table 2-2 below lists the types and properties of the core materials used in this study. Since the properties and geometry of the core will play an important role in the vibrational properties, core materials with varying shear moduli and densities were used in this study. The “specific shear modulus”, that is, shear modulus divided by density (\( \rho \)), will be an important parameter and is discussed further in Chapter 3.

Table 2-2: Core material properties

<table>
<thead>
<tr>
<th>Core Material</th>
<th>Shear Modulus (( G_{13} ), GPA)</th>
<th>Density (kg/m(^3))</th>
<th>( G/\rho )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rohacell 51 WF</td>
<td>24</td>
<td>52</td>
<td>0.473</td>
</tr>
<tr>
<td>Rohacell 110 IG</td>
<td>50</td>
<td>110</td>
<td>0.455</td>
</tr>
<tr>
<td>Rohacell 110 WF</td>
<td>70</td>
<td>110</td>
<td>0.636</td>
</tr>
<tr>
<td>Nomex Honeycomb</td>
<td>30.1</td>
<td>29</td>
<td>1.038</td>
</tr>
<tr>
<td>Kevlar Honeycomb</td>
<td>148.0</td>
<td>108</td>
<td>1.370</td>
</tr>
<tr>
<td>Pine Wood</td>
<td>409–605 (Measured)</td>
<td>332</td>
<td>1.521</td>
</tr>
<tr>
<td>Balsa Wood</td>
<td>201–230 (Measured)</td>
<td>475</td>
<td>0.453</td>
</tr>
<tr>
<td>Cork Agglomerate</td>
<td>2.4</td>
<td>120</td>
<td>0.020</td>
</tr>
</tbody>
</table>

Note that the shear moduli for the pine and balsa wood were measured by utilizing the methods in ASTM D7250 [46]. Such methods in the ASTM make it possible to
determine the core’s shear modulus based upon the flexural deformation (load-displacement) data and geometry of the sandwich composite.

Figure 2.7 below shows one sampling of the materials in the study; Figure 2.7(a) is a carbon-fiber face sheet with cork agglomerate core, while Figure 2.7(b) contains the carbon-fiber face sheet with Rohacell 110 IG core.

Figure 2.7: Close up photographs of (a) carbon-fiber face sheet with cork agglomerate core beam and (b) carbon fiber face sheet with Rohacell 110 IG core beam (Courtesy of H. Kim)

Another comparison which can be made is the microstructure of the cork and foam cores. From the SEM images shown in Figure 2.8 below, there are clear differences in the microstructures.
While the Rohacell foams have cell sizes ranging from 309-824μm in diameter, cork has a significantly smaller cell size of approximately 41μm in diameter. Cork also boasts a smaller cell wall thickness of 1.04μm compared to 18.8μm and 26.3μm for 110WF and 110IG, respectively. In another comparison, Figure 2.9 shows the cotton face sheet with (a) pine core and the (b) bamboo face sheet with balsa core. Note how the face sheets are thicker than the carbon fiber face sheets seen in Figure 2.7, as well as the cores having different dimensions; this will be discussed in Chapter 3.
**Figure 2.9:** Close up photographs of (a) cotton face sheet with pine wood core and (b) bamboo face sheet with balsa wood core (Courtesy of H. Kim)
Chapter 3
RESULTS

Chapter 3 discusses the results from the series of experimental investigations. At first, for each investigation, the coincidence frequency and wave number amplitudes are discussed. Investigations include a study of the core thickness effect, core material affect, and the use of natural materials on the acoustic performance of sandwich composites. Following the acoustic analysis, each study is then analyzed in terms of structural damping, and correlations between this and acoustic performance are discussed.

3.1 Wave Number Analysis: Experimental Results

Several different analyses were performed utilizing the materials mentioned in Chapter 2. The first two sets of experiments were designed to test the effect of certain properties on the overall wave number/acoustic performance of the structure. Subsequent experimentation was utilized to understand the structure-property relationships with respect to acoustic and damping performances in sandwich composite beams. Finally, these same relationships were explored in sandwich composite beams composing of natural materials.
3.1.1 Core Thickness/Bending Stiffness Effect

As mentioned, by altering the core material’s thickness one can substantially change the sandwich structure’s bending stiffness. Thus sandwich beams with carbon fiber-epoxy face sheets and Rohacell 51 WF cores were fabricated, having core materials of four different thicknesses (5.9mm, 8.5mm, 10.7mm and 18.4mm).

