THE BUILDING AND CHARACTERIZATION OF A STRETCHED-PULSE ADDITIVE PULSE MODE-LOCKING FIBER RING LASER WITH 2 MODES OF OPERATION AND AN AMPLIFICATION STAGE

by

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ABSTRACT

Ultrafast lasers have been researched over 40 years now. Fiber lasers is a newer area with almost 30 years of development at a point where it has matured enough to be present in many industry applications nowadays, where the generation of shorter and more energetic pulses is possible thanks to the development of rare earth doped fibers. This is a great deal of importance for telecommunication applications, medical applications and materials processing in industrial environments.

In the present work, we present a stretched-pulse additive-pulse mode-locking fiber ring laser capable of producing sub 100 fs optical pulses. The cavity is capable of operating in two regimes called Noise-like soliton regime and Gain-guided soliton regime, whereas maintaining small positive net dispersion on a cavity build with normal and anomalous dispersion segments of fiber with a cavity less than 10 m long.

Additionally a pulse optimization and amplification stage is studied and characterized in a second stage for amplification and compression of the output, using a large-mode area Erbium-doped fiber amplifier based prechired amplifier and compressor proposed in the literature, where these two modes can be enhanced to increase the output power from the laser.
Chapter 1

INTRODUCTION

“I liked things better back in the year 3000, when my best friend was a robot and people ate lasers.”

-Phillip J. Fry, “Futurama”-

Now days there are relatively few experiments in physics that don’t make use of a laser in some manner. From very accurate material manipulation, laser surgery, passing through high energy devices used for military purposes, lasers have become an important part in our daily lives.

The telecommunication explosion in the 1990’s opens a new window for research and development using lasers for communications. In this particular area, pulsed lasers found a profound bolster to look for better and more reliable ways to produce stable, efficient and controlled high energy pulsed laser sources. Other applications as laser micromachining and laser surgery took advantage of this boom and accounted for greater improvements thanks to the intense research done over the last 15 years.

Pulsed lasers can be found in various flavors, from big bulky configurations as the Ti:Sapphire laser, smaller packed devices with the capability of produce high energy and low repetition rate as the Nd:YAG laser and fiber lasers that still do not provide as much power as compared to the first two, but its portability and reliability compares closely to semiconductor lasers that is the main target for comparison. In this group where fiber optics lasers are, we find devices that are small
and compact enough to be packed in a box presentation, incredible performance for such simplistic layout to the point where no bulk parts are needed for its operation but still subject to the instabilities of the large systems as temperature variations, pressure and the inherent fragility of the glass.

Despite the factors mentioned above, extensive research in pulsed fiber lasers is done nowadays to overcome these limitations. Even now, there are commercial systems available that offer excellent performance built on sturdy presentations that meet high industry standards. The main reason to justify the ongoing research on fiber lasers is that when the above mentioned difficulties are solved they will be able to outperform their semiconductor counterparts with the advantage of much more lower cost of fabrication and similar portability.

Fiber optics possesses the capacity of transporting solitons, an electromagnetic pulse that can travel over very long distances, thanks to a delicate balance between nonlinear and linear effects present in the traveling media. Temporal solitons will be the result of balancing nonlinear effects with dispersion and thus the pulse will travel unchanged without using any kind of repeaters or pulse regenerators. This property makes fiber optics perfectly suited for applications in the telecomm area where rather than use fibers only for transport the information, the whole fiber laser system could be used as a non-expensive generation and modulation source.

This Master’s thesis depicts a small improvement on performance for pulsed fiber lasers. Small compared to all the things that are yet to be improved. We present a configuration were two operability characteristics known as noise like and gain guided soliton operation, are improved to a better point by optimizing the fabrication process of the cavity.
1.1 Thesis Goals and Organization

They main focus of this thesis is to provide an understanding of the mode-locked fiber lasers, with a particular application on two operation states that are possible to reach with the cavity presented.

This document follows in Chapter 2 with a review on the historical evolution of the laser achievements that gave rise to the pulsed fiber laser configuration. Chapter 3 provides an analysis on the theory behind the operation of the mode-locked all-fiber laser is presented with the corresponding model in Chapter 4. Then a detailed description on the setup used and the results obtained are shown in Chapter 5. A follow up discussion is presented in Chapter 6 and a closing conclusion and future directions will be enounced in Chapter 7 to finalize this thesis.
Chapter 2

HISTORICAL BACKGROUND REVIEW

The starting point to talk about fiber lasers is to talk about fiber optics itself. Every phenomenon that takes place inside a fiber laser relies on the waveguiding characteristic of fiber optics. Waveguiding is known since late 19th century, when it was studied and tested by important personalities in Physics as J. J. Thomson in 1893, Oliver Joseph Lodge in 1894 and afterwards the study of propagating modes within a hollow metal cylinder was first performed by Lord Rayleigh in 1897.

Although uncladded glass fibers were fabricated in the 1920s, the field of fiber optics was not born until the 1950s when the use of a cladding layer led to considerable improvement in the fiber characteristics [1]. The first fiber laser was built in 1961 using a Neodymium-doped (Nd-doped) fiber with a large core [2], and a decisive contribution was made in 1973 by Hasegawa and Tappert when they suggested that optical fibers can support the transmission of short optical pulses as a result of the interplay between the dispersive and nonlinear effects [3].

Real attention was brought into this area in the 1980’s with the development of rare-earth doped fibers using materials such Neodymium, Ytterbium and Erbium. The last one has an important preference on industry-oriented applications due its emission at telecommunication wavelengths.

With the development of the Erbium Doped Fiber Amplifier (EDFA), fiber lasers have been mainly developed for telecommunication applications using
different mode-locking techniques to produce very short optical pulses, ranging from 1ps down to a few femtoseconds, at repetition rates of up to 200GHz [4]. Despite all these good characteristics, fiber lasers are still highly sensitive to temperature, pressure and vibrations, facts that have limited its massive use in real-life telecomm applications, still yet extensive research is done in order to overcome these limitations and improve its performance.

2.1 A Historical Introduction to Mode-locked Fiber Lasers

The first mode-locked fiber laser was demonstrated by M.I. Dzhibaev, et. al. in 1983 [5] using a neodymium glass-fiber laser and a flashlamp that acted as a saturable absorber producing transient color centers, passively mode-locking the laser.

Mollenauer and Stolen proposed a configuration where incorporating a length of single-mode, polarization-preserving fiber into the feedback loop of a mode locked laser were able to get what they denominated the Soliton Laser. They obtained pulses as short as 219fs directly from the cavity [6]. This soliton laser used a technique known as self-phase modulation (SPM) in the fiber to produce an intensity-dependent phase-shift over the pulse traveling along the laser cavity. The chirped pulse product of this effect was again introduced in the cavity whose length was carefully adjusted to guarantee interference with the un-chirped version of the pulse. This allowed constructive interference in the pulse center and destructive interference in the pulse wings compressing the final output pulse duration and increasing its intensity, as a result.

A new mechanism for pulse shortening where the compression does not rely on dispersion on an auxiliary cavity was first demonstrated by J. Mark, et.al. [7]. This mechanism was called additive pulse mode-locking (APM) and it seemed to
explain with further accuracy the previous results showed in [6] whereas before, self-phase modulation alone was enough to obtain mode-locking in laser systems. However, soliton formation is not always necessary and short lasers pulses can be achieved even with positive dispersion in the external fiber cavity. The model presented in [7] showed clearly that for optimum pulse shortening only the cavity length is to be adjusted and dispersion is not required in the process.

A further deep study on APM was presented by E. P. Ippen, et.al. [8] where also was stated that mode-locking was possible in fiber laser with positive net cavity dispersion. In APM pulse shortening occurs by the coherent addition of self-phase-modulated pulses. This pulse shortening mechanism produces a mode-locking similar to a fast saturable-absorber mode-locking. Then the pulse parameters are determined by the gain, the gain dispersion, the group velocity dispersion (GVD) and self-phase modulation [9].

Passively mode locked lasers based on artificial fast saturable absorbers are the most common type in the fiber optics laser area. The interplay between nonlinear phase modulation and dispersion lead to strong soliton like behavior, where each effect often make large individual contribution per round trip in the cavity, but still both opposite actions balance each other, and overall changes are small in the end. This is due the large amount of material present inherently in the cavity [10].

With the development of fiber amplifiers such as the erbium amplifier, all-fiber lasers were the next step. The first approach to an all-fiber erbium-laser with no bulk components was presented by I. N. Duling with the figure-eight laser (F8L). This configuration consists in a nonlinear amplifying loop mirror that has been turned into a laser by feedback of the output to the input through a fiber-pigtailed optical isolator.
Using a nonlinear amplifying loop mirror (NALM) comprising a 50% fiber coupler, an erbium doped fiber and a wavelength division multiplexer (WDM) pump coupler. The output of the NALM was connected to an output coupler and then to a pig-tailed optical isolator which was fed into the input of the NALM. The schematic of the F8L is showed in figure 1.

Figure 1. General schematic of the F8L. PC accounts for Polarization Controller.

By properly adjusting the cavity length down to 1.2 m, Duling’s group reported pulses of 314 fs assuming a sech² pulse shape, characteristic that is very common in soliton-like lasers. In this configuration again, interference between two versions of the same pulse occurs in order to do the desired pulse shaping. Another
F8L was reported by the same group, presenting a 320 fs pulse width but using lower pump power [12].

Another important all-fiber configuration that has lasted for several years since its report and is the configuration used to report the results in this document, is been introduced as the all-fiber ring configuration was first constructed by L. F. Mollenauer [13] and deeper study was done by V. J. Matsas, et.al. [14-15] using a birefringent fibers.

K. Tamura, et.al. reported a cavity using no birefringent fibers, which reflected on a shorter cavity of a few meters compared to the lengthy cavity shown in [14-15] where over a hundred meters of birefringent fiber was used. This latter model presented a fully integrated polarization-APM erbium ring laser, which generated stable 452 fs pulses at 42 MHz of repetition rate [16]. The basic setup is comprised of a fiber ring, where a segment is erbium doped fiber, a polarizer/isolator, WDM for coupling the pump in and the output coupler. Polarization controllers are used before and after the polarizer to set the desired field polarization in the ring.

The operation mechanism presented in is known as nonlinear polarization rotation (NLPR) which is a particular application of APM using superposition of two circular modes of the pulse generated in the cavity after the first PC. This theory will be further explained in the next chapter. The unidirectional ring geometry facilitates self-starting, whereas other configurations and external impulse or modulation had to be introduced in the cavity to start the mode-locking process [16].
Looking to obtain shortest and more energetic pulses, some improvements have been reported using as the starting point, the configuration proposed in [16]. On a further publication, the same group led by Tamura, Haus, Ippen and Nelson, reported what they called the stretched-pulse fiber laser in 1993 [17].