3.1.1.1 Coincidence Frequency Analysis

Since bending stiffness governs the vibrational response at low frequencies, the first set of experiments was designed to see the bending stiffness effect on wave number properties via a change in core thickness. As one can see from Equations 9 and 10 in Chapter 2.6, there are several variables which could change a beam’s bending stiffness; however, it was assumed that the materials could not be changed, as may often be the case in engineering design. Thus changing the core’s thickness would have the most drastic effect on bending stiffness. Here, these experiments utilized the carbon fiber epoxy face sheets with Rohacell 51 WF core, which is commonly used in aerospace applications such as aircraft cabins. Figure 3.1 below displays the dispersion curves for the beams with core thicknesses of 5.9 mm, 8.5 mm, 10.7 mm, and 18.4 mm:
Figure 3.1: Dispersion plot for core thickness study

From the dispersion plot, the coincidence frequency of the beams can be determined by finding the intersection of the speed of sound line and the dispersion plot data points. From Figure 3.1 it is observed that core thicknesses of 5.9 mm, 8.5 mm, 10.7 mm and 18.4 mm correspond to coincidence frequencies of 3400 Hz, 2700 Hz, 1200 Hz and 900 Hz, respectively. For each beam (in this analysis and subsequent ones), a minimum of two samples were fabricated and tested, with the results of the experiments being near identical. Also, there is generally little error in determining coincidence frequency, which may arise from reading the intersection of the dispersion curve data points and the speed of sound line. However this error is no
greater than ± 50 Hz which is, at most, approximately 5% relative to the coincidence frequency.

In previous chapters, it was mentioned that bending stiffness dominates low-frequency vibrational responses. From Figure 3.1, one can see that the beams with a thinner core, hence lower bending stiffness, have greater wave number values at the same frequency compared to beams with thicker cores. Thus, this provides the improvement in coincidence frequency by essentially “shifting” the dispersion plots higher. Figure 3.2 below displays the relation between bending stiffness and core thickness, while Figure 3.3 shows coincidence frequency as a function of core thickness.

**Figure 3.2:** Relation between bending stiffness and core thickness
From Figure 3.3 it is important to note the non-linear effect that core thickness has on coincidence frequency. If the core thickness decreases from 18.4mm to 10.7mm, which is a 42% reduction, only a 33% improvement in coincidence frequency is observed. However, reducing the thickness from 10.7mm to 5.9mm, which is a 45% reduction, provides a 125% increase in acoustic performance. Therefore in this non-linearity, there is some critical point in which substantial improvements in acoustic performance can be achieved for very minor sacrifices in core thickness.
3.1.1.2 Wave Number Amplitude Analysis

As mentioned, not only is a structure’s coincidence frequency an indicator of acoustical performance, but so is the amplitude of the wave number response data. Thus a structure can also mitigate noise by having reduced wave number amplitudes. Figures 3.4 below shows the wave number amplitude results for the beams with 5.9 mm and 18.4mm cores. Note these amplitudes are just an amplitude-frequency axis projection from the surface plot, such as one seen previously in Figure 2.2.
Figure 3.4 Wave number amplitude projection for the carbon fiber beam with (a) 5.9mm and (b) 18.4mm Rohacell 51 WF core
Note, while there were four beams used in this study, only two are shown here, since the 5.9mm and 8.5mm beams had similar amplitudes, and likewise the 10.7mm and 18.4mm beams had similar amplitudes. There are several conclusions which can be reached from such a comparison. The first thing to observe is that the amplitude for frequencies less than 1000 Hz is substantially reduced in the 18.4mm core compared to the 5.9mm core. This can be directly attributed the increase in bending stiffness, which dominates the low frequency response. Secondly, for frequencies above 1000 Hz, it is observed that the amplitudes are roughly constant between 1000 and 2000 G/lbf from both beams. Such a result is to be expected, since this frequency range is dominated by the core’s mechanical properties, and not its geometry; in these experiments all of the beams have identical core materials and thus, the same properties. Thus although the thicker beams have a reduced coincidence frequency, their wave number amplitudes are reduced for frequencies less than 1000 Hz, helping to reduce the level of noise radiated in this frequency range.

3.1.2 Core Material Effect

To understand the effect that the core material plays on vibrational and acoustical performance, carbon fiber-epoxy beams of identical geometry with five different core materials were fabricated. Core materials included Rohacell 51 WF, 110 WF, and 110 IG, along with Kevlar and Nomex honeycomb cores.
3.1.2.1 Coincidence Frequency Analysis

Since it is known that the shearing of the core material can affect vibrational properties at higher frequency [42], experimentation was performed with several different beams of identical geometry, but core materials with different properties. Figure 3.5 below shows the dispersion curve results:

![Dispersion curves for the core material effect experimentation](image)

**Figure 3.5:** Dispersion curves for the core material effect experimentation

By first analyzing the response in the low frequency region (less than 1000 Hz), two trends are observed. All five beams show typical dispersion data for the bending stiffness dominated frequency range. However it is seen that the Rohacell 110 IG and 110 WF beams have slightly higher wave number values compared to the other beams.
Such an increase can be attributed to their increase in density, causing the wave speeds to slow down and thus increase their wave numbers. While Kevlar also boasts a high density, it does not follow this trend, which could be a result of the honeycomb core structure or anisotropic properties [15]. The second observation is that the beams within these two “groups” have identical wave number values in this low frequencies range, as a result of their near-identical bending stiffness values; see Appendix B for these values.