In this cavity setup, a segment of large normal dispersion fiber is introduced on an all-anomalous cavity, providing an ingenious control over the overall cavity dispersion, making it almost independent of the factor length, which is only further used to determine the repetition rate of the pulses in the cavity. This model gets its name from the fact that the pulses undergo a strong temporal stretching and compressing while traveling along the laser cavity.
Erbium-doped fiber amplifiers were first developed in 1987 by R. J. Mears et. al. [23] presenting a GaAs laser diode pumped erbium-doped fiber amplifier with a high gain amplification of 28 dB at a wavelength of 1.54 µm. Deep studies and modeling done by P.R Morkel and R.I. Laming in 1989 [24] provided a numerical solution of a general rate-equation model of an erbium-doped fiber amplifier, setting the base for the design of the most recent erbium-doped fiber amplifier presented by Gong-Ru Lin, et. al., [21].

The amplifier proposed in [21] works as a pulse compressor and amplifier at the same time, reaching pulse widths of 29 fs with 2.3 nJ of pulse energy. This stage is attached directly to the laser output, with a cavity characteristics similar to the one presented in [18] making a modular two-stage amplification/compression system.

Additional analysis into the model presented in [17] is explained by K. Tamura, et. al. in [18]. New operation states that are reachable in the all-fiber ring configuration. A clear distinction was made between operating the laser in nonsoliton mode, by using the stretched pulse technique or soliton mode, briefed in the additive-pulse mode-locking technique.

The nonsoliton mode is attained by setting the net cavity dispersion to a positive value by carefully adjusting the lengths of the positive and negative dispersion segments within the cavity. Sub-100 fs pulses were obtained with higher pulse energies than before [17] due the small value of positive net group velocity dispersion (GVD) in the cavity which favor avoiding nonlinear saturation. The output segment of the cavity was adjusted to form a negatively dispersive delay line to compensate the output’s positive chirp and obtain the shortest pulse duration possible [18].
To the date the shortest pulse reported directly from a stretched pulse fiber laser has been reported by D. Y. Tang and L. M. Zhao [20] using a laser configuration where the polarization controllers were changed for bulk components as quarter and half waveplates and without using any second amplification/compression stage.
Chapter 3

MODE-LOCKING IN FIBER LASERS

When referring to ultrashort or ultrafast pulsed light we are talking about very short time duration pulses of light that are generated in a laser. The accepted scale for ultrafast pulses is applied to pulses with duration of 1 picosecond (1 ps) and below. To put it in perspective, if we could take a snapshot of a 1-s light pulse, this pulse will stretch over a distance of 300,000 Km, likewise a snapshot of a pulse of 1 ps of duration will be 0.3mm wide, close to the thickness of a study card. For this project, pulses as short as 0.1 ps or 100 fs are generated from an all-fiber laser cavity.

Ultrashort pulses present several useful characteristics that make them desirable in many applications [10]. Time resolution: under the picosecond scale, ultrafast pulses deliver extremely accurate time resolution for excitation and characterization of physical process.

Spatial resolution: The spatial extent of an ultrashort pulse of light is specified by the pulse duration times the speed of light. The spatial extent for ultrashort pulses is on the order of micrometers. This makes ultrashort light pulses suited for microscopy and imaging applications.

Bandwidth: The product of the pulse-width times the optical bandwidth must be of order unity (or higher). This relation is specified by the uncertainty principle. If the pulse duration decreases the bandwidth has to increase to maintain validity. A pulse of 10 fs has a bandwidth on the order of 100 terahertz (THz). This
large bandwidth is of great value for optical communications as well as other applications.

*Potential for high intensity:* Femtosecond pulse sources deliver high peak intensities at moderate energy levels.

One way to obtain ultrashort pulses is by the use of mode-locked lasers. The two most common types of mode-locked lasers for ultrafast pulse generation are the Ti:Sapphire laser and fiber lasers. Although Ti:Sapphire lasers are of great use in research environments due the high power produced and proven stability over many days, their portability is very limited, requires a high monetary investment to acquire one piece of equipment and a fully vibration-isolated optical bench to guarantee the permanent alignment of the bulk components. Fiber lasers, on the contrary, are highly portable and although reliable, a fiber laser can only stay mode-locked over several hours.

Fiber optics is a well-developed technology that has reached the point where acquire a spool of standard SMF-28 telecommunications fiber is under 1 dollar per meter. This is a very decisive factor when looking for a femtosecond source when not much power is needed.

### 3.1 Erbium-doped Fiber Amplifier (EDFA)

Fiber lasers and fiber amplifiers are composed of segments of glass fiber and segments of doped glass fiber. The dopants used for these fibers are rare earth ions which are present in the fiber core. These ions act as active atoms in the host material which is the glass fiber and are responsible for the gain associated in the fiber.
The rare earth metal erbium, discovered in 1842 in a Swedish town called Ytterby, is one of the few materials that are used for light amplification in telecommunications.

On an erbium-doped amplifier, light amplification occurs via stimulated emission. The gain media possesses a particularly high efficiency due to the strong optical confinement inherent to the fiber’s structure. The emission wavelength for the ion Er$^{3+}$ exhibits an optical transition at 1.5 µm to 1.6 µm, which matches the optical window where the least attenuation exists in commercially available optical fibers [26].

Gain has been demonstrated over practically the entire emission spectrum for Er$^{3+}$ doped fibers, with values over 20 dB easily obtainable between 1530 and 1560 nm. A critical factor in the success of Er$^{3+}$ doped fiber amplifiers in optical communications is the long lifetime of the metastable state which permits the required population inversion levels to be obtained under steady state conditions, over several milliseconds depending on the host material [27].
Figure 3 shows the absorption spectrum for an Er\textsuperscript{3+}-doped silicate glass. Each absorption band between 500 and 1500 nm has been employed to successfully pump fiber amplifiers, although those which match the output of diode lasers are preferred, as it is for the 980 nm pump laser.

3.2 Mode-locking

A laser operates at discrete frequencies, called longitudinal or axial modes. These modes are standing waves that travel within the cavity, either by bouncing back and forth if the resonator is a Fabry-Perot etalon or through a
circulation over a close cavity, as is the case of the fiber ring laser. In the frequency domain, the modes present in the cavity are restricted to the discrete values given by

\[ \nu_q = \frac{q c}{L} \]  

with \( q = 1, 2, 3 \ldots \), which are the resonance frequencies of the cavity. Adjacent modes are separated by a constant frequency difference,

\[ \nu_f = \frac{c}{L} \]  

where \( L \) is length necessary to make a round trip and \( c \) is the speed of light in the medium present in the cavity \( c = c_0/n_0 \) (\( c_0 = 3 \times 10^8 \text{ m/s} \)). The round trip distance must be precisely equal to a multiple integer of the resonant wavelength present in the cavity.

One limit situation of laser oscillation is the single-frequency or single-mode laser, in which only one axial mode is oscillating, so that the circulating pattern has a constant amplitude and frequency everywhere within the cavity, and the number of round trips is infinitely large.

Another limiting situation is a very high-gain, when the entire laser oscillation burst only lasts for two or perhaps three round trips, the circulating energy within the cavity can be mostly random noise with a bandwidth approaching the line width inherent to the gain medium of the laser [22].

There is always a fixed time delay or time shift \( T \) between successive round trips that makes the axial mode to look like the laser output signal itself, independently of the waveform or the carrier signal. The total number of modes
circulating the cavity will essentially be \( N_{\text{modes}} \approx T/\tau_p \), where \( T \) is the time to do a round trip of the signal and \( \tau_p \) is the duration of the signal. In the case of the fiber laser, a pulse shape will be assumed where \( \tau_p \) is given by its full-width half-maximum measure (FWHM).

Let’s assume the case where there are two oscillating axial modes. These modes will interfere with each other and the output field in the time domain can be expressed as

\[
\mathcal{E}(t) = \text{Re}\left[ E_1 e^{i(\omega_1 t + \phi_1)} + E_2 e^{i(\omega_2 t + \phi_2)} \right]
\]  

(3.3)

\( \omega_1, \omega_2 \) are the sidebands for each mode respectively. The intensity as a function of time becomes

\[
I(t) = |\mathcal{E}(t)|^2 = E_1^2 + E_2^2 + 2E_1E_2\cos\left[(\omega_1 - \omega_2)t + \phi_1 - \phi_2\right]
\]

(3.4)

where \( \phi_1 \) and \( \phi_2 \) are the phase angles of the two phasor amplitudes \( \hat{E}_1 \) and, \( \hat{E}_2 \) respectively. From equation 3.4 can be seen that the output intensity will have a sinusoidal shape with a beat frequency defined by the difference \( \omega_1 - \omega_2 \). This feature is known as mode beating, and illustrates the presence of two or more modes within a laser cavity, also is the starting point for the mode-locking mechanism.

The important concept derives from the fact that changes in the relative phases \( \phi_1 \) and \( \phi_2 \) will change the occurrence in time of the mode-beating peak, however mode beating will still happen unchanged. “Mode-locking” is a direct consequence of this factor, where it doesn’t seem to be much meaningful with only two modes, with three or more the concept will be clarified.
“Mode-locking” means to lock together the phases of sinusoidal signals so they can be coupled, that is, they can interfere constructively and generate a pulsed output. Wherever else the modes are not locked, the interference will be destructive and the modes will cancel themselves out. The more modes are in the cavity the strong this effect will be, and the pulses will be more defined.

Figure 4 shows three sine waves with equally spaced frequencies and initial amplitudes. At $t=0$ the three modes are completely in phase, at this time the resultant field amplitude will be three times the amplitude of any single mode, and hence the peak intensity is nine times the intensity of any single sideband.

![Figure 4. Superposition of three equally spaced frequency components which are all exactly in phase at $t=0$ [22]. © University Books 1986.](image)

Even with only three modes in perfect phase there can be already an acceptable “mode locked” pulse.
Now, extending this reasoning to \( N \) equally spaced axial-mode frequencies, all of them with the same amplitude. Taking the time \( t=0 \) for the time where all the modes are in phase, the phasor description in 3.3 and 3.4 can be further extended

\[
E(t) = \sum_{n=0}^{N-1} e^{j(\omega_0 t + n\omega_{ax})} = \frac{e^{iN\omega_{ax}t} - 1}{e^{i\omega_{ax}t} - 1} e^{i\omega_0 t}
\]

(3.5)

and so, the intensity of this series of modes interfering together

\[
I(t) = |E(t)|^2 = \frac{1 - \cos(N\omega_{ax}t)}{1 - \cos(\omega_{ax}t)} = \frac{\sin^2(N\omega_{ax}t)/2}{\sin^2(\omega_{ax}t)/2}
\]

(3.6)

where \( \omega_{ax} = 2\pi f \). At every multiple of \( t=\pm T, \pm 2T, \ldots \), there is an amplitude peak. The result is a short intense “mode locked” pulse. Its FWHM is approximately \( \tau_p \approx T/N \) and the full width at its base is given by \( 2T/N \).

### 3.3 Mode-locking Techniques

Modulation plays a key role in initiating and maintaining mode-locked laser operation. Active mode-locking refers to the case where the modulator is externally driven. Passive mode-locking refers to situations in which the modulator forms its own modulation through nonlinearities; both amplitude and phase nonlinearities can be important [10].

This theses focus on the use of passive mode-locking to produce a special case of ultrashort pulses. However active mode-locking will be explained first for comparison purposes, then a development in passive mode locking will follow.
3.3.1 Active Mode-Locking

![Diagram of an active mode-locked cavity](image)

$\omega_m = 2\pi/T$

Figure 5. Theoretical schematic for an active mode-locked cavity [10]. © This material is reproduced with permission of John Wiley & Sons, Inc. 2009.

The most simple description of an active mode-locked laser is depicted in figure 5. There is a laser cavity, a gain media and a modulator that will vary its transmission coefficient based on a control signal that is externally driven. One device widely used for modulators in active mode-locking is an acousto-optic modulator. Based on the control signal, the modulator will act as a beam deflector or as high transmission device.

Let’s assume that the control signal for the acousto-optic modulator is given by

$$V_{AO}(t) = V_0 \cos(\omega_c t) \cos\left(\frac{1}{2} \omega_m t\right)$$  \hspace{1cm} (3.7)
where $\omega_c$ is the carrier frequency and $\omega_m$ is the modulation frequency. The modulator works the following way

$$\left| \cos\left(\frac{1}{2} \omega_m t \right) \right| = \begin{cases} 1 & \text{Beam deflection/low transmission} \\ 0 & \text{High transmission} \end{cases}$$  \hspace{1cm} (3.8)$$

The beam deflection happens via diffraction, and is minimal when the modulation is equal to zero. The loss modulation $\Gamma_0$ in the cavity will be then proportional to the modulation signal

$$\Gamma_0(t) \propto \left| V_0 \right|^2 \cos^2\left( \frac{1}{2} \omega_m t \right)$$  \hspace{1cm} (3.9)$$

and the internal cavity loss is less than gain available every modulation period $T=2\pi/\omega_m$, and this occurs only for a short time window. The photons arriving at the modulator during this short time window will be amplified and therefore pulses will be generated as shown in figure 6.
From the point of view of the circulating pulse, a more quantitative analysis can be made: first consider a pulse circulating inside a laser cavity without a modulator and its representation in the frequency domain, $p=2L$ for a round trip

$$E_r(\omega) = g_r(\omega) \times E_0(\omega) = e^{(\alpha_m p_m - \Gamma_0)} \cdot e^{\left(-j\frac{\omega}{c}p\right)} \cdot E_0(\omega) \quad (3.10)$$

which is described by the initial characteristics of the pulse $E_0(t)$ times the total gain inside the cavity per round trip described by $g_r(\omega)$, including the phase change due to propagation, the field gain coefficient $\alpha_m$ and the length of the gain medium $p_m$. Right now the total phase change due to nonlinearities is being ignored in equation 3.10. The intensity of the pulse will be
For a pulse to propagate unchanged inside the cavity the value for the term \( g_{rt}(\omega) \) in equation 3.10 should be equal to 1, which means that the pulse will be the same after one round trip. For many mode-locked lasers, the actual pulse bandwidth is much smaller than the available gain bandwidth, so for a pulse centered at a frequency \( \omega_a \), and assuming a Lorentzian gain profile

\[
\alpha_m(\omega) = \alpha_{\text{max}} \left( \frac{1}{1 + \left( \frac{\omega - \omega_a}{\Delta \omega_a / 2} \right)^2} \right) \tag{3.11}
\]

so equation 3.12 can be approximated to

\[
\alpha_m(\omega) = \alpha_{\text{max}} \left[ 1 - \frac{4}{\Delta \omega_a^2} (\omega - \omega_a)^2 \right] \tag{3.12}
\]

and plugin equation 3.12 into \( g_{rt}(\omega) \), one can notice that there will be a frequency dependence in the round-trip total gain given by the factor that follows the minus sign in equation 3.12. This frequency dependence makes \( g_{rt}(\omega) \) to act as a band-pass filter, narrowing the pulse spectrum and broadening the pulse duration as well, after one round trip where the pulse has been subject to amplification. This effect is called gain narrowing and that’s why \( g_{rt}(\omega) \) cannot be equal to 1 on a cavity without the modulator.

Assuming a Gaussian pulse shape now, in order to show the gain narrowing effect
\[ E(\omega) = E_0 \sqrt{\frac{\pi}{a}} e^{-\frac{(\omega - \omega_0)^2}{4a}} \] 

(3.13)

and after one round trip, the intensity is given by

\[
|E_n(\omega)| = E_0 \sqrt{\frac{\pi}{a}} e^{\left(\frac{\sigma_{\text{max}} P_n}{\Delta \omega^2} - \frac{(\omega - \omega_0)^2}{4a} \right)} e^{\left(\frac{1}{4} \left(1 + 16\frac{\sigma_{\text{max}} P_n}{\Delta \omega^2} \right)(\omega - \omega_0)^2 \right)}
\]

(3.14)

From the develop in equation 3.15, it can be concluded that in the time domain, the field will be proportional to \( a' \)

\[ E_n(t) \propto e^{-a't^2} \] 

(3.15)

For a Gaussian pulse, the pulse duration is given by the relation

\[ \tau = \sqrt{\frac{2 \ln 2}{a}} \] 

(3.16)

and having the new pulse duration \( \tau' \) as a function of \( a' \), it can be derived that

\[
\tau' = \sqrt{\frac{2 \ln 2}{a} \left(1 + \frac{16\alpha_{\text{max}} P_m}{\Delta \omega_a^2} \right)} = \tau_0 \sqrt{1 + \frac{16\alpha_{\text{max}} P_m}{\Delta \omega_a^2}}
\]

(3.17)

\[ \tau' > \tau_0 \]
Gain narrowing also will prevent steady state pulse oscillation inside the cavity; therefore steady state pulse oscillation requires the loss modulator to oppose this effect.

The effect of the loss modulator is better analyzed in the time domain where the effect over the original pulse duration will show a clear pulse shortening. For mathematical convenience, a Gaussian pulse will be assumed as the input pulse again. Also, the initial duration of the input pulse is much shorter than the low loss time windows depicted in figure 6. Now, including the loss modulation as a function of time, the modulated field $E_{\text{mod}}(t)$ will be the result of the input field under the effect of the modulated loss $G_{\text{mod}}(t)$.

$$E_{\text{mod}}(t) = G_{\text{mod}}(t) \cdot E_{\text{in}}(t)$$  (3.18)

Assuming that $G_{\text{mod}}(t)$ in 3.19 has the following form

$$G_{\text{mod}}(t) = e^{-\Delta_{\omega}(1-\cos \omega_{m}t)}$$  (3.19)

and only we are interested in times near the maxima of $G_{\text{mod}}(t)$, therefore small angle approximation can be taken for $\cos(\omega_{m}t) \approx 1 - \frac{1}{2}(\omega_{m}t)^2$, so equation 3.19 becomes

$$G_{\text{mod}}(t) \sim e^{-\Delta_{\omega} \frac{1}{2}(\omega_{m}t)^2}$$  (3.20)

now replacing on equation 3.18 for a Gaussian pulse shape

$$E_{\text{mod}}(t) = e^{-\Delta_{\omega} \frac{1}{2}(\omega_{m}t)^2} E_{0} e^{-\omega^2 t^2} e^{i\omega_m t} = E_{0} e^{i\omega_m t} e^{-(\omega + \Delta_{\omega} \omega_{m}t)^2}$$

$$E_{\text{mod}}(t) = E_{0} e^{-\omega^2 t^2} e^{i\omega_m t}$$  (3.21)
and doing the same treatment done in 3.17 for the new pulse duration $\tau''$ as a function of $a''$

$$\tau' = \sqrt{\frac{2 \ln 2}{a}} \left( \frac{1}{1 + \frac{1}{2} \frac{\Delta_m \omega_m}{\omega_m^2}} \right) = \tau_0 \frac{1}{\sqrt{1 + \frac{1}{2} \frac{\Delta_m \omega_m}{\omega_m^2}}}$$

$$\tau' < \tau_0$$

(3.22)

this result confirms that the loss modulator shortens the pulse duration, offering the possibility of compensate for the gain narrowing effect present in active mode-locking. Now $a'$ can be approximated to

$$a' \approx a - \frac{16 \alpha_m p_m a^2}{\Delta \omega_m^2}$$

(3.23)

and so, to balance both effects

$$a' = a - \frac{16 \alpha_m p_m a^2}{\Delta \omega_m^2} + \frac{1}{2} \frac{\Delta_m \omega_m^2}{\omega_m^2}$$

(3.24)

the two terms after a in equation 3.25 must cancel each other, therefore must be zero be able to maintain pulse duration

$$a = \frac{\omega_m \Delta \omega_m}{4} \sqrt{\frac{\Delta_m}{2 \alpha_m p_m}}$$

(3.25)
with this conclusion the pulse duration as a function of the parameters of the cavity, plus the balance between gain narrowing and loss modulation inside for active mode locking can be calculated for a Gaussian pulse shape.

\[
\tau_p = \sqrt{\frac{2 \ln 2}{a}} = \left( \frac{8 \sqrt{2 \ln 2}}{\omega_m \Delta \omega_a} \right)^{\frac{1}{2}} \left( \frac{\alpha_m P_m}{\Delta_m} \right)^{\frac{1}{4}}
\]  

(3.26)

Some important conclusions can be drawn from equation 3.27:

- Larger modulation depth \( \Delta_m \) produces shorter pulses.
- Higher modulation frequency, \( \omega_m \), yields to shorter pulses. This effect is called harmonic mode-locking, where \( \omega_m = 2 \Delta \omega_a \).
- Increasing the modulation frequency to much allows multiple pulses to circulate inside the cavity.
- A lower loss cavity gives shorter pulses, based on the fact that the round-trip gain equals the cavity loss to maintain steady state.
- Larger \( \Delta \omega_a \) gives shorter pulses because the gain narrowing effect gets reduced by the same amount.
- Active mode locking is not suitable for generating femtosecond pulses because of the much wider bandwidth of femtosecond pulses compared to the gain bandwidth, they can be comparable.