In comparing their coincidence frequencies, the Rohacell 110 IG and 110 WF beams have values between 2200 and 2500 Hz, which is a substantial improvement over the values of 1200 Hz, 1200 Hz and 900 Hz for Nomex, Rohacell 51 WF and Kevlar, respectively. Such a result is derived from the greater density of the Rohacell 110 beams, which causes the vibrational wave speeds to slow down as they traverse the beam.

For frequencies above 1000 Hz, it is known that the core material’s properties will begin to influence vibrational responses due to transverse shear deformation. However, as the core materials’ properties vary, they are best compared by taking the ratio of shear modulus to density, or commonly called specific shear modulus. Materials with high specific properties boast excellent mechanical properties, providing strong mechanical performance at low weight. For beams which have a core material with a high specific modulus, such as Kevlar (G/ρ = 1.37), a low slope of wave number responses is observed. On the contrary for low specific shear modulus, such as the Rohacell cores (0.455 to 0.636), greater slopes are observed.
However, the beam with the Nomex core does not seem to follow this trend; again, this may be due to the honeycomb geometry of the core structure, and further investigation is necessary to clarify this trend. But in general it can be concluded that wave number performance and thus, acoustic performance, is inversely proportional to a core material’s specific shear modulus.

3.1.2.2 Wave Number Amplitudes Analysis

As in the previous analysis, it is necessary to analyze the amplitudes of the responses in the wave number domain as a second metric of characterizing acoustic performance. Figure 3.6 below compares the wave number amplitudes for the beams in this particular study:
Upon first observation, it is seen that the Rohacell 110 IG core has the lowest amplitudes over nearly the entire frequency range; similar results were seen for Rohacell 110 WF as well and will be shown in a different comparison. Coupled with its higher coincidence frequency of roughly 2200 Hz, Rohacell 110 IG provides substantially improved acoustic performance in both criteria. However, such a result is not without sacrifice, as the Rohacell 110 IG has a density twice that of Rohacell 51 WF, and roughly 3 times that of Nomex Honeycomb. Both Rohacell 51 WF and Nomex honeycomb core beams have similar wave number amplitude responses.

Figure 3.6: Wave number amplitudes for the (a) Kevlar core beam and (b) Nomex core beam (c) Rohacell 51 WF core and (d) Rohacell 110 IG Core
3.1.3 Natural Materials

The next study was focused on using natural material based sandwich composites. Again, the purpose of such experiments is to see how these structures behave acoustically, something which has yet to be studied in literature. The fibers used in this study, such as cotton and bamboo, can be essentially grown for sandwich composite structures, providing significantly reduced energy consumption in their fabrication process compared to synthetic materials. This provides a reduction in carbon emissions and, coupled with their recyclability and being biodegradable, will be an environmentally friendly alternative.

3.1.3.1 Coincidence Frequency Analysis

Figures 3.7 shows the dispersion curve results of the wave number analysis for the bamboo and cotton face sheet beams with Rohacell 51 WF, balsa wood and pine wood as core materials.
Figure 3.7: Dispersion curves for natural sandwich beams with (a) bamboo fiber face sheets and (b) cotton face sheets, each compared to carbon fiber with Rohacell 51 WF core.
It is important to compare these wave number characteristics (coincidence frequency and amplitude) to a baseline structure. Thus, these results are compared with the carbon fiber with Rohacell 51 WF core beam, which is composed entirely of synthetic materials and often used in aircraft and aerospace structures. This sandwich composite beam has the lowest coincidence frequency of any beam investigated in this specific analysis, having a value of approximately 1200 Hz. Utilizing an “all-natural” beam, such as a cotton or bamboo face sheet combined with either balsa or pine core, the coincidence frequency is improved about 100% to 2400 Hz. Coupling a natural fiber face sheet (cotton or bamboo) with Rohacell 51 WF core provides the best improvement in coincidence frequency, which reaches a value of roughly 4000 Hz corresponding for a 233% increase.