### 3.4 Master Equation for Active Mode-locking

This equation developed by Herman Haus [9] can describe active and passive mode-locking. A simplified derivation will be enough to describe the actors in active mode-locking. The first assumption to be made is to assume a ring cavity
configuration instead of a two mirror resonator. The second assumption is that in steady state, each cavity element has a *small* effect on the circulating pulse.

This differential equation describes the evolution of the pulse envelope inside the cavity, $E(t)$. A general scheme of what is happening inside the cavity for active mode-locking is depicted in figure 7.

**Figure 7. General scheme for the processes happening in active mode-locking.**

Let’s start first with gain narrowing, from equation 3.15, the initial field is subject of the cavity gain and loss features as shown in figure 7, that will provide the expression for $\mathcal{E}'(\omega)$

$$\mathcal{E}'(\omega) = e^{(a_{\max}p_m - \Gamma_0)} \cdot e^{-4a_{\max}p_m \left(\frac{(\omega - \omega_m)^2}{\Delta \omega_a^2}\right)} \cdot \mathcal{E}_m(\omega)$$

(3.27) 

thanks to the second assumption the exponential terms can be approximated as follows.
\[ \mathcal{E}'(\omega) = \left[1 + \left(\alpha_{\text{max}} p_m - \Gamma_0\right) \right] \left[1 - 4\alpha_{\text{max}} p_m \frac{(\omega - \omega_a)^2}{\Delta \omega_a^2}\right] \mathcal{E}_\infty(\omega) \]

\[ \begin{align*} 
\mathcal{E}'(\omega) &= \left[1 + \delta_0\right] \cdot \mathcal{E}_\infty(\omega) \\
\mathcal{E}'(\omega) &= \left[1 + \delta_0 - \delta_\infty \right] \cdot \mathcal{E}_\infty(\omega) 
\end{align*} \] (3.28)

\[ \begin{align*} 
\mathcal{E}'(\omega) &= \left[1 + \delta_0 \right] \cdot \mathcal{E}_\infty(\omega) 
\end{align*} \]

the second order term \( \delta \epsilon \) can be dropped because its contribution is negligible. Now, going back to the time domain, \( \epsilon \) depends on \( \omega \) so the equation 3.28 has to be rearranged

\[ \mathcal{E}'(\omega) = \left[1 + \delta_0 \right] \cdot \mathcal{E}_\infty(\omega) - \frac{4\alpha_{\text{max}} p_m}{\Delta \omega_a^2} (\omega - \omega_a)^2 \mathcal{E}_\infty(\omega) \]

(3.29)

assume that \( \mathcal{E}_\text{in}(t) = E(t)e^{j\omega t} \rightarrow \mathcal{E}_\text{in}(\omega) = E(\omega - \omega_a) \), then

\[ \mathcal{E}'(t) = \left(1 + \delta_0\right) E_\infty(t) e^{j\omega t} + \frac{4\alpha_{\text{max}} p_m}{\Delta \omega_a^2} \frac{\partial^2}{\partial t^2} E_\text{in}(t) e^{j\omega t} \] (3.30)

the value for \( \mathcal{E}'(t) \) given in equation 3.30 will now undergo the process of the loss modulator

\[ \begin{align*} 
\mathcal{E}''(t) &= e^{-\frac{1}{2} \Delta_n \omega_a^2 t^2} \mathcal{E}'(t) \\
\mathcal{E}''(t) &= \mathcal{E}'(t) - \left(\frac{1}{2} \Delta_n \omega_a^2 t^2\right) \mathcal{E}'(t) \\
\mathcal{E}''(t) &= \mathcal{E}'(t) - \left(\frac{1}{2} \Delta_n \omega_a^2 t^2\right) \left(1 + \delta_0\right) E_\text{in}(t) e^{j\omega t} + \left(\frac{4\alpha_{\text{max}} p_m}{\Delta \omega_a^2} \frac{\partial^2}{\partial t^2} E_\text{in}(t) e^{j\omega t}\right) \right] \] (3.31)

In order of contribution the last term in equation 3.32 can be neglected.

Plugin equation 3.30 into 3.31 for steady state operation, where \( \mathcal{E}''(t) = \mathcal{E}_\text{in}(t) \) gives the following equation for the field envelope
since $\delta_0=\alpha_{\text{max}} p_m \Gamma_0=\text{gain (g) }-\text{loss (l)}$, then replacing the terms in equation 3.32, the master equation for active mode-locking is derived

$$\left[\delta_0 + \frac{4\alpha_{\text{max}} p_m \partial^2}{\Delta \omega^2_a} \right] - \frac{1}{2} \Delta_m \omega^2_a t^2 \right] E_\omega(t) = 0$$

(3.32)

the term multiplying to the second derivative over time operator, $4g/\Delta \omega^2_a$ corresponds to the gain narrowing process and the term next to squared time variable, $\Delta_m \omega^m / 2$ corresponds to the loss modulation process. The $g - l$ term is a scalar multiplication and basically does not affect the pulse shape.

The gain narrowing term is proportional to the second derivative of the envelope over time, that is, there is a small attenuation of the peak of the pulse, likewise, the loss modulation term, being proportional to modulation depth over time cuts the wings of the pulse.

The pulse duration is ultimately determined by the curvature of the loss modulation as stated at the beginning of this section, therefore it can be concluded that the fastest modulation is obtained if the pulse modulates itself: self-amplitude modulation (SAM) which is key for passive mode-locking. The pulse shortening process finds its limit by the available gain spectrum, in theory one could be able to mode-lock the entire gain bandwidth of the laser medium.
Passive mode-locking requires self-amplitude modulation to function. This is usually accomplished by means of a *saturable absorber*, which is an absorber whose optical absorption coefficient depends on the intensity of the light being absorbed. If the incident light has a weak intensity the absorption is high and therefore, low transmission through the absorber. If the incident light has a strong intensity, the absorber gets saturated, which represents a high transmission through. Once a pulse reaches certain threshold limit only the pulse’s peak “burns through” and gets further amplification and compression in the cavity. There are two types of saturable absorbers, slow and fast.

### 3.5.1 Passive Mode-locking: Slow Saturable Absorber

The response time of the absorber is much slower than the pulse duration. The pulse is integrated with time, followed by a slow recovery from the absorber. Dye lasers and semiconductor lasers exhibit this type of absorbers.
In figure 8, it can be seen that in steady state the gain is smaller than the loss then, after the pulse passes there is a time window where the loss is smaller and the pulse gets amplified.

This can be interpreted as a shutter that opens swiftly as soon as the pulse saturates the absorber. This clips successfully the leading edge of the pulse but the trailing edge does not get much clipping. To compensate for this distortion, once the pulse is amplified, the gain also saturates to the point where it comes back to be smaller than the losses. This completes the shutter mechanism that opens once the pulse saturates the losses and closes once the amplified pulse saturates the gain. The speed for the shutter to open or close is due the short optical pulse, it does not depend on the material response time.
A good way to describe a slow saturable absorber is through the analysis of the rate equations for a two level atomic system. On a two level atomic system there are two types of electronic transitions that can be present between the energy levels. The rate equations describe the rate of population change in state $N_i$ is equal to the rate of atoms entering that state minus the rate of atoms leaving it. The number of transitions per second denominated by $W_{\text{sig}}$ is the stimulated emission generated from an external signal for a two level system is the same, and spontaneous emission has to be included as follows,

$$
\frac{dN_2}{dt} = -W_{\text{sig}} (N_2 - N_1) - \frac{1}{\tau_{21}} N_2
$$

$$
\frac{dN_1}{dt} = +W_{\text{sig}} (N_2 - N_1) + \frac{1}{\tau_{21}} N_2
$$

(3.34)

The constraint for equation (35) is given by $N_1 + N_2 = N$. In this case, the main goal for this analysis is not steady state, on the contrary, the interest is that $\Delta N(t) \neq 0$. Assuming that the pulse duration $I_{\text{sig}}(t)$ is much shorter than the spontaneous emission rate $\tau_{21}$, equation 3.34 can be approximated to

$$
\frac{dN_2}{dt} \approx -W_{\text{sig}} (N_2 - N_1)
$$

$$
\frac{dN_1}{dt} \approx +W_{\text{sig}} (N_2 - N_1)
$$

(3.35)

and plug the definition of $W_{\text{sig}}(t)$ in equation 3.35

$$
\frac{d\Delta N}{dt} = -2W_{\text{sig}}(t)\Delta N(t)
$$

(3.35)
where \( \sigma_s \) is the absorption cross-section and \( \hbar \omega_s \) is the incoming photon energy. Now separating terms on equation 3.36 to integrate on both sides

\[
\int \frac{1}{\Delta N} d\Delta N = -\frac{2\sigma_s}{\hbar \omega_s} \int J_{\text{sig}}(t) dt = -\frac{2\sigma_s}{\hbar \omega_s} \cdot U(t)
\]  

(3.37)

\( U(t) \) is defined as the pulse fluence, and expresses energy density over time. Also the saturation fluence will be defined as \( U_{\text{sat}} = \frac{\hbar \omega_s}{2 \sigma_s} \). The solution to equation 3.37 is

\[
\Delta N(t) = \Delta N(-\infty) e^{-\frac{U(t)}{U_{\text{sat}}}}
\]  

(3.38)

The value of \( \Delta N(-\infty) = -N \), that is just the steady state value before the pulse, which is all the concentration is in level \( N_1 \). Just after the pulse there is going to be some new value for \( \Delta N = N_2 - N_1 = N \) because just after the pulse passes \( N_2 \) will increase its concentration. However this state will not last forever. Spontaneous emission will restore the population of the system to its initial steady state. So after the pulse the rate equations for the two levels will be

\[
\begin{align*}
\frac{dN_2}{dt} &\approx -\frac{1}{\tau_{21}} N_2 \\
\frac{dN_1}{dt} &\approx +\frac{1}{\tau_{21}} N_2 \\
\frac{d\Delta N}{dt} &\approx -\frac{2}{\tau_{21}} N_2
\end{align*}
\]  

(3.39)

relating the value of \( \Delta N = N_2 - N_1 \) with the initial constraint \( N = N_1 + N_2 \), \( N_2 \) can be derived as
\[ N2 = \frac{\Delta N + N}{2} \]  \[ \frac{d\Delta N}{dt} = \frac{1}{\tau_{21}} \Delta N - \frac{1}{\tau_{21}} N \]  (3.40)

and the solution for the differential equation for \( \Delta N \) from equation 3.41

\[ \Delta N = -N + N \frac{U}{U_{\text{sat}}} e^{-t/\tau_{21}} \]  (3.41)

so the recovery time in a slow saturable absorber is given by the time constant \( \tau_{21} \).

This confirms the assumption that a femtosecond pulse will be much shorter than the recovery time of the absorber which is in the order of nanoseconds.

This behavior is called Self Amplitude Modulation (SAM), where the amplitude of the pulses will self-modulate its ability to travel through the cavity, and the more intense the pulse is more amplified gets. Two conditions are required to have a slow saturable absorber: first, the absorption cross-section of the absorber has to be greater than the absorption cross-section of the gain, that is \( \sigma_{\text{abs}} > \sigma_{\text{gain}} \), so the absorber saturates easily than the gain. Second, the recovery time of the absorber is faster than the recovery time of the gain. This will prevent formation of extra pulses in the cavity.

SAM acts as the modulator in active mode-locking, but the pulse modulates itself the losses in the cavity. The accounted effect for SAM will replace the modulation effect in the mode-locking master equation from equation 3.34,

\[ \left( g - I + \frac{4g}{\Delta \omega^2} \frac{\partial^2}{\partial t^2} \right) + \tilde{g}(t) - \bar{I}(t) \right] E(t) = 0 \]  (3.42)

where is \( \tilde{g}(t) \) the saturable gain and \( \bar{I}(t) \) is the saturable loss.
3.5.2 Passive Mode-locking: Fast Saturable Absorber

In a fast saturable absorber the recovery time is much shorter than the optical pulse [28]. At any time during the pulse, $\Delta N$ is equal to the steady state value at the present value of $I_{\text{sig}}$. This means that over time, the value of $\Delta N$ follows $I_{\text{sig}}(t)$

$$\Delta N(t) = \Delta N_0 \frac{1}{1 + \frac{I_{\text{sig}}(t)}{I_{\text{sat}}}} \approx \Delta N_0 \left[ 1 - \frac{I_{\text{sig}}(t)}{I_{\text{sat}}} \right]$$ (3.43)

The constant $\Delta N_0$ corresponds to the small signal value. The absorption coefficient of the absorber can be then related to the approximation done in 3.43 and expressed as a function of the $I_{\text{sig}}(t)$.

$$\alpha_{\text{abs}} = \sigma_s \Delta N = \sigma_s \Delta N_0 \left[ 1 - \frac{I_{\text{sig}}(t)}{I_{\text{sat}}} \right] = \alpha_0 \left[ 1 - \frac{I_{\text{sig}}(t)}{I_{\text{sat}}} \right]$$ (3.44)

Inside a fast saturable absorber, the pulse intensity also self-modulates the loss in the cavity hence there is a component of self-amplitude modulation (SAM). Let’s assume that the output of the field after passing through the saturable absorbing is $\mathcal{E}'(t)$ and is expressed as

$$\mathcal{E}'(t) = \mathcal{E}_{\text{in}}(t)e^{-\alpha_{\text{abs}}P_{\text{abs}}} \approx \mathcal{E}_{\text{in}}(t)[1 - \alpha_{\text{abs}}P_{\text{abs}}]$$ (3.45)

and plugin equation 3.44 into equation 3.45

$$\mathcal{E}'(t) = \mathcal{E}_{\text{in}}(t) \left[ 1 - \alpha_0 P_{\text{abs}} + \frac{\alpha_0 P_{\text{abs}}}{I_{\text{sat}}} I(t) \right] = \mathcal{E}_{\text{in}}(t) \left[ 1 - \alpha_0 P_{\text{abs}} + \frac{1}{2} \varepsilon_0 cn \frac{\alpha_0 P_{\text{abs}}}{I_{\text{sat}}} |E(t)|^2 \right]$$ (3.46)
the factor $\alpha_0 p_{abs}$ can be lumped into the overall cavity loss in the master equation. The factor that accompanies the magnitude of the envelope $|E(t)|^2$, is called the self-amplitude modulation coefficient $\gamma$

$$\gamma = \frac{1}{2} e_0 c h \frac{\alpha_0 P_{abs}}{I_{sat}}$$  \hspace{1cm} (3.47)

then the master equation becomes,

$$\left[ g - l + \frac{4g}{\Delta \omega_n^2} \frac{\partial^2}{\partial t^2} + \gamma |E(t)|^2 \right] E(t) = 0$$  \hspace{1cm} (3.48)

So far, equation 3.48 accounts for the contributions of gain narrowing and SAM. The master equation for mode-locking has solutions of the form $E(t) = E_0 sech(t/\tau)$. This type of solution is the same that describes solitons travelling in nonlinear media and just like solitons, $E(t)$ has constraints in pulse duration and amplitude,

$$\gamma |E_0|^2 = \frac{8g}{\Delta \omega_n^2 \tau^2}$$  \hspace{1cm} (3.49)

$$g - l + \frac{4g}{\Delta \omega_n^2 \tau^2} = 0$$  \hspace{1cm} (3.50)

the first constraint from equation 3.49 can be re-written as,

$$\frac{1}{\tau^4} = \frac{\Delta \omega_n^2 \gamma |E_0|^2}{8g \tau^2}$$  \hspace{1cm} (3.51)
and recalling from active mode-locking in equation (27), \(a^2\) is inversely proportional to \(\tau^4\), as follows,

\[
\frac{1}{\tau^4} \propto a^2 = \frac{\Delta \omega_m^2}{8g}, \frac{\Delta_m \omega_m^2}{4}
\]

(3.52)

from comparison between equation 3.51 and 3.52, two equivalencies can be drawn between active and passive mode-locking: the self-amplitude modulation coefficient factor \(\gamma|E_0|^2\) is the equivalent to the modulation depth \(\Delta_m\), and, the inverse of the pulse duration \(1/\tau\) is equivalent to the modulation frequency \(\omega_m\). From the second equivalency, an important fact arises: the huge advantage of passive mode-locking is the extremely high effective modulation frequency, on the order of Terahertz.

The second constraint in equation 3.51 can be re-written as well,

\[
g - l = -\frac{4g}{\Delta \omega_m^2 \tau^2} < 0
\]

(3.53)

which means that in steady state operation the gain is less than the loss. This feature is shown in figure 9. The modulation is completely according to the pulse shape, and the loss modulation recovers as soon as the pulse passes.
Figure 9. Gain and loss dynamics in fast saturable absorber mode-locking. The shaded region corresponds to positive net gain [10].© This material is reproduced with permission of John Wiley & Sons, Inc. 2009.

Now, passive mode-locking is the mechanism better suited for to produce ultrashort pulses, which require higher peak intensities. With more intensity, nonlinearities arise and two artifacts show:

- Group Velocity Dispersion (GVD): Shorter pulses mean larger bandwidths and hence dispersion compensation is required to maintain the pulse.
- Self-phase modulation (SPM): High peak intensities cause nonlinear propagation due to the nonlinear nature of the, intensity dependent, index of refraction inside the fibers.
In fact, when taking advantage of the existence of the nonlinearities, GVD and SPM can enhance the soliton-like nature of passive mode-locking and produce shorter, more energetic pulses.

GVD is originated mostly due to the gain media. Dispersion refers to the dependence of group velocity on frequency. In the case of nonzero GVD, different frequencies have slightly different cavity round trips, and this can be an important pulse broadening mechanism [10]. To include GVD in the master equation for mode-locking, let’s start first by propagating the pulse through a dispersive media. Again, the assumption that for every round-trip the changes in the pulse are small is maintained.

Let \( \mathcal{E}'(t) \) be the output of a field after propagating through a dispersive media. If only dispersion is present, the phase change in the frequency dependent propagation constant is of second order, since the zero, \( \phi_0 \), and first order, \( \phi_0' \), phase components correspond to a constant phase shift and a constant delay due propagation respectively,

\[
\begin{align*}
\mathcal{E}'(\omega) &= \mathcal{E}_{in}(\omega)e^{-j\phi(\omega)} = \mathcal{E}_{in}(\omega)e^{-j\phi_0' (\omega-\omega_0)^2} \\
\mathcal{E}'(\omega) &\approx \mathcal{E}_{in}(\omega) \left[ 1 - j\frac{1}{2}\phi_0' (\omega-\omega_0)^2 \right] \quad (3.54) \\
\mathcal{E}'(\omega) &\approx \mathcal{E}_{in}(\omega) + j\frac{1}{2}\phi_0' \left[ - (\omega-\omega_0)^2 \mathcal{E}_{in}(\omega) \right]
\end{align*}
\]

and taking the inverse Fourier transform from the result in equation 3.54

\[
\mathcal{E}'(t) = \mathcal{E}_{in}(t) + j\frac{1}{2}\phi_0' \frac{\partial^2 \mathcal{E}_{in}}{\partial t^2} e^{i\omega t} \quad (3.55)
\]
the factor $j^{1/2}\phi_0$ acts on the phase of the pulse and the next part of the second term of equation 3.55 similar to the definition of gain narrowing. Including the GVD effect on the master mode-locking equation in 3.48, it becomes

$$\left[ \frac{4g}{\Delta\omega_n^2} + j\phi''_0 \right] \frac{\partial^2}{\partial t^2} + \gamma |E(t)|^2 E(t) = 0 \quad (3.56)$$

Now, to include SPM in the master equation, a similar procedure as for GVD will be done. In SPM, a time-varying nonlinear phase shift which itself modifies the pulse appears as a consequence of the intensity-dependent nonlinear phase shift. For a medium with the nonlinear index of refraction $n_2 > 0$, SPM gives rise to red shifts on the leading edge of the pulse and blue shifts on the trailing edge. This variation of instantaneous frequency as a function of time is called a *chirp*.