As both bending stiffness and mass contribute to acoustic and mechanical performance, these quantities were measured, and their ratios are reported in Table 3-1. This will help to qualitatively and fairly compare each beam; it is noticed that the natural material based sandwich composites had higher bending stiffness values (Appendix B), but they also have a higher weight. Also note Appendix B gives the core thickness values for each beam. Due to the lower elastic moduli of the natural fiber face sheets (~3GPa compared to the carbon fiber epoxy’s 100 GPa), the core materials and face sheets of these beams are thicker to help increase their bending stiffness.
### Table 3-1: Natural fiber beam stiffness-to-mass ratio analysis

<table>
<thead>
<tr>
<th>Sandwich Composite Beam</th>
<th>Stiffness-to-Mass Ratio</th>
<th>Percent Decrease from Carbon Fiber with Rohacell 51 WF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon Fiber-Rohacell 51 WF (18.4mm)</td>
<td>1794</td>
<td>N/A</td>
</tr>
<tr>
<td>Cotton-Rohacell 51WF</td>
<td>903</td>
<td>49.67%</td>
</tr>
<tr>
<td>Cotton-Balsa Wood</td>
<td>537</td>
<td>70.07%</td>
</tr>
<tr>
<td>Cotton-Pine Wood</td>
<td>870</td>
<td>51.50%</td>
</tr>
<tr>
<td>Bamboo-Rohacell 51 WF</td>
<td>1449</td>
<td>19.23%</td>
</tr>
<tr>
<td>Bamboo-Balsa Wood</td>
<td>754</td>
<td>57.97%</td>
</tr>
<tr>
<td>Bamboo-Pine Wood</td>
<td>1288</td>
<td>28.20%</td>
</tr>
</tbody>
</table>

From Table 3-1, beams with the highest ratios are desirable for structural applications. All of the beams are compared to the carbon-fiber Rohacell 51 WF beam, which is the synthetic baseline sandwich composite. Each percent decrease in stiffness-to-mass ratio is with respect to this beam. Some beams such as the cotton and balsa, and bamboo and balsa, have substantially reduced ratios, and thus may not be suited for structural applications which require high specific stiffness. However, the bamboo face sheets with Rohacell 51 WF core only sacrificed a 19% decrease in stiffness to mass ratio, for a 100% improvement in coincidence frequency along; similarly, the bamboo-pine beam had a 28% decrease in stiffness to mass ratio for a 233% improvement in coincidence frequency. It can be expected that for smaller decreases in stiffness to mass ratios, acoustic performance can still be improved, just not as
much as reported in this study. Thus for some applications which have flexibility in mechanical performance or weight, natural material based sandwich composites could be a viable alternative and provide improvements in coincidence frequency, while also utilizing materials which are environmentally friendly.

As previously mentioned in Chapter 3.1.2, the core material’s specific shear modulus plays an important role in determining coincidence frequency; it was concluded that the two are inversely proportional to each other. It should be noted that when comparing beams of identical face sheets with different cores, say cotton face sheets with Rohacell 51 WF, balsa, and pine, this trend still holds true. Coincidence frequency improves with a decrease in the core’s specific shear modulus. Those beams have cores with specific shear moduli of 0.473, 0.663 and 1.053, corresponding to coincidence frequencies of 4000 Hz, 2500 Hz, and 2000 Hz, respectively.

In a separate comparison, the wave number data was measured for the cork agglomerate core beam and compared to cores of similar density (Rohacell 110 IG and 110 WF) along with the aluminum reference beam. All of these sandwich composite beams utilized the carbon fiber-epoxy face sheets, and are of identical geometry (10.7mm core thickness, 25.4 mm width). Figure 3.9 below shows the dispersion plots; note here the maximum vibrational frequency is 10,000 Hz.
Figure 3.8: Dispersion plot containing results for cork agglomerate core, compared to beams with cores of similar density as well as the aluminum reference beam

As one can see, the vibrational limit was raised to from 4 kHz to 10 kHz to capture the response of the beam with cork agglomerate core as best as possible. From Figure 3.9, it is observed that the carbon fiber beams with Rohacell 110 IG and 110 WF cores have coincidence frequencies in the range of 2200-2500 Hz, which is less than the half of the coincidence frequency for an aluminum beam, and seen before in the core material study (Chapter 3.1.2). On quite the contrary, the coincidence frequency of the carbon fiber-natural cork agglomerate core sandwich composite is not able to be
measured in the 10 kHz range. Its amplitudes are so low at frequencies above 5 kHz that wave number peaks could not be observed for the wave number amplitudes (such a phenomenon will be analyzed in the next section). But for the dispersion curve, it is impossible to determine the dispersion curve above this frequency. Such a result could imply that the coincidence frequency may not be identified even within the audible frequency range of humans, which is from 20 Hz to 20 kHz [50]. Therefore, utilizing a cork-agglomerate core results in substantially improved acoustic performance with no compromise in bending stiffness (see Appendix B) or weight.

3.1.3.2 Wave Number Amplitude Analysis

The wave number amplitude discussion will be separated in similar fashion to the coincidence frequency analysis. Figure 3.10 shows the results for the beams with cotton face sheets, while Figure 3.11 shows the results for beams with bamboo face sheets. In each figure, the results for the Rohacell 51 WF beam is shown for comparison.
Figure 3.9: Wave number amplitude plots of (a) Carbon fiber with 18.4mm Rohacell 51 WF core compared to cotton face sheets with (b) pine core (c) Rohacell 51 WF core and (d) Balsa core
There are many trends and analyses which can be made from Figures 3.10 and 3.11. First, perhaps most obvious is that the baseline sandwich composite, again which is carbon fiber epoxy with Rohacell 51 WF core, has the highest wave number amplitudes over the entire frequency range. Switching from a carbon fiber-epoxy face sheet to a natural fiber face sheet provides substantially reduced wave number amplitudes, especially in higher frequencies. Reductions can be observed up to 40% in amplitudes, whereas if the core material is changed to use a pine or balsa core, this
reduction is enhanced even further up to 75% compared to the carbon fiber-Rohacell 51WF beam.

The second study in this natural materials selection involved the cork-agglomerate core beam. In analyzing the cork agglomerate core’s wave number amplitudes, the results are quite intriguing. Figure 3.11 below compares the carbon-fiber with Rohacell 110 WF core with the cork-agglomerate core; again, this comparison is taken since both beams have identical face sheets, and cores of roughly equivalent density. One can immediately see how the cork-agglomerate core beam has reduced wave number amplitudes over the entire frequency range, and showing “no peaks” in frequencies above 5000 Hz; this is why the dispersion plot data cannot be determined in this frequency range. Thus, even if the wave speeds do become supersonic at some vibrational frequency above 5000 Hz, the level of noise which would be emitted after this frequency would be difficult for humans to audibly hear.

As the cork-agglomerate core beam has the highest improvements in acoustic performance without any sacrifice in bending stiffness or acoustic performance, it may be an environmentally friendly solution to the sandwich structure-noise radiation problem.
Figure 3.11: Wave number amplitudes comparison between carbon-fiber epoxy beams with (a) cork agglomerate core and (b) Rohacell 110 WF core
3.2 Wave Number Analysis: Analytical Results

The theory developed by Kurtze and Watters was applied to several beams in the study to understand, and confirm, their model’s application. Figure 3.12 below shows one sample of the analytical model, this one applied to the carbon fiber beam with 5.9mm thick Rohacell 51 WF core.

![Wave Number Analysis](image)

**Figure 3.12:** Kurtze and Watters model applied to the carbon fiber beam with 5.9mm thick Rohacell 51 WF core.

There are several important implications that were observed from the use of this model. First is that the model cannot predict the actual dispersion curves; rather, it provides information on the wave numbers for each individual response, i.e. sectional...
bending, core shear and face sheet bending. Secondly, for frequencies less than 1000 Hz, the flexural bending wave number curve (from the model) corresponds well with the experimental data points. However as frequency increases, the core shear wave number begins to have an impact on the dispersion curve/experimental data. Note how for higher frequencies, the data falls in between the sectional bending and core shear wave numbers. Thus, the two wave numbers do not act independently. Rather, both the flexural bending wave number and core shear wave number will influence the structure’s overall wave number response. Another observation is that there is a geometrical limitation for the theory, since in the analysis the assumption of “thin beam theory” is applied. That is, the length to thickness ratio is at least 20:1 [46]. Thus, as this limit is approached, the theory becomes less accurate. In Figure 3.13 below, the model is applied to the carbon fiber beam with 18.4mm thick Rohacell 51 WF core. Note how the data points do not correlate well to the model.
One final observation is that the face sheet bending response is well out of the frequency range of interest, and does not pertain to this frequency range where the coincidence frequency is found. In summary, the analytical model can be helpful in estimating the wave number response of a thin, symmetric, sandwich beam. However it does not provide a method for determining coincidence frequency, or any insight into the amplitudes of the wave number domain.