Again, let’s assume that $E'(t)$ is the output of a field that just passed through a nonlinear media, and ignore dispersion for simplicity,

$$E'(t) = E_{in}(t)e^{-j\phi_{in}} = E_{in}(\omega)[1 - j\phi_{nl}]$$

$$E'(t) \approx E_{in}(t) - j\left[ \frac{2\pi}{\lambda} n_2 p_m I(t) \right] E_{in}(t)$$

$$E'(t) \approx E_{in}(t) - j\left[ \frac{2\pi n_2}{\lambda} p_m \frac{1}{2} e_0 cn \right]|E_{in}|^2 \cdot E_{in}(t) \quad (3.57)$$

replacing everything inside the square brackets for the parameter $\delta$ called the SPM coefficient,

$$E'(t) \approx E_{in}(t) - j\delta |E_{in}|^2 \cdot E_{in}(t) \quad (3.58)$$
the result in equation 3.58 can now be included into the master mode-locking equation. Since \( j\delta \) is multiplying the magnitude of the envelope, the effect of SPM will modify the effect of SAM over the pulse magnitude,

\[
\left[ g - l + \left( \frac{4g}{\Delta \omega_n^2} + j \frac{\phi_0''}{2} \right) \frac{\partial^2}{\partial t^2} + \left( \gamma - j\delta \right) \left| E(t) \right|^2 \right] E(t) = 0 \tag{3.59}
\]

the factor \((\gamma - j\delta)\) is known as the Kerr factor, because by definition, the Kerr effect is how the field intensity affects its propagation over a nonlinear media. In general, equation 3.59 describes the evolution of the envelope of a mode-locked pulse inside the cavity, and it accounts for the effects of Gain Narrowing, Group Velocity Dispersion, Self-Amplitude Modulation and Self-Phase Modulation on a fast saturable absorber mode-locked laser.

Passive mode-locking possesses a soliton-like nature in order to maintain pulses inside the cavity, from equation 3.59, derives that \( \phi_0'' < 0 \) in order to balance SPM and GVD and allow the pulse to travel unchanged and to produce a chirp free pulse as a soliton-like.

In [9], Haus graphically summarizes the effects contemplated by the master equation in a very useful schematic intended for a unidirectional ring cavity. In the ring configuration, an isolator is used to ensure unidirectional direction of traveling of the light, as shown in figure 10.
3.6 Additive-Pulse Mode-locking in Fiber Lasers

Haus, Ippen and Tamura make a specific analysis of APM in [29] using an all-fiber ring cavity operating in the negative groups velocity regime. Nonlinear polarization rotation is used on a fiber ring cavity to obtain APM effect. The configuration is similar to that showed in figure 2. The ring is composed of an erbium-doped fiber segment spliced to a standard Corning SMF-28 fiber. The polarization controllers and the polarizer produce APM action by first transforming linear polarization into elliptic polarization. SPM in the fiber rotates the ellipse. The polarizer transforms the rotation of the ellipse into amplitude modulation. Two
polarization controllers are used to compensate for the uncontrollable polarization transformations happening on a non-polarization-maintaining fiber.

This configuration is said to be self-starting mode-locked. When the laser pulse builds up from an initial amplitude fluctuation corresponding to the excitation of a few adjacent modes, the APM action has to provide injection signals into the different modes to produce phase coherence.

Figure 11, explains the nonlinear polarization rotation (NLPR) effect that takes place inside the cavity and facilitates the start of SAM.

![Figure 11. Summary of the nonlinear effects present on a cavity that lead to the nonlinear polarization rotation technique to start [30]. © IEEE 1995](image)

First the linearly polarized light leaves the polarizer where encounter the first polarization controller that induces a birefringence in the fiber by applying physical pressure to it. This pressure changes the relative axial indexes of refraction in
the fiber, inducing a change in the polarization of the light: it becomes elliptically polarized with the axes aligned with the fiber birefringence. This is a superposition of left- and right-hand circular modes of different intensity.

Due to the Kerr effect present in the whole cavity, the travelling elliptically polarized light will experience a phase change that will rotate the ellipse as it travels through. Because a component of the ellipse that carries more energy and hence more intensity will see stronger nonlinear effects in the fiber, the nonlinear phase change induced over this component will be stronger and the ellipse aligned with the strong birefringence component will rotate faster while propagating over the Kerr medium. The modes then will be interfered to produce APM action.

The weaker component will also rotate, but a slower speed. By the time the light does a round trip on the ring, if the birefringence in the fiber has been carefully adjusted to guarantee that only the strong component will pass again through the polarizer, this will generate mode-locking through favoring the amplification the stronger modes inside the cavity. The weaker components that did not experience enough Kerr rotation will be absorbed by the artificial saturable absorber action.

The second polarization controller biases the system to a point where higher intensities experience lower loss. In theory, for an isotropic Kerr medium, only the first polarization controller is necessary, however for the experimental system a second controller was inserted to counteract any effects of residual birefringence in the fibers [29].

The fast saturable absorber action is inherent to the polarizer action, guaranteeing that the loss will always smaller when the stronger components cross through and “fast” enough to keep up with the ultrashort pulse duration. The pulse
compression effect happens in the polarizer as well, when the wings of the pulses are clipped per pass and only the intense pulse center will remain.

The master equation for APM, includes the gain bandwidth of the gain media,

\[
-\mathbf{l} + g \left(1 + \frac{1}{\Omega_g^2} \frac{\partial^2}{\partial t^2}\right) + jD \frac{\partial^2}{\partial t^2} + \left(\gamma - j\delta\right)|\mathbf{E}(t)|^2 \right] \mathbf{E}(t) = j\phi \mathbf{E}(t) \tag{3.60}
\]

sets the sum of changes equal to the net phase-shift \( \phi \) around the loop. Here \( \mathbf{l} \) is the loss, \( g \) is the gain per pass, \( \Omega_g \) the gain bandwidth, \( D \) the net GVD, \( \delta|\mathbf{E}(t)|^2 \) is the SPM, \( \phi \) the phase shift per pass, and \( \gamma|\mathbf{E}(t)|^2 \) the contribution of SAM. The solution of the equation 3.61 is

\[
\mathbf{E}(t) = \mathbf{E}_0 \left[ \text{sech} \left( \frac{t}{\tau} \right) \right]^{[1+j\beta]} \tag{3.61}
\]

gives dependences of pulse width and chirp parameter on dispersion, with the SPM coefficient as a parameter. If \( \beta=0 \) the pulse is soliton like. Although this soliton-like pulse may experience large attenuation and amplification over a round trip, the net phase-shift after one round trip is small, so the master equation holds up correctly [29]

### 3.7 Stretched Pulse Mode-locked Fiber Laser

The operation of the NLPR-APM laser falls into the soliton operation regime, which constraints the single-pulse energy to that of a fundamental soliton. APM clamps the peak power because of its interferometric nature. Once the peak
power is clamped, its duration and energy are determined by the soliton area theorem [17].

M. E. Fermann, et.al. [33] proposed a mode-locked laser, by using a combination of erbium doped fiber for the gain medium, a prism pair and an acousto-optic modulator. The fiber possessed a positive GVD parameter while the intracavity dispersion was adjusted using the prism pair to be a negative GVD. The third order dispersion coefficient was also adjusted to counter the positive value of $\beta_3$ in the fiber.

Femtosecond pulses were generated as a result of passive amplitude modulation’s arising from NLPR in the Er-doped fiber. Due to the absence of third order dispersion in the cavity, the output spectrum of the laser was relatively smooth.

Having a dispersion-compensated cavity, allows for chirp linearization through the interaction of dispersion and self-phase modulation in the fiber. Mode-locking achieved by NLPR in such cavity allows for the power dependence of the nonlinear loss to be tuned by adjusting the polarization controller, and the ratio of the nonlinear loss to the nonlinear phase shift is smaller [33].

The configuration presented in [17] is all-fiber ring geometry. This configuration employs positive dispersion fibers, on where the soliton effects are not present and negative dispersion fibers on the same cavity.

The positive dispersion segment will time-stretch the circulating pulse, effect that will help to reduce the nonlinearity and prevents the saturation of the APM mechanism. In this cavity the chirp will be highly linear and can be compensated by use if a proper length of output coupling fiber as a dispersive delay line [17]. The ring configuration is shown in figure 12.
Due to the large GVD of the fibers, the pulse is temporally stretched and compressed in one transit. This serves to lower the average peak power compared to that which would be experienced by a transform-limited pulse of the same bandwidth. The output pulse is, in general, chirped [30]. The stretching reduces the nonlinearity which is always excessive in fiber systems longer than ~1 m in length [18].

Figure 12. Layout for the stretched pulse APM fiber laser. [10]. © This material is reproduced with permission of John Wiley & Sons, Inc. 2009.

The principle of operation starts the same way as in APM, where nonlinear polarization rotation resolves an artificial saturable absorber action. The function of the components already mentioned in the last section remains the same. The erbium-doped segment provides both the gain and a positive dispersion medium.
The overall cavity net dispersion after compensation of both segments ranges from negative to positive values between -0.1 to +0.01 ps$^2$ [17,30-31]. The net cavity dispersion can be easily adjusted by changing the length of each segment to the desired value and can be even adjusted to be zero.

When the net cavity dispersion was adjusted to be a negative value, the laser operated in a clear soliton mode with power spectra with sidebands typical of soliton operation. Adjusting the net cavity dispersion to a positive small value, the spectrum showed a smooth wider range with the correspondingly pulse compression [17].
Chapter 4

EXPERIMENTAL SETUP AND RESULTS

A Er-doped stretched-pulse fiber ring configuration is used. This cavity tuned in such way that is capable of operating either in the noise-like regime [33,34] and in the self-similar [35] regime independently, in a stable way for several hours.

The pump used is a JDS Uniphase 980nm laser diode. The diode is driven by a Thorlabs LDC-120C Laser Controller and a ILX Lightwave LDT-5412 temperature controller. The equipment used to perform the measures,

- Optical Spectrum Analyzer (OSA) by Hewlett-Packard, model HP-70951B.
- Frequency Spectrum Analyzer (RFSA) by Advantest, model R-4131A.
- Oscilloscope by Tektronix, model TDS380.
- Autocorrelator by Femtochrome, model FR103-PD.

Laser components used to build this cavity,

- SMF-28 WDM: Sifam L2WM98/15X.
- Thorlabs Erbium-doped fiber: M12-980-125. GVD = +20.28±0.46 ps2/Km, value provided by the company upon request.
- SMF-28 fiber pigtailed 1x2 coupler with a splitting ratio (%) of 90/10 at 1550nm: E-TEK SWBC2PS0CRV29. Used to couple the output of the laser.
- Polarizer/Isolator: JDSU C-band Polarization Isolator SPFI211WTSS02.
- Polarization Controllers (PC’s): OCT In-Line Polarization Controllers.
- Standard telecommunications SMF-28 fiber is used with a GVD = -23 ps²/Km.