**Figure 3.13:** Analytical model applied to the carbon fiber beam with 18.4mm thick Rohacell 51 WF core.
3.3 Damping Properties

As previously mentioned, damping properties are essential to structures. They directly correlate to a structure’s ability to withstand dynamic and vibrational loads, thus improving fatigue properties and operational lifetime. Since a wave number approach was used to determine acoustic performance via vibrational data, and vibrational data determines damping properties, it is essential to determine if there exists a correlation between wave number and damping performance. It should be noted here that, according to ASTM E756-05 [46], the damping results from the first natural frequency are ignored. Such results are often influenced by other factors (i.e. air resistance) and are not a true value of the structure’s damping capabilities.

3.3.1 Core Thickness/Bending Stiffness Effect

Figure 3.14 below shows the loss factors as a function of frequency for the core thickness experimentation, along with 95% confidence intervals.
Two conclusions can be drawn from Figure 3.14. First, and the most obvious, is that all four beams share a similar, parabolic trend. Such a trend has been seen in previous studies [9], and the above data shows small errors associated with each data point, with the exception of the 5.9mm thick beam at about 1500 Hz. Secondly, a correlation exists between the damping data in Figure 3.14 and the wave number amplitudes in Chapter 3.1.1.2, where it was concluded that high core thickness/bending stiffness values reduce the amplitudes in the wave number domain. Observing Figure 3.14 above for frequencies less than 1000 Hz, the beams with the highest damping values have the thicker cores, whereas beams with thinner cores have lower damping values.
Thus in a low frequency range, having thicker cores in a sandwich structure will not only improve damping performance and fatigue properties at frequencies under 1000 Hz, but also lead to reductions in amplitudes of noise radiation.

### 3.3.2 Core Material Effect

Figure 3.15 shows the results of the damping experiments for the beams in the core material study;

**Figure 3.15**: Damping results for the core material study with 95% confidence intervals
Once again, the parabolic trend of the damping curves is observed for each beam in the study, having a maximum value between 1800 and 2500 Hz. Error bars associated with 95% confidence intervals are generally low, with the exception of the Nomex and Rohacell 110 IG cores at about 3500 Hz. While the correlations between structural properties and loss factors across the 4000 Hz range are weak, one interesting trend appears at approximately 1000 Hz, which is the beams second resonant frequency (again, the first resonant frequency is ignored). Those beams which a high specific shear modulus (Nomex and Kevlar at 1.038 and 1.370) had damping values of .029 and .021, whereas the beams with low specific shear moduli (ranging from 0.455-0.636) had damping values ranging from .027-.050. Thus, beams with a low specific shear modulus had higher damping values at approximately 1000 Hz. It is also noted that, with the exception of the Rohacell 110 IG beam, all of the values become similar after 3000 Hz.

Once again, it is important to determine the correlation between damping properties and wave number amplitudes, as they are both products of vibrational response. One observation is that the high loss factors observed for the Rohacell 110 WF and 110 IG beams correlate to the low wave number amplitudes across the entire frequency range. In another observation, as all of the beams have their highest damping values between 1800 Hz and 3000 Hz, this is also the frequency range with the lowest wave number amplitudes. Thus a similar correlation is observed, in that by improving structural damping, a reduction in the level of noise radiation can be obtained.
3.3.3 Natural Material Based Sandwich Composites

Figure 3.16 shows the results for the natural fiber beams with cotton and bamboo-vinyl ester face sheets. Note that the error bars represent 95% confidence intervals for the data points. Even with some of the beams having larger errors than others, the trends for these beams follow the parabolic profiles observed in earlier works, as well as fall within a similar range of values (0.02 to 0.06). Thus, structural damping is not sacrificed when using a natural material based sandwich composite.

One interesting thing of note is that the data for the bamboo beams (Figure 3.16(a)) is more dispersion than the cotton face sheet beams. In comparing the two plots, it is interesting to note that the Rohacell core beams (four in total) all have their maximum loss factor around 2000 Hz. Since this frequency is in the core material dominated region, such a result is expected. However if one compares the balsa and pine cores between the different face sheets, it is noted that the frequency where the maximum damping value is obtained is not the same. It is concluded that the reason for this is the non-homogeneity of the wood cores, as the properties and grains can vary from point to point. This fact may also lead to the higher error values in sandwich composite beams with the wood cores.
Figure 3.16: Damping results for the (a) bamboo face sheet and (b) cotton face sheet sandwich composite beams, shown with 95% confidence intervals
Next, the damping values of the cork-agglomerate core beam are compared to Rohacell 110WF and 110IG core beams. This can be seen in Figure 3.17 below, showing the damping values for the cork agglomerate, Rohacell 110 IG and Rohacell 110 WF cores over the frequency range from 0 to 5000 Hz.