4.1 Laser Cavity Dimensions

The net cavity dispersion is designed to be +0.04 ps². The EDF segment length is 208” (5.28m) and the total cavity length is 352” (8.94m). A negative dispersion single mode fiber (SMF-28) compensation segment was calibrated at the output of the laser to ensure shortest pulse output with a length of 46” (1.16m). The laser geometry is the same as of figure 2 and figure 12.

4.2 Laser Results

In the following sections various practical measures are presented as part of this project. First, in section 4.2.1 an optimization of the output of the laser is presented, then in the section 4.2.2 the use of an amplifying system to compress/amplify the pulse and finally, with careful adjustment on the polarization controllers it’s possible to operate in two different mode-locked states in section 4.2.3.

4.2.1 SMF Output Segment Optimization

The process started using a long segment of SMF-28 that will compensate for the strong chirping at the laser output and at the same time compress the output pulse. The measures were made cutting approximately 3 inches per cut from that segment. Table 1 shows the results for the process.
Table 1. Optimization process of the output delay line to compress the output pulse.

<table>
<thead>
<tr>
<th>output length (in.)</th>
<th>C.F. [ps/ms] ((2^*x)/(0.3^*s)) [x]=mm [s]=ms</th>
<th>FWHM from autocor. [s]</th>
<th>FWHM real p.w. [ps]</th>
<th>Δt/ΔT</th>
<th>Pulse Width [ps]</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>2.656042497</td>
<td>6.795E-05</td>
<td>0.180466564</td>
<td>0.648</td>
<td>1.169E-01</td>
</tr>
<tr>
<td>62</td>
<td>2.4600246</td>
<td>7.040E-05</td>
<td>0.173185805</td>
<td>0.648</td>
<td>1.122E-01</td>
</tr>
<tr>
<td>59</td>
<td>2.49500998</td>
<td>6.672E-05</td>
<td>0.166463897</td>
<td>0.648</td>
<td>1.079E-01</td>
</tr>
<tr>
<td>56</td>
<td>2.465483235</td>
<td>6.874E-05</td>
<td>0.169484454</td>
<td>0.648</td>
<td>1.098E-01</td>
</tr>
<tr>
<td>53</td>
<td>2.476473502</td>
<td>7.019E-05</td>
<td>0.173823325</td>
<td>0.648</td>
<td>1.126E-01</td>
</tr>
<tr>
<td>49</td>
<td>2.458210423</td>
<td>6.970E-05</td>
<td>0.171328868</td>
<td>0.648</td>
<td>1.110E-01</td>
</tr>
<tr>
<td>46</td>
<td>2.472799209</td>
<td>7.079E-05</td>
<td>0.17504484</td>
<td>0.648</td>
<td>1.134E-01</td>
</tr>
<tr>
<td>46</td>
<td>2.472799209</td>
<td>6.778E-05</td>
<td>0.167608224</td>
<td>0.648</td>
<td>1.086E-01</td>
</tr>
</tbody>
</table>

The result obtained for the shortest pulse duration was a segment length of 59”. The progression of the compression process can be seen in figure 13.

Figure 13. Output pulse width compression by changing the length of the output SMF segment.
The noise-like regime is reached due to internal birefringence of the laser cavity combined with a nonlinear transmission element and the gain response of the fiber amplifier that generates high-intensity broadband noise-like pulses.

4.2.2 Comparison Between Amplified and Non-amplified Output Characteristics

A large-mode area (LMA) Er-doped fiber amplifier based prechirped amplifier and compressor is used to compress/amplify the output pulse form the laser. This configuration is proposed by Gong-Ru Lin, et. al.in [21]. Figure 14 shows the detailed schematic used for the amplifier.

Figure 14. Detailed schematic of the large-mode area EDFA amplifier/compressor presented in [21]. © IEEE 2007
Once the pulse leaves the laser, it is goes into a 1.32 m length of EDF which is bidirectionally pumped with two 980 nm laser diodes through WDM couplers. The EDF length is adjusted to prevent the condition for power saturation [21] and to avoid stimulated Raman scattering induced during the high-power amplification process originated from the nonlinear effects. The EDF segment is very short but highly doped LMA Er-doped fiber. The pre-chirped SMF segment is for chirp controlling of the pulse before being launched into the LMA-EDF and the last compression stage [36].

The results are presented in the following figures based on the type of measure which includes optical spectrum, frequency spectrum of the pulse train, and autocorrelation trace, comparing non-amplified version vs. its amplified counterpart.

The optical spectrum is shown in figure 15. It can be seen the amplification given by the LMA-EDFA. The spectrum is more symmetric for the amplified version which will represent less chirp of the optical pulse at the output. The optical width for the regular (non-amplified) version is 52 nm whereas for the amplified version is 50 nm, measured from the raw data obtained.
Figure 15. Optical spectrum of the output. Blue corresponds to the amplified version, green corresponds to the regular version.

Although the optical widths are almost the same, the amplification done to the pulse does remove a lot of the chirp and by removing this chirp compression of the pulse duration in time is possible as it will be shown later with the autocorrelation measures.

The frequency spectrum is shown in figure 16. The repetition rate for the cavity is 22.1MHz. It can be seen that the separation of each pulse is very stable and it’s maintained after the amplification process. The top plot is the amplified version of the bottom plot in figure 16. The output of the laser before amplification shows noise-bands pedestals that can be attributed to noise inside the cavity, while the sharp pulse shapes over the sidebands represents the ideal mode-locked pulse train [37].
The jitter in the pulse is successfully removed in the amplifier, although the frequency width increases because of the amplification process that firsts stretch out the pulse, then amplifies it and the recompress it back.

**Figure 16.** RF spectrum for the output after amplification (top) and before amplification (bottom).

The pulse train is showed in figure 17. Given the short pulse duration, the detector was not able to detect much noticeable difference between the amplified and the non-amplified version of the pulse train, just a slight variation of amplitude, but besides that the structure remains.
Figure 17. Pulse train in time. The red plot corresponds to the non-amplified version of the pulse, the green plot corresponds to the amplified version.

Watching carefully the repetition rate in figure 17, it is double the frequency measured in figure 16, the happen sometimes when the detector shows pulses of double or triple the pulse height, but this can be generally attributed to the response time of the detector, which registers two pulses spaced in time by less than the original repetition rate of the pulse train, in this case the pair high-short pulse repeats the laser’s native repetition rate. It’s been claimed that this is due to a quantization of soliton pulses in an APM system that has excessive gain [29].

The autocorrelation traces are presented in figures 18 and 19. It was assumed a sech^2 pulse shape for both of them. For the figure 18, the full-width half maximum (FWHM) of the pulse is calculated to be 101 fs.
Figure 18. Autocorrelation trace for the non-amplified pulse.

For the figure 19, that corresponds to the amplified version the FWHM calculated was 96 fs. It can be seen how the pulse possesses less chirp since the interference pattern for the autocorrelation trace can be seen.
It is possible to generate noise-like pulses within a cavity with low birefringence. It has been shown that the noise-like emission was caused by the peak-power clamping effect of the laser cavity on the gain-guided soliton. The laser cavity is designed with total positive net dispersion, using erbium-doped fiber for the gain media and standard telecommunications components for its operation.

4.2.3 Gain-guided and Noise-like Operation Regimes

The cavity built has the ability to operate in two different regimes: the noise-like (NL) regime [33] and the gain-guided (GG) [39] or self-similar regime [35]. Each mechanism has a well-defined operability characteristic: NL regime depends on the balance between the fiber nonlinear Kerr effect and the linear cavity dispersion. GG regime is caused by the laser gain saturation and dispersion [41].
The terms gain-guided and self-similar are not interchangeable, the optical spectrum for both cases is similar in shape and characteristics [39]; however the pulse shape assumed for the self-similar regime is parabolic [35] and for gain-guided is sech^2. It’s been found in the literature, that both terms are used when presenting optical spectrums that fulfill the characteristics of having steep edges on the side bands due the limit imposed by the gain bandwidth [41, 45], but the parabolic pulse shape is a clear distinction.

Self-similar propagation of intense pulses is interrupted if the pulse meets a limitation to its spectral bandwidth [35] therefore the correct way to refer the operation mode is gain-guided since the gain bandwidth has a major role in it.

We believe that this two modes are possible in this cavity at such defined differentiation because the cavity net GVD is close to zero but no close as the values presented in [17] which is +0.004 ps^2, our cavity net dispersion is 0.04 ps^2. Far away from the point where stretched pulse operation can be fully achieved, the laser is operating more in the soliton regime.

The optical spectrum comparison for the two operation regimes is shown in figure 20.
The maximum optical width obtained for the noise-like regime 71nm according to figure (20-b). The trace presents the characteristics of a typical noise-like regime where the trace is smooth and broad, with no ripple or side-band peaks. Figure (20-a) shows the optical spectrum for the gain-guided regime. The steep edges that gradually broaden when reaching the bottom of the spectrum are easily appreciated. The optical width measured is 38nm taking a strict 3dBm limit, and about 50nm measuring edge-to-edge the top of the square-like shape at ~6dBm limit [46]. The pump power was set to 210mW at 980nm and only the polarization controllers were set to enter in either operation regime.

Given the characteristics of operation of each regime, being GG limited by the gain bandwidth, makes it less sensitive to noise fluctuations inside the cavity.
Comparing that to NL, where as its name suggests, the pulses resemble a bunch of amplified noise released a very short window of time [33]. It is expected to see some jitter effects on the frequency pulse spectrum for the NL regime, as shown in figure 21.

![Figure 21. Frequency spectrum for GG regime (top) and NL regime (bottom).](image)

It’s important to note that regarding the operation regime the repetition rate of the pulse emission remains constant, since it depends on the cavity parameters solely. The fundamental cavity repetition rate is 22.3 MHz.

The intensity autocorrelation is obtained by assuming sech² pulse shape. From figure 20 it can intuitively be seen that for NL regime the pulse width will be
shorter since its spectrum is wider and generates more power. Both autocorrelation traces are shown in Figure 22.