![Figure 3.17: Damping values for carbon-fiber beams with cork agglomerate, Rohacell 110 IG and 110 WF cores.](image)

It is clear to see that the cork agglomerate core beam has significantly higher damping values at frequencies up to 4000 Hz; also note that the errors of all three beams are significantly less than other natural sandwich composites. This can be contributed to the homogeneity and isotropic properties of the core materials. At frequencies below
2000 Hz, the cork values are 200% greater than those of the Rohacell foam core beams. Such high damping values are a result of two main factors; the energy dissipated between granule-granule friction, and the smaller cell microstructure. The cork agglomerate core is composed of cork granules bonded together with polyurethane, as the vibrational load is applied to the sandwich beam, energy could be dissipated through friction at the interfaces between neighboring granules. Moreover, the substantially smaller cell wall size and diameter can lead to more surface area for this internal friction. It is also important to note that the relation between loss factors and wave number amplitudes holds same. As mentioned, the cork agglomerate core beam boasts the lowest wave number amplitudes of all the beams tested, as well as having the highest damping values.
4.1 Bending Stiffness Effect

4.1.1 Acoustic Performance

The first and most dominating characteristic which influences vibrational properties is bending stiffness, and as such, it has the most drastic effect on acoustic performance. While experimentation in this study controlled bending stiffness via the core material’s thickness, it is possible to control stiffness through other methods, such as changing the modulus of the face sheet or beam width. However, it was concluded that relationship between core thickness/bending stiffness and coincidence frequency is quite non-linear. At high values of bending stiffness, the coincidence frequency reaches some limiting value. However at lower bending stiffness values, there seems to be some critical point, and around this bending stiffness value large improvements in coincidence frequency can be achieved with minimal sacrifices in mechanical performance (i.e. bending stiffness). In terms of wave number amplitudes, a thicker core material, (corresponding to a higher bending stiffness value), substantially reduces the amplitudes at low vibrational frequencies.
4.1.2 Damping Performance

Damping values were measured and were similar to previous works, as the trends were parabolic in nature with a maximum values around 2000 Hz. Loss factor values ranged from 0.02 to 0.06. It was determined in this specific study that increased damping performance in the low frequency range (less than 1000 Hz) lead to lower wave number amplitudes, which in turn leads to reductions in the level of noise radiation.

4.2 Core Material Effect

4.2.1 Acoustic Performance

As vibrational frequency increases, the influence of transverse shear deformation from the core material plays an important role in the vibrational response. Consequently, experimentation was conducted on sandwich composite beams with varying core materials. Each was compared by the core material’s specific shear modulus. It was determined that an improvement in coincidence frequency can be achieved through a decrease in the core materials’ specific shear modulus. An increase in the core material’s density also shifts the dispersion curves higher and consequently can lead to improvements in coincidence frequency. For materials with higher specific shear moduli, reductions in wave number amplitudes were observed, corresponding to lower levels of noise radiation.
4.2.2 Damping Performance

Unfortunately correlations between mechanical properties and damping performances were not able to be determined; however, two key trends were observed. First, materials with lower specific shear moduli tended towards having higher damping values at approximately 1000 Hz. Secondly, regardless of the core material, all of the beams have their highest damping values between 1800 and 3000 Hz, which was the region of lowest wave number amplitudes. Thus, there is still a strong correlation between damping values and wave number amplitudes.

4.3 Natural Materials

4.3.1 Acoustic Performance

Acoustic performance in the wave number domain was explored and characterized for several different sandwich composite beams composing of various natural materials. Improvements in acoustic performance were achieved compared to a synthetic sandwich composite beam (carbon fiber-epoxy with Rohacell 51 WF core). Coincidence frequencies improved up to 233%, and in some cases saw sacrifices as little as 19% in stiffness to mass ratio, while also seeing reductions in wave number amplitudes (ranging up to 75%). Therefore this study shows promise for the use of sandwich composites structures composed of natural materials for improvements in acoustic performance, while also being renewable, recyclable and biodegradable.
However, the best improvements in acoustic performance were observed using a natural cork agglomerate core with a carbon fiber-epoxy face sheet. Such a coupling of synthetic and natural materials provided unprecedented improvements in both coincidence frequency and wave number amplitudes, leading towards a conclusion that such a structure may be “noise free”. Moreover, this improvement was achieved without sacrificing bending stiffness or weight, and could truly be a novel and environmentally friendly solution to the sandwich structure-noise radiation problem.