![Autocorrelation traces for GG (a) and NL (b) regimes.](image)

**Figure 22. Autocorrelation traces for GG (a) and NL (b) regimes.**

The pulse width for the GG regime is calculated to be 191 fs with a pulse energy of 386 pJ and the output power was measured to be 7.03 mW. The pulse width for the NL regime is calculated to be 95 fs with a pulse energy of 373 pJ and output power was measured to be 8.55 mW.
Chapter 5

DISCUSSION

On a Stretched-pulse additive mode-locking fiber laser, as long as cavity design concerns, it’s been found a strong dependence on the dispersion management process, rather than the cavity length itself [31]. The correct balance of the normal and anomalous segments establishes first, the generation of a clean pulsed output and second, the ability to operate either in the noise –like regime or the gain-guided regime.

Soliton lasers in general are limited to low pulse energies, below 100 pJ [11], which is in a way, a confirmation that this laser is operating in the stretched-pulse regime due to its higher output energy. Dispersion managed solitons can tolerate higher nonlinear phase shifts and pulse energies that surpasses the soliton energy by an order of magnitude [35].

Both operation regimes have been introduced in the past using different cavities, varying different parameters as net dispersion, cavity length, full positive dispersion fibers cavities [39,44,45] or using different components within the cavity to provide a net negative cavity dispersion as reported in [34]. Also a method to obtain broad bandwidth without using the stretched-pulse technique on a full-anomalous dispersion fiber ring has been presented [38].

Horowitz, et. al. [33] explained the noise like regime due the internal birefringence of the laser cavity combined with an element of nonlinear transmission and the gain response of the amplifier at the fiber. However the cavity presented was a
long one with 15m of length. In agreement with the work mentioned, we conclude that thanks to the net positive cavity dispersion, noise-like mode is achieved on a stable manner.

Polarization dependent delay (PDD) due to strong birefringence reduces the capacity of the laser to produce shorter pulses [33]. The way our setup is physically arranged, might confirm this effect, where the fibers are spooled in order to fit the table bench. However polarization measures at the output could not be performed, leaving the door open to future discussion when this factor can be measured.

In general, NL regime has a bandwidth much wider than the gain bandwidth, which in contrast is very important for the GG regime. The relatively smooth spectrum is also indicative of the absence of any effects that are due third-order dispersion, which typically leads to pulse breakup and a split spectrum [32]. Under the NL regime there is always a single mode-locked pulse traveling within the cavity [38] and this state can be reached over the whole range of pumping power available for the pump laser diode.

The output intensity depends directly in the pump power, however ranges for pumping power were found, that is, starting from just above the threshold necessary to mode-lock the laser and adjusting the polarization controllers to reach the NL operation. Once the pump power is increased, the mode-locked state will be maintained until certain value where the output becomes unstable and mode-locking eventually is lost. We attribute this to the dependence of the nonlinear polarization rotation on the pulse intensity that will make it rotate longer over the same distance, to
the point where an over-rotation was reached. This called for readjusting of the polarization controllers in order to reach mode-locked state again.

The spectral width for the NL regime could be further improved by using a longer portion of anomalous dispersion fiber in the cavity. The nonlinearities accumulated in the anomalous dispersion fibers can induce the creation of new spectral components, but that will represent a change in the overall GVD of the cavity as it is now, and can introduce instabilities in the stretched-pulse operation [47] and this laser is operated on the dispersion managed scheme [48].

The output pulse train of the laser is not fully stable in time, fact that is confirmed by the design on the cavity. This cavity is designed to work in the stretched pulse regime, where the anomalous dispersion segment is shorter than the normal dispersion segment. This means that most of the nonlinear effects take place on the anomalous dispersion fiber segment, and this brings unwanted instabilities in the cavity overall performance, leading to the noise-like operation, but sacrificing output pulse stability [34]. So far we have been constrained by the output power of the laser diode pump, but we believe that wider spectrum is achievable by further increase in the pump power with the same stability behavior.

Soliton collapse is a general property of the gain-guided solitons and fundamentally is a consequence of excessive nonlinearity where high intensities produce large nonlinear phase shifts [35]. It is also, a feature present in all passively mode-locked fiber soliton lasers [38,44]. The gain-guided soliton state is usually reached very close to the noise-like pulse generation regime due gradual broadening of the optical spectrum observed. The transition between GG to NL is attributed to the peak clenching effect of the cavity over the gain-guided soliton [44]. This effect
describes how, on GG regime, increasing the pump power will not increase proportionally the output energy, because the gain bandwidth is limiting this evolution. In exchange, dispersive waves and background noise will get amplified by increasing the pump power, which inevitably will lead to the NL regime by interference of the amplified dispersive waves with the GG soliton.

Operation mode competition is a clear characteristic of the laser, and depending on the polarization controller settings, one mode will be favored over the other. As long as the net cavity dispersion is kept large positive or large negative, both operation regimes will be possible within the same cavity, even if it is a dispersion-managed cavity [41].

The spectral edges on the mode-locked pulse spectrum in the gain-guided operation regime are a clear confirmation that the laser has reached this regime in for a stretched-pulse cavity [49]. It’s been found that at the highest pump power, the shape of the optical spectrum resembles a parabolic trace atop the spectrum [35] and lowering the pump power, the shape becomes more of a rectangular-like shape as presented in figure (20-b). Also the spectral edge-to-edge width varies with the pump variation.

Gain-guided operation state also presents range of stability based on the pump power, this ranges are longer than the measured NL stability ranges. When the pump power is increased the spectrum shows a slight broadening in the edge-to-edge width and increases the flat’s top amplitude until starts showing some parabolic curvature, followed by small- amplitude ripples atop the spectra that disappear with further pumping. These ripples can be attributed to nonlinear effects of SPM with normal dispersion but strongly limited by the gain bandwidth.
Mode-locking state can be lost when the pumping power is gradually increased in either operation regime. This can attributed to extra pulses formation inside the cavity that will compete and interfere with the fundamental pulse [40]. These extra pulses can be seen in our setup; however its randomness in amplitude and phase makes them unsuitable for further analysis. Just by readjusting the cavity parameters with the polarization controllers, solitary pulse operation was reached again by enhancing the effect of NPR technique.

Independently on the pump power only one pulse was leaving the cavity at all times [39]. The frequency spectrum for the GG regime showed a cleaner well-defined pulse train, without showing any sign of sidebands at the base of the pulses. On the contrary, side bands can be appreciated in the NL regime and are a clear observation of the noise that is present inside the cavity that originates the jitter seen. While being gain-bandwidth limited, GG regime avoids the amplification of unnecessary noise by the sacrifice of the final pulse width and pulse energy due to weaker nonlinear effects.

The fact that our cavity and the cavity presented in [35] have positive net dispersion justifies the similarities in the optical spectrum between the self-similar and the stretched-pulse regimes.

In order to mode-locking to start the pump power has to exceed a threshold between 60 to 70 mW, depending on the particular set of the polarization controllers. This feature has been discussed in [40] showing a similar rectangular shape in the optical spectrum but in a twice long the cavity length presented in this project. Their spectral width is 17.61 nm while ours easily doubles this value at higher pumping values without entering in a multi-pulse generation regime [40].
Other operation modes carried along in this type of cavity have been achieved, as the stable 8th harmonic generation in communion with the noise-like regime [43].

Our configuration presents a balance with laser cavity length, using stretched-pulse APM, where there is a cavity composed of normal and anomalous dispersion that is able to operate in both modes on a stable fashion for both of them, while maintaining a similar configuration to the one proposed in [17] without the use of bulk components or extra ones. The only substantial change was to increase in an order of magnitude the net cavity dispersion, in order to be on a state where mode competition was possible.
Chapter 6
CONCLUSIONS AND FUTURE APPROACHES

In the end, one of the most important motivations behind the research in fiber optics is that despite its advantages in compact size, excellent stability and low cost, fiber laser still cannot compete with semiconductor lasers in power generation. Once this limit is reached, fiber optics lasers will provide with a low cost alternative to industry in important applications as optical coherence tomography, and optical surgery thanks to the use of wavelengths on a safe-eye region. The door remains open to find new ways to improve fiber optics lasers in this direction.

In this theses it’s been presented a deep study in mode-locked fiber lasers, from its starting point using self-amplitude modulation until the understanding of a very sophisticated mechanism called stretched-pulse additive-pulse mode-locking.

Experimental results were shown that match the theory regarding this devices, and going further contributing to enhance the operative capabilities of this configuration. Stretched-pulse additive pulse mode locking is probably the simplest and yet most effective starting point of producing ultrashort pulses under 100 fs in fiber optics lasers.

The amplifier can be used as a secondary stage in any kind of stretched-pulse fiber laser, whose output delivers a strong chirp parameter. It successfully reduce the chirp of ultrashort pulses while amplifying and compression the pulse width. Further work will be focused in optimize this characteristics to our own study case.
By adjusting the cavity net dispersion to an appropriate value, enhanced operation modes can be reached. Each operation regime can be achieved by adjusting the polarization controllers in the cavity without changing the pump power. The noise-like regime presents a broad optical spectrum, that has potential applications in optical spectrum slicing and fiber optics sensing integrated systems. Gain guided regime is more suited for telecommunications applications where clean and very stable pulse trains are always required.

Our laser cavity characteristics, present a contribution to the field of looking for better and simpler sources of high optical bandwidth. In either operation regime, it surpass in performance to similar configurations proposed while maintaining the simplicity of the stretched-pulse fiber ring laser cavity.

Any improvement in this configuration has become an engineering problem where optimizing certain parameters will lead to better results. Further work with this configuration include the analysis of increasing the positive net cavity dispersion parameter to improve the overall performance and maximize the general parameters in the cavity.

We believe that through carefully adjusting the cavity net dispersion by reducing the net value on the stretched-pulse scheme, the ability of supporting the two operation modes will be lost. This suggests the existence of a cavity net dispersion threshold over which the laser will remain in strict operation on the stretched pulse regime, before jumping into supporting multiple operation regimes. Discovering this threshold and understanding its characteristics will enhance the contribution of this work.
REFERENCES


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Fig. 2 Absorption spectrum of Er3+-doped L22 silicate glass. The peaks of the string bands at 380 and 520 nm are 1.5 and 1.0 respectively.

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Luis Felipe Gerlein

--

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2 messages

Luis Felipe Gerlein Reyes <gerlein@udel.edu>  Wed, Apr 27, 2011 at 2:24 PM
To: copyright@osa.org

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Fig 1. Schematic of ring cavity with gain, gain dispersion, SPM, GVD, fast saturable absorption, and linear loss and phase shift.

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325 DuPont Hall
Newark, De
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Bembia, Hannah <hbembia@osa.org>  Wed, Apr 27, 2011 at 3:11 PM
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Fig 27.5 Supersposition of three equally spaced frequency components which are all exactly in phase at t=0

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---

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325 DuPont Hall
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