4.3.2 Damping Performance

Non-cork agglomerate beams should similar damping trends to the synthetic beams, showing that structural damping is not sacrificed in switching to a natural fiber based sandwich composite. However, the cork core showed superior improvements in loss factor values, improving up to 200% in certain frequency ranges. Again, this confirms the prior conclusions in that high damping values correlate to low wave number amplitudes and thus, low levels of noise radiation.
Chapter 5

FUTURE WORK

This work reports potential solutions and characterization methods to the sandwich structure-noise radiation problem. However, more mechanical analyses could be helpful in determining the use of such materials in industry, such as the cork-agglomerate core beams. While bending stiffness was characterized for the cork-core beam, and shown to be similar to synthetic-core beams, analyses involving fatigue and impact resistance would be important in various applications. It is expected though, that these properties would be superior to traditional materials from the core agglomerate’s excellent damping and energy absorbing capabilities.

A second path which could be taken would be to develop this approach for other structures, such as plates, panels and shells. These are other commons structures used in a wide variety of applications, and applying the wave number approach to these structures would again be helpful in determining acoustic performance experimentally.

Next, the use of nanoparticle or nanotube enhanced composites would be an excellent foundation for future work. Such materials have shown to have excellent internal energy dissipation [51], thus they could provide excellent damping and acoustic performance when used in sandwich composite materials. Thus perhaps utilizing engineered nano-architecture as a core material could help to improve vibrational performance.
Finally, developing appropriate finite element methods to determine wave number responses would help to provide more insight and useful design guidelines in engineering acoustic performance of sandwich composites. However for precisely predicting the performance, the accurate material properties of constituent materials need to be characterized appropriately, including vibrational data such as damping loss factors.
REFERENCES


47. sendpielaudio.com


Appendix A

MATLAB CODE FOR WAVE NUMBER CALCULATION

% Wavenumber Calculation

results=zeros(4000,64);  % Creates the matrix. If zeroes(x,y) is the matrix, then x
                         % represents the number of frequency data points, and y
                         % represents the number of spatial data points

for x=1:64  % end of for loop must match y in above

  % reading the excel files. Must be named “data” and then each successive excel file
  % is numerically increased, i.e. Data1, Data2, ...Data64

  [nums,~,~]=xlsread(['Data (' num2str(x) ').csv','J2:K4001']);
  results(:,x)=sqrt(nums(1:4000,2)./nums(1:4000,1));
  end

% create wave number matrix
Z = abs(fft(results'));

Appendix B

BENDING STIFFNESS CALCULATION RESULTS

<table>
<thead>
<tr>
<th>Face Sheet Material</th>
<th>Core Material (Thickness)</th>
<th>Bending Stiffness (10^6 Nmm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon Fiber Epoxy</td>
<td>Rohacell 51 WF (5.9mm)</td>
<td>5.2</td>
</tr>
<tr>
<td>Carbon Fiber Epoxy</td>
<td>Rohacell 51 WF (8.5mm)</td>
<td>12.7</td>
</tr>
<tr>
<td>Carbon Fiber Epoxy</td>
<td>Rohacell 51 WF (10.7mm)</td>
<td>16.24</td>
</tr>
<tr>
<td>Carbon Fiber Epoxy</td>
<td>Rohacell 51 WF (18.4mm)</td>
<td>50.0</td>
</tr>
<tr>
<td>Carbon Fiber Epoxy</td>
<td>Rohacell 110 WF (10.7mm)</td>
<td>23.6</td>
</tr>
<tr>
<td>Carbon Fiber Epoxy</td>
<td>Rohacell 110 IG (10.7mm)</td>
<td>28.7</td>
</tr>
<tr>
<td>Carbon Fiber Epoxy</td>
<td>Cork Agglomerate (10.7mm)</td>
<td>24.6</td>
</tr>
<tr>
<td>Carbon Fiber Epoxy</td>
<td>Kevlar Honeycomb (10.7mm)</td>
<td>28.8</td>
</tr>
<tr>
<td>Carbon Fiber Epoxy</td>
<td>Nomex Honeycomb (10.7mm)</td>
<td>24.6</td>
</tr>
<tr>
<td>Cotton-Vinyl Ester</td>
<td>Rohacell 51 WF (17.0mm)</td>
<td>145</td>
</tr>
<tr>
<td>Cotton-Vinyl Ester</td>
<td>Pine Wood (9.5mm)</td>
<td>201</td>
</tr>
<tr>
<td>Cotton-Vinyl Ester</td>
<td>Balsa Wood (18.5mm)</td>
<td>153</td>
</tr>
<tr>
<td>Bamboo-Vinyl Ester</td>
<td>Rohacell 51 WF (17.0mm)</td>
<td>158</td>
</tr>
<tr>
<td>Bamboo-Vinyl Ester</td>
<td>Pine Wood (9.5mm)</td>
<td>228</td>
</tr>
<tr>
<td>Bamboo-Vinyl Ester</td>
<td>Balsa Wood (18.2mm)</td>
<td>159</td>
</tr>
</tbody>
</table